

Transient Wear Simulation in Sliding Contacts of Spur Gear Teeth

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Abstract: Gear transmission is important in engineering due to its high efficiency in transferring both power and motion. As a surface phenomenon, wear may change the gear geometry, cause a non-uniform gear rate and increase dynamic effects, all of which can lead to reduced efficiency and even severe tooth failure. In numerical predictions of wear, the conventional method, where the contact pressure over the slip distance is integrated, will cause a computation bottle-neck. To obtain an accurate integration of the wear within the small, fast moving contact area, the finite element model needs to be meshed very finely, and the time increments must be sufficiently small to resolve the contact pressure field. As proposed in recent work, the so-called CForce method has the potential for fast wear computation for two-dimensional Hertzian contacts. The CForce method replaces the pressure integration by the contact force and eliminates the need to determine the contact pressure field. In this work, the CForce method was used to predict the wear in spur gears with a focus on the transition of the contact load from one pair of teeth to the next. Predictions of wear for two different torques show that the CForce method can achieve robust results with higher efficiency compared with that of the conventional pressure integration approach.

Keywords: Wear, Wear prediction, Spur gear, Contact mechanics, Numerical simulation

1 Introduction

Gear transmissions, which transfer both power and motion with a high efficiency, are becoming more important in modern technology. In addition to the well-known applications for spur gears, which have been known for many decades, micro-system technology, wind energy and e-mobility have created new fields for their

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application, accompanied with their own specific challenges concerning their design and life time. Wear can critically affect the interactions between the gear teeth in a transmission. As shown in (Litak and Friswell 2003), excessive wear will increase backlash, which will lead to a loss in contact and cause non-uniform motion transmissions. Wear changes the tooth profile geometry of the gear pair and leads to non-uniform rotation, which would severely influence the gear system, such as decrease the efficiency, create a non-uniform gear rate, and could even cause failure of the gear teeth. Thus, the ability to predict wear and the corresponding lifespan is important in the development of reliable gear components.

The pioneering work in gear wear prediction was described in (Andersson and Eriksson 1990; Flodin and Andersson 1997). A single-point observation method was proposed that determined the relative sliding distance for a contact pair of two mating gear teeth, obtained the contact pressure using the Winkler model, and finally determined the wear of the spur gear surface using a modified generalised Archard's wear model (Archard 1953) with an Euler integration method. For wear on a helical gear, the helical wheel is treated as several thin, independent spur gear teeth with a common central axis that gradually turned relative to each other at a small angle, which corresponded to the helix angle (Flodin and Andersson 2001); the contact pressure between the two gear teeth was simply assumed to be constant and equal to the Hertzian mean pressure. Additionally, a wear prediction methodology for spur gears has been used to investigate the influence of material properties on gear tooth wear under unlubricated conditions; a Matlab program was developed to account for the changes in the gear contact per mesh cycle as the wear occurs (Dhanasekaran and Gnanamoorthy 2008). The contact pressure used in the wear calculation was analytically obtained assuming Hertzian contact. Pit formation and life prediction of gears using numerical tools have also been reported by (Aslantas and Tasgetiren 2004; Glodez, Abersek, Flasker and Ren 2004). Recently, an effort has been made to capture the multi-scale behaviour for wear simulation with a very finely meshed finite element model (Hegadekatte, Hilgert, Kraft and Huber 2008). The discrete nature of the finite element method indicated that the time increments must be sufficiently small to resolve fast moving contacts and to obtain the correct integration of the wear. However, this approach results in an immense computational cost to predict the wear of a gear train.

In all the aforementioned works, the contact pressure field was determined from analytical/empirical formulae or directly from a finite element numerical simulation. As soon as dynamic effects of a complex system and multiple contacts are included, the resulting multi-scale problem in space and time forms a computation bottle-neck, which is particularly true in predicting the life-span for multiple wear steps because the wear continuously changes the local contact geometry for a long

time scale. Using a changing contact radius distribution along the tooth profile is still an unsolved problem in hybrid Finite Element (FE)/analytical approaches. Thus, it is essential to develop an efficient approach for wear prediction that can simply accept coarse finite element meshes and large time increments and maintain a high prediction accuracy.

