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A Review of Metamaterial Invisibility Cloaks

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Abstract: The exciting features of metamaterial in conjunction with transformation optics leads to various applications in the microwave regime with such examples as invisible cloak, frequency selective surfaces (FSS), radomes, etc. The concept of electromagnetic invisibility is very much important in aerospace platform. Hence to study the feasibility of implementation of this concept for stealth, an extensive literature survey of metamaterial cloaks has been carried out and reported in this paper along with the basic concept of cloaking. To make the review more effective, the technical papers are classified into three broad sections viz. mathematical modeling, design and simulations, and fabrications and experimental demonstration. Further the design and simulation is focused on different techniques implemented such as finite difference time domain (FDTD), finite element method (FEM), finite integration technique (FIT), inductor-capacitor representation of metamaterial (LC MTM) etc. The review also reports the methods implemented for analysis of metamaterial cloaks with possibility of application to the specific frequency range.

Keywords: Cloaking, Metamaterial, FDTD, FEM, LC MTM

1 Introduction

Recently, a novel approach to the design of electromagnetic structures has been proposed, in which the electromagnetic waves are controlled within a material by introducing a prescribed spatial variation in the constitutive parameters. The emerging concept of metamaterials attracted the attention of researchers for this type of exciting applications in different frequency regions corresponding to the applications in microwave, terahertz (THz) region, infrared, optics, acoustics, etc.

In aerospace applications, invisibility implies preventing information about an object from reaching radar-like detectors. This has been done by reducing the cross-section of the object using various stealth techniques. The goal of cloaking phe-

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nomenon leads to the optimal version of stealth technique, i.e. no reflection or absorption of energy.

A perfectly invisibility cloak has the scattering property of vacuum. As stealth technology is an important part of aerospace engineering, an initiative has been taken here to study the feasibility of metamaterial invisibility cloak towards stealth technology. An extensive literature survey of metamaterial invisibility cloaks has been carried out and reported in this work. The methods implemented for analysis of metamaterial cloaks with possibility of application to the specific frequency range have been reviewed. The classifications of the papers have been done in three sections, viz., design and simulation, fabrication and experimental demonstration and mathematical analysis. Further, the design and simulation section is focused on different techniques implemented such as the finite-element method (FEM), finite-difference time-domain (FDTD), finite integration technique (FIT), and inductor-capacitor representation of metamaterial (LC MTM) etc.

2 Basic Concept of Cloaking

Hiding an object from detection by radar or any other detecting object is the prime motive behind cloaking. This can be achieved when an electromagnetic wave incident on an object to be concealed, comes out of the cloak without being scattered or reflected by that object, i.e., electromagnetic field should bend around the object. The design of a cloak uses transformation optics, in which a conformal coordinate transformation is applied to Maxwell's equations to obtain a spatially distributed set of constitutive parameters that defines the cloak. The permeability and permittivity tensors (Cui *et al.*, 2010) of the cloak material are derived in such a way that the material becomes spatially invariant, anisotropic and inhomogeneous, which is the property required to achieve cloaking.
Permittivity tensor,

$$\varepsilon' = \Lambda\, \varepsilon\, \Lambda^T / \det(\Lambda) \qquad (1)$$

Permeability tensor,

$$\mu' = \Lambda\, \mu\, \Lambda^T / \det(\Lambda) \qquad (2)$$

where, ε and μ are permittivity and permeability in free space respectively, and Λ is the Jacobian transformation tensor with components,

$$\Lambda_{ij} = \mathrm{dx}_i' / \mathrm{dx}_j \qquad (3)$$

Then a volume of free space is transformed into a shell type region using coordinate transformation, which can hide the object inside the shell from incident electromagnetic waves. Figure 1 shows schematic of a typical cloak. The cloak is a multilayer structure of radius *b*, and the object to be cloaked is shown as a PEC cylinder having radius *a*.

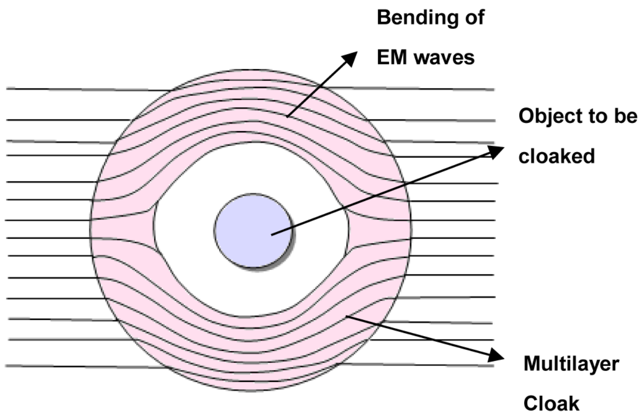

Figure 1: Illustration of cylindrical cloak.

3 Design and Simulation of Metamaterial Invisibility Cloak

The 3-dimentional conformal multilayer structure of cloaking device makes the analysis very complex and no analytical techniques exists for the same. Hence, numerical techniques are essential for design and simulation of an invisibility cloak. The technical papers based on method used for simulation are categorized into four subsections, viz., finite-element method (FEM), finite-difference time-domain method (FDTD), finite integration technique (FIT), and inductor-capacitor representation of metamaterial (LC MTM). The FEM, FDTD and FIT are well-known numerical techniques. In contrast LC MTM is a type of equivalent circuit analysis method, where the analysis is done using the inductive and the capacitive components that represent the metamaterial.

3.1 Cloaking by Finite Element Method (FEM)

The finite element method (FEM) is a numerical technique, which gives approximate solutions to the partial differential as well as integral equations (Atluri, 2005). FEM is a special case of the Galerkin method with polynomial approximation functions. It is observed that this method has been most frequently used for design and simulations of invisibility cloak.

The first practical 2D metamaterial cloak was realized by Schurig *et al.* (2006). The cloaked object was a copper cylinder and the cloak was made up of 10 concentric layers of cylinders, each of which had 3 unit cells (SRR) arranged in diagonal basis. The frequency of operation of the cloak was 8.5 GHz. The working of the cloak was proved by comparing the measured fields to simulated (by COMSOL Multiphysics S/w) fields, and the results showed that cloak reduces both the forward and backscatter. The invisibility was imperfect due to approximations and material absorption. The coordinate transformation used here compresses the cylindrical region $0 < r < b$ into $a < r' < b$. The transformation that helps to attain this objective is,

$$r' = \frac{b-a}{b} r + a \tag{4}$$

$$\theta' = \theta \tag{5}$$

$$z' = z \tag{6}$$

The expression for the permittivity and permeability tensor is,

$$\varepsilon_r = \mu_r = \frac{r-a}{r} \tag{7}$$

$$\varepsilon_\theta = \mu_\theta = \frac{r}{r-a} \tag{8}$$

$$\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \tag{9}$$

Further Leonhardt (2006) introduced a conformal mapping technique to reduce the imperfections in perfect invisibility within the accuracy of geometrical optics for objects that are larger than the wavelength of operation. This technique also proved that perfect invisibility cannot be achieved (because waves are not only refracted at the boundary but also reflected), but the reflection can be reduced by anti-reflection coating.

Coordinate transformation method was also used by Cai *et al.* (2007*a*) for the design of a non-magnetic cylindrical cloak at optical frequency. This cloak was successful in hiding macroscopic objects by compressing a cylindrical region into concentric cylinders. The permittivity could be achieved using metal wires of sub-wavelength size and the wavelength of operation was 632.8 nm. The field distribution with and without cloak were shown using COMSOL Multiphysics S/w. The wave-fronts flew around the cloaked region with little perturbation in presence of the cloak, and the object casted a shadow in the absence of cloak. The co-ordinate transformation used here is similar to the one used by Schurig as given by eqs.

