Copyright © 2013 Tech Science Press CMC, vol.36, no.2, pp.155-176, 2013

Thermal-Cyclic Fatigue Life Analysis and Reliability Estimation of a FCCSP based on Probabilistic Design Concept

Yao Hsu[1], **Chih-Yen Su**[2], **Wen-Fang Wu**[3,4]

Abstract: To study the fatigue reliability of a flip-chip chip scale package (FCCSP) subject to thermal cyclic loading, a Monte Carlo simulation-based parametric study is carried out in the present study. A refined procedure as compared with the recently released Probabilistic Design System (PDS) of ANSYS is proposed and employed in particular. The thermal-cyclic fatigue life of the package is discussed in detail since it is related directly to the reliability of the package. In consideration of the analytical procedure as well as real manufacturing processes, a few geometric dimensions and material properties of the package are assumed random. The empirical parameters used in the fatigue life prediction formula are also assumed random to account for their uncertainties. Numerical calculation is performed following the standard finite element analysis procedure. The result indicates that PDS can indeed be employed to find the cumulative thermal-cyclic fatigue life distribution of the electronic package owing to various uncertainties. The proposed refined design procedure can further improve the accuracy of the quantitative reliability estimation.

Keywords: Electronic package, Probabilistic design, Fatigue life, Reliability.

1 Introduction

The reliability of electronic packages has become an important issue because it would directly determine the life expectancy of the electronic products. A variety of topics on the reliability of packages has been studied by many researchers

---

[1] Department of Business and Entrepreneurial Management, Kainan University, Taoyuan 33857, Taiwan, R.O.C.

[2] Department of Mechanical Engineering, National Taiwan University, Taipei 10617, Taiwan, R.O.C.

[3] Department of Mechanical Engineering and Graduate Institute of Industrial Engineering, National Taiwan University, Taipei 10617, Taiwan, R.O.C.

[4] Corresponding author, Tel.: +886-2-33662727. E-mail address: wfwu@ntu.edu.tw

[Chen, Lin and Chiang (2005, 2006); Hung (2007); Lee, Chang and Yang (2009); Lin, Kung, Chen and Liang (2006)]. When considering the thermal cyclic loadings, the reliability or fatigue life of the electronic package can be predicted based on the extreme strain or strain energy of the package along with an empirical or semi-empirical equation or model such as Coffin-Manson equation, Norris-Landzberg model, Engelmaier model, and Darveaux model [Anand (1985); Coffin (1954); Collins (1981); Darveaux (2000); Engelmaier (1990); Manson (1966); Norris and Landzberg (1969)]. The extreme strain or strain energy of the package is usually obtained by performing finite element analysis. The fatigue life prediction equations or models stated above are generally established from experimental data through the application of simple statistics such as linear regression analysis. In general, both the extreme strain or strain energy and the fatigue life obtained in this customary reliability analysis of electronic packages are of deterministic values. However, the real outcomes of package-life observed from laboratories always appear to have certain degree of scatter and are frequently described by probability distributions such as Weibull probability distribution. The objective of this paper is to investigate possible causes for this discrepancy.

Many works discussing about parameter uncertainties and how they affect the life of electronic package have been published [Evans Evans Ghaffarian, Mawer, Lee and Shin (2000); Wu and Barker (2010); Yang Liang, Li, Ernst and Zhang (2004); Zulkifli, Jamal and Quadir (2011)]. For example, Evans *et al.* (2000) applied Monte Carlo simulation to evaluate fatigue life distributions in consideration of variations of material property and manufacturing capabilities Wu and Barker (2010) found that material properties affect life of the ball grid array (BGA) package considerably but the geometric dimensions affect little due to their relatively tight distributions. After reviewing these articles, it is found that geometric and material uncertainties were usually considered separately and have seldom been considered together when investigating their effects on the package-life. How the effect would be when both uncertainties are considered jointly will be studied in this paper. In fact, this kind of uncertainty consideration and analysis has also been employed in many other structural analyses such as Gao, Song and Tin-Loi (2009), Kaminski (2011), and Wang, Gao, Yang and Song (2011).

The first part of the work will be a finite element analysis performed through ANSYS PDS under the assumption that certain geometric parameters and material properties are random variables. The purpose is to investigate the combined effect of these random geometric parameters and material properties on the fatigue life of a package. Moreover, the present paper suggests that the coefficients appeared in the fatigue life prediction formula or model also have certain influences on the fatigue life distribution and should be considered random as well to account for their uncertainties. Therefore, the second part of the work will concentrate on the fatigue life prediction formula or model and investigate how random coefficients influence the fatigue life of a package.

Although the output of ANSYS PDS includes a cumulative distribution function (CDF), it is derived based on empirical analysis of data and only simple statistical test is carried out therein. The result is that curves of the CDF are not smooth enough, which may affect the accuracy of reliability prediction of packages when fatigue life is concerned. To improve the prediction accuracy, a refined probabilistic design procedure will be employed. The procedure follows a more rigorous statistical analysis of data, which includes probability density fits of the data and statistical test of their goodness-of-fits. The result from the refined procedure will be checked with that obtained from ANSYS PDS and comparison will be made between them.

2 Finite element analysis

2.1 Model description

The flip-chip chip scale package (FCCSP) is chosen for study in the present paper. A FCCSP is generally composed of components such as silicon chip, solder balls, Au-stud bumps, substrate, passivation, under-bump metal (UBM), underfill, solder mask and printed circuit board (PCB). To simplify the analytical process, modeling on layers of passivation, solder mask and UBM are neglected. The FCCSP model to be analyzed contains a 7 × 7 × 0.41 mm^3 silicon chip which is interconnected to a 7.45 × 7.45 × 0.25 mm^3 substrate through 100 Au-stud bumps in a perimeter and a layer of underfill. The diameter and standoff of an Au-stud bump are 0.1 mm and 0.025 mm, respectively. The solder balls have radii of 0.15 mm and heights of 0.297 mm, and are linked to the substrate and the PCB in a 10 × 10 array with a ball-pitch of 0.65 mm. The dimensions of the PCB are 8 × 8 × 1 mm^3. Figure 1 shows the schematic diagram of the FCCSP. All materials except the solder balls in the finite element model are assumed to be isotropic and linear elastic. Their material properties are listed in Table 1 [Jong Chen Tsai, Chiu and Chang (2006); Lau and Dauksher (2005)] Since the solder ball made of 95.5Sn3.9Ag0.6Cu would show the creep behavior, the generalized Garofalo–Arrhenius equation characterizing the steady-state creep mechanism is adopted in the present study. The equation has the form as

$$\frac{d\varepsilon_{cr}}{dt} = C_1 \left[\sinh\left(C_2\sigma\right)\right]^{C_3} \exp\left(-\frac{C_4}{T}\right) \tag{1}$$

where σ indicates the equivalent stress, $d\varepsilon_{cr}/dt$represents the equivalent creep strain rate, T denotes the absolute temperature, $C_1, C_2, C_3 and C_4$ are all coefficients and their values are given in Table 2 [Lau and Dauksher (2005)].