As stated in (Chen and Huber 2010), the CForce method has the potential to replace the pressure integration by the contact force and eliminates the need to determine the contact pressure field for generally shaped two-dimensional tribosystems. For a twin-wheel model and a simplified camshaft-follower configuration, the proposed CForce method has already achieved excellent results with a greater efficiency and robustness compared with the conventional pressure integration approach. Furthermore, the CForce method adopts an interpolation algorithm, which ensures that the correct wear can be assigned to un-contacted nodes and thus, it is still feasible for wear prediction when the time increment is increased in the contact simulation.

The prediction of wear on gear teeth is quite challenging due to the complex geometry, which results in a nonlinear force transfer from one pair of teeth to the next. In this paper, the wear of a gear system was studied using the CForce method. The theory of the CForce method will be summarised in section 2. The implementation and computation of the wear for spur gears will be presented in detail in section 3.

2 Wear prediction by the CForce method

Archard's wear model (Archard 1953) is the most popular model for wear simulation and has been adopted by many authors (Williams 2001; Brauer and Andersson 2003; Hegadekatte, Huber and Kraft 2005a; Hegadekatte, Huber and Kraft 2005b; Dhanasekaran and Gnanamoorthy 2008; Hegadekatte, Hilgert, Kraft and Huber 2008; Nürnberg, Nürnberg, Golle and Hoffmann 2008). Essentially, Archard's wear law represents a linear relationship between the incremental wear, local contact pressure and sliding distance. It was originally formulated for the global component scale in the form

$$\frac{V_w}{s} = k_D F_N, \quad (1)$$

where V_w is the wear volume, k_D is the dimensional wear coefficient, s is the sliding distance, and F_N is the applied normal force.

Archard's wear law can be rewritten as an initial value problem, which can be solved by integrating the contact pressure over the sliding distance. The wear at each surface point is a result from the solution of the integral, denoted here as the

Ipdsdt method, of the form

$$h = k_D \int p ds = k_D \int p(t) \frac{ds}{dt} dt, \quad (2)$$

where p is the contact pressure and ds/dt is the slip rate.

With a pressure-force transformation, the wear integration can be simplified to an algebraic equation (Chen and Huber 2010), the so-called CForce method, which has the form

$$h(x) = k_D \int p \frac{ds(x)}{dt} \frac{dX}{v_c(x)} = k_D \frac{ds(x)}{dt} \frac{1}{v_c(x)} \int p dX, \quad (3)$$

where $v_c(x)$ is the average contact velocity when the contact moves through a point x . The pressure-force transformation modifies the remaining integral in Eq. (2) in such a way that it reduces to $F_N = \int p dX$, where X is the tangential spatial coordinate on the contacting surface and F_N is equivalent to the total contact force.

As a result, the wear at an arbitrary location is directly related to the current values of the slip rate, average contact velocity and contact force. Eq. (3) is only valid for two-dimensional Hertzian contacts, where the pressure profile is moving completely over a surface point with a non-zero contact velocity. This assumption includes a number of technically important components, such as camshaft follower systems and gears, which belong to the class of plane rotating tribosystems.

3 Wear prediction for spur gear-teeth

3.1 Wear Processor

A description of the software, Wear Processor 5, working with the fil-file of Abaqus 5 and its modifications of the recent version of Wear Processor 6.0 with Python control and interfaced with the odb-file (introduced in Abaqus 6) can be found detail in (Hegadekatte, Huber and Kraft 2005a,b) and (Chen and Huber 2010), respectively. Compared with the two-dimensional twin-wheel model or the camshaft-follower model discussed in (Chen and Huber 2010), the contact force on the gear tooth is redistributed when the adjacent gear teeth comes in contact or loses contact. To reduce the total model size and improve the computational efficiency, a “master tooth” technique will be introduced in this paper.