(4) to (2). The constitutive parameter tensors of the cloak are reduced, *i.e.* one of the three parameters is kept constant and the other two are varied. Hence the permittivity and permeability tensors may be expressed as,

$$\varepsilon_r = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r} \tag{10}$$

$$\varepsilon_\theta = \left(\frac{b}{b-a}\right)^2 \tag{11}$$

$$\mu_z = 1 \tag{12}$$

The power reflection is predicted as

$$\Gamma = \left(\frac{R_{ab}}{2-R_{ab}}\right)^2 \tag{13}$$

where, $R_{ab} = b/a$.

Using this method cloaking can also be achieved in infrared and microwave regions.

Further Cai *et al.* (2007*b*) demonstrated the use of higher order transformation to eliminate undesired scattering from the cloaked object at a wavelength of 632.8 nm. The radiation pattern of the scattered field was plotted for four cases, viz., bare cylinder, an ideal linear cloak, a linear non-magnetic cloak and an optimal quadratic cloak. It was demonstrated using COMSOL Multiphysics S/w that the scattered field outside an ideal cloak is zero. Compared to the usual linear non-magnetic cloak, the scattering was reduced by almost an order of magnitude in a cloak with optimal quadratic transformation.

Coordinate transformation was also used by Zolla *et al.* (2007) in the finite element analysis of a diffraction problem involving a coated cylinder, which enabled cloaking of a lossy object with sharp wedges. It was revealed that in electromagnetism, a change in the co-ordinates results in a different material. The response of the cloak is calculated using this phenomenon. The electromagnetic field radiated by an electric line source and a magnetic dipole located at an arbitrary point inside the cloak in presence of an F-shaped object was calculated. Mirage effects were observed while shifting the position of the source.

A square electromagnetic cloak and an omnidirectional field concentrator were designed by Rahm *et al.* (2007) using the permittivity and permeability tensors derived from the form-invariant transformations of Maxwell's equations. The frequency of operation of the cloak was 8.5GHz. Based on simulation (COMSOL Multiphysics) results obtained from a finite element solver, it was concluded that cloaks of any arbitrary shape can be designed using this method. An example of a square cloak is shown in Figure 2. In case of concentrator, reflections were reduced by impedance matching and hence enhanced the EM energy density of incident waves in a given area.

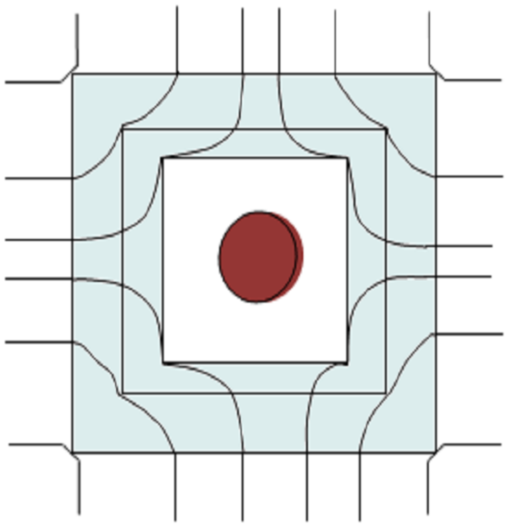

Figure 2: Schematic of the square shaped EM cloak design.

Isic *et al.* (2007) investigated strictness of conditions imposed on the parameters of metamaterial cloaks by calculating the degree of wave scattering when these parameters (non-singular) have variations *w.r.t* ideal theoretical value. The object to be hidden was a perfect electric conductor. The analytical results obtained using transformation optics was compared with finite element simulations of Helmholtz equation, which were in agreement with one another.

A cloak was designed by Chen *et al.* (2007) based on coordinate transformation to make an object invisible to a wide range of frequencies instead of a single frequency. The cross section was sacrificed to achieve broad bandwidth. A dispersive cloak was designed, in which continuous cloaking shell was broken into 10 shells in the radial direction and the parameters of the split ring resonators (SRR) for each shell were determined over a frequency range of 8.5 GHz to 8.75 GHz. The scattering cross section for a bare PEC, dispersive cloak and a single frequency cloak were plotted (by COMSOL Multiphysics S/w), in which it was reported that the dispersive cloak worked for a broader frequency.

The parameters for the 2-D cylindrical cloak were determined by Zhang *et al.* (2008a) for multilayered and gradually changing (inhomogeneous) background.

The effect of a Gaussian beam in gradually changing media and the bending of the beam in presence of cloak is shown. The frequency of operation was 2GHz. Theoretical analysis was carried out based on finite element and coordinate transformation. Results proved that cloaking with proposed parameters performs well in the inhomogeneous medium. The method could also be extended to spherical cloaks.

A novel idea of non-singular mapping, instead of singular mapping was used by Isic *et al.* (2008) to design an imperfect cylindrical cloak. The cloaking efficiency depended on the medium that filled the hidden region. The cloak reduced the size of the concealed object making use of the form invariance of Maxwell's equation instead of hiding it. Optimization in the design of cloak could be done by considering different mappings and finding the most appropriate one. Scattering and radiation from the imperfect cylindrical cloak was solved analytically to obtain the electric field intensity in terms of Bessel's and Hankel's functions, and the results were confirmed by full-wave finite element simulation (COMSOL Multiphysics).

An effort to investigate the challenges of realizing a cloak in practice using full wave simulation of cylindrical cloaking using ideal and non-ideal EM parameters was reported by Cummer *et al.* (2008*a*). The electric field distribution and electromagnetic power flow were plotted (using COMSOL Multiphysics S/w) for various configurations such as, cloak with simplified parameters, with ideal parameters, with and without loss, and cloak with 8-layer approximation. Cloaking effect was clear in ideal case. However, the performance of cloak deteriorated with increase in loss.

A scheme for cloaking objects on a dielectric half-space was proposed by Zhang *et al.* (2008*b*), in which two vertical matching strips at the bottom surfaces of a semi-cylindrical cloaking cover were introduced to achieve the cloaking effect. The cloaking effect was tested by placing a PEC scatterer on the axis of the cloaking cover. The matching strips eliminated the non-uniformity of intensity distribution making the reflected cylindrical wave uniform resulting in cloaking.

Co-ordinate transformation approach was further used by W. X. Jiang (2008*a*) for design of cloaks of arbitrary shape. The anisotropic, inhomogeneous permittivity and permeability tensors were derived for an elliptical cloak, which could be extended to spherical cloak. The cloaking phenomenon was illustrated using an elliptical cylindrical cloak operating at a frequency of 2 GHz. The object to be hidden was a perfect electric conductor (PEC). The electric field distributions and power flow lines were plotted for lossy and lossless cloaking material, which showed that fields in the interior region were zero, and the power flow lines propagated smoothly around the PEC. Outside the cloak, the waves however were unchanged thus achieving cloaking. Loss degraded the performance of the cloak in the forward scattering direction but the invisibility effect was good for the backward and other direction. The computational domain used for full wave simulation of elliptical cylindrical cloaks is as shown in Figure 3.