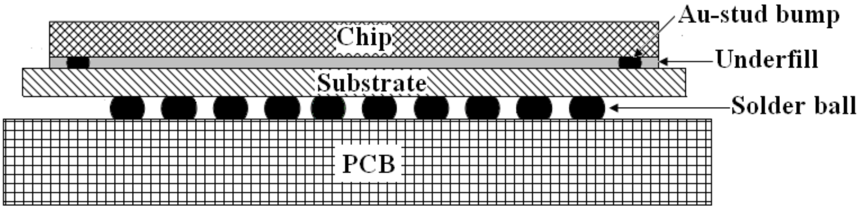

Figure 1: Schematic diagram of the FCCSP

Table 1: Material properties [Jong Chen Tsai, Chiu and Chang (2006); Lau and Dauksher (2005)]

Component	Young's modulus (MPa)	Poisson's ratio	CTE (ppm/K)
Chip	131,000	0.3	2.8
Au-stud bump	88,790	0.44	14
Underfill	14,470	0.28	20
Substrate	18,200	0.25	19
Solder ball (95.5Sn3.9Ag0.6Cu)	49000-70T_c	0.35	21.3+0.017T_c
PCB	27,000	0.39	18
Note: T_c is Celsius temperature in ^{o}C.			

Table 2: Coefficients of creep equation [Lau and Dauksher (2005)]

Parameter	Value
C_1 (1/sec)	500,000
C_2 (1/MPa)	0.01
C_3	5
C_4 (K)	5,802

Since higher accuracy of the calculated extreme stress/strain in finite element analysis is required for subsequent reliability analysis, simulation model with large number of fine meshes is necessary. However, as the number of meshes increases, the computation time increases substantially, making the numerical analysis costly and inefficient. To cope with this problem, the sub-modeling technique [Cheng, Yu and Chen (2005); Kao, Lai and Wang (2007); Lai and Wang (2005); Lee Chou, Chiu, Hsia and Chiang (2008); van Driel van Silfhout and Zhang (2008); Wang and Lai (2005)] is adopted in the present study. In the finite element analysis with the sub-modeling technique, the whole model with coarser meshes is analyzed first to obtain the displacement field. The displacements are then imposed on the boundary of the sub-model as its boundary conditions for the following detailed analysis. Because of geometric symmetry, the whole electronic package model can be simplified to be stripe-typed model with appropriate boundary conditions setting for saving computational time. As shown in Figure 2, the symmetric planes are frontal and left sides of the model, and the nodes located at the intersection of the two symmetric planes are totally fixed to prevent right body motion during analysis. Furthermore, all nodes on the plane of Z = –pitch/2 are coupled at the displacement in Z direction (UZ) to restrain out-of-plane movement during thermal loading condition. Figure 3 shows the global model with coarse meshes of the package and the location of the corresponding sub-model. The width and thickness (X and Z directions as indicated in Figure 3) of the sub-model are the ball-pitch and one-half of the ball-pitch, respectively. In fact, the sub-model can be further reduced in its height (Y direction) to decrease the computational time to some extent [Lai and Wang (2005)]. After several trials, without affecting the simulated results and violating the Saint-Venant's principle, the optimal height is found to be one-third of the original height. This refined sub-model is called the 1/3 sub-model in the present paper and is illustrated in Figure 4.

