 CMC, vol.40, no.1, pp.21-34, 2014

Nonlinear Symmetric Free Vibration Analysis of Super Elliptical Isotropic Thin Plates

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Abstract: Nonlinear symmetric free vibration analyses are first presented for super elliptical isotropic thin plates with simply supported edge and clamped edge based on classical plate theory. Approximate solutions of super elliptical thin plates are obtained by Ritz method, and the validity can be confirmed by comparison with related researchers' results. Numerical results confirm that the characteristics of nonlinear vibration behaviors are significantly influenced by different boundary conditions, vibration amplitudes, the power of the super ellipse, as well as ratio of major to minor axis.

Keywords: Super elliptical plates, Nonlinear vibration, Ritz method.

1 Introduction

Super elliptical plates which are defined by shapes between an ellipse and a rectangle have a wide range of use in engineering applications, and it is more difficult to analyze nonlinear behaviors of super elliptical plates than rectangular, circular and elliptical plates.

Some studies for linear behaviors of super elliptical plates are available in the literature, for example, Wang et al. (1994) presented accurate frequency and buckling factors for super elliptical plates with simply supported and clamped edges by using Rayleigh-Ritz method. Lim (1998) investigated free vibration of doubly connected super-elliptical laminated composite plates. Then Chen et al. (1999) reported a free vibration analysis of laminated thick super elliptical plates. Liew and Feng (2001) studied three-dimensional free vibration analysis of perforated super elliptical plates. Zhou (2004) analysed three-dimensional free vibration of super elliptical plates based on linear elasticity theory using Chebyshev-Ritz method. Altekin (2008) gave out free linear vibration and buckling of super-elliptical plates resting on symmetrically distributed point-supports. Altekin and Altay (2008) calculated

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static analysis of point-supported super-elliptical plates, then Altekin (2009; 2010) discussed free vibration and bending of orthotropic super elliptical plates on intermediate supports. Çeribaşı et al. (2008) gave out static linear analysis of super elliptical clamped plates based on the classical plate theory by Galerkin's method. Çeribaşı and Altay (2009) investigated free vibration of super elliptical plates with constant and variable thickness by Ritz method, then Çeribaşı (2012) investigated static and dynamic linear analyses of thin uniformly loaded super elliptical clamped functionally graded plates. Jazi and Farhatnia (2012) discussed buckling of functionally graded super elliptical plate based on the classical plate theory using Pb-2 Ritz method. Tang et al. (2012) presented upper and lower bounds of the solution for the superelliptical plates problem using genetic algorithms.

Many studies for nonlinear vibration of rectangular and circular plates are available in the literature. For example, Ostiguy and Sassi (1992) investigated the influence of initial geometric imperfections on the dynamic behavior of simply supported rectangular plates subjected to the action of periodic in-plane forces. Saniei and Luo (2001) presented the natural frequency and responses for the nonlinear free vibration of heated rotating disks when non-uniform temperature distributions pertaining to the laminar and turbulent airflow induced by disk rotation were considered. Haterbouch and Benamar (2003; 2004) examined the effects of large vibration amplitudes on the axisymmetric mode shapes and natural frequencies of clamped thin isotropic circular plates, then Haterbouch and Benamar (2005) investigated nonlinear free axisymmetric vibration of simply supported isotropic circular plates by using the energy method and a multimode approach. Allahverdizadeh et al. (2008) studied vibration amplitude and thermal effects on the nonlinear behavior of thin circular functionally graded plates. Bakhtiari-Nejad and Nazari (2009) calculated nonlinear vibration analysis of isotropic cantilever plate with viscoelastic laminate. Shooshtari and Razavi (2010) presented a closed form solution for linear and nonlinear free vibrations of composite and fiber metal laminated rectangular plates. Alijani et al. (2011) investigated geometrically nonlinear vibrations of FGM rectangular plates in thermal environments via multi-modal energy approach. Ma et al. (2012) reported nonlinear dynamic response of a stiffened plate with four edges clamped under primary resonance excitation. Xie and Xu (2013) applied a simple proper orthogonal decomposition method to compute the nonlinear oscillations of a degenerate two-dimensional fluttering plate undergoing supersonic flow.

