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An Enhanced Formulation of the Maximum Entropy Method for Structural Optimization

S. Hernández[1,2], A. Baldomir[1], J. Díaz[1] and F. Pereira[1]

Abstract: A numerical optimization method was proposed time ago by Templeman based on the maximum entropy principle. That approach combined the Kuhn-Tucker condition and the information theory postulates to create a probabilistic formulation of the optimality criteria techniques. Such approach has been enhanced in this research organizing the mathematical process in a single optimization loop and linearizing the constraints. It turns out that such procedure transforms the optimization process in a sequence of systems of linear equations which is a very efficient way of obtaining the optimum solution of the problem. Some examples of structural optimization, namely, a planar truss, a spatial truss and a composite stiffened panel, are presented to demonstrate the capabilities of the methodology.

Keywords: numerical optimization, maximum entropy method, structural design.

1 Introduction

It is commonly accepted that the seminal contribution to the field of structural optimization in its modern form was made by Lucien Schmit (1960). Nevertheless, an initial period, and that interval lasted more than two decades, was marked by the dispute between two methodological lines regarding to the concept of the optimum structural design.

One of these lines was based upon the mentioned Schmit's work that formulated the problem as the optimization of a function F(**X**) subject to a set of constraints $g_j(\mathbf{X})$ (j=1,...,n), which depended on a vector of design variables **X**, and could be written as

$$opt\ F(\mathbf{X}) \quad (1a)$$

[1] Structural Mechanics Group, School of Civil Engineering, University of La Coruña.
[2] Corresponding author: hernandez@udc.es

subject to

$$g_j(\mathbf{X}) \leq 0 \tag{1b}$$

Constraints could be related to any kind of structural response and the problem defined by Eq.1 could be solved by any optimization algorithm already existing [Kirsh (1981); Vanderplaats (2009); Hernández (1990); Haftka and Gurdal (1996)].

The other line of reasoning was based on the assumption that engineers, due to their experience, could know in advance what makes optimum a given design, in other words what would be the solution of the problem defined by Eq.1. Because of that, they were coined as optimality criteria methods [Berke and Venkayya (1974)] and amongst them the more popular were the simultaneous failure modes [Shanley (1952)] and the fully stressed design [Gellatly and Berke (1971)]. Other methods were created to deal with displacement constraints [Venkayya (1971)] or buckling [Khot (1984)]. Some of the drawbacks of these approaches were solved by using more scientific basis [Khot, Berke and Venkayya (1979)] and some researchers made many improvements to these methodologies [Rozvany (1989)].

It is worthy to mention that another approach, entitled as the dual method, was instrumental in the controversy between the mathematical programming methods and the optimality criteria techniques. It showed that the latter approach corresponded exactly to a linear approximation of the actual mathematical programming problem. This circumstance was extensively discussed and led the debate to an end [Fleury (1979)].

A more sophisticated technique used the principle or formulism of the maximum entropy [Jaynes (1957)] to solve the optimization problem. The main idea in this procedure was to find out the subset of active constraints at the optimum without biasing the process, in other words, without introducing any subjective judgment and hence by using only the information about constraints behaviour provided in the previous steps of the optimization process. This technique was proposed by Templeman (1987) and some other alternatives or modifications of the original method have been presented afterwards.

For instance, Xingsi (1992) presented an aggregate method for solving nonlinear minimax optimization problems, which converted a minimax problem to an unconstrained minimization on a differentiable aggregate function. Chen and Templeman (1995) showed further theoretical analysis and developments of entropy-based methods for mathematical programming and also some new methods for minimax and constrained nonlinear programming problems are discussed. Das et al. (1999) presented an information-theoretic algorithm for solving constrained nonlinear programming problems based upon the principle of minimum cross-entropy and surrogate mathematical programming.

2 Probabilistic identification of the active constraints using the maximum entropy principle

2.1 Procedure with double optimization loop

The approach set up by Templeman (1987) used the Kuhn-Tucker (1951) condition to identify the active constraints at the optimum design. As it is well known, that condition states that, if a point is an optimum of a constrained problem, the gradient of the objective function and the gradients of the subset of m_a active constraints are related by the following expression:

$$\sum_{j=1}^{m_a} \lambda_j \nabla \mathbf{g}_j = -\nabla \mathbf{F} \quad \lambda_j \geq 0 \quad (j = 1, ..., m_a) \tag{2}$$

The former expression can be written in a more general way as

$$\sum_{k=1}^{m_p} \lambda_k \nabla \mathbf{g}_k + \sum_{n=1}^{m_a} \lambda_j \nabla \mathbf{g}_j = -\nabla \mathbf{F} \tag{3a}$$

being

$$\lambda_k = 0 \quad (k = 1, ..., m_p) \quad \lambda_j \geq 0 \quad (j = 1, ..., m_a) \tag{3b}$$

where m_p is the subset of passive constraints, which is complementary of m_a.

Consequently, at the optimum design for each constraint $g(\mathbf{X})$ and component λ the following condition will happen

$$\lambda g(\mathbf{X}) = 0 \tag{4}$$

Because of:

- the constraint is active and $g(\mathbf{X}) = 0$

- the constraint is passive and $\lambda = 0$

Thus, Templeman substituted the formulation of Eq. 1 for the following

$$opt\ F(\mathbf{X}) \tag{5a}$$

subject to

$$\sum_{j=1}^{m} \lambda_j \nabla \mathbf{g}_j (\mathbf{X}) = 0 \tag{5b}$$

The unknowns of the new formulation were the $\mathbf{X}$ and $\boldsymbol{\lambda}$ vectors.

The procedure worked by fixing the values of λ_j and optimizing the design variables $\mathbf{X}$ and then updating the $\boldsymbol{\lambda}$ vector keeping constant the values of $\mathbf{X}$ obtained previously. Both steps were repeated until convergence was achieved.