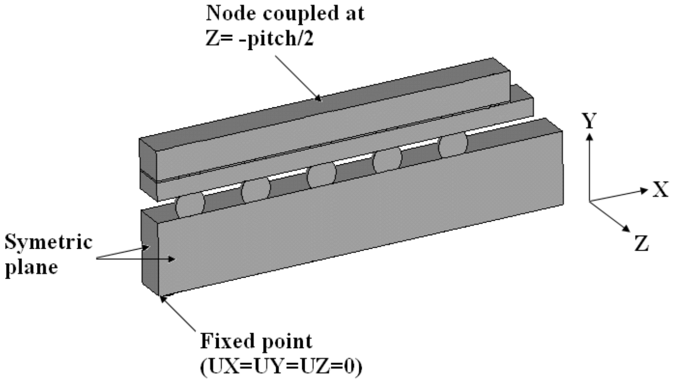

Figure 2: Global model and boundary conditions

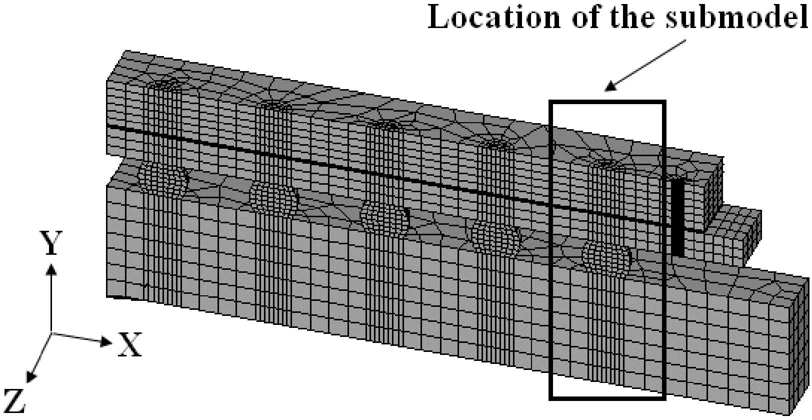

Figure 3: Meshes of global model

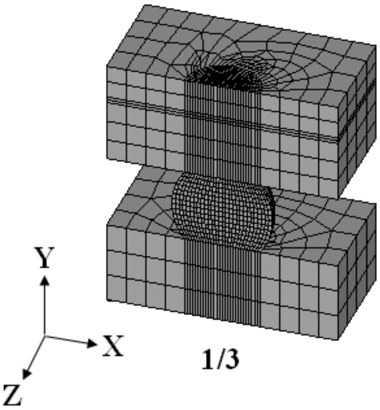

Figure 4: Meshes of the 1/3 sub-model

2.2 Loading condition

According to test condition G in JESD 22-A 104C of the JEDEC STANDARD, the thermal cyclic test analysis on electronic packages for the reliability evaluation is performed at temperatures ranging from -40 to 125^{o}C and the reference temperature is 25^{o}C [JEDEC (2005)]. Time for ramp up, dwell, ramp down and dwell loading is 900, 600, 900, 600 sec, respectively, and therefore one complete thermal cycle test is 3,000 sec in total. Figure 5 shows the temperature profile of four complete thermal loading cycles during the temperature cyclic test analysis.

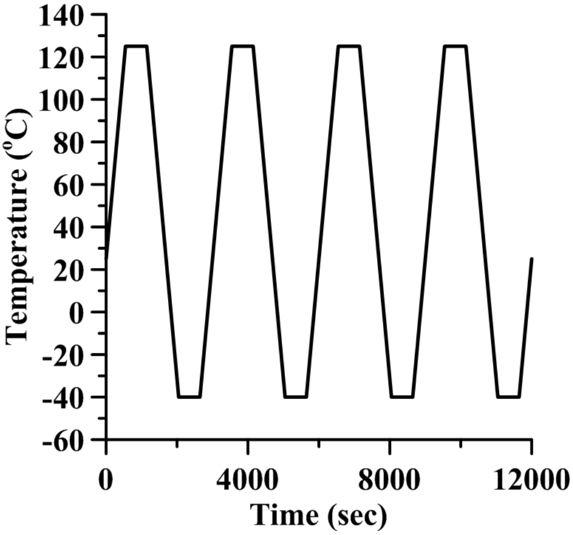

Figure 5: Cyclic temperature loading

3 Random consideration

In the prediction of the fatigue life of electronic packages subject to thermal cyclic loading, the present paper proposes an analytic procedure based on a probabilistic viewpoint that takes uncertainties into consideration. The uncertainties include geometric parameters and material properties of the electronic package as well as coefficients in the empirical fatigue life prediction formula. They are all treated as random variables

3.1 Random geometric parameters and material properties

In the present study, seven geometric parameters and material properties of the package, namely radius of the solder ball, heights of the chip and the substrate, Young's moduli of the chip and the substrate, coefficients of thermal expansion of the chip and the substrate are chosen as design variables. All of these seven random variables are assumed to be distributed normally and each one has a mean (nominal) value and a coefficient of variation (c.o.v.) of 3%. Their mean values and corresponding standard deviations are tabulated in Table 3.

A Monte Carlo simulation-based probabilistic design system, PDS, provided by ANSYS finite element software is adopted to analyze the strain/stress fields of electronic packages when design variables have randomness. A random sample of size 40 of the design variable group is generated within ANSYS PDS, and forty outcomes of the maximum equivalent strain range are obtained from running ANSYS. After substituting these strain ranges into the fatigue life prediction formula, forty fatigue lives of the electronic package are obtained. To further interpret these sample results, ANSYS PDS provides users with the empirical CDF plot which refers to the equation proposed by Kececioglu (1991). The empirical CDF associated with sample value x_i is

$$\sum_{k=i}^{n} \frac{n!}{(n-k)!k!} F_i^k (1-F_i)^{n-k} = 50\% \tag{2}$$

in which F_i is the cumulative probability, n is the sample size and i is the order number.

Although the CDF obtained from ANSYS PDS can provide us information about the reliability of an electronic package, the result is an empirical distribution curve which is not accurate enough. More detailed reliability analysis including probability density fit, parameter identification and testing of validity can be carried out furthermore. In the present study, following procedures documented in Ang and Tang (1975) and Ebeling (1997), the simulated forty fatigue lives are plotted in various probability papers and chi-square test is applied to verify the goodness-of-fit to those distributions. The most favorable distribution and its parameters are identified. The distribution would provide us more accurate reliability estimation of the package.

Table 3: Statistical characteristics of geometric parameter and material property

Variable	Mean (nominal) value	Standard deviation
Solder ball radius (mm)	0.15	0.0045
Chip height (mm)	0.41	0.0123
Substrate height (mm)	0.25	0.0075
Young's modulus of the chip (MPa)	131,000	3,930
Young's modulus of the substrate (MPa)	18,200	546
Coefficient of thermal expansion of the chip (ppm/K)	2.8	0.084
Coefficient of thermal expansion of the substrate (ppm/K)	19	0.57

3.2 Random coefficients in modified Coffin-Manson equation

In electronic package researches, a frequently applied fatigue life prediction formula is the so-called 'modified Coffin-Manson equation', which can be written as:

$$N_f = A\,(\Delta\varepsilon)^B \tag{3}$$

in which N_f indicates the fatigue life, $\Delta\varepsilon$ represents the maximum equivalent strain range, and A and B are empirical coefficients. Herein, the two empirical coefficients are treated as random variables to take uncertainties into account. Under this consideration and given the maximum equivalent strain range, the fatigue life predicted for a package based on Equation (3) is a random variable as well. To understand how the degree of randomness of coefficients A and B affect that of the fatigue life, ANSYS PDS is employed and the analytic procedure is similar to that described in Section 3.1.