A literature review of works on the nonlinear vibration of rectangular and circular plates is given by Chia (1980), Sathyamoorthy (1987) and Chia (1988), while investigations on nonlinear vibration of super elliptical plates haven't been reported. Zhang (2013) first investigated non-linear bending of super elliptical thin plates based on classical plate theory, and presented approximate solutions and convergence studies by Ritz method. The present paper extends the previous works [Zhang (2013)] to the case of nonlinear vibration analysis for super elliptical thin plates, and approximate solutions are also obtained by Ritz method.

2 Basic formulations of thin plates based on classical plate theory

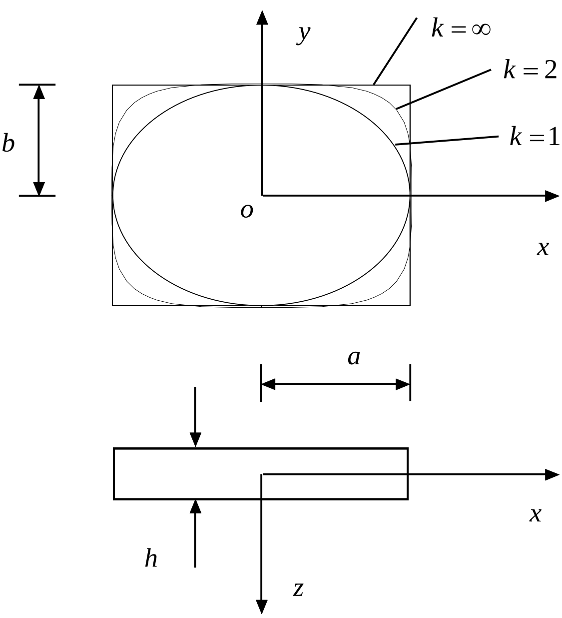

Figure 1: Geometry and coordinates of a super elliptical plate.

Consider a super elliptical plate of major axis $2a$, minor axis $2b$ and thickness h, and the coordinate system is illustrated in Fig. 1. The boundary shape equation of the super elliptical plates can be represented by

$$\frac{x^{2k}}{a^{2k}}+\frac{y^{2k}}{b^{2k}}-1=0 \tag{1}$$

k is the power of the super ellipse, and if $k=1$, the shape becomes an ellipse, if $k=\infty$, the shape becomes a rectangle. According to classical plate theory, the displacement fields are

$$u=u_0-z\frac{\partial w}{\partial x}, \tag{2a}$$

$$v = v_0 - z\frac{\partial w}{\partial y}, \tag{2b}$$

$$w = w(x,y,t) \tag{2c}$$

in which$u$, $v$and w are total displacements, u_0 and v_0 are mid-plane displacements in the x and y directions, respectively. Considering nonlinear von Kármán strain-displacement relationships, the strains can be expressed by

$$[\varepsilon] = [\varepsilon_x, \varepsilon_y, \gamma_{xy}]^T = \left[\varepsilon^{(0)}\right] + z\left[\varepsilon^{(1)}\right] \tag{3}$$

in which

$$\left[\varepsilon^{(0)}\right] = \left[\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial w}{\partial x}\right]^T \tag{4a}$$

$$\left[\varepsilon^{(1)}\right] = \left[-\frac{\partial^2 w}{\partial x^2}, -\frac{\partial^2 w}{\partial y^2}, -2\frac{\partial^2 w}{\partial x \partial y}\right]^T \tag{4b}$$