2.1.1 *Phase 1: Optimization of design variables* $\boldsymbol{X}$

The problem formulation is:

$$opt\ F(\mathbf{X}) \tag{6a}$$

subject to

$$\sum_{j=1}^{m} \lambda_j \mathbf{g}_j(\mathbf{X}) = 0 \tag{6b}$$

$$\lambda_j \geq 0 \tag{6c}$$

Values of λ_j are interpreted as the possibility for the corresponding constraints to be active at the optimum. This is coherent with the fact, that when a constraint $g_j(\mathbf{X})$ is passive, its associated λ_j is zero. Initially all components of $\boldsymbol{\lambda}$ vector have the same value equal, namely

$$\lambda_{1m} = \frac{1}{m} \tag{7}$$

After that, any optimization algorithm allows to obtain the optimum values of the design variables $\mathbf{X}_1^*$. As some constraints could have become active this is inconsistent with the values of λ_j previously selected, so the next step is to update the $\boldsymbol{\lambda}$ vector.

2.1.2 *Phase 2: Updating of* $\boldsymbol{\lambda}$ *vector*

The principle of maximum entropy is used to update $\boldsymbol{\lambda}$ vector in an unbiased mode. This leads to formulate that the next set of λ_j would be those that maximize the following optimization problem

$$\max = -\sum_{j=1}^{m} \lambda_{2j} \ln \lambda_{2j} \tag{8a}$$

subject to

$$\sum_{j=1}^{m} \lambda_{2j} = 1 \tag{8b}$$

$$\sum_{j=1}^{m} \lambda_{2j} g_j(\mathbf{X}_1^*) = \varepsilon_k \tag{8c}$$

Constraint Eq.8.b is used to normalize values of λ_j, while Eq.8.c recalls the fact that as the components of $\boldsymbol{\lambda}$ and $\mathrm{g}_j(\mathbf{X})$ do not belong to the same iteration, the whole set of products do not cancel out, on the contrary a decreasing error ε_k appears. Because of that, a usual expression is $\varepsilon_k = e^{-k}$.

Solution of Eq.8 provides the new values of $\boldsymbol{\lambda}$ and afterwards Phase 1 and Phase 2 can be repeated until convergence.

2.2 Procedure with a single optimization loop

The problem defined by Eq.8 is an optimization problem with equality constraints that can be solved by using the Lagrange multipliers method as follows:

$$\mathscr{L} = -\sum_{j=1}^{m} \lambda_{2j} \ln \lambda_{2j} + (1+\delta)\left(\sum_{j=1}^{m} \lambda_{2j} - 1\right) + \mu\left(\sum_{j=1}^{m} \lambda_{2j} g_j(\mathbf{X}) - \varepsilon_k\right) \tag{9}$$

Imposing the conditions of a stationary point

$$\frac{\partial \mathscr{L}}{\partial x_i} = 0 \quad \frac{\partial \mathscr{L}}{\partial \delta} = 0 \quad \frac{\partial \mathscr{L}}{\partial \mu} = 0 \tag{10}$$

Solving the system of equations, the values of λ_j may be written as

$$\lambda_j = \frac{e^{\mu g_j}}{\sum\limits_{j=1}^{m} e^{\mu g_j}} \tag{11}$$

And the multiplier μ is the solution of the equation

$$\sum_{j=1}^{m} \frac{g_j(\mathbf{X})\, e^{\mu g_j}}{\sum\limits_{k=1}^{m} e^{\mu g_k}} = \varepsilon_k \tag{12}$$

Having in closed form the expression of the elements of $\boldsymbol{\lambda}$ vector, the problem defined by Eq.2 can be reformulated as

$$opt\ F(\mathbf{X}) \tag{13a}$$

subject to

$$\sum_{j=1}^{m} \frac{g_j e^{\mu g_j}}{\sum\limits_{k=1}^{m} e^{\mu g_k}} = 0 \tag{13b}$$

The value of μ should be obtained in Eq.12, but that is a difficult step, so the usual procedure is to solve Eq.13 several times for different values of μ until convergence is achieved [Templeman (1987)].

3 New procedure for solving the single loop optimization problem

Problem defined by Eq.13 may be solved by any non linear optimization algorithm, but its computational implementation has shown that numerical instabilities often occur. For instance, Templeman (1993) related that the procedure worked quite efficiently at the beginning but then it slowed up as the value of μ became larger after a few iterations.

Therefore, taking in account that it is an optimization problem with equality constraints, a Lagrangian function can be created

$$\mathscr{L} = F + \gamma \sum_{j=1}^{m} \frac{e^{\mu g_j}}{\sum\limits_{k=1}^{m} e^{\mu g_k}} g_j \tag{14}$$

and the conditions for the stationary point can be imposed

$$\frac{\partial \mathscr{L}}{\partial x_i} = \frac{\partial F}{\partial x_i} + \gamma \frac{\partial}{\partial x_i} \left(\sum_{j=1}^{m} \frac{e^{\mu g_j}}{\sum\limits_{k=1}^{m} e^{\mu g_k}} g_j \right) = 0 \quad (i = 1, \ldots, n) \tag{15a}$$

$$\frac{\partial \mathscr{L}}{\partial \gamma} = \sum_{j=1}^{m} \frac{e^{\mu g_j}}{\sum\limits_{k=1}^{m} e^{\mu g_k}} g_j = 0 \tag{15b}$$

Carrying out some calculations

$$\frac{1}{\gamma} \frac{\partial F}{\partial x_i} = -\sum_{j=1}^{m} (1 + \mu g_j) \frac{\partial g_j}{\partial x_i} \frac{e^{\mu g_j}}{\sum\limits_{k=1}^{m} e^{\mu g_k}} = -\sum_{j=1}^{m} (1 + \mu g_j) \frac{\partial g_j}{\partial x_i} \lambda_j \quad (i = 1, \ldots, n) \tag{16a}$$

$$\sum_{j=1}^{m} g_j \lambda_j = 0 \tag{16b}$$

Linearizing the constraints

$$g_j(\mathbf{X}) \approx g_j(\mathbf{X}_0) + \sum_{l=1}^{n} \frac{\partial g_j}{\partial x_l} \Delta x_l \quad (j = 1, \ldots, m) \tag{17}$$