The finite element analysis of a diffraction problem involving a coated cylinder for cloaking a lossy object with sharp wedges was carried out by Zolla *et al.* (2007). Nicolet *et al.* (2008) then extended this analysis to a finite conducting object with sharp wedges. The EM wave picture in the case of a 2D cylindrical object with a cloak of circular shape was given and then the results were extended to an elliptical cloak. From the computations it was observed that invisibility should be restricted to some approximations of the material properties.

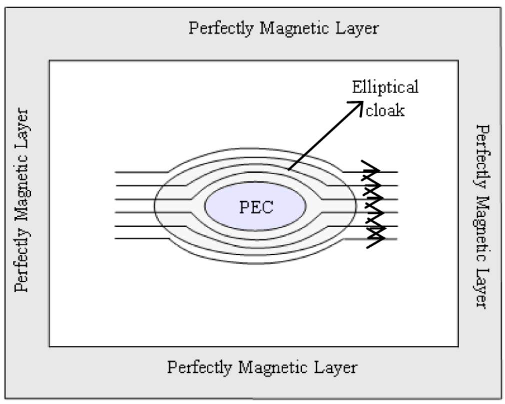

Figure 3: Full-wave simulations domain of elliptical-cylindrical cloaks. The cloaking object is an elliptical cylindrical PEC, and the cloak was made up of anisotropic material.

Design of different types of invisibility cloaks were presented by Ribeiro and Paiva (2008) using the equivalence principle of electromagnetics. A new metamaterial transformation called the left translation in a group structure was used. A metric perturbation g, which corresponds to the cloaking topology, was determined. The geometric interpretation was converted to material interpretation by using this metric perturbation. Then the equivalence principle was used to determine the metamaterial that creates the new geometry. Illustration was done for spherical and elliptical cylindrical co-ordinates.

A general transformation method was presented by Li *et al.* (2008*c*) to design cylindrical cloak with arbitrary regular or irregular cross-sections. The strategy was to introduce a new co-ordinate system conformal to the surface of the scatterer to be concealed. The EM properties of a 2D cloak with irregular geometry was analyzed and designed. The frequency of the harmonic wave considered was 1 GHz. The design was validated using full wave simulations based on finite element method (FEM) and the performance was evaluated based on Huygens's principle.

Cloaks of conical shape with increasing cross-section along the axial direction were introduced by Luo *et al.* (2008*b*). Full wave scattering model was used to analyze the EM characteristics of this cloak, which showed that perfect conical cloak can guide any kind of EM wave. It was shown that for azimuthally invariant incoming field, simplified conical cloak of wide working frequency with permittivity and permeability tensors greater than one could be constructed. The design was also used to achieve a polarization rotator.

A general transformation method was used by Li and Li (2008*b*) to design cylindrical cloaks with arbitrary regular or irregular cross-section. This method was extended to design 2D cloaks with non-conformal inner and outer boundaries (Li *et al.*, 2008*c*). The frequency of operation was 1GHz. The EM waves were guided around the inner region by the cloak with non-conformal inner and outer boundaries. The field transformation and medium parameters were verified numerically based on finite element method inconjunction with Huygens's principle. The numerical values were in good agreement with the theoretical results. The interactions of the cloak with EM waves from different orientations were also studied.

Soon after the analysis of transformation approach for cylindrical cloaks with arbitrary regular or irregular cross-section, Li *et al.* (2008*d*) presented the design of 2D electromagnetic cloak devices with polygon cross-section. An irregular polygon cloak was designed to show the flexibility of the method. Full wave simulations were performed to verify the invisibility of the irregular polygon cloak to external incident waves.

Co-ordinate transformation was also used to design an elliptical invisibility cloak by Jiang *et al.* (2008*b*). To avoid singularities in parameters, the cloak crushed the object to a line segment instead of a point. The operational frequency of the designed cloak was 9 GHz. The elliptical cloak was validated using full wave simulations based on finite element method. The result showed that reduced parameter cloak had a better performance than the full parameter cloak.

A method was proposed by Zhang *et al.* (2008*e*) to generate reduced 2D cloaks from perfect 3D cloaks to avoid singularity of the constitutive parameter tensors (CPT). Elliptic and bow-tie cloaks were designed. FEM simulations showed that reduced 2D model works for any polarization and the irrelevant components of CPT could be discarded. Simulation results showed that proposed reduced 2D cloak could make PEC cylinders nearly invisible. Figure 4 shows the schematic of the bow-tie cloak.

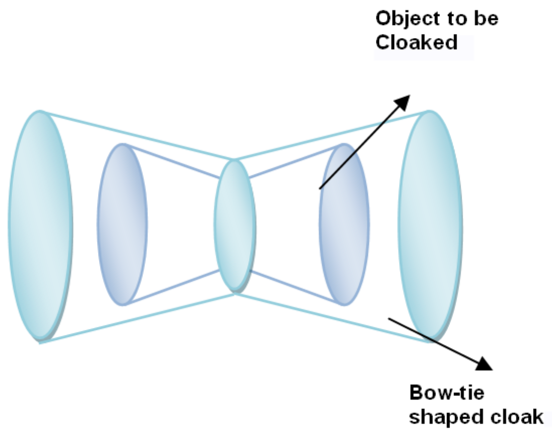

Figure 4: Schematic of the bow-tie cloak which can conceal a bow–tie type of object.

Farhat *et al.* (2009*a*) introduced a cylindrical cloak to control the bending of waves, propagating inside thin plates. It was shown that a heterogeneous orthotropic cloaking could be performed by an anisotropic Young's modulus and a radially dependent isotropic mass density. The response of a cloak surrounding a clamped obstacle in presence of a cylindrical excitation was analyzed. By computing the diffracted vibration amplitude, it was shown that the total electric field was identical to the incident field thereby achieving cloaking. Further, the same principle was used (Farhat *et al.*, 2009*b*), to study the EM and acoustic cloaking mechanisms to the domain of flexural waves propagating within thin infinite elastic plates. The design of a broadband multilayered cloak consisting of a large number of thin homogeneous isotropic layers was also proposed.

Xi *et al.* (2009) proposed a one-directional perfect cloak made of homogeneous anisotropic materials. The principle of this cloak was that, there would be no scattering when transverse magnetic (TM) polarized waves were incident on PEC surface. A diamond shaped cloak was designed by co-ordinate transformation. One-directional non-magnetic cloak with simplified parameters was also proposed. FEM simulations were used to prove cloak effect.

The cloaks proposed so far hid objects within the cloaking shell, whereas Lai *et al.* (2009) proposed a cloak that could conceal objects external to the cloak. The object as well as the surrounding space was optically cancelled out by using a complementary media layer with an embedded complementary image of the object called anti-object. Then the correct optical path in the cancelled space was restored by a dielectric core material. Hence the total system was equal to a piece of empty space fitted into a cancelled space thus achieving invisibility.

A strategy to reduce the parameters of a 2D cloak meanwhile restoring the refractive index was proposed by Chen *et al.* (2009b). Reduced cloak could be derived from mapping a virtual cylindrical space of isotropic inhomogeneous constitutive parameters, to a cylindrical annular region of constitutive parameters using transfer matrix method. The wavelength of operation of the designed cloak was 0.1 nm. COMSOL Multiphysics was used for numerical calculations.

An infrared optical carpet cloak was proposed using a co-ordinate transformation, which required only homogeneous anisotropic dielectric material by Xu *et al.* (2009). The cloak was designed by fabricating a uniform silicon grating structure over silicon on insulator wafer. The wavelength of operation was 1372-2000 nm. Working of cloak was tested by mapping 2D fields and calculating the near-field magnetic field distributions for four different cases.