According to Hooke's law, the stresses can be determined as

$$[\sigma] = [\sigma_x, \sigma_y, \tau_{xy}]^T = \left[\tilde{Q}\right][\varepsilon] \tag{5}$$

where $[\tilde{Q}]$ is the stiffness matrix transformation, defined by

$$\left[\tilde{Q}\right] = \begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} & 0 \\ \tilde{Q}_{21} & \tilde{Q}_{22} & 0 \\ 0 & 0 & \tilde{Q}_{66} \end{bmatrix} \tag{6}$$

and

$$\tilde{Q}_{11} = \tilde{Q}_{22} = \frac{E}{1-\nu^2}, \tag{7a}$$

$$\tilde{Q}_{12} = \tilde{Q}_{21} = \frac{\nu E}{1-\nu^2}, \tag{7b}$$

$$\tilde{Q}_{66} = \frac{E}{2(1+\nu)} \tag{7c}$$

The constitutive equations can be deduced by proper integration.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^{(0)} \\ \varepsilon_y^{(0)} \\ \gamma_{xy}^{(0)} \end{bmatrix}, \tag{8a}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^{(1)} \\ \varepsilon_y^{(1)} \\ \gamma_{xy}^{(1)} \end{bmatrix} \tag{8b}$$

In Eq. (8) A_{ij} and D_{ij} are the plate stiffnesses, defined by

$$(A_{ij}, D_{ij}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tilde{Q}_{ij} \left(1, z^2\right) dz \quad (i, j = 1, 2, 6) \tag{9}$$

In the following analysis, all edges of plate are assumed to be simply supported and clamped with no in-plane displacements, i.e. prevented from moving in the x- and y-directions.

$$u_0 = v_0 = w = M_n = 0, \quad \text{(for simply supported edge, referred to as Case 1)} \tag{10a}$$

$$u_0 = v_0 = w = \frac{\partial w}{\partial n} = 0, \quad \text{(for clamped edge, referred to as Case 2)} \tag{10b}$$

where n refers to the normal directions of the plate boundary.

3 Ritz method for approximate solutions of nonlinear vibrations of super elliptical thin plates

Ritz method is adopted in this section to obtain approximate solutions of super elliptical plates. The key issue is first to assume the deflection and mid-plane displacements of the plate

$$w = \sum_{m=1,2,\cdots}^{M} W_m, \tag{11a}$$

$$u_0 = \sum_{m=1,2,\cdots}^{M} \bar{U}_m, \tag{11b}$$

$$v_0 = \sum_{m=1,2,\cdots}^{M} \bar{V}_m \tag{11c}$$

where M is total number of series. For symmetrical problems about the plate with *Case* 1, it can be assumed that

$$W_m = \sin \omega t \left(1 - \frac{x^{2k}}{a^{2k}} - \frac{y^{2k}}{b^{2k}}\right) \sum_{i=0,2,\cdots}^{2(m-1)} a_{ij} \frac{x^{2i}}{a^{2i}} \frac{y^{2j}}{b^{2j}} \tag{12a}$$

$$\bar{U}_m = \sin^2 \omega t \left(1 - \frac{x^{2k}}{a^{2k}} - \frac{y^{2k}}{b^{2k}}\right) \frac{x}{a} \sum_{i=0,2,\cdots}^{2(m-1)} d_{ij} \frac{x^{2i}}{a^{2i}} \frac{y^{2j}}{b^{2j}} \tag{12b}$$

$$\bar{V}_m = \sin^2 \omega t \left(1 - \frac{x^{2k}}{a^{2k}} - \frac{y^{2k}}{b^{2k}}\right) \frac{y}{b} \sum_{i=0,2,\cdots}^{2(m-1)} e_{ij} \frac{x^{2i}}{a^{2i}} \frac{y^{2j}}{b^{2j}} \tag{12c}$$