It turns out that

$$\frac{1}{\gamma} \frac{\partial F}{\partial x_i} = -\sum_{j=1}^{m} \left[1 + \mu \left(g_j + \sum_{l=1}^{n} \frac{\partial g_j}{\partial x_l} \Delta x_l \right) \right] \frac{\partial g_j}{\partial x_i} \lambda_j \quad (i = 1, \ldots, n) \tag{18a}$$

$$\sum_{j=1}^{m}\left(g_j+\sum_{l=1}^{n}\frac{\partial g_j}{\partial x_l}\Delta x_l\right)\lambda_j=0 \tag{18b}$$

The previous expression can be written in a more condensed form as

$$\boldsymbol{\lambda}^T=|\lambda_1,...,\lambda_m| \quad \mathbf{g}^T=|g_1,...,g_m| \quad \Delta\mathbf{X}^T=|\Delta x_1,...,\Delta x_n| \tag{19a}$$

$$\nabla\mathbf{G}=\begin{pmatrix}\frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_m}{\partial x_1}\\ \vdots & & \vdots\\ \frac{\partial g_1}{\partial x_n} & \cdots & \frac{\partial g_m}{\partial x_n}\end{pmatrix}=|\nabla\mathbf{g}_1,\nabla\mathbf{g}_2,..,\nabla\mathbf{g}_m| \tag{19b}$$

$$\mathbf{F}^T=\left|\frac{\partial F}{\partial x_1},...,\frac{\partial F}{\partial x_n}\right| \quad \mathbf{G}=\begin{pmatrix}g_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & g_m\end{pmatrix} \quad \Lambda=\begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \ddots & 0\\ 0 & 0 & \lambda_m\end{pmatrix} \tag{19c}$$

So the problem can be presented as

$$\mu\nabla\mathbf{G}\cdot\boldsymbol{\lambda}\cdot\nabla\mathbf{G}^T\cdot\Delta\mathbf{X}=-\frac{1}{\gamma}\nabla\mathbf{F}-(\nabla\mathbf{G}+\mu\nabla\mathbf{G}\cdot\mathbf{G})\boldsymbol{\lambda} \tag{20a}$$

$$\mathbf{g}^T\boldsymbol{\lambda}+\Delta\mathbf{X}^T\cdot\nabla\mathbf{G}\cdot\boldsymbol{\lambda}=0 \tag{20b}$$

Eq.20.a represents a system of linear equations with the same size as the number of design variables. Given an initial design $\mathbf{X}_0$, a new point $\mathbf{X}$ can be found out by solving the system and obtaining $\Delta\mathbf{X}$ and thus

$$\mathbf{X}=\mathbf{X}_0+\Delta\mathbf{X} \tag{21}$$

Solving independently the system of equations for $\nabla\mathbf{F}$and for the remaining leads to

$$\Delta\mathbf{X}=-\frac{1}{\gamma}\Delta\mathbf{X}_F-\Delta\mathbf{X}_\lambda \tag{22}$$

substituting in Eq.20.b

$$\mathbf{g}^T\boldsymbol{\lambda}-\left(\frac{1}{\gamma}\Delta\mathbf{X}_F^T+\Delta\mathbf{X}_\lambda^T\right)\nabla\mathbf{G}\lambda=0 \tag{23}$$

After some calculations

$$\gamma=\frac{\Delta\mathbf{X}_F^T\nabla\mathbf{G}\lambda}{\mathbf{g}^T\boldsymbol{\lambda}-\Delta\mathbf{X}_\lambda^T\nabla\mathbf{G}\lambda} \tag{24}$$

Back substituting the expression of γ in Eq.22 and going to Eq.21, the new design point $\mathbf{X}$ is obtained.

In summary the steps of the procedure are:

1. Define the initial value of μ
2. Define the initial design $\mathbf{X}_0$
3. Evaluate constraints g_j, sensitivities $\frac{\partial g_j}{\partial x_i}$ and multipliers $\lambda_j = \frac{e^{\mu g_j}}{\sum\limits_{k=1}^{m} e^{\mu g_k}}$
4. Arrange vectors and matrices $\boldsymbol{\lambda}, \boldsymbol{\lambda}, \mathbf{g}, \mathbf{G}, \nabla\mathbf{G}, \nabla\mathbf{F}$
5. Solve the system of linear equations to obtain $\Delta\mathbf{X}$
6. $\mathbf{X}$=$\mathbf{X}_0 + \Delta\mathbf{X}$
7. If each component of $\Delta\mathbf{X}$ is not small enough, go to 3). Else, check convergence of $\mathbf{X}$ with the one obtained in the former iteration. If convergence is not achieved modify μ and go to 3).
8. If convergence is achieved, then $\mathbf{X}$ is the optimum design.

The method presented can be interpreted from several perspectives:

1. From the point of view of $\boldsymbol{\lambda}$, it is a probabilistic approach of the optimality criteria methods.
2. From the point of view of $\Delta\mathbf{X}$ it is a gradient based design algorithm.
3. From the point of view of $\nabla\mathbf{G}$ it is a sequence of linear problems.
4. For linear problems, it is a new algorithm that can be an alternative to simplex algorithm.
5. From the global point of view, the optimization problem is converted into the task of solving several systems of linear equations.

The criterion for defining the initial value of μ and the rule for modifying it at each iteration deserves further research. In this work the value of μ_k at the k-esime iteration was chosen as μ_k= $k\mu_1$, being μ_1 the initial value.

