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A New Iterative Regularization Method for Solving the Dynamic Load Identification Problem

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Abstract: In this paper, a new iterative regularization method (ITR) is presented to solve the reconstruction of multi-source dynamic loads acting on the structure of simple supported plate. Based on a quadratical convergence method, this method is used to compute the the approximate inverse of square matrix. The theoretical proofs and numerical test show that the proposed method is very effective. Finally, the present method is applied to the identification of the multi-source dynamic loads on a surface of simply supported plate. Numerical simulations of two examples demonstrate the effectiveness and robustness of the present method.

Keywords: Load identification; Inverse problem; Iterative regularization; Multi-source dynamic loads

1 Introduction

It has been well recognized that the loads have important impacts on the dynamic performances of practical engineering structure, and the accurate load is very important for the design and dynamical analysis of engineering structure system. Also, to accurately evaluate the damage extent and the residual life of the structure, the first task of an efficient and reliable structural health monitoring system is to detect and identify the loads which act on the structure system. After knowing these loads, we maybe assess the damage of structure by the fatigue, strength, and reliability of structures. But in fact, it is impossible or difficult to directly measure the external loads in most cases of practical applications. Thus it could be

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beneficial to indirectly reconstruct the expected loads using the response data of structure. So it is very necessary to develop some inverse analysis techniques for load identification by the measured response which are easily obtained.

It has received much attention to developing the inverse techniques for the identifying the force histories in developing the structural health monitoring systems [1-7]. Recently, Wang et al. presented a procedure to identify the modal loads by the structural response [8]. Sato et al. [9] and Onozaki and Sekine [10-12] proposed the identification methods exploiting acceleration responses at points on composite laminated plates. Various methods were developed to solve the inverse problems associated with indirect force measurement [13-16]. Chan et al. [17] first formed the equations of motion under a set of independent moving forces by modelling the bridge, and then identified the loads acting on the beam. Law et al. [18] modeled the moving forces acting on a simply supported Euler beam as the step functions in a small time interval and exploited the modal superposition principle to identify the loads on the structure in time domain. Unfortunately, these inverse problems mentioned above are complex and inherently ill-posed. Regularization methods usually control well a level of numerical accuracy to these inverse problems. For these above-mentioned studies, much attention was paid to the complicated technical problems in mathematics, especially in the ill-posedness and regularization methods. In this article, a new iterative regularization method is proposed to solve the dynamic load identification problem of simply supported plate.

This paper is organized as follows. In Section 2 we establish a new iterative regularization method and prove its regular property. In Section 3 we demonstrate the effectiveness of the proposed method using a numerical test, and then apply this method to identify the multi-source dynamic loads acting on simply supported plate in Section 4 and we give a conclusion in Section 5.

2 A new regularization method for the approximate inverse of square matrix

It is well-known that there exist large scale of continuous ill-posed problems such as Fredholm integral equations of the first kind, and their discretization will generate the discrete ill-posed problems of the following form

$$Ax = y, \tag{2.1}$$

where A is an ill-conditioned matrix.

The right data y is usually contaminated by the measurement errors and noise, i. e.
,

$$y = \tilde{y} + e, \tag{2.2}$$

where the noise-free vector $\tilde{y}$ is unavailable and e represents the perturbations and measurement errors. Our aim is to compute a good approximation of the solution of the noise-free linear system (2.1) even in the presence of noisy data y.

Let V_0 be an initial approximation to the inverse of A. In the following part, we will get a sequence $\{V_q\}$ to approximate its inverse, and correspondingly construct a new regularization method for approximating the inverse of a square matrix of the following form:

$$V_{q+1} = V_q[\alpha^2(AV_q)^2 - 3\alpha(AV_q) + 3I]. \tag{2.3}$$

Then we immediately have the following results:

Theorem 2.1. If $\|E_0\| = \|I - \alpha AV_0\| < 1$, then the iterative formula (2.3) is convergent and V_q converges cubically to the inverse of A.