A 3D axiolitic cloak was proposed by Zhang *et al.* (2009*b*) based on coordinate transformation. Permittivity and permeability tensors of the cloak were derived using the form invariant property of Maxwell's equation in the transformed space. Simulations (COMSOL Multiphysics) were carried out for arbitrary 3D axiolitic cloaks at 2 GHz. Results showed that outside the cloaking shell, EM waves were unaltered and inside the cloaking material, the phase fronts were bent smoothly around cloaking shell thus proving the invisibility.

Maxwell's equations were solved by Luo *et al.* (2009) in elliptical cylindrical coordinate system to study the electromagnetic characteristics of the elliptical cylindrical cloak. General boundary conditions were derived and it was proved that an ideal elliptical cylindrical cloak could not achieve perfect invisibility when it was exposed to an EM wave directly. The effect of non-ideal parameters on cloaking performance was analyzed, which confirmed that adding loss to material parameters could suppress the backward scattering.

A two step co-ordinate transformation was used by Li *et al.* (2009) to achieve a near-perfect, non-singular cylindrical cloak with diamond cross section. Singularity in material parameters was avoided. The space was compressed into two orthogonal directions resulting in a diamond shaped cloak operating at 0.15 GHz. Simulation results showed that the cloak had a good performance for all incident directions of plane waves and could work at multiple frequencies.

An invisibility cloak that contained two or more cavities was proposed by Chen and Weng (2009*a*). The cloaked region was simulated by using a hippopedal curve. The equation that generates a hippopedal curve (Figure 5) is given by

$$r^2 = 4b\left(a - b\sin^2\theta\right) \tag{14}$$

Two objects were hidden simultaneously without the knowledge of their mutual

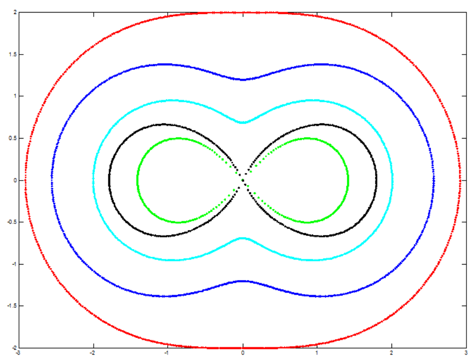

Figure 5: The hippopedal curves creating two cavities.

presence. The numerical simulations based on finite element method showed that the fields were bent around the cloaking region and returned to their original trajectory outside the curved cloak. The operational frequency of this cloak was 3GHz. In case of connected bounded oval structure, the incoming rays passed in and out of the cloaking region twice.

An ellipsoidal cloak placed in inhomogeneous medium for layered as well as gradually changing medium was studied by Han *et al.* (2010) by deriving the constitutive parameter tensors based on co-ordinate transformation theory. The frequency of operation was 1.5GHz. The simulation results showed that the ellipsoidal cloak could deflect the arbitrarily incident waves and guide them to propagate into the inner object. Loss introduced in the cloak degraded the performance in the forward scattering region but invisibility was good for backscattering region.

A 3D broadband ground plane cloak was realized by Ma and Cui (2010*b*). It had the ability to conceal a 3D object located under a curved conducting plane from all viewing angles. This cloak was designed by drilling holes in layered dielectric plates making it a broadband, low-loss cloak. A high gain lens antenna was designed as the transmitter of the measurement system to measure the cloaking properties.

Transformation optics and conformal mapping were used by Zhang *et al.* (2010) to design a 2D carpet cloak to hide an object in a dielectric half-space. Dielectric halfspaces with and without loss were simulated based on the finite element method. In the lossy case, the incident beam was scattered over a wide range of directions and the reflected beam was restored to an expected direction. In the lossy case, the reflection was restored and refracted waves were attenuated. This cloak was characterized by non-singular material parameters and hence broadband cloaking was achieved at both the microwave and the optical frequencies.

The EM characteristic of metamaterial cylindrical cloak, which conceals a dielectric cylinder illuminated by electric line source, was demonstrated by Wu *et al.* (2010). The general solution of the EM field in the cloak was given and the distribution of electric field was calculated by *Mathematica* based on analytical expressions. Based on the near and far field analyses, it was seen that the cloak bent the EM waves and guided them to propagate around the inner region thereby achieving invisibility.

An efficient FEM solver was used by Zhai *et al.* (2010) to overcome the memory and time requirement in conventional commercial software analyzing 3D axisymmetric EM devices such as invisibility cloak and concentrators designed by transformation approach. An edge-based vector basis function was used to expand the transverse field components and node-based scalar basis function was used to expand the angular component. Rotators, hyper-lens, etc. could also be analyzed by this method.

In order to overcome the lateral shift of the scattering fields in ground plane invisibility cloaks based on isotropic design through quasi-conformal transformation, an alternative method was used by Xu *et al.* (2010) to design the cloak using EM beam modulation through co-ordinate transformation. It was shown that the proposed co-ordinate transformation could overcome the problem of EM beam lateral shift by plotting the near and far field distributions over a spread of different incident angles.

A homogeneous invisibility carpet cloak with uniform silicon grating structure was fabricated by Zhang *et al.* (2011*b*) by electron beam lithography (EBL). Reactive etching process was used to cloak an object of comparatively larger size and to overcome the requirement of spatially varying refractive index. The wavelength of operation was 1480 nm to 1580 nm. The simulation results showed that reflection from the cloaked surface resembled that reflected by a flat surface thereby achieving the cloaking effect, whereas from the bare surface, two largely separated beams emerged as a result of protrusion of the ground surface.

3.2 Cloaking by FDTD Method

Finite-difference time-domain (FDTD) method is a widely used computational technique for various EM applications. FDTD is a systematic approach where a new structure to be modeled, is reduced to a problem of mesh generation rather than reformulation of an integral equation, thereby reducing complexity (Taflove & Hagness, 2005; Young *et al.*, 2005). It has a simple implementation for a full-wave solver. Since it is a time-domain method, solutions can cover a wide frequency range, with a single simulation run. FDTD belongs to the grid-based differential time-domain numerical modeling methods.

A novel transmission line network method was used by Alitalo *et al.* (2008) to simulate the wave propagation in the medium surrounding a cloaked object. By coupling the EM fields outside the medium into the transmission line network, the object was hidden. The effect of non-ideal propagation characteristics on cloaking were studied using FDTD code. Scattering cross section yielded a measure of the performance of the cloak as given below:

$$\sigma_s = 2\pi R \frac{|E_s|^2}{|E_{inc}|^2} \tag{15}$$

where, R is the distance from the center of the cloaked object to the boundary of the cloak, E_s is the scattered electric field and E_{inc} is the incident electric field. The scattering cross section was calculated for an idealized cloak and an array of vertical rods made of PEC. In both the cases, it was observed that the cloaking efficiency improves after 4 GHz. The simulations were verified using Ansoft HFSS software package.

Co-ordinate transformation was employed by Zharova *et al.* (2008) to initiate the idea of inside-out (inverse) cloak, which could be employed as a non-reflecting boundary layer in numerical simulations of EM wave propagation. The concept of inverse cloak was employed for creating a new type of perfectly matching layer for the FDTD simulations in one and two dimensions. The advantage of this cloak was that they worked simultaneously for all polarization and could be employed in any kind of numerical simulations.