4 Application examples

The previously described procedure has been applied to two well known examples in the optimization literature and a stiffened composite panel.

4.1 Ten bar planar truss

It is a planar truss presented in Fig. 1 with two vertical loads of value Q=1 kN and ten design variables corresponding to the cross sectional areas of the bars [Berke (1974)]. The elastic modulus of the material is E=210 GPa and the material density is $\rho = 7.85 t/m^3$. The optimization problem is to minimize the structure mass and only stress constraints are imposed to the design

$$\min F(\mathbf{X}) = \sum_{i=1}^{25} \rho x_i l_i \tag{25a}$$

subject to

$$-250MPa \leq \sigma_i \leq 250MPa \quad (i = 1, ..., 10) \tag{25b}$$

$$x_i \geq 0.1\ cm^2 \quad (i = 1, ..., 10) \tag{25c}$$

Evolution of the objective problem, stress in each element and values of the design variables appear in Figs. 2 to 4. The numerical values of the results are shown in Tab. 1 and they are consistent with those reported in the literature.

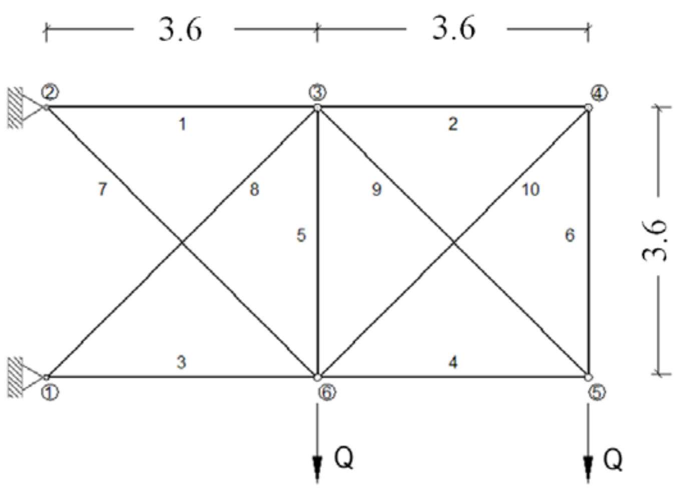


Figure 1: Ten-bar truss structure. Dimensions in metres

4.2 Twenty five bar space truss

This space truss is composed by twenty five elements whose cross sectional areas are the design variables [Thoft-Christensen and Baker (1982)]. Geometry appears

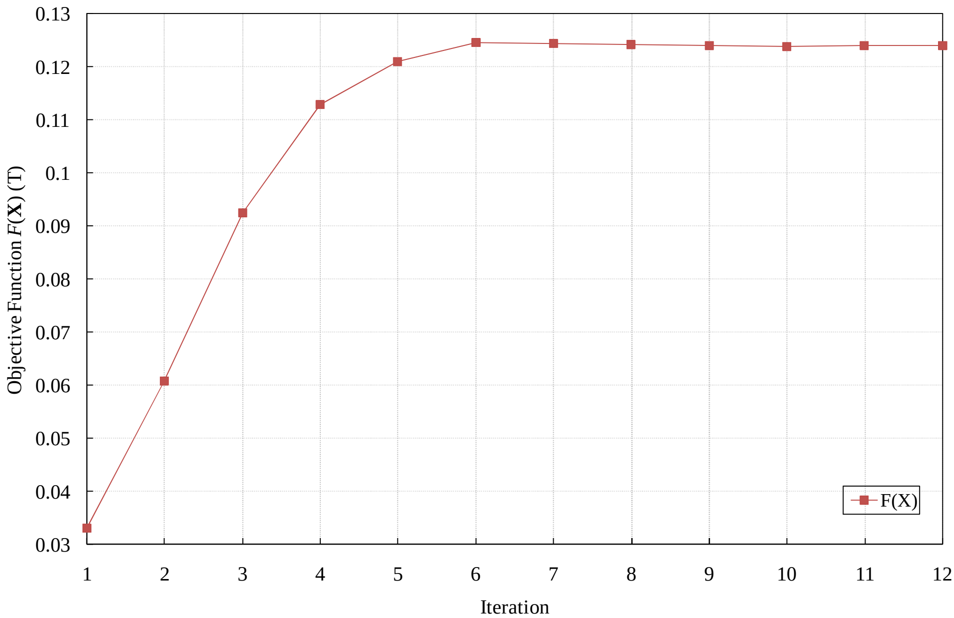


Figure 2: Evolution of objective function

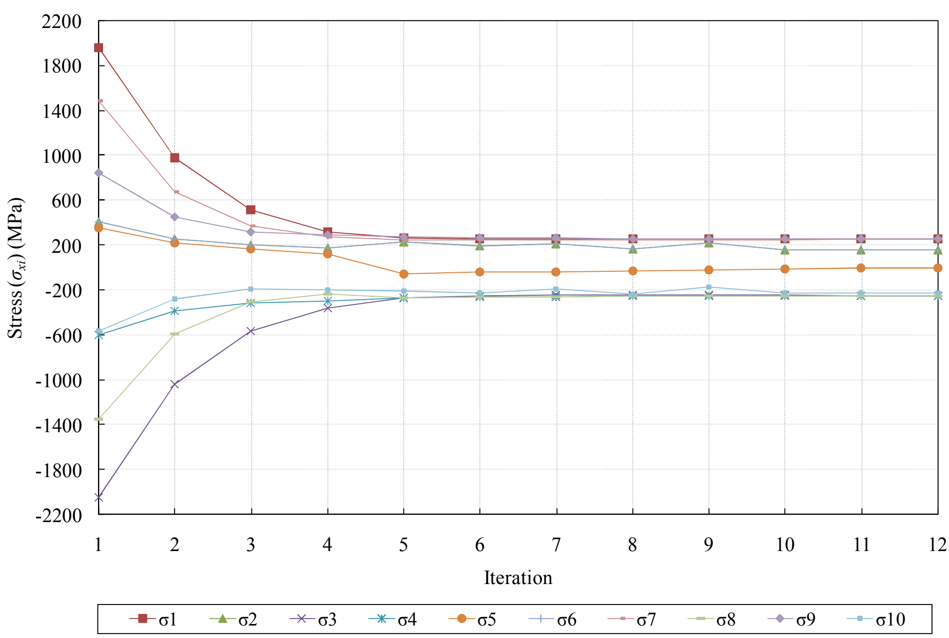


Figure 3: Evolution of bar stresses

Table 1: Cross sectional area (cm^2) and stress values (MPa) of each bar at the optimum design