Proof. Let $E_q = I - \alpha AV_q$, which together with (2.3), we have

$$\begin{aligned} E_{q+1} &= I - \alpha AV_{q+1} \\ &= I - \alpha AV_q[3 - 3\alpha AV_q + \alpha^2(AV_q)^2] \\ &= I - 3\alpha AV_q + 3\alpha^2(AV_q)^2 - \alpha^3(AV_q)^3 \\ &= (I - \alpha AV_q)^3 \\ &= E_q^3 \end{aligned} \tag{2.4}$$

Noticing that

$$\|E_0\| = \|I - \alpha AV_0\| < 1,$$

then we have

$$\|E_q\| \le \|E_{q-1}\|^3 \le \cdots \|E_0\|^{3^q} \to 0.$$

It is obviously easy to check that

$$\lim_{q\to\infty}(I - \alpha AV_q) = 0,$$

i. e.,

$$\lim_{q\to\infty} V_q = \frac{1}{\alpha}A^{-1}.$$

Setting

$$e_q = A^{-1} - \alpha V_q,$$

we can obtain

$$Ae_q = I - \alpha AV_q = E_q,$$

Exploiting the formula (2.4), we can obtain

$$(Ae_q)^3 = E_q^3 = E_{q+1}.$$

According to the equality

$$Ae_{q+1} = E_{q+1},$$

we can obtain

$$e_{q+1} = e_q(Ae_q)^2$$

Therefore

$$\|e_{q+1}\| \le \|e_q(Ae_q)^2\| \le \|A\|^2\|e_q\|^3.$$

Then we complete the proof of valid iteration scheme (2.3) which is convergent and at least cubically converges to the inverse of a square matrix.

We summarize the algorithm as follows:

Algorithm (Iterative method).

Step 1: input $(A_{n\times n}, y, A)$, and choose α such that $\|E_0\| = \|I - \alpha A^2\| < 1$.

Step 2:

$$V_1 = V_0[3I - 3\alpha AV_0 + \alpha^2(AV_0)^2], q = 1;$$

Step 3: while ($\|V_q - V_{q-1}\| \succ \varepsilon$) do

$$V_{q+1} = V_q[3I - 3\alpha AV_q + \alpha^2(AV_q)^2], q = q+1;$$

Step 4: Obtain the approximate solution

$$x = \alpha V_q b.$$

3 Benchmark test

To demonstrate the effectiveness of the proposed method, we consider the first kind of Fredholm integral equation as follows:

$$\int_0^1 e^{ts}x(s)ds = \frac{e^{t+2}-1}{t+2}, \quad t \in [0,1]. \tag{3.1}$$

It is easy to check that the true solution of Eq.(3.1) is $x(s) = e^{2s}$. In general terms, we usually consider the perturbed equation

$$\int_0^1 e^{ts}x(s)ds = y^\delta(t), \quad t \in [0,1]. \tag{3.2}$$

Discretizing Eq. (3.2), we can obtain

$$\frac{1}{N}\sum_{j=1}^{N} e^{t_i s_j}x(s_j) = y_i^\delta, \quad i,j = 1,2,\cdots,N, \tag{3.3}$$

where

$$t_i = \frac{i-1}{N}, s_j = \frac{j-1}{N}, y_i^{\delta} = y(t_i) + \theta_i \delta,$$

θ_i is a random number and satisfies $|\theta_i| \leq 1$.

To analyze the performance of the present method, we choose the noisy level $\delta = 0.0001, N = 34, \alpha = 0.5$. Applying PC-MATLAB environment, we obtain the following results.