The dynamical process of a cloaking structure was investigated by Liang *et al.* (2008) using finite-difference time-domain (FDTD) simulations and introducing Lorentzian dispersion into the permittivity and permeability models. The dispersive effect of dynamical process was investigated by tuning the dispersion parameters. It was shown that the cloaking structure with weaker dispersion would scatter lesser field (better cloaking effect) in the dynamical process.

A new type of cloak, in which all the cloaked objects appeared as a flat conducting sheet was proposed by Li and Pendry (2008*a*). The permittivity and permeability tensors were designed to have singular values. The anisotropy of the cloak was minimized to a small value by choosing a suitable coordinate transform thereby avoiding the usage of metamaterials. The field pattern was obtained using FDTD simulations.

Further, coordinate transformation was used by Kallos *et al*. (2009) to create a cloak that covered a triangular shaped conducting object placed on a ground plane. A 2D mesh that surrounded the domain above the object was found using conformal mapping. The frequency of operation was 400 THz. To test the cloak, a new approximate cloak with 16 evenly distributed values of permittivity was chosen. Scattering field distribution from the object for a TM pulse using non-orthogonal FDTD and 2D FDTD were plotted. The comparison confirmed that both results were similar.

3.3 Cloaking by Finite Integration Technique (FIT)

The finite integration technique (FIT) is used for full range of electromagnetics and optics applications and is the basis for commercial simulation tools. This numerical method uses a spatial discretization scheme to solve EM field problems both in the time and frequency domains. The basic approach is to apply the Maxwell equations in integral form to a set of staggered grids. This method is well-suited for highly flexible geometric modeling and boundary handling problems of arbitrary material distributions and material properties (Clemens and Weiland, 2001).

The possibility of using a multilayered plasmonic shell as a cloak for reducing the total scattering cross section of a particle (impenetrable), simultaneously at different frequencies in the optical domain was suggested theoretically by Alu and Engheta (2008). By exploiting the property of negative polarizability and frequency dispersion of plasmonic material, the effect of covering a dielectric or conducting object with multilayered cloak was investigated. This cloak reduced the visibility at different frequencies. The variation of total scattering efficiency, the magnitude and phase of near zero electric field distribution were plotted for three different cases using Mie scattering and validated with FIT software. The total scattering cross section of the system was reduced by 99.2%. The results showed that cloaking performance was better at larger wavelength.

The experimental realization of 2D plasmonic cloaking and measurement at microwave frequencies was reported by Edwards *et al*. (2009) at a frequency of 2 GHz. The total electric field distribution was compared using finite integration technique for the air-filled case and metamaterial based plasmonic layer around the object. The results showed that there was a shadow behind and standing wave in front of the object, produced by large amount of scattering from the central cylinder for the air-filled case. The scattering width was decreased by 75% for the plasmonic layer.

Cylindrical and spherical cloaks were designed at visible frequencies by Bilotti *et al.* (2010) based on the principle of employing alternate layers of plasmonic and non-plasmonic materials to obtain epsilon near zero (ENZ) behavior at optical frequencies. The frequency of operation was 600 THz. Two different cases were analyzed for cylindrical cloak, one surrounded by a homogeneous cover and the other surrounded by a cloak made of concentric shells of Ag (Silver) and SiO_2(silicon-dioxide). The designs were done using full-wave numerical simulations performed with a numerical code based on finite integration technique.

3.4 Cloaking by LC MTM Method

Metamaterials can be represented by distributed L-C circuit by spatially discretizing Maxwell's equation, thereby arriving at Kirchhoff's voltage and current laws. The lumped-element model of the transmission line can be determined by applying quasi-static field conditions to Maxwell's equations and obtaining the circuit equations thereby drawing the analogy between permeability, inductance, permittivity, and capacitance. The L-C circuit network representation lies in the fact that the capacitive and inductive elements directly determine the constitutive parameters. For a 2D transmission line network, the series inductance $Z(\omega)$ and the shunt admittance $Y(\omega)$ with conductance g, represent the permeability and permittivity, given by

$$\mu\ (\omega) = (1/\mathrm{g})(\mathrm{Z}\ (\omega)/\mathrm{d}/\mathrm{j}\omega) \tag{16}$$

$$\varepsilon(\omega) = \mathrm{g}(\mathrm{Y}\ (\omega)/\mathrm{d}/\mathrm{j}\omega) \tag{17}$$

A transmission line approach was used by Liu *et al.* (2009*b*) to realize a 2D cylindrical invisible cloak using planar anisotropic metamaterials at 50 MHz. It was shown that an inhomogeneous and anisotropic medium, with permittivity and permeability diagonalized in cylindrical basis, can be artificially synthesized by non-periodic transmission line (TL) networks. The voltage distribution of a conducting cylinder with and without cloak was simulated. Scattering was observed in case of bare cylinder, whereas in case of cylinder with cloak, the back scattering and the forward scattering were insignificant.

The use of LC components to achieve ENZ or MNZ (permittivity and permeability negative or close to zero) behavior, which is a requirement for cloaking, was demonstrated by Hrabar *et al.* (2010). The cloak (operational frequency: 1 GHz) showed broader bandwidth than the split ring resonator (SRR) based cloak due to different type of dispersion. The values of capacitance and inductance were found using coordinate transformation. Simulation was done using ADS circuit simulator. Results showed that the cloaking effect was achieved but it worsened with the change in frequency. The relation between tensor elements and the values of lumped elements are given by:

$$L_r = \mu_\phi d \frac{\Delta r}{r\Delta\phi} \tag{18}$$

$$L_\theta = \mu_r d \frac{r\Delta\phi}{\Delta r} \tag{19}$$

$$C_z = \frac{\varepsilon_z r\Delta r\Delta\phi}{d} \tag{20}$$

where L and C are the capacitive and inductive components, d is the thickness of the cloak and Δr, $\Delta\phi$ are the dimensions of the metamaterial unit cell.

Malcic *et al.* (2011) showed that the bandwidth in a volumetric SRR based cloak and a planar transmission line based cloak were restricted by basic physics and not by applied technology. The constitutive parameter tensors derived were anisotropic, which ensured polarization independence of the cloak. The cloak was constructed using SRR like inclusions of 10 cylindrical layers. The structure was illuminated by an open-ended waveguide and the measured field distribution proved cloaking at a single frequency.

3.5 Cloaking by Other Simulation Techniques:

The similarity between anisotropic soft surface and isotropic artificial magnetic conductor (band gap surface) is explained by Kildal *et al.* (2007). These authors demonstrated the application of hard surfaces to cloak cylindrical objects. A circular cylinder of 54 mm was cloaked over a frequency range of 6-11 GHz. By coating the cylinder with an ideal PMC, the blockage width was reduced by 47 mm. With the 10 layer metamaterial coating, the blockage width was further reduced and varied between 20-37 mm. Results showed that hard surface cloaks performed better than the metamaterial cloaks.

A 2D electromagnetic cylindrical cloak was designed by Huang *et al.* (2007) that did not require metamaterials with subwavelength structured inclusions to realize the anisotropy or inhomogeneity of the material parameters. The low-reflection and power-flow bending properties of the EM cloaking structure was obtained by proper design. The far-field scattering pattern was plotted, which showed that concentric layered structure could be used to realize a cylindrical shell. The magnetic field distribution showed that in presence of cloaking structure, the wave-fronts were deflected and guided around the cloaked region and returned to the original propagation direction with little perturbation. In the absence of the cloaked structure, the waves were scattered by the object.