Bar	1	2	3	4	5	6	7	8	9	10
Area	7.93	0.1	8.07	3.92	0.1	0.1	5.75	5.51	5.52	0.1
σ	250	157.6	-249.9	-250	-5.4	157.6	250	-250	250	-222.8

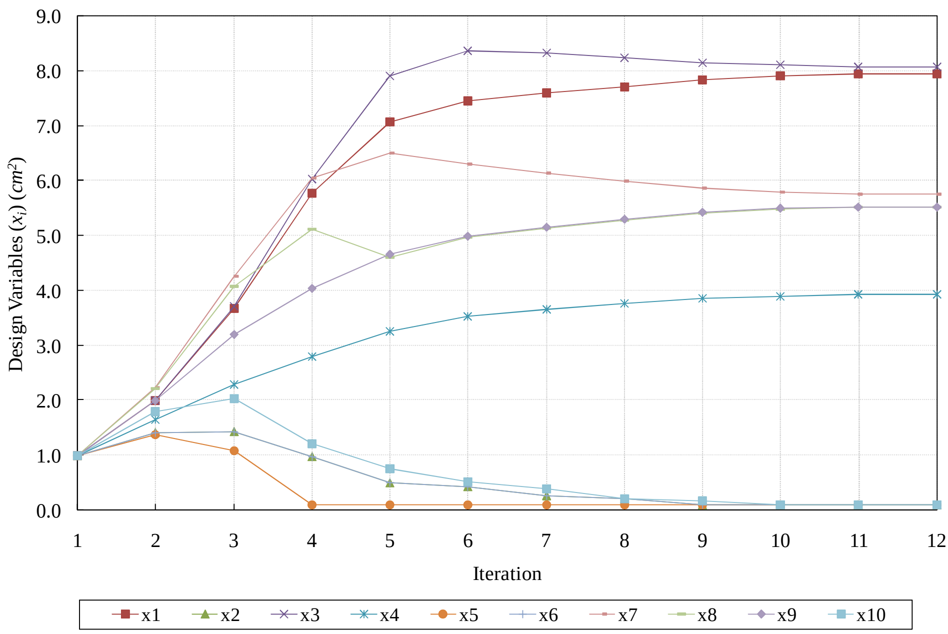


Figure 4: Evolution of design variables

in Figs. 5 to 7 and the external loads are constituted by two isolated forces of P_1=127.74 kN applied on nodes 1 and 2 and two forces of value P_2=125.72 kN on nodes 3 and 5 (Fig. 5). The numerical values of the force components are shown in Tab 2.

Elastic modulus of the material is E=70.6 GPa. Objective function is the volume of material and only stress constraints and the lower limit of the design variables are considered. Problem formulation turns out to be

$$\min F(\mathbf{X}) = \sum_{i=1}^{25} x_i l_i \tag{26a}$$

subject to

$$-276MPa \leq \sigma_i \leq 276MPa \quad (i = 1, ..., 25) \tag{26b}$$

$$x_i \geq 0.1\ cm^2 \quad (i = 1, ..., 25) \tag{26c}$$

The total number of design variables is thirteen due to the symmetry of the geometry and loads.

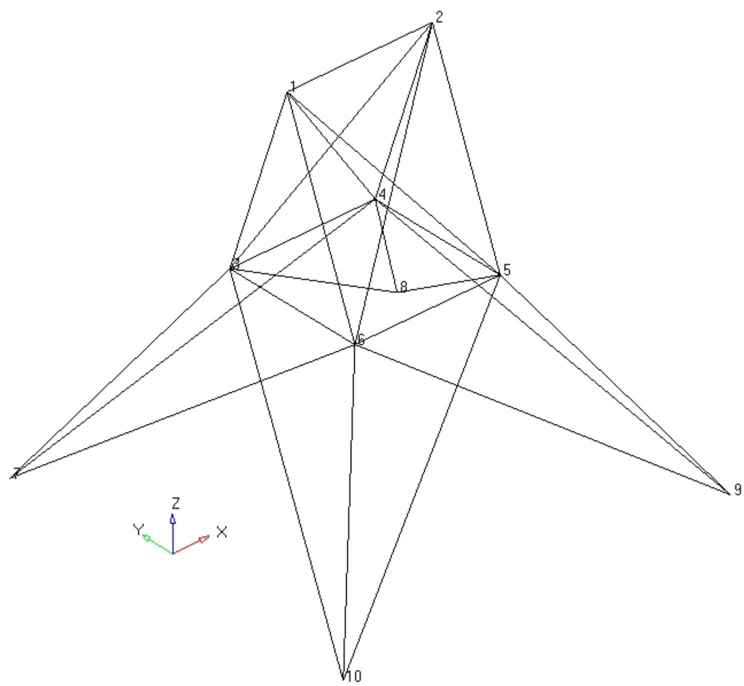


Figure 5: Geometry of space truss. Node identification

Table 2: Force components (kN)

Node	F_X	F_Y	F_Z
1	-88.9	88.9	-22.6
2	88.9	-88.9	-22.6
3	-88.9	88.9	0.0
5	88.9	-88.9	0.0

Similarly to the previous example, the objective function, stress in each element and values of the design variables appear in Figs. 8 to 10. These results are presented numerically in Tab. 3.