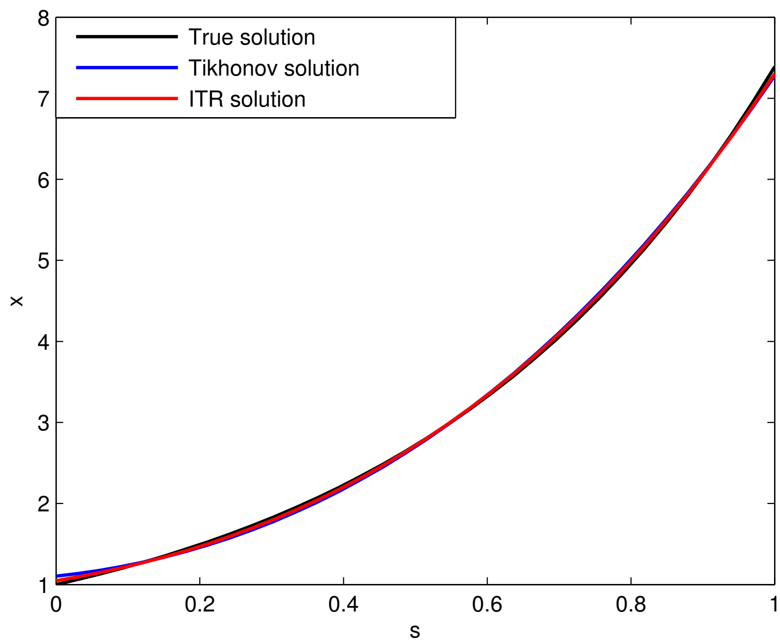

Figure 1: Numerical results of equation (3.1)

Figure 1 indicates that the present method and Tikhonov regularization method are stable and effective in identifying the true solution of the first kind of Fredholm integral equation (3.1). In fact, more informative results on the performances of these methods are shown in Table 1. It can be shown that the maximal deviation and average deviation of the present method are $0.6334\%, 0.2979\%$, respectively, obviously smaller than Tikhonov regularization method. It means that the present method is more precise and effective than Tikhonov regularization method, and the numerically optimal convergence rate of the regularized solution roughly coincides with the theoretical analysis.

Table 1: Identification errors of two regularization methods

	Noise level 0.0001		
	Maximum Error (%)	Average Error (%)	Iterative number
Present method	0.6334	0.2979	22
Tikhonov regularization method	1.3849	0.5863	0

4 Application

To illustrate the present methodology for use in determining the unknown time-dependent multi-source dynamic loads acting on simply supported plate, we need to know the following knowledge for a linear elastic structure.

Here we consider the multi-source dynamic load identification problem for a linear and time-invariant dynamic system. The response at an arbitrary receiving point in a structure can be expressed as a convolution integral of the forcing time-history and the corresponding Green's kernel in time domain :

$$y(t) = \int_0^t G(t-\tau)p(\tau)d\tau, \tag{4.1}$$

where $y(t)$ is the response which can be displacement, velocity, acceleration, strain, etc. $G(t)$ is the corresponding Green's function, which is the kernel of impulse response. $p(t)$ is the desired unknown dynamic load acting on the structure.

By discretizing this convolution integral, the whole concerned time period is separated into equally spaced intervals, and the equation (4.1) is transformed into the following system of algebraic equation:

$$Y(t) = G(t)P(t), \tag{4.2}$$

or equivalently,

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} g_1 & & & \\ g_2 & g_1 & & \\ \vdots & \vdots & \ddots & \\ g_m & g_{m-1} & \cdots & g_1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{pmatrix} \triangle t,$$

where y_i, g_i, and p_i are response, Green's function matrix and input force at time $t = i\triangle t$, respectively. $\triangle t$ is the discrete time interval. Since the structure without applied force is static before force is applied, y_0 and g_0 are equal to zero. All the elements in the upper triangular part of G are zeros and are not shown. The special form of the Green's function matrix reflects the characteristic of the convolution integral.

To recover the time history $P(t)$, the knowledge of $y(t)$ and $G(t)$ are required. In fact, the response at a receiving point and the numerical Green's function of a structure can be obtained by finite element method. However, the problem of identifying the dynamic load $P(t)$ by $y(t)$ and $G(t)$ is usually ill-posed, and cannot be solved by inverse matrix method. In the following, our method will be suggested to solve this problem.