For symmetrical problems about the plate with *Case* 2, it can be assumed that

$$W_m = \sin \omega t \left(1 - \frac{x^{2k}}{a^{2k}} - \frac{y^{2k}}{b^{2k}}\right)^2 \sum_{i=0,2,\cdots}^{2(m-1)} a_{ij} \frac{x^{2i}}{a^{2i}} \frac{y^{2j}}{b^{2j}} \tag{13a}$$

$$\bar{U}_m = \sin^2 \omega t \left(1 - \frac{x^{2k}}{a^{2k}} - \frac{y^{2k}}{b^{2k}}\right) \frac{x}{a} \sum_{i=0,2,\cdots}^{2(m-1)} d_{ij} \frac{x^{2i}}{a^{2i}} \frac{y^{2j}}{b^{2j}} \tag{13b}$$

$$\bar{V}_m = \sin^2 \omega t \left(1 - \frac{x^{2k}}{a^{2k}} - \frac{y^{2k}}{b^{2k}}\right) \frac{y}{b} \sum_{i=0,2,\cdots}^{2(m-1)} e_{ij} \frac{x^{2i}}{a^{2i}} \frac{y^{2j}}{b^{2j}} \tag{13c}$$

in which $j = 2m - i - 2$ in Eqs. (12-13). Note that a_{ij}, d_{ij} and e_{ij} are undetermined coefficients, and Eqs. (12-13) satisfy displacement boundary conditions. In addition, Eqs. (12-13) are adapt to analysis of symmetrical nonlinear fundamental vibration modes, but not adapt to other modes, so other modes are not discussed in this paper.

Nonlinear algebraic equations about a_{ij}, d_{ij} and e_{ij} can be obtained by substituting w, u_0 and v_0 into the following expression.

$$\int_0^{\frac{2\pi}{\omega}} \frac{\partial \Pi}{\partial a_{ij}} dt = 0, \tag{14a}$$

$$\int_0^{\frac{2\pi}{\omega}} \frac{\partial \Pi}{\partial d_{ij}} dt = 0, \tag{14b}$$

$$\int_0^{\frac{2\pi}{\omega}} \frac{\partial \Pi}{\partial e_{ij}} dt = 0 \tag{14c}$$

in which $\Pi = K - U$, and the strain energy is

$$U = \frac{1}{2} \int_{\Omega} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy})\, d\Omega \tag{15a}$$

The kinetic energy may be expressed by

$$K = \frac{1}{2} \int_{\Omega_0} \rho h \left(\frac{\partial w}{\partial t}\right)^2 d\Omega_0 \tag{15b}$$

where Ω denotes domain of plates and Ω_0 denotes mid-plane of plates.

As for plates with given nonlinear fundamental frequency ω_{NL} and other known coefficients, a_{ij}, d_{ij} and e_{ij} can be solved by Newton-Raphson method or other equivalent methods. For the sake of brevity, nonlinear algebraic equations and the solving process are omitted. Substituting these coefficients back into Eqs. (12-13), w, u_0 and v_0 may then be completely determined. In addition, linear fundamental frequency ω_L can be easily obtained by making solutions of coefficients a_{ij}, d_{ij} and e_{ij} approach to zero.

4 Results and discussion

4.1 Comparison studies

The present paper extends convergence studies of the previous works [Zhang (2013)] to nonlinear vibrations analysis of super elliptical plates, and $M = 5$ is used in al-l the following calculations in consideration of both simplicity and convergence. To ensure the accuracy and effectiveness of the present method, two examples are solved for nonlinear vibration analysis of isotropic circular and rectangular plates.

Example 1. The nonlinear-to-linear fundamental frequency ratio (ω_{NL}/ω_L) for the isotropic circular plates with *Case* 2 is calculated and compared in Table 1 with results of Haterbouch and Benamar (2003) and Allahverdizadeh et al. (2008). In this example, the plates have $a/h = 136.59$ and $\nu = 0.3$.

Table 1: Comparisons of nonlinear-to-linear fundamental frequency ratio for isotropic circular plates with *Case* 2.