In fact, from probability theory, the statistical properties of the fatigue life related to randomness of A and B can be derived mathematically. The probability distribution can even be found analytically under certain assumptions. If taking logarithm on both sides of the Equation (3), one obtains

$$\ln[N_f] = \ln[A] + B\ln[\Delta\varepsilon] \tag{4}$$

If, for example, both A and B are treated as random variables and $\Delta\varepsilon$ is deterministic, the mean value and standard deviation of $\ln[N_f]$ become, respectively, as

$$\mu_{\ln[N_f]} = \mu_{\ln[A]} + \ln[\Delta\varepsilon]\,\mu_B \tag{5}$$

$$s^2_{\ln[N_f]} = s^2_{\ln[A]} + (\ln[\Delta\varepsilon])^2\, s^2_B \tag{6}$$

in which μ and s are the mean value and standard deviation of the associated quantity denoted by the subscript. For simplicity, it is assumed that A is not correlated to B in the derivation of Equations (5) and (6). If one further assumes that A is a lognormal random variable and B is a normal random variable, the mean value and standard deviation of $\ln[A]$ can be written as

$$\mu_{\ln[A]} = \ln[\mu_A] - \frac{1}{2}\ln\left[1 + \frac{s^2_A}{\mu^2_A}\right] \tag{7}$$

$$s^2_{\ln[A]} = \ln\left[1 + \frac{s^2_A}{\mu^2_A}\right] \tag{8}$$

From probability theory, N_f will also be a lognormal random variable. Its probability density function can be written as

$$f(n_f) = \frac{1}{\sqrt{2\pi} s_{\ln[N_f]} n_f} \exp\left[-\frac{1}{2} \left(\frac{\ln[n_f] - \mu_{\ln[N_f]}}{s_{\ln[N_f]}} \right)^2 \right] \tag{9}$$

and its mean value and standard deviation can be derived to be

$$\begin{aligned} \mu_{N_f} &= \exp\left\{ \mu_{\ln[N_f]} + \tfrac{1}{2} s^2_{\ln[N_f]} \right\} \\ &= \exp\left\{ \ln[\mu_A] + \ln[\Delta\varepsilon]\, \mu_B + \tfrac{1}{2} (\ln[\Delta\varepsilon])^2 s_B^2 \right\} \end{aligned} \tag{10}$$

and

$$\begin{aligned} s^2_{N_f} &= \exp\left\{ 2\mu_{\ln[N_f]} + 2s^2_{\ln[N_f]} \right\} - \exp\left\{ 2\mu_{\ln[N_f]} + s^2_{\ln[N_f]} \right\} \\ &= \exp\left\{ 2\ln[\mu_A] + \ln\left[1 + \tfrac{s_A^2}{\mu_A^2} \right] + 2\ln[\Delta\varepsilon]\, \mu_B + 2 (\ln[\Delta\varepsilon])^2 s_B^2 \right\} \\ &\quad - \exp\left\{ 2\ln[\mu_A] + 2\ln[\Delta\varepsilon]\, \mu_B + (\ln[\Delta\varepsilon])^2 s_B^2 \right\} \end{aligned} \tag{11}$$

In the present paper, the following three cases are studied: (i) A is considered a lognormal random variable with other parameters deterministic; (ii) B is considered a normal random variable with other parameters deterministic; and (iii) A is a lognormal random variable and B is a normal random variable. The results of the first two cases can, in fact, be deduced from that of the last case. In particular, if A is considered a lognormal random variable, then the derived mean value and standard deviation of the fatigue life become

$$\mu_{N_f} = \exp\left\{ \mu_{\ln[N_f]} + \frac{1}{2} s^2_{\ln[N_f]} \right\} = \exp\{ \ln[\mu_A] + B \ln[\Delta\varepsilon] \} = \mu_A (\Delta\varepsilon)^B \tag{12}$$

and

$$\begin{aligned} s^2_{N_f} &= \exp\left\{ 2\mu_{\ln[N_f]} + 2s^2_{\ln[N_f]} \right\} - \exp\left\{ 2\mu_{\ln[N_f]} + s^2_{\ln[N_f]} \right\} \\ &= \exp\left\{ 2\ln[\mu_A] + \ln\left[1 + \tfrac{s_A^2}{\mu_A^2} \right] + 2B\ln[\Delta\varepsilon] \right\} - \exp\{ 2\ln[\mu_A] + 2B\ln[\Delta\varepsilon] \} \\ &= \mu_A^2 \left(1 + \tfrac{s_A^2}{\mu_A^2} \right) (\Delta\varepsilon)^{2B} - \mu_A^2 \left(\tfrac{s_A^2}{\mu_A^2} \right) (\Delta\varepsilon)^{2B} \\ &= s_A^2 (\Delta\varepsilon)^{2B} \end{aligned} \tag{13}$$

For the other case, if B is considered a normal random variable, then the mean value and standard deviation of the fatigue life have the expressions of

$$\begin{aligned}\mu_{N_f} &= \exp\left\{\mu_{\ln[N_f]} + \tfrac{1}{2}s^2_{\ln[N_f]}\right\} \\ &= \exp\left\{\ln[A] + \ln[\Delta\varepsilon]\,\mu_B + \tfrac{1}{2}\left(\ln[\Delta\varepsilon]\right)^2 s_B^2\right\}\end{aligned} \tag{14}$$

and

$$\begin{aligned}s^2_{N_f} &= \exp\left\{2\mu_{\ln[N_f]} + 2s^2_{\ln[N_f]}\right\} - \exp\left\{2\mu_{\ln[N_f]} + s^2_{\ln[N_f]}\right\} \\ &= \exp\left\{2\ln[A] + 2\ln[\Delta\varepsilon]\,\mu_B + 2\left(\ln[\Delta\varepsilon]\right)^2 s_B^2\right\} \\ &\quad - \exp\left\{2\ln[A] + 2\ln[\Delta\varepsilon]\,\mu_B + \left(\ln[\Delta\varepsilon]\right)^2 s_B^2\right\}\end{aligned} \tag{15}$$

Aside from the analytical derivation, samples of size 40 of the assumed probability distributions of A and/or B are generated by ANSYS PDS for each of the above three cases. Also, for each case, forty fatigue lives are calculated based on Equation (3) and under the assumption that $\Delta\varepsilon$ is a deterministic value calculated without considering randomness or uncertainty of geometric parameters and material properties. Statistical analysis is performed for these fatigue lives and comparison is made between the result and that from the analytical derivation.