A practical engineering problem is to determine vertical forces acting on simply supported plate, which is shown in Figure 2. The material properties of the plate are are as: $\rho = 7.8 \times 10^{-9} kg/m^3, E = 2.0 \times 10^5 MPa, \nu = 0.3$.

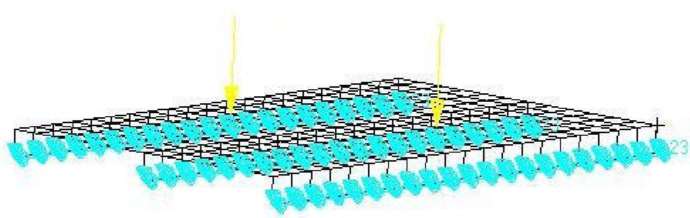

Figure 2: The finite element model of simply supported plate

The vertical concentrated load is applied to the outside surface, and the measured response is the vertical displacement. Three straight members of simply supported plate are fixed, and the others is free. We establish its finite element model as shown in Figure 2. The arrow in Figure 2 denotes the action point of dynamic force.

The concentrated loads are defined as follows:

$$F_1(t) = \begin{cases} q_1 \sin(\frac{2\pi t}{t_d}), & 0 \le t \le 2t_d \\ 0, & t < 0 \quad and \quad t > 2t_d \end{cases}$$
$$F_2(t) = \begin{cases} 4q_2 t/t_d, & 0 \le t \le t_d/4 \\ 2q_2 - 4q_2 t/t_d, & t_d/4 < t \le 3t_d/4 \\ 4q_2 t/t_d - 4q_2, & 3t_d/4 < t \le t_d \\ 0, & t > t_d \end{cases}$$

where t_d is the time cycle of sine force, and $q_i (i = 1,2)$ is a constant amplitude of the force. When $t_d = 0.004s, q_1 = 1000N$, and $q_2 = 800N$, the sine force and triangle force are shown in Figures 3-4.

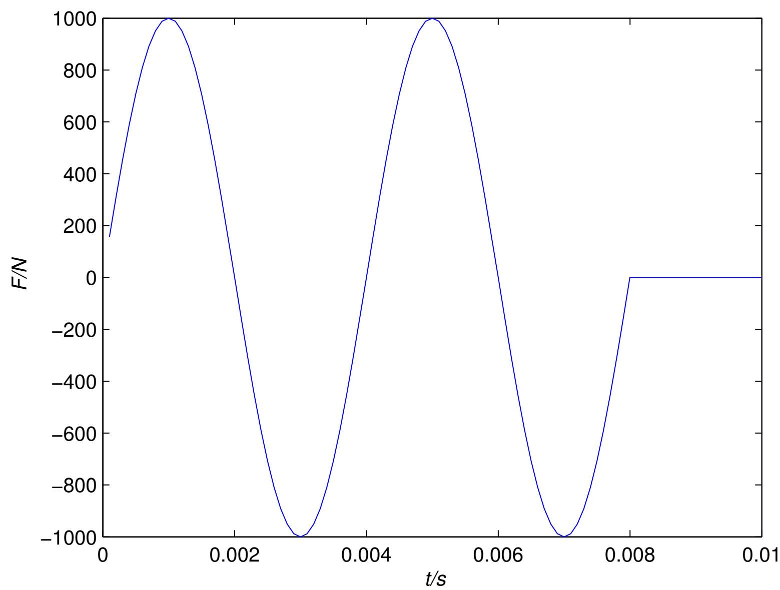

Figure 3: The vertical concentrated sine load acting on the outside surface

Herein, the experimental data of response is simulated by the computed numerical solution, and the corresponding vertical displacement response can be obtained by finite element method, as shown in Figures 5 and 6. Furthermore, a noise is directly added to the computer-generated response to simulate the noise-contaminated measurement, and the noisy response is defined as follows:

$$Y_{err} = Y_{cal} + l_{noise} \cdot std(Y_{cal}) \cdot rand(-1,1),$$

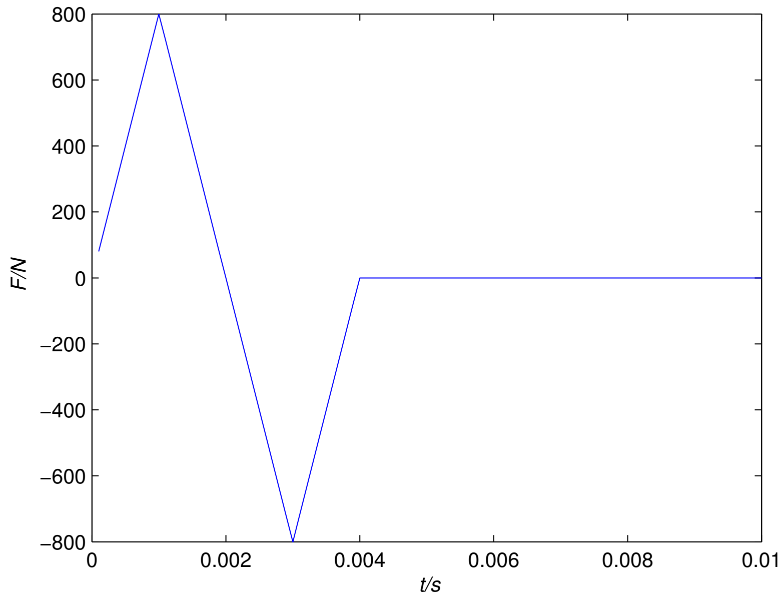

Figure 4: The vertical concentrated triangle load acting on the outside surface

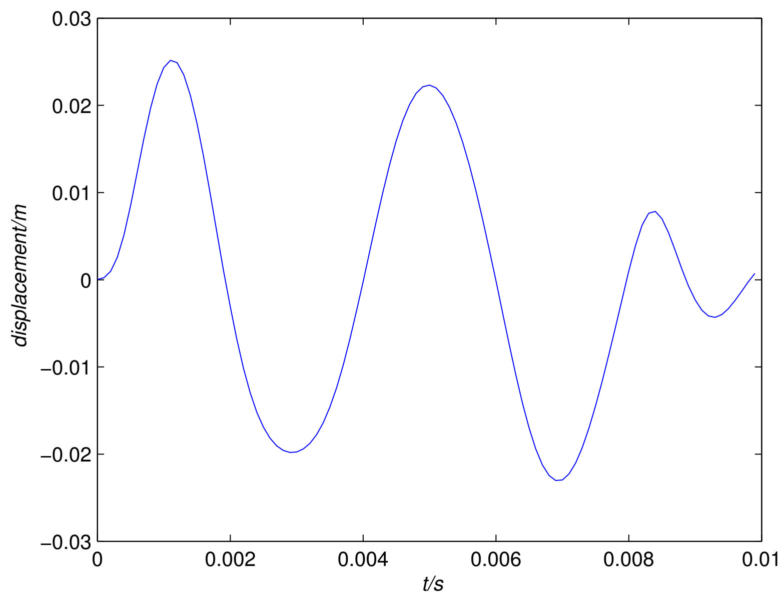

Figure 5: The corresponding vertical displacement response at one point

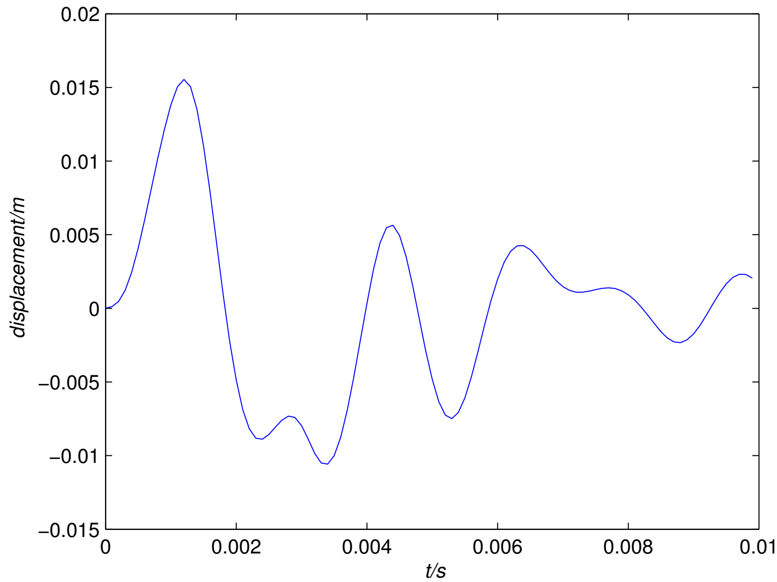

Figure 6: The corresponding vertical displacement response at the other point

where Y_{cal} is the computer-generated response; $std(Y_{cal})$ is the standard deviation of Y_{cal}; $rand(-1,1)$ denotes the random number between -1 and +1; l_{noise} is a parameter which controls the level of the noise contamination.

In order to investigate the effect of measurement error on the accuracy of the estimated values, we consider the case of noise level namely 5%, and the present method is adopted to determine the dynamic forces. By using a similar argument in Section 3, so the optimal solution obtained by the present method will be compared to those by Tikhonov regularization method. The comparison will be made quantitatively by way of the relative estimation error:

$$\tilde{F} = \left|\frac{F_{Real} - F_{Identified}}{F_{Real}}\right|. \tag{4.3}$$

To evaluate the effectiveness of two regularization methods mentioned above, five time points are selected, and for each point the identified force will be compared with the corresponding actual force.

The results of numerical simulations are as follows:

From Figures 7-8, it can be shown that the present method and Tikhonov regularization methods can both stably and effectively identify the multi-source dynamic loads by the measured noisy responses. Moreover, the more detailed results by them at five time points are listed in Table 2. It can be found that at these five