Ren *et al.* (2011) proposed a novel 2D isotropic acoustic metamaterial towards acoustic cloaking applications. They introduced conformal mapping based transformation acoustics to realize the isotropic metamaterials for easy implementation of the cloak design.

A cylindrical wave expansion method was developed by Ruan *et al.* (2007) to obtain the scattering field for an ideal 2D cylindrical invisibility cloak. The scattering co-efficients could not be obtained directly by rigorous scattering theory. Therefore a small perturbation called near ideal cloak was defined. By systematically studying the change of the scattering coefficients from the near ideal to ideal case, it was confirmed that the cloak with ideal material parameter was a perfect invisibility cloak.

The conditions required for matter-wave cloaking were determined and confirmed by particle and wave approaches by Zhang *et al.* (2008*c*). An invariant transformation of the Schrodinger equation for quantum waves was studied and it was shown that matter cloaking could be achieved by spatially controlling the potential and effective mass of a particle as it travelled inside the medium. The results were verified for a spherical cloak, by calculating the scattering of quantum waves incident upon the cloak and the trajectories of classical particles inside such a cloaking system.

An approach to broadband cloaking of light waves was analyzed by Kildishev *et al.* (2008) for a simplified design, which involved concentric cylindrical layers using scaling transformation. Wavelength multiplexing was used as the basic concept for achieving broadband cloaking. A number of different inner boundaries and transformations were used to provide the broadband capability. It was also shown that pseudo-broadband cloaking could be realized when there was amplification at the invisibility wavelength and at other wavelengths the signal should be suppressed.

Coordinate transformation approach was used by You *et al.* (2008) towards design of 3D rounded cuboids, and 3D rounded cylindrical cloaks. The electric field distributions and the scattering cross-sections associated with cloaked object, was simulated using discrete dipole approximation (DDA) method. Simulated fields showed satisfactory cloaking effect for planar incidence.

Two designs for optical cloaking based on non-linear transformation for a non-magnetic cylindrical cloak for TM incidence and a magnetic cylindrical cloak for transverse electric (TE) incidence were provided by Cai and Shalaev (2008). The magnetic cloak was realized with a metamaterial consisting of periodic arrays of high-permittivity material. The simulation results showed that for a non-magnetic cylindrical cloak with any transformation function, the radial component of permittivity varied from 0 at the inner boundary, to 1 at the outer surface, while the azimuthal component was a function of radius with varying positive values.

A perfect cylindrical invisibility cloak with spatially invariant axial material parameters was proposed by Luo *et al.* (2008*a*) in order to reduce the difficulties in the experimental realizations of the cloak. Only radial and azimuthal components of permittivity or permeability were spatially invariant. The scattering properties of this cloak with a perturbation at the inner boundary were examined by cylindrical wave expansion method. The results showed that backward scattering and total cross section of the cloak could be suppressed by introducing loss appropriately.

An electrically large cylindrical metamaterial cloak was designed and manufactured by Liu *et al.* (2008). The simulation results showed that the cloak had two resonant frequencies at 8.5 GHz and 24 GHz due to the resonant characteristic of the structure. The scattering parameters were determined using the below mentioned formulae.

$$S_{11} = \frac{\Gamma(1-T^2)}{1-\Gamma^2 T^2}, \quad S_{21} = \frac{T(1-\Gamma^2)}{1-\Gamma^2 T^2} \tag{21}$$

where, Γ and T are reflection and transmission coefficient respectively.

It was found that the RCS of the metal cylinder surrounded by metamaterial cloak was lower than the bare metal cylinder over a wide range of frequencies, which indicated that metamaterial cloak had low scattering characteristics.

A perfect cylindrical cloak with only two spatially variant parameters was proposed by Luo *et al.* (2008*c*). The scattering properties of this cloak with a perturbation at the inner boundary were studied with cylindrical wave expansion method, indicating that both backward scattering and total scattering cross-section of the imperfect cloak could be suppressed by introducing loss. The effect of loss in metamaterial on the performance of the cloak was analyzed. The cloak could also reduce the cross-section when a tiny perturbation was introduced in the azimuthal component of permittivity.

The solution of EM field, near and far field properties in the cloak illuminated by electric line source were investigated by Wu *et al.* (2008*b*). The numerical results contrasted with the results obtained through COMSOL Multiphysics. Simulation results showed that outside the cloak, the wave was almost unaltered as if no scatterer were present while inside the cloak, the phase fronts were bent smoothly around the cloaked object.

A method was proposed by Popa and Cummer (2009) to reduce the scattering from arbitrary objects by surrounding them with layers of anisotropic homogeneous shell designed through an optimization procedure. The optimization was done using Broyden-Fletcher-GoldFarb-Shanno (BFGS) method. It was found that an optimized 3-layer shell could perform much better than a 100-layer approximation to the analytical cloak.

Zhang *et al.* (2009*a*) used full wave analysis method on transmission of Gaussian wave within an invisibility cloak. By calculating the instantaneous field and Poynting vector at each point in space, it was shown that the local group velocity and energy transport velocity could not be defined due to local distortion of the signal.

A wideband low-loss cloak was demonstrated by Tretyakov *et al.* (2009). The cloak used metal layers instead of metamaterial layers. The idea behind this cloaking was to reduce the volume occupied by the fields by gradually decreasing the distance between each layer. Cloaking effect was observed at 3.2 GHz. The experimental demonstration for a rectangular waveguide and simulation results for cloaking in the visible region were provided.

Simplified non-magnetic cloak parameters were provided by Zhang *et al.* (2010*a*), which could be optimized at a given frequency to minimize scattering. The frequency of operation was 200 THz. The incident field, scattered field and the fields inside the cloak were expressed in terms of Bessel's function and a parameter in the constants of this function was found, which optimized the performance of the cloak. It was also shown that the total cross section increased as the thickness of the cloak decreased.

A reduced parameter cloak was designed by Kundtz *et al.* (2010). The scattering cross section (SCS) measurement was done using a planar waveguide apparatus on a 50 mm diameter metal cylinder with and without the cloak. The apparatus consisted of a waveguide, antenna and a cloak. The cloaking occurred at a frequency of 10.06 GHz since the scattering cross section decreased at this resonant frequency.

A non-metallic cloak was implemented by Semouchkina *et al.* (2010) for the infrared range from identical chalcogenide glass resonators. The air fractions were used to obtain spatial dispersion of the effective permeability. The frequency of operation was 300 THz. The resonance responses of glass resonators were simulated using a full-wave software package viz. CST Microwave Studio. The results showed that cylindrical resonators with diameters twice as large as their height performed well.

Zhang *et al.* (2011*a*) constructed a macroscopic volumetric invisibility carpet cloak from natural bi-refringent calcite crystal. The cloak operated at visible frequencies and was capable of hiding 3D objects of size of 3 orders of magnitude larger than the wavelength of light. The cloaking effect was observed for red and green laser beams without microscope. Ray tracing calculations were performed to verify the design. The results showed that beams with TM polarization incident at two different angles reflected without distortion, thus verifying the cloak design.

A macroscopic invisibility cloak was realized by Chen *et al.* (2011*c*) at broadband visible wavelength by applying transformation optics design into optical lens fabrication. The 2D anisotropic cloaking principle was implemented at optical frequencies with light polarized such that the magnetic field was parallel to the horizontal reference plane. The object to be hidden was a steel wedge, a pattern 'MIT' was printed on an opaque plastic plate. A charge coupled device (CCD) camera and a mirror as a ground plate were used. For the wedge to be hidden, the patterns 'MIT' were to be captured by the camera without disturbance. The letters were captured at their original altitude thus hiding the wedge.