4 Numerical case study

For the case stated in Section 3.1, after finite element analysis, a random sample of forty fatigue lives are calculated based on Equation (3) in which $A = 0.04535$ and $B = -2.4489$ are adopted. The relative frequencies of fatigue life are plotted by ANSYS PDS in the form of histogram as that shown in Figure 6. Although the distribution shape of fatigue life is obtained, one cannot judge which type of commonly-known probability distributions such as normal, lognormal or Weibull distribution it is. Further statistical analyses are therefore needed. To this end and in contrast to that of ANSYS PDS, the present paper employs a refined analytical procedure to determine the best theoretical distribution and its corresponding parameters for the data. In the refined procedure, fatigue lives are plotted in normal, lognormal and Weibull probability papers, respectively. The chi-square test is used to determine each of their goodness-of-fits. Parameters of these three probability distributions are estimated by least-squared regression and curve fitting techniques. To determine the best fit probability distribution, the simulation data are grouped into several intervals. In each interval I, there is a value of observed number of occurrence O_i resulting from sampled data and a corresponding value of expected number of occurrence E_i deduced from the hypothesized distribution. The chi-square test is performed based on values of O_i and E_i. In this study, the test statistics of normal, lognormal and Weibull distributions are found to be 4.1372, 2.7632 and 4.4925, respectively. Because the lognormal distribution has the smallest value of test statistic, it is asserted to fit the fatigue life data the best. For saving space, the detail of the statistical analysis is not shown herein but can be referred to Lin (2005).

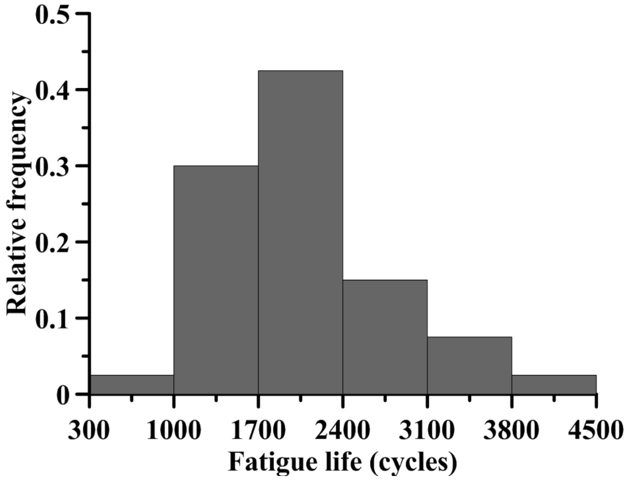

Figure 6: Histogram of fatigue lives

Figure 7 demonstrates the CDF curve of the fatigue life for the fitted lognormal distribution and the empirical CDF curve produced from ANSYS PDS. It can be seen that the two curves are quite consistent with each other. However, when observing Figure 7 carefully, it is found that the empirical CDF curve of ANSYS PDS is not as smooth as the other continuous curve. It is owing to the inherent statistical method used in ANSYS PDS. This phenomenon would be more obvious when the number of data is smaller. This deficiency of ANSYS PDS would adversely affect the accuracy of the subsequent reliability analysis of electronic packages even if ANSYS PDS declares itself an efficient probabilistic design tool. With the help of the refined procedure proposed in the present study, even with limited data, not only a smooth CDF curve can be constructed, but also more statistical information can be revealed. It indicates more accurate reliability estimation can be achieved.

To understand how and to what degree each of those geometric parameters and material properties affects the fatigue life, sensitivity analysis is conducted and the result is summarized in Table 4. Based on values of Spearman rank-order correlation coefficient, it is found that radius of the solder ball affects the fatigue life of the studied package the most, and it is in a positive sense. The substrate height comes the second. The result coincides with the observation that solder ball radius dominates the entire deformation and warp of the structure. It is also compatible with the understanding that solder ball geometry should be paid more attention to when designing an electronic package. In general, parameters and properties about the chip have less effect on the package life.

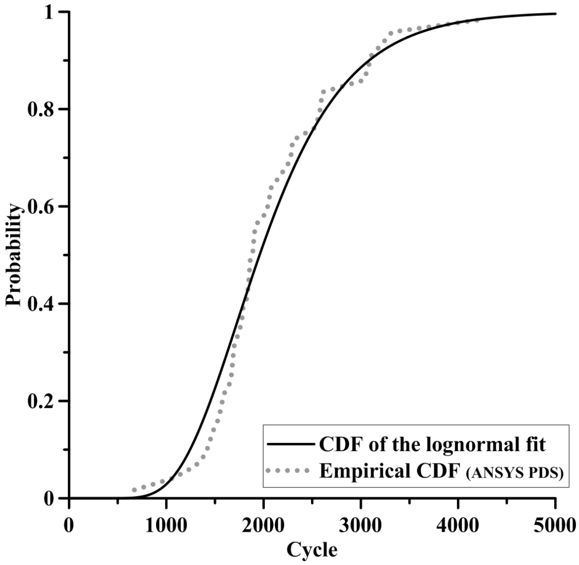

Figure 7: CDF of fatigue life due to random geometry and material properties

In the second part of the study, coefficients in the modified Coffin-Manson equation are assumed to be random, but the maximum equivalent strain range is kept as a deterministic value calculated from the nominal geometric and material property values stated previously. Under normal and/or lognormal assumptions made in Section 3.2 for those coefficients, their random effects on dispersive fatigue life can be obtained from two different tools, ANSYS PDS and the above derived equations. For example, when $\Delta\varepsilon$ is equal to 0.01265, under the assumption that A has a mean value of 0.04535, B has a mean value of -2.4489, and each random variable has a coefficient of variation of 3%, Figure 8 compares the CDF curves of the fatigue life produced from ANSYS PDS and the derived equations respectively for the case that (i) A is a lognormal random variable, (ii) B is a normal random variable, and (iii) A is a lognormal and B is simultaneously a normal random variable. From Figure 8, it is observed that result of case (iii) is almost the same as that of case (ii) but

Table 4: Sensitivity analysis of fatigue life

Variable	Spearman rank-order correlation coefficient
Solder ball radius	0.728
Substrate height	0.336
Coefficient of thermal expansion of the substrate	0.239
Young's modulus of the substrate	0.201
Young's modulus of the chip	-0.175
Coefficient of thermal expansion of the chip	0.092
Chip height	-0.015

very different from case (i). This is expected since coefficient B is an exponent in the modified Coffin-Manson equation and, therefore, affects the fatigue life more than coefficient A. Furthermore, Figure 8 shows that curves calculated from the two different methods match well with each other, but curves from equations we have derived in this study are smoother than those generated from ANSYS PDS. As stated before, since the smoothness can lead to more accurate statistical and reliability analysis of electronic packages, the derived equation is used instead of ANSYS PDS for subsequent investigation.