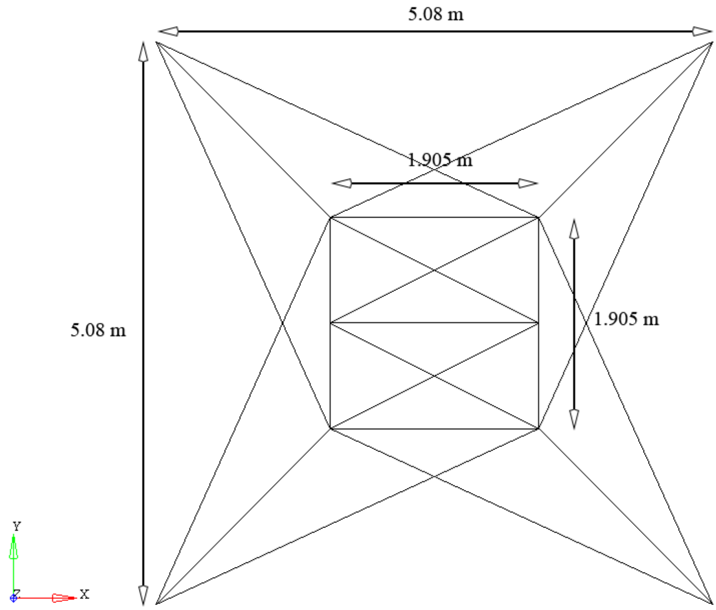


Figure 6: Dimensions of twenty five bar space truss (top view)

Table 3: Cross sectional area (cm^2) and stress values (MPa) of each bar at the optimum design

Bar	1	2	3	6	7	10	12	14	15	18	19	22	23
Area	0.58	0.44	5.04	0.10	4.67	0.58	0.10	2.27	0.59	0.10	0.10	0.10	2.33
σ	27.6	-27.6	27.6	8.2	-27.6	27.6	12.7	27.6	-27.6	3.1	-9.6	5.2	-27.6

4.3 Stiffened composite panel

In this example, a curved panel designed with composite skin and aluminum stiffeners is optimized. In Fig. 11 the geometry, dimensions of the panel and shape of the vertical and horizontal stiffeners are presented. The elastic properties of the materials are shown in Tab 4.

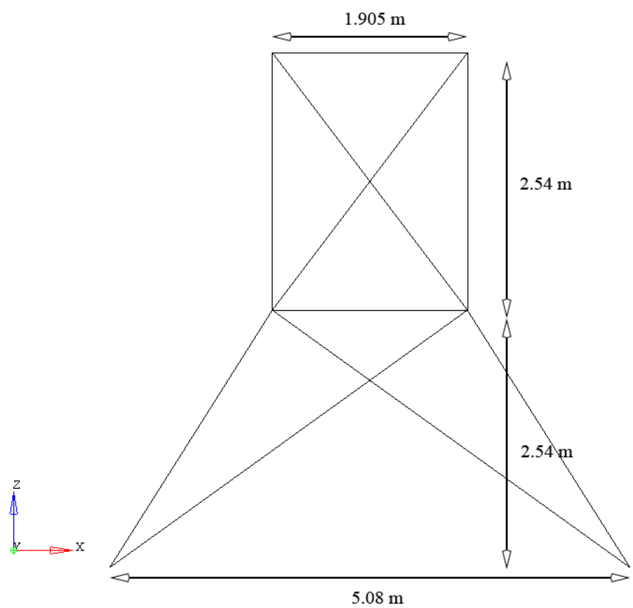


Figure 7: Dimensions of twenty five bar space truss (front view)