A graphene monolayer was used by Chen *et al.* (2011*a*) to realize the thinnest possible mantle cloak operating in the far field and terahertz frequency range. It was shown that the graphene monolayer, which possessed good conductivity and wide bandwidth properties, suppressed the scattering of planar and cylindrical objects. Dynamically tunable invisibility cloaks from tunable graphene were analyzed. It was reported that cloaking was achieved using graphene but a slight difference existed between the graphene cloak and an ideal reactive surface due to the material dispersion and absorption. Effect of loss and tunability were also studied.

Lee *et al.* (2012) investigated the guided modes in a five layered slab waveguide with double cladding of metamaterials. They used a graphical method to derive the dispersion equations both in the symmetric and asymmetric five layered slab. The dispersion equations for multilayer structures can be a input for analysis of cloaking device.

Dispersion on EM characteristics of reduced and ideal parameter cloaks were studied and compared using Drude dispersion model by Liu *et al.* (2011). The frequency of operation was 2.6 GHz and the boundaries of the cloak were made of perfectly matched layer. It was observed from the simulation, that the electric field within the inner radius of the cloak was zero irrespective of the dispersion model and when the dispersion model was introduced, the scattering strength in back scattering direction surpassed the strength in other directions thus proving the invisibility effect.

Recently a novel method of analysis namely TLTMM method is used for analysis of multilayer metamaterial structures for both the TE and TM polarizations (Narayan *et al.*, 2012). This method can take care of the normal incidence as well as oblique incidence. This method can be implemented for analysis of the scattering properties of a invisibility cloak.

A diamond shaped reciprocal cloak with perfect invisibility in a particular direction based on linear optical transformation was proposed by Yang *et al.* (2011). The frequency of operation of the designed cloak was 2 GHz. The object hidden inside could receive information from the outer region in contrast to the conventional cloaks. The results were simulated for a configuration, in which the complementary and recovering layers were optically cancelled and the transformation was equal to the layer of air. This proved successful cloaking.

There are three cloaking modes: ideal cloaking, sensor cloaking and resonance. Transformation optics was used by Chen and Uhlmann (2011*b*) to cloak 3D objects in sensor mode by degraded shielding. The configuration of the cloak, sensor, procedure for removal of singularity, and the analytical results for EM fields were provided. It was reported that a moderate loss could shift the resonance mode to sensor mode and vice-versa.

The emerging trends in soft computing techniques are the suitable choice to optimize the parameters of a multilayer cloaking structure. Choudhury *et al*. (2012) have reported the suitability of the soft computing techniques for design and optimization of metamaterial structures those including invisibility cloaking applications.

4 Fabrication and Measurement of Metamaterial Invisibility Cloak

Flexible metamaterials on free standing polyimide substrate operating at terahertz (THz) frequency range has been presented by Tao *et al.* (2008). Terahertz time domain spectroscopy has been used to characterize the metamaterial response. In this case, the functional frequency region was 0.25-2.5 THz. The thickness of the polyimide substrate was 5.5 μm. The transmission of the THz electric field was measured for the SRR sample and for air as the reference sample. The electric field spectral amplitude and phase were determined through Fourier transformation. The dielectric response was determined from the simulated transmission, in which all samples show strong resonances and negative dielectric constant above the resonant frequency. The cloak was simulated in COSMOL Multiphysics S/w using hybrid-mode solver. The results showed that the cloak clearly reduces scattering in both the forward and backward directions. Figure 6 shows the mechanical flexibility of the fabricated metamaterial wrapped around a glass bottle.

Liu *et al.* (2009*a*) experimentally realized a broadband ground plane cloak that could hide an object. An optimization technique was used for the transformation space, in which a quasi-conformal coordinate map was generated. The cloak was embedded in a medium of refractive index 1.331. An additional impedance matching layer was used around the cloak. It was reported that this cloaking eliminated backscattering and restored the reflected beam. In this design, the entire sample region was divided into 2 mm by 2 mm squares, requiring 10000 elements of which 6000 were unique. The generation of the regression curve and final layout of the elements were performed using Matlab. The behavior of the ground plane cloak was verified by making use of a phase-sensitive, near field microwave scanning system to map the electric field distribution inside a planar waveguide. The realized cloaking structure showed a very low loss over the entire operational frequency range of 13 to 18 GHz.

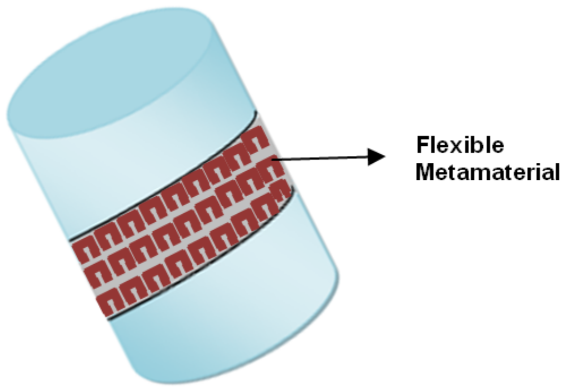

Figure 6: A flexible metamaterial designed and fabricated for cloaking applications

An optical invisibility cloak that hid an object under a carpet with the help of a reflective surface was demonstrated by Gabrielli *et al.* (2009). The design and fabrication of a cloaking device at optical frequencies that was capable of reshaping the reflected image and providing the observer with the illusion of looking at a plane mirror was provided. The wavelength of operation was 1550 nm. The device was triangular shaped and had an area of 225 μm^2 and hid a region of 1.6 μm^2.The reflective surface consisted of a distributed Bragg reflector (DBF), which consisted of alternating region of SiO_2 and crystalline silicon. The propagation of light in the device was simulated using FDTD method. The results showed that, due to the presence of the cloak, the image of the light incident on the deformation (the region, under which an object could be hidden) resembled the image of a wave propagating in a homogeneous medium without the deformation. Using the transformation optics, opposite of cloaking, i.e., concentration of light in an area could also be done.

Design and realization of a cylindrical cloak with a non-uniform LC network was done by Li *et al.* (2010). The background medium was designed to be a conformal LC network. The LC network consisted of three branches, which independently controlled the material parameters μ_r, μ_θ and ε_z. The response of this cloak to a broadband EM pulse was investigated experimentally. It was also shown that an inhomogeneous and anisotropic medium with material tensors in cylindrical basis could be synthesized artificially by a non-periodic transmission line network. The frequency of operation was 40 MHz. The performance of the cloak was demonstrated using Agilent E5071C vector network analyzer. The transient response was obtained by transforming the frequency-domain response to time domain response. The time delay effect, pulse broadening effect, and the scattered energy reduction ability of the cloak were also investigated. The results showed that the non-resonant nature of the unit cell led to a very low dispersion of cloak parameters, which resulted in invisibility of the cloak to broadband EM pulse.

A non-magnetic cloak was proposed by Kante *et al.* (2009) by direct mapping of magnetic field. The diameter of the concealed region was 4.4 wavelengths. The structure was based on the electric response of split ring resonator (SRR), designed at a frequency of 11 GHz. The dielectric spacer used was resin, whose thickness decreased from the outer layer to the inner layer of the cloak. Magnetic field was mapped using a loop antenna consisting of a circular coil made up of an inner conductor of SMA cable. The height of the cloak was 2.25 cm. A schematic of this novel arrangement of the cloaking structure is shown in Figure 7.