To investigate the influence of degree of randomness of coefficients A and B on the fatigue life distribution and compare the result with that of combined random geometric and material parameters stated in Section 3.1, Figures 9 to 11 are constructed. Among them, Figure 9 reflects the influence of random parameter A, Figure 10 reflects the influence of random parameter B, and Figure 11 reflects the combined influence of A and B. Three different coefficients of variation are considered in each case. Since B is more sensitive to the fatigue life than A as stated before, smaller coefficients of variation are considered in preparing Figures 10 and 11. From these figures, it is observed that for coefficient B, a coefficient of variation of 3% results in almost the same amount of fatigue life variation as those obtained previously for random geometric parameters and material properties. As for parameter A, this same amount of fatigue life variation would be caused by a coefficient of variation greater than 10%. It should be noted that, when the coefficient of variation of A is great than 10% or coefficient of variation of B is great than 3%, it reflects data used in constructing the modified Coffin-Manson equation scattering to a certain extent, which might not be realistic.

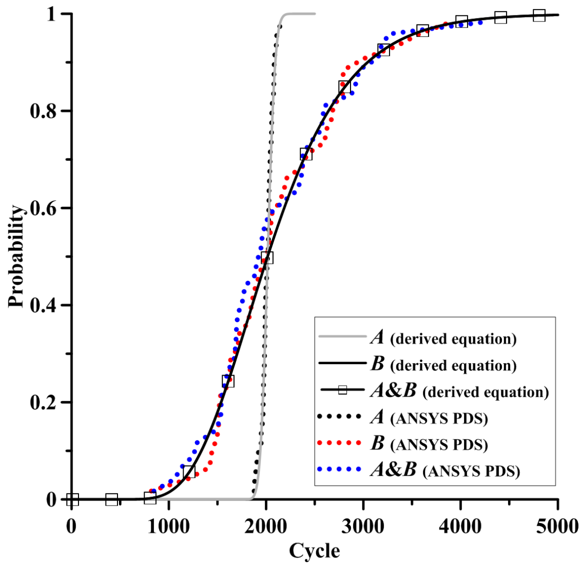

Figure 8: CDF of fatigue life due to random coefficients in modified Coffin-Manson equation

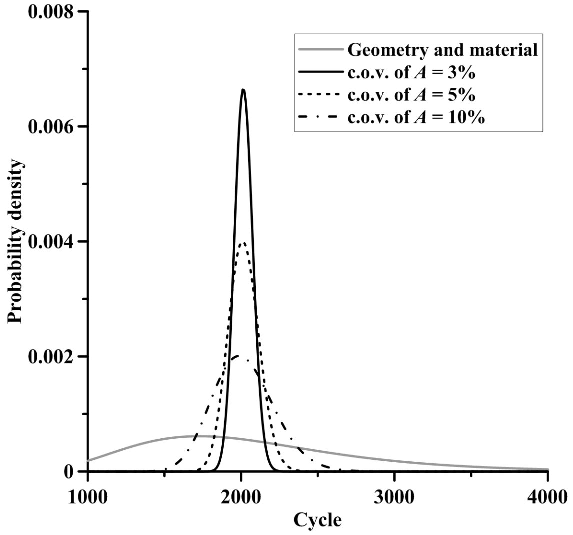

Figure 9: Influence of random coefficient A

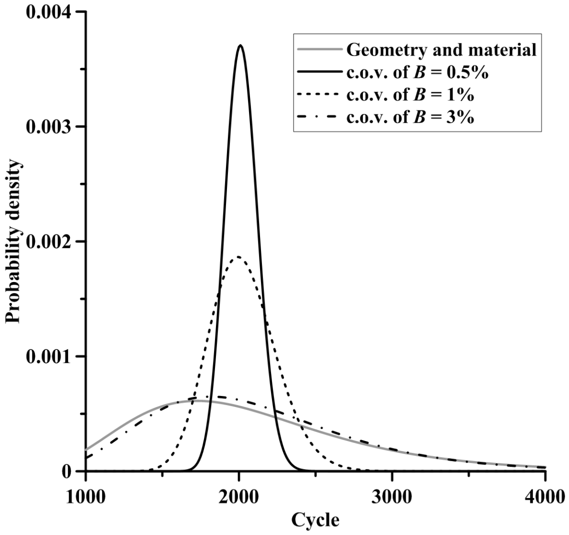

Figure 10: Influence of random coefficient *B*

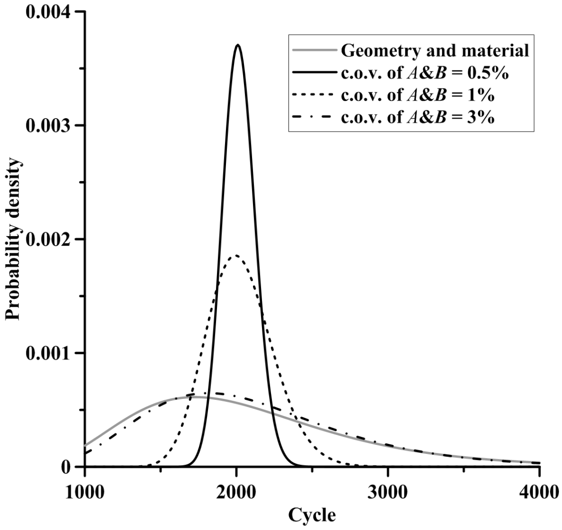

Figure 11: Combined influence of random coefficients *A* and *B*

To summarize results for all cases studied, Table 5 is constructed. By recognizing that the fatigue life of the studied package is a random variable resulted from consideration of different random variables, its mean value and standard deviation for each investigated case are listed, respectively, in the second and third columns of the table, and its coefficient of variation in the fourth column. The quantitative reliability of the package after usage of 1,800 thermal cycles is shown in the fifth column. To reflect scatter of the package life furthermore, the percentage of fatigue life distributed between 1,815 and 2,219 (2,017±10%) thermal cycles is shown in the last column of the table. It should be mentioned that the 'original fatigue life' in the table indicates the result obtained by an ordinary engineering approach assuming all parameters in the simulation and calculation are deterministic values, i.e. those mean or nominal values.