$w_{\max}/h$	Haterbouch and Benamar (2003)	Allahverdizadeh et al. (2008)	Present
0.2	1.0072	1.0075	1.007
0.4	1.0284	1.0296	1.030
0.5	1.0439	1.0459	1.046
0.6	1.0623	1.0654	1.065
0.8	1.1073	1.1135	1.113
1.0	1.1615	1.1724	1.172
1.5	1.3255	1.3567	1.357
2.0	1.5147	1.5789	1.578

Example 2. The nonlinear-to-linear fundamental frequency ratio (ω_{NL}/ω_L) for the isotropic rectangular thin plates with *Case* 1 is calculated and compared in Fig. 2

with results of Shooshtari and Razavi (2010). In this example, the super elliptical plate with $k = 10$ represents a shape similar to a rectangular plate, $a/h = 50$ and $\nu = 0.3$.

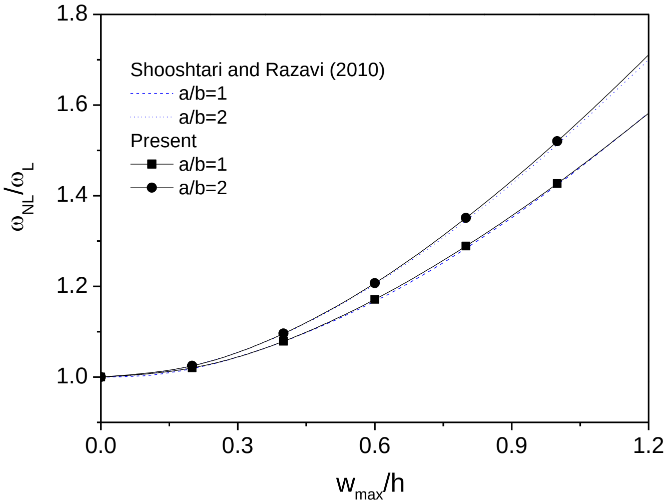

Figure 2: Comparison of nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for isotropic rectangular plates with *Case* 1.

These two comparisons show that the present results agree well with existing results, and thus the validity can be confirmed.

4.2 Parametric studies

Numerical results are presented in this section for linear and nonlinear vibration of isotropic super elliptical thin plates with *Case* 1 and *Case* 2, the plates have $\nu = 0.3$, ratio of major to minor axis $a/b = 1, 1.5, 2, 3$ and the power of the super ellipse $k = 1, 2, 3, 4$. The non-dimensional fundamental frequency is defined by $\omega_L^* = 4\left(\omega_L a^2/\pi^2\right)\sqrt{\rho h/D}$ in this section.

The relations of nonlinear-to-linear frequency ratio ω_{NL}/ω_L and non-dimensional vibration amplitudes $w_{\max}/h$ for the plate are calculated in Table 2-9. It can be observed that the present results agree well with existing results of Wang et al. (1994) for linear vibration of isotropic super elliptical thin plates. It can be concluded that nonlinear vibration frequencies increase significantly with increasing the value of vibration amplitudes and ratio of major to minor axis a/b. It can be also observed that the characteristics of nonlinear vibration are significantly influenced by different boundary conditions and the power of the super ellipse k.

Table 2: Nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for super elliptical thin plates with *Case* 1 and $k = 1$.

a/b	ω_L^*		$w_{\max}/h$				
	Wang et al. (1994)	Present	0.2	0.4	0.6	0.8	1
1	2.000	2.000	1.027	1.103	1.220	1.369	1.540
1.5	3.357	3.357	1.027	1.103	1.220	1.369	1.541
2	5.355	5.355	1.027	1.103	1.220	1.371	1.545
3	10.976	10.976	1.027	1.104	1.226	1.384	1.569

Table 3: Nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for super elliptical thin plates with *Case* 1 and $k = 2$.

a/b	ω_L^*		$w_{\max}/h$				
	Wang et al. (1994)	Present	0.2	0.4	0.6	0.8	1
1	1.878	1.878	1.024	1.093	1.199	1.334	1.490
1.5	3.100	3.100	1.025	1.097	1.208	1.350	1.515
2	4.872	4.873	1.027	1.105	1.225	1.380	1.561
3	9.995	9.997	1.029	1.115	1.250	1.427	1.634