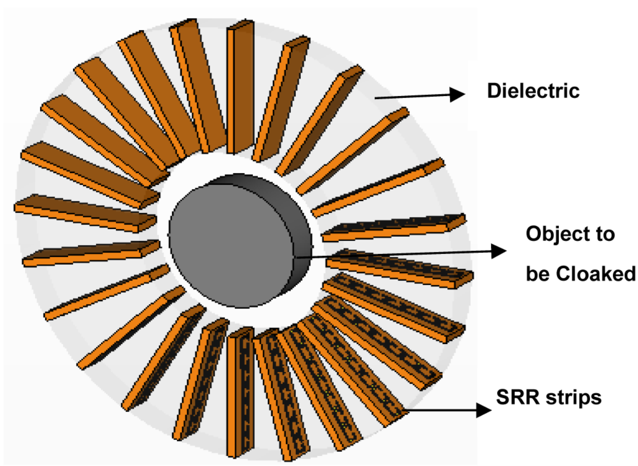

Figure 7: Schematic of the designed and fabricated cloak with variable dielectric resin spacer.

A homogeneous invisibility carpet cloak with uniform silicon grating structure fabricated by electron-beam lithography (EBL), and reactive etching process was demonstrated by Zhang *et al.* (2011*b*). It was reported that this cloak overcomes the requirement of spatially varying refractive index and also the size of the cloaked object was large compared to conventional cloaks. The broadband invisibility behavior was verified at near infrared frequencies. The cloak was designed using a nano-structured artificial anisotropic material and the reflecting surface was coated with gold and its parameters were homogeneous. The cloak restores the path of the reflecting wave from the surface thereby creating an illusion of a flat plane for a triangular bump on the surface. The results were compared for a Gaussian beam reflected from a cloaked plane and bare plane at the wavelength ranging from 1480 nm to 1580 nm. The reflection from the cloaked surface resembles that reflected by a flat surface, thereby achieving cloaking effect. Experimental results for several frequencies (1480 nm, 1550 nm, 1580 nm) were also provided.

5 Mathematical Analysis of Metamaterial Invisibility Cloak

Although transformation optics and geometrical coordinate transformation are introduced by most authors, their details such as the plane wave interaction with cloak, cloaking conditions, analysis and propagation of EM wave to realize cloaking etc. are considered in this section.

The interactions between plane EM wave and the cylindrical cloak were demonstrated by Wu *et al.* (2008*a*) analytically and expressions of the field inside the cloak were derived. These expressions were in terms of Bessel's functions, and the unknown co-efficients were solved using boundary conditions. These results enabled analysis and design of the cylindrical cloaks.

Analysis of EM wave propagation in a metamaterial cloak was presented by Yuan *et al.* (2008). By Cardan's formula, it was proved that the refractive index in a metamaterial cloak was a pure imaginary quantity, which implied that any EM wave would be excluded from penetrating the metamaterial structure regardless of its propagation direction. Fresnel's equation of wave normals was derived from Maxwell's equation by studying the characteristics of a monochromatic plane wave propagated in a metamaterial cloak.

A coordinate transformation was used by Kallos *et al.* (2009) to create a cloak that covered a triangular shaped conducting object placed on a ground plane. A 2D mesh that surrounded the domain above the object was found using conformal mapping at a operational frequency of 400 THz. To test the cloak, a new approximate cloak with 16 evenly distributed values of permittivity was chosen. Scattering field distribution from the object for a TM pulse using non-orthogonal FDTD and 2D FDTD were plotted. The comparison confirmed that both results were similar.

A new cloaking method for 2D quasi-statics and 2D Helmholtz equation was presented by Vasquez *et al.* (2009). For 2D quasi-statics, it was proven that a single active exterior cloaking device would suffice to shield an object from surrounding fields. For the 2D Helmholtz equation, it was numerically shown that 3 exterior cloaking devices placed around the object were sufficient to hide it.

Single-frequency cloaking conditions that were sufficient to ensure cloaking for arbitrary shapes and arbitrary illumination was presented by Maci *et al.* (2009). The conditions were derived in terms of general vector potentials through transformation of incident fields. Volumetric equivalence theorem was formulated for a bi-anisotropic metamaterial scatterer. The constitutive parameter tensors were given as a function of two vector potential obeying simple boundary conditions.

A bi-anisotropic material was defined theoretically by S. Maci (2010). It used a linear inhomogeneous field transformation, which did not involve space compression. Duality conditions were applied to select appropriate shape of the constitutive dyads, which resulted in a metamaterial defined by two real differentiable functions of space η and σ. It was shown that when these functions satisfied the condition $\sigma^2 + \eta^2 = c$ (constant) on the medium contour, the medium became globally lossless. Poynting vector, volumetric density of losses and global losses inside metamaterial were derived. Analytical results were given for an invisible sphere and for a spherical cloak.

6 Conclusions

In this work, an attempt has been made to review the metamaterial invisibility cloaks, along with the feasibility of simulation and fabrication techniques. An extensive literature survey has been carried out and the technical papers are classified into three broad sections, viz., design and simulations, fabrications and experimental demonstration, and mathematical analysis. It has been observed that maximum number of technical papers are based on design and simulation and very few are on fabrications and experimental demonstration. Hence the design and simulation section is focused further on the actual techniques implemented like finite-element method (FEM), finite-difference time-domain method (FDTD), finite integration technique (FIT), and inductor-capacitor representation of metamaterial (LC MTM), etc. It is observed that FEM has been most frequently used for the design and simulations of the invisibility cloaks. The review also reports the methods implemented for analysis of metamaterial cloaks with the possibility of application to the specific frequency ranges. A few inferences obtained from the review are listed below.

The first practical 2D metamaterial cloak realized by Schurig *et al.* (2006) was imperfect due to approximations and material absorption. Introduction of conformal mapping technique reduced these imperfections in perfect invisibility within the accuracy of geometrical optics.

Coordinate transformation method was used to design non-magnetic cylindrical cloak at optical frequency for hiding macroscopic objects. Further, higher-order transformations were used to eliminate undesired scattering from the cloaked object optical frequency. Coordinate transformation could also be used to design broadband cloaking, although performance of cloak deteriorated with the increase in loss.

It is possible to design a cloak that can hide objects external to the cloak, in which the object as well as the surrounding space can be optically cancelled by using a complementary media layer with an embedded complementary image of the object, called the anti-object. This could also be considered as an inverse cloak. The advantage of this cloak is that they worked simultaneously for all polarizations.

Broadband cloaking of light waves can be obtained by concentric cylindrical layers using scaling transformation. Wavelength multiplexing was used as the basic concept for achieving broadband cloaking.

Towards reduction of RCS, an electrically large cylindrical metamaterial cloak was designed and manufactured, which indicated that metamaterial cloak has low scattering characteristics.

A graphene monolayer can be used to realize the thinnest possible mantle cloak operating in the far field and terahertz frequency range. The good conductivity and the wide bandwidth properties of the graphene monolayer suppressed the scattering of planar and cylindrical objects. Dynamically tunable invisibility cloaks were also analyzed using tunable graphene.

Flexible metamaterials on free-standing polyimide substrate operating at the terahertz (THz) frequency range were used for fabrication of metamaterial cloak. The COSMOL Multiphysics S/w in the hybrid-mode solver was used by most of the researchers towards analysis of cloaking effect compared to HFSS and CST Microwave studio.

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