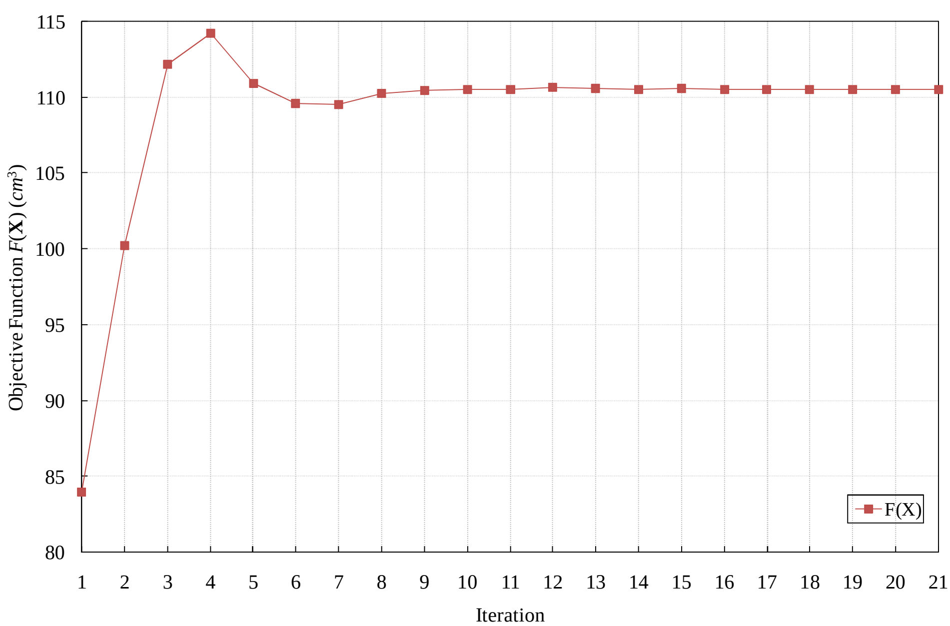


Figure 8: Evolution of the objective function

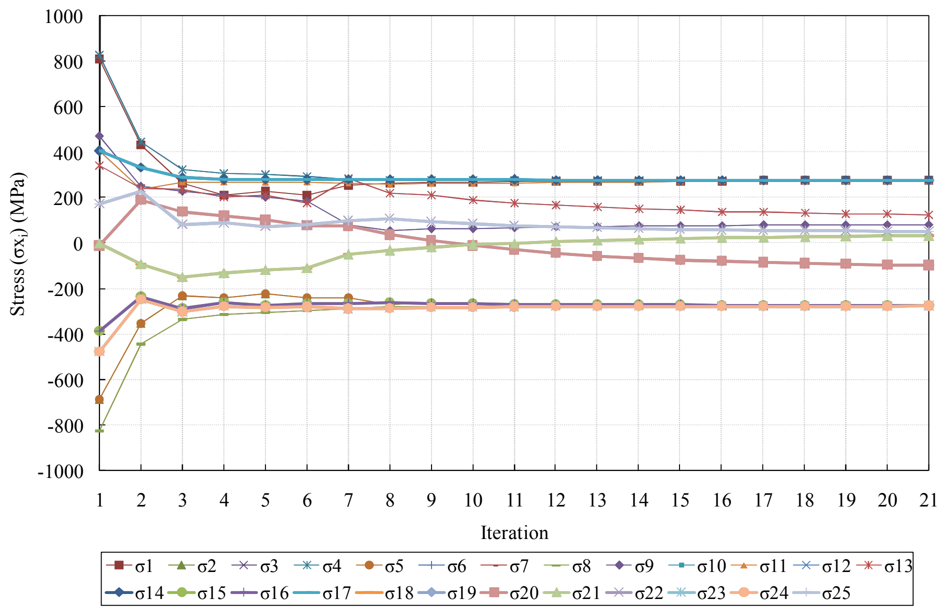


Figure 9: Evolution of the stress in the bars

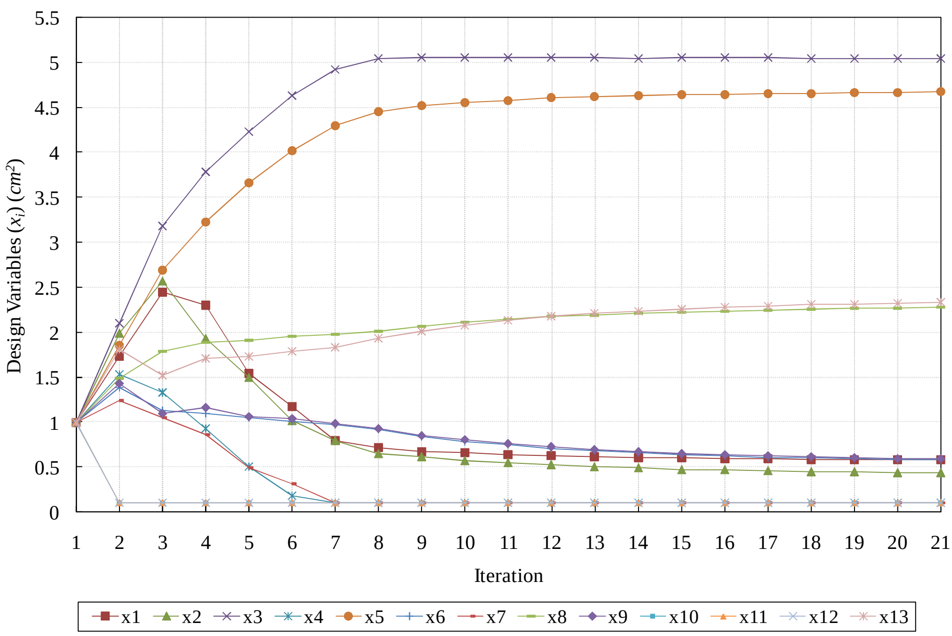
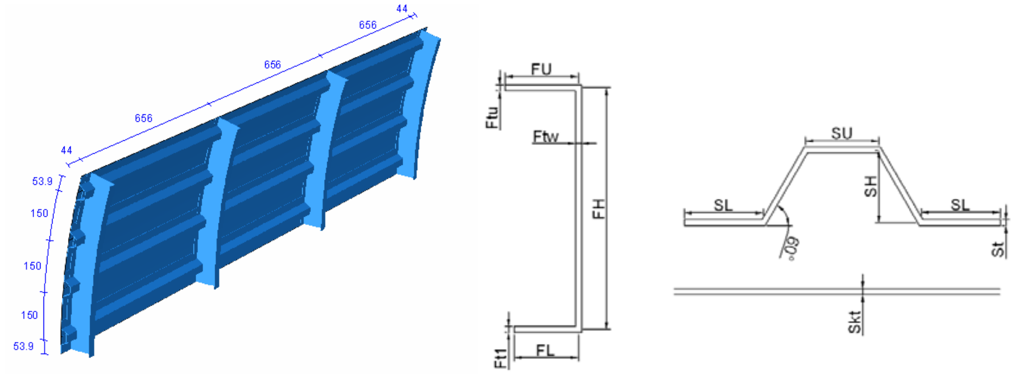


Figure 10: Evolution of the design variables



a) Geometry and dimensions (mm) b) Vertical stiffener c) Horizontal stiffener

Figure 11: Stiffened composite panel

Table 4: Material properties

Aluminium stringers					
E (Mpa)	Nu				
72000	0.3				
Composite-skin					
E1 (Mpa)	E2 (Mpa)	Nu12	G12 (Mpa)	G13 (Mpa)	G23 (Mpa)
157000	8500	0.35	4200	4200	4200

A shear buckling analysis of the panel has been performed considering the load specified in Fig. 12. A unit load has been applied so that the eigenvalue is directly the critical buckling load. In addition, a static analysis has been performed to evaluate the affect of the internal pressure in the fuselage. In this case a maximum pressure of value of 0.0621 MPa was considered, as presented in the data specified in the paper by Rouse, Ambur, Bodine and Dopker (1997).