For the case considering randomness of geometry and material, all of their coefficients of variation are assumed to be 3%. Thus, if coefficient of variation of 3% is used as a bench mark and comparison is made among all cases studied in Table 5, the effect of combined geometric parameters and material properties is found to be slightly greater than that of combined random coefficients of A and B. The latter, however, results in more dispersive fatigue life than that when either A or B is considered random only. It should be noted that the reliability also shows its rapid decrease due to the random consideration. Moreover, it is found that coefficient A is the least sensitive factor to the evaluation of random fatigue life and quantitative reliability of the package.

It is also noted that without enough knowledge on random quantities A and B in the modified Coffin-Manson equation, the assumption of lognormal and normal distribution should be reasonable from central limit theory. Nevertheless, the real coefficient of variation for each coefficient or one of the coefficients should be obtained from regression analysis of the experimental data. It should be mentioned that even if coefficients A and B in the modified Coffin-Manson equation do not follow normal and/or lognormal distributions which prevents us to derive close-form expressions of mean and variance of the fatigue life, the investigation on variability of the fatigue life resulted from variations of coefficients A and B can still be conducted through the application of ANSYS PDS. As for the case of random geometric parameters and material properties, it is reasonable to assume them to be normally distributed from the viewpoint of statistical process control since the process parameters generally have normal distributions. Concerning their coefficients of variation, values can be estimated through statistical process control in real manufacturing processes.

Table 5: Comparison among various random considerations

		μ_{N_f} (cycle)	s_{N_f} (cycle)	c.o.v. of N_f	Reliability (1800 cycles)	Probability in [1815, 2219]
Original fatigue life		2,017	-	-	100%	100%
Combination of geometric parameter and material property with c.o.v. 3%		2,085	762	0.3654	59.40%	22.25%
Variation of A with c.o.v.	3%	2,017	60	0.0297	99.99%	99.91%
	5%	2,017	100	0.0495	98.84%	95.65%
	10%	2,017	201	0.0996	86.33%	68.71%
Variation of B with c.o.v.	0.5%	2,018	108	0.0535	98.26%	93.86%
	1%	2,027	217	0.1070	85.52%	65.23%
	3%	2,122	699	0.3294	63.76%	24.54%
Variation of A & B with c.o.v.	0.5%	2,018	108	0.0535	98.26%	93.86%
	1%	2,027	218	0.1075	85.40%	65.01%
	3%	2,122	702	0.3308	63.66%	24.44%

5 Conclusions

Under assumptions made in the present paper, several conclusions can be drawn as follows:

1. ANSYS PDS is shown to be an efficient tool for probabilistic design of electronic packages. The CDF of fatigue life and reliability of electronic packages subject to thermal-cyclic loading can be evaluated from running ANSYS PDS. However, the non-smooth CDF curves provided by ANSYS PDS would adversely affect the accuracy of reliability analysis. The refined probabilistic design procedure proposed in the present study can improve the smoothness of CDF curves so as to perform quantitative reliability analysis more accurately.

2. Through the refined probabilistic design procedure, it is found that the fatigue life of the studied electronic package has a lognormal distribution rather than Weibull distribution as commonly assumed by most researchers. This probability distribution is relevant to quantitative reliability analysis of electronic packages.

3. The sensitivity analysis result reveals that the solder ball radius is the most sensitive parameter to the fatigue life of the studied package. The sub-

strate height comes in second, followed by thermal expansion coefficient and Young's modulus of the substrate. The solder ball radius should be designed carefully since it not only affects fatigue life and reliability the most, but also influences the thermal deformation configuration of the package more than other parameters.

4. Randomness of coefficients in the modified Coffin-Manson equation reflecting its uncertainty also contributes to random fatigue life and reliability of the package to some extent. The random exponential coefficient has more influence on the fatigue life distribution than the other.

5. The degree of the fatigue-life dispersion owing to uncertainties resulting from both geometry and material properties of the electronic package is approximately equal to that resulting from uncertainties of coefficients in the modified Coffin-Manson equation. It indicates that engineers must carefully consider these uncertainties as well as their influences on the prediction of fatigue life and reliability of electronic packages.

Acknowledgements

This work is supported by the National Science Council of Taiwan, R.O.C. under Grant No. NSC 98-2221-E-002-003-MY2. The authors appreciate this financial support. They were also delighted to participate in the Symposium in Honor of Professor Wen-Hwa Chen on the Occasion of his Receiving the ICCES Life-Time Achievement Medal held in ICCES'13 Seattle during May 24-28, 2013. The corresponding author appreciates, in particular, Professor Chen's guidance and helps in all aspects during his career development. This paper contains part of the material presented in ICCES'13.

References

Anand, L. (1985): Constitutive equations for hot-working of metals. *International Journal of Plasticity*, vol. 1, pp. 213–231.

Ang, A. H. S.; Tang, W. H. (1975): *Probability Concepts in Engineering Planning and Design, Vol. I Basic Principles*. John Wiley & Sons, Inc.

Chen, W. H.; Lin, S. R.; Chiang, K. N. (2005): Stability of solder bridging for area array type packaging. *CMC: Computers, Materials & Continua*, vol. 2, no. 3, pp. 151–162.

Chen, W. H.; Lin, S. R.; Chiang, K. N. (2006): Predicting the liquid formation for the solder joints in flip chip technology. *Journal of Electronic Packaging, Transactions of the ASME*, vol. 128, no. 4, pp. 331–338.

Cheng, H. C.; Yu, C. Y.; Chen, W. H. (2005): An effective thermal-mechanical modeling methodology for large-scale area array typed packages. *CMES: Computer Modeling in Engineering & Sciences*, vol. 7, no. 1, pp. 1–17.