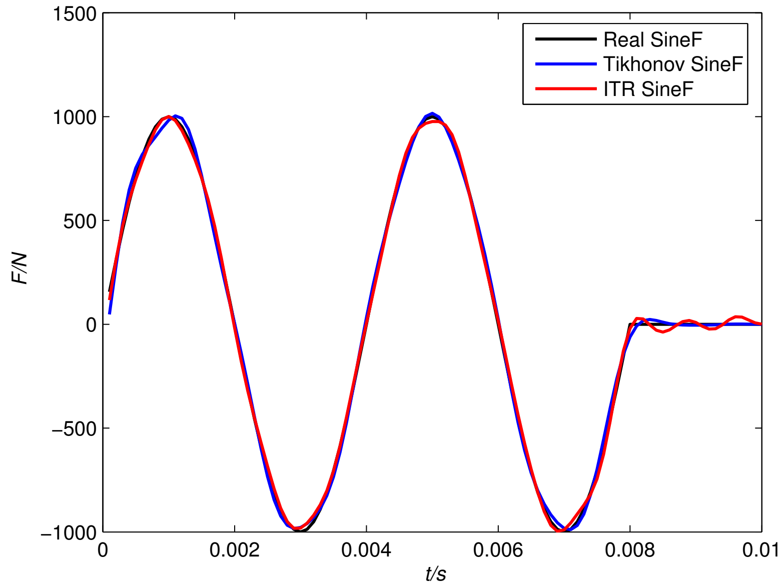

Figure 7: The identified sine force at noise level 5%

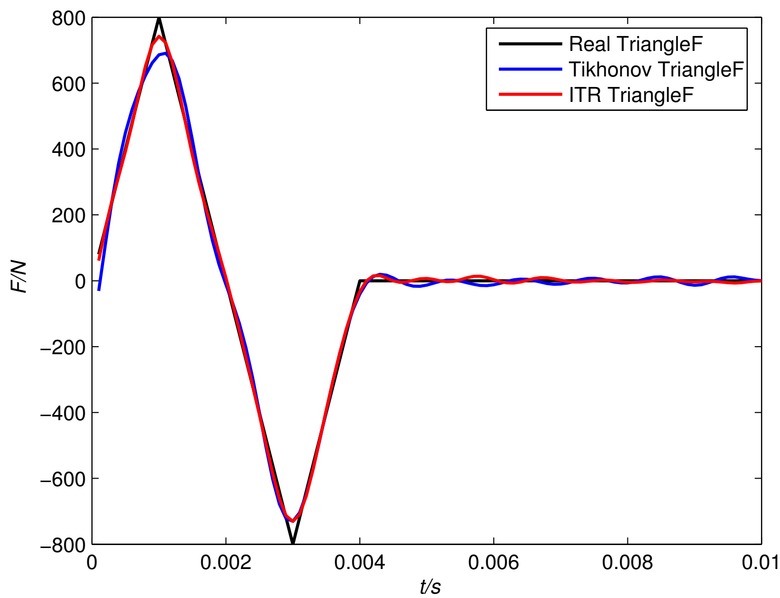

Figure 8: The identified triangle force at noise level 5%

Table 2: The identified force at five time points at noise level 5%

	Time point	Real force	Tikhonov Identified force	Tikhonov Error (%)	Present method Identified force	Present method Error (%)
Sine	0.001	1000	983.06	1.69	1000.4	0.04
Triangle	0.0006	480	520.06	5.01	475.03	0.62
Sine	0.003	-1000	-978.18	2.18	-982.52	1.75
Triangle	0.001	800	686.09	14.24	742.29	7.21
Sine	0.0045	707.11	677.83	2.93	715.76	0.87
Triangle	0.0016	320	317.93	0.26	296.33	2.96
Sine	0.0063	-453.99	-471.43	1.74	-434.56	1.94
Triangle	0.0033	-560	-573.3	1.66	-574.61	1.83
Sine	0.0073	-891.01	-914.17	2.32	-870.57	2.04
Triangle	0.0038	-160	-148.84	1.40	-146.18	1.73
	Error (%)		Maximum	Average	Maximum	Average
	Sine		10.91	1.95	3.99	1.56
	Triangle		14.24	2.08	8.65	1.01

time points for noise level $\pm 5\%$, most of the deviations of the identified loads by the present method are smaller than Tikhonov regularization method, which dues to better efficient identification. It can be also found that the most deviations by Tikhonov regularization method and the present method concentrate in the range of $15\%, 9\%$, respectively. In addition, for the identification of sine force, the maximal deviation and average deviation by the present method are $3.99\%, 1.56\%$, respectively, obviously smaller than the later. Furthermore, we can find that the maximal deviation and average deviation of the identification of triangle force by the present method are $8.65\%, 1.01\%$, respectively, both smaller than Tikhonov regularization method. In a word, the present algorithm achieves an excellent estimation, and also gives satisfactory results when recovering the loading time function.

5 Conclusion

In this paper, a new iteration regularization method is proposed and considered as an alternative to approximate the solutions of linear ill-posed problems or ill conditioned matrix equations. The theoretical proofs and numerical experiments show that our iterative method is very effective. Also, more importantly, it has been found that these methods can be used to compute the inner inverse and their convergence proofs are given by fundamental matrix tools. At the same time, it is validated by numerical example test and suggested to identify the multi-source dynamic loads acting on simply supported plate by the noisy responses. Numerical simulations have shown that the proposed method is effective and accurate in solving the load identification problems of the practical engineering.

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