Boundary conditions are established along the edges of the panel. Firstly, all displacements and rotations are constrained for one of the short edges. Secondly, remaining edges are defined as follows: constraining the normal displacement with respect to the plane defined by the four panel corners and constraining all rotations.

The design variables of the optimization problem are the thickness of the skin and the thickness and the internal dimensions of the stiffeners. In overall a number of 11 design variables are considered.

The design constraints included in the optimization problem are the followings:

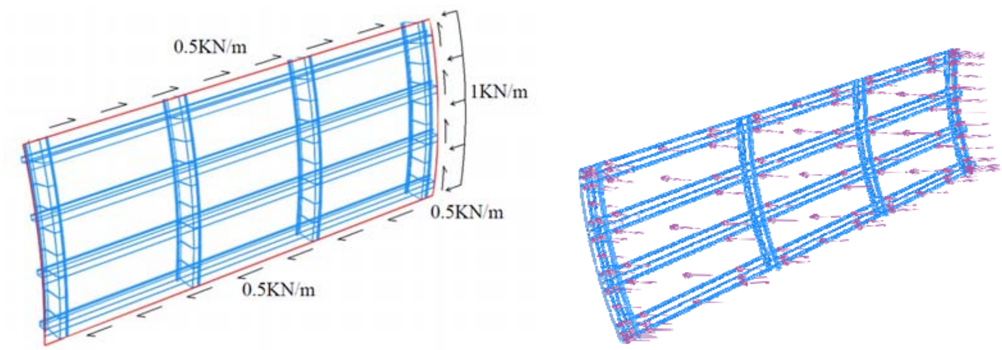


Figure 12: Load cases on the panel. Shear forces (left) and pressure forces (right)

1. Buckling load $\lambda \geq \lambda_{\min} = 110.46$

2. Slenderness' constraints to prevent local buckling in stringer sections, according to Eurocode 9 (2007) for aluminum structure:

$$\frac{Su}{St} \leq 15.714; \quad \frac{SH}{St} \leq \frac{15.714}{\sin(\pi/3)}; \quad \frac{Fu}{Ftu} \leq 3.571; \quad \frac{FH}{Ftw} \leq 15.714$$

1. The displacement of the stiffeners is constrained to 5% of the maximum panel displacement in buckling analysis. This constraint is defined to prevent global panel buckling. This constraint guarantees that the buckling occurs between two stiffeners.

2. Upper and lower value of the strain in the static analysis in which pressure forces are applied: $-0.003 \leq \varepsilon \leq 0.003$

The objective function selected was the Mass Ratio (kg/m^2), in other words the total mass divided by the panel surface that was to be minimized.

To solve this problem, the procedure explained in this paper was implemented in a Matlab code. In Tab. 5 appear the initial values of the design variables and the final values at the optimum. In Fig. 13 the evolution of the objective function is presented.

5 Conclusions

The following conclusions can be drawn from this work:

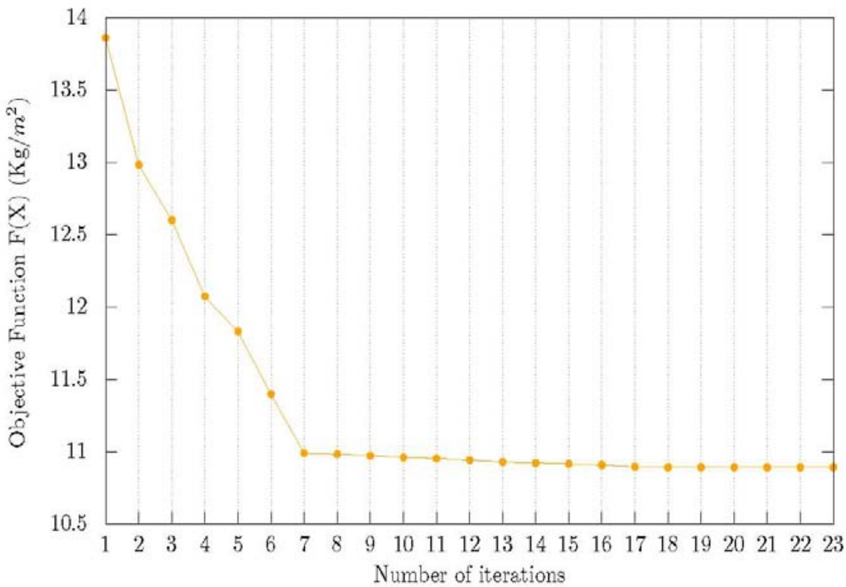


Figure 13: Evolution of the objective function

Table 5: Initial and optimum values of the design variables (mm)

Variable	Initial	Optimum
FL	22	18.75
FU	17	13.03
FH	70	53.64
Ft1	2	1.53
Ftu	5	3.83
Ftw	5	3.83
SL	27	20.69
SU	25	29.14
SH	30	30.62
St	3	2.30
skt	2	1.99

1. A procedure based on a probabilistic approach of the optimality criteria has been revisited and enhanced.

2. The new method can be understood as a sequence of linear problems.

3. For linear problems, this work constitutes a version of the so called interior point methods, which requires very few iterations and, therefore, it is an alternative to the *simplex* method.

4. As the method needs the use of first order derivatives it can be also seen as a new gradient based procedure.

5. The new algorithm transforms the optimization problem into the task of solving several times a system of linear equations. The number of unknowns in this system is equal to the number of design variables and it is independent of the number of constraints.

6. The algorithm requires a parameter μ which is problem dependent. In the examples presented an initial value of $\mu_{xg}250$ was selected for the truss structures and $\mu_{xg}0.1$ for the stiffened panel. Future research is needed to better understand the dependency of this parameter on the optimization problem.

7. The formulation created has been applied to several structural optimization problems and the accuracy of numerical results is very promising.

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