Coffin, L. F., Jr. (1954): A study of the effects of cyclic thermal stress on a ductile metal. *Transactions of the ASME*, vol. 76, pp. 931–950.

Collins, J. A. (1981): *Failure of Materials in Mechanical Design.* John Wiley & Sons, Inc.

Darveaux R. (2000): Effect of simulation methodology on solder joint crack growth correlation. *Proceedings of the 50th IEEE Electronic Components and Technology Conference*, Las Vegas, NV, USA, pp. 1048–1058.

Ebeling, C. E. (1997): *An Introduction to Reliability and Maintainability Engineering*. McGraw-Hill, Inc.

Engelmaier, W. (1990): The use environments of electronic assemblies and their impact on surface mount solder attachment reliability. *IEEE Transactions on Components, Hybrids, and Manufacturing Technology*, vol. 13, no. 4, pp. 903–908

Evans J. W.; Evans, J. Y.; Ghaffarian R.; Mawer A.; Lee, K. T.; Shin C. H. (2000): Simulation of fatigue distributions for ball grid arrays by the Monte Carlo method. *Microelectronics Reliability*, vol. 40, no. 7, pp. 1147–1155.

Gao, W.; Song, C.; Tin-Loi, F. (2009). Probabilistic interval response and reliability analysis of structures with a mixture of random and interval properties. *CMES: Computer Modeling in Engineering and Science*s, vol. 46, no. 2, pp. 151–190.

Huang, C. Y.; Huang, H. H. (2011): Comparison of results from empirical ALT test to CAE simulation for wireless clients. *Microelectronics International*, vol. 28, no. 3, pp. 51–59.

Hung, Y. H. (2007): Optimal process parameters design for a wire bonding of ultra-thin CSP package based on hybrid methods of artificial intelligence. *Microelectronics International*, vol. 24, no. 3, pp. 3–10.

JEDEC Solid State Technology Association (2005): *JESD22-A104C: Temperature Cycling*. Joint Electronic Devices Engineering Council.

Jong, W. R.; Chen, S. C.; Tsai, H. C.; Chiu, C. C.; Chang, H. T (2006): The geometrical effects of bumps on the fatigue life of flip-chip packages by Taguchi method. *Journal of Reinforced Plastics and Composites*, vol. 25, no. 1, pp. 99–114.

Kaminski, M. M. (2011): Probabilistic analysis of transient problems by the least squares stochastic perturbation-based finite element method. *CMES: Computer Modeling in Engineering and Sciences*, vol. 80, no. 2, pp. 113-140.

Kao, C. L.; Lai, Y. S.; Wang, T. H. (2007): Numerical evaluation of thermal cycling reliability of high performance flip-chip package assembly using submodeling analysis. *Journal of the Chinese Institute of Engineers*, vol. 30, no. 2, pp. 349–352.

Kececioglu, D. (1991): *Reliability Engineering Handbook, Vol. 1.* Prentice-Hall Inc.

Lai, Y. S.; Wang, T. H. (2005): Verification of submodeling technique in thermomechanical reliability assessment of flip-chip package assembly. *Microelectronics Reliability*, vol. 45, no. 3-4, pp. 575–582.

Lau, J.; Dauksher, W. (2005): Effects of ramp-time on the thermal-fatigue life of SnAgCu lead-free solder joints. *Proceedings of the 55th IEEE Electronic Components and Technology Conference*, Lake Buena Vista, FL, USA, pp. 1292–1298.

Lee, C. C.; Chang, K. C.; Yang, Y. W. (2009): Lead-free solder joint reliability estimation of flip chip package using FEM-based sensitivity analysis. *Soldering & Surface Mount Technology*, vol. 21, pp. 31-41.

Lee, C. C.; Chou, T. L.; Chiu, C. C.; Hsia, C. C.; Chiang, K. N. (2008): Cracking energy estimation of ultra low-*k* package using novel prediction approach combined with global-local modeling technique. *Microelectronic Engineering*, vol. 85, no. 10, pp. 2079–2084.

Lin, H. C.; Kung, C.; Chen, R. S.; Liang, G. T. (2006): An Optimization analysis of UBM thicknesses and solder geometry on a wafer level chip scale package using robust methods. *CMC: Computers, Materials & Continua*, vol. 3, no. 2, pp. 55–64.

Lin, Y. Y. (2005): *Mechanics analysis and quantitative reliability assessment of electronic packages*. Master thesis, National Taiwan University, Taipei, Taiwan.

Manson, S. S. (1966): *Thermal Stress and Low-Cycle Fatigue*. McGraw-Hill, Inc.

Norris, K. C.; Landzberg, A. H. (1969): Reliability of controlled collapse interconnections. *IBM Journal of Research and Development*, vol. 13, no.3, pp. 266–271.

van Driel W. D.; van Silfhout R. B. R.; Zhang G. Q. (2008): Reliability of wirebonds in micro-electronic packages. *Microelectronics International*, vol. 25, no. 2, pp. 15–22.

Wang, C.; Gao, W.; Yang, C. W.; Song, C. M. (2011). Non-deterministic structural response and reliability analysis using a hybrid perturbation-based stochastic finite element and quasi-Monte Carlo method. *CMC: Computers Materials & Continua*, vol. 25, no. 1, pp. 19–46.

Wang, T. H.; Lai, Y. S. (2005): Submodeling analysis for path-dependent thermomechanical problems. *Journal of Electronic Packaging, Transactions of the ASME*, vol. 127, no. 2, pp. 135–140.

Wu, M. L.; Barker, D. (2010): Rapid assessment of BGA life under vibration and bending, and influence of input parameter uncertainties. *Microelectronics Reliability,* vol. 50, no. 1, pp. 140–148

Yang D. G.; Liang J. S.; Li Q. Y.; Ernst L. J.; Zhang G. Q. (2004): Parametric study on flip chip package with lead-free solder joints by using the probabilistic designing approach. *Microelectronics Reliability*, vol. 44, no. 12, pp. 1947–1955.

Zulkifli, M. N.; Jamal, Z. A. Z.; Quadir, G. A. (2011): Temperature cycling analysis for ball grid array package using finite element analysis. *Microelectronics International*, vol. 28, no. 1, pp. 17–28.