Table 4: Nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for super elliptical thin plates with *Case* 1 and $k = 3$.

a/b	ω_L^*	$w_{\max}/h$				
		0.2	0.4	0.6	0.8	1
1	1.920	1.022	1.086	1.185	1.312	1.460
1.5	3.146	1.023	1.091	1.196	1.331	1.488
2	4.895	1.026	1.101	1.217	1.367	1.543
3	9.930	1.029	1.114	1.250	1.428	1.638

Table 5: Nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for super elliptical thin plates with *Case* 1 and $k = 4$.

a/b	ω_L^*		$w_{\max}/h$				
	Wang et al. (1994)	Present	0.2	0.4	0.6	0.8	1
1	1.947	1.947	1.021	1.083	1.179	1.302	1.446
1.5	3.179	3.180	1.022	1.088	1.191	1.322	1.476
2	4.925	4.926	1.025	1.099	1.213	1.361	1.535
3	9.934	9.938	1.029	1.113	1.249	1.427	1.636

Table 6: Nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for super elliptical thin plates with *Case* 2 and $k = 1$.

a/b	ω_L^*		$w_{\max}/h$				
	Wang et al. (1994)	Present	0.2	0.4	0.6	0.8	1
1	4.141	4.140	1.007	1.030	1.065	1.113	1.172
1.5	6.944	6.942	1.007	1.030	1.065	1.113	1.172
2	11.100	11.096	1.007	1.030	1.065	1.113	1.173
3	23.023	23.020	1.007	1.030	1.065	1.115	1.176

Table 7: Nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for super elliptical thin plates with *Case* 2 and $k = 2$.

a/b	ω_L^*		$w_{\max}/h$				
	Wang et al. (1994)	Present	0.2	0.4	0.6	0.8	1
1	3.688	3.687	1.007	1.029	1.064	1.112	1.170
1.5	6.233	6.230	1.007	1.030	1.065	1.113	1.173
2	10.094	10.087	1.008	1.030	1.067	1.117	1.180
3	21.445	21.432	1.008	1.031	1.070	1.124	1.192

Table 8: Nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for super elliptical thin plates with *Case* 2 and $k = 3$.

a/b	ω_L^*	$w_{\max}/h$				
		0.2	0.4	0.6	0.8	1
1	3.654	1.007	1.029	1.063	1.109	1.166
1.5	6.172	1.007	1.029	1.064	1.112	1.170
2	9.990	1.008	1.030	1.066	1.116	1.178
3	21.226	1.008	1.031	1.071	1.126	1.196

Table 9: Nonlinear-to-linear frequency ratio (ω_{NL}/ω_L) for super elliptical thin plates with *Case* 2 and $k = 4$.

a/b	ω_L^*		$w_{\max}/h$				
	Wang et al. (1994)	Present	0.2	0.4	0.6	0.8	1
1	3.648	3.650	1.007	1.028	1.063	1.110	1.167
1.5	6.161	6.163	1.007	1.029	1.065	1.112	1.171
2	9.971	9.974	1.007	1.030	1.066	1.116	1.178
3	21.179	21.184	1.008	1.031	1.071	1.126	1.196

5 Conclusions

In this paper, nonlinear vibration analyses are first presented for super elliptical plates based on classical plate theory. Ritz method is employed to analyze nonlinear vibration behaviors. Numerical results confirm that the characteristics of nonlinear vibration behaviors are significantly influenced by different boundary conditions, vibration amplitudes, the power of the super ellipse k, as well as ratio of major to minor axis.

Acknowledgement: This research was supported by a grant of the Fund of the National Natural Science Foundation of China (Nos. 11172285, 10802082) and the Natural Science Foundation of Zhejiang Province (Nos. LR13A020002). The authors would like to express their sincere appreciation to these supports.

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