Due to the geometrical symmetry, the wear on the outer surface of each tooth should be the same for one gear. Thus, only a representative part of the whole gear train needs to be modelled (see Figure 1) to obtain the wear for the entire gear. To increase the robustness, the wear distribution of several teeth is computed, and the

resulting average distribution is then applied to each tooth by the Wear Processor before the next wear step begins. To use the CForce method as given in Eq. (3), an extension in the programming of the Wear Processor was required, which allowed the total contact force F_N for each individual tooth to be treated separately.

3.2 FE Model

The FE model of the gears, shown in Figure 1, represents a part of a micro-planetary gear train, which was developed and produced within the collaborative research centre SFB 499 at KIT, Germany, using micro powder injection moulding of a ceramic feedstock and subsequent sintering of the ceramic parts (Giezelt 2003; Piotter 2011). Details of the measurements of the gear shape after production can be found in (Albers 2010) and their referenced literature. In this work, the ideal shapes of the original micro-gears are scaled geometrically similar by a factor of 100. Plane strain four-node isoparametric quadrilateral elements and three-node triangular elements are used. Considering the time-consuming contact simulation, the mesh is only refined at the contact regions to capture the correct local contact deformation, and a very coarse mesh is applied for the remaining part of the gears. The model consists of 37626 nodes and 35790 elements. In the first case, the applied torque on the planet gear is 8000 Nmm.

Each tooth in the FE model needs to be generated with its individual contact surface to provide the required data for the Wear Processor. In the model, the sun gear and the planet gear carry the slave and master surfaces, respectively. In Figure 1(a), the definition of the individual teeth is given by the adjacent number beside each tooth. The variable L , given in Figure 1(b), represents a local coordinate for each individual tooth of the gear along its outer surface. For each tooth of the sun gear or planet gear, the origin of L is located at the intersection of the symmetry line between two teeth and the outer surface. The positive direction is defined along the anti-clockwise direction with regard to the centre of the gear. The distance measured from the origin along the outer surface of the tooth defines the value of L .

The sun and planet gears were made of ceramics with elastic properties taken from a previous work (Hegadekatte, Hilgert, Kraft and Huber 2008). The parameters for the gear model are shown in Table 1. The dynamic effects, as they were studied in the mentioned reference, are not considered here. All the simulations are quasi-static, and a time step 1 s was chosen for simplicity.

The FE model was solved using Abaqus (V8.1) on an SGI Altix 4700 Blade server running Linux. A single run of the Abaqus simulation turns the sun gear by 83 degrees in 2872 increments. The required computation time was 10 CPUh for the Abaqus simulation, and the following wear step took 72 CPUh with the Wear

Processor. The major part of the post-processing time was used for reading the field output from the 2GB odb file using Python. Clearly, it is obvious that a further refinement of the finite element mesh would lead to immense computation times.

Table 1: Parameters for the sun gear and planet gear

	Sun gear	Planet gear
Total number of teeth	24	16
Pressure angle	20	20
Module (mm)	4.5	4.5
Tip radius (mm)	58.5	40.5
Number of teeth in gear model	5	5
Load torque (N mm)	–	8000 (case 1), in clockwise direction 4000 (case 2), in clockwise direction
Angular velocity (radian/s)	1.45, in clockwise direction	–
Young's modulus (N/mm ²)	304000	304000
Poisson's ratio	0.24	0.24
Wear coefficient (mm ² /N)	13.5E-9	13.5E-9

3.3 Wear of gear teeth

3.3.1 Contact force

The total contact force for the sun tooth and planet tooth as a function of time is shown in Figure 2 (arbitrary scale, where 1 s corresponds to a rotation of 83° of the sun gear). The curves clearly show that there are phases with a constant total contact force of 238 N, which is when there is only one pair of teeth in contact. There, the contact force is simply defined by the applied torque and the lever from the gear centre to the line of action.

When the next pair of teeth comes in contact, the contact force on the previous pair decreases gradually and increases on the incoming teeth in the same manner; their sum equals the constant contact force required to maintain the applied torque. The redistribution of the contact force within a contact pair is nonlinear as seen, e.g., for slave tooth 2, 3 and 4 in Figure 2. For an unworn gear, the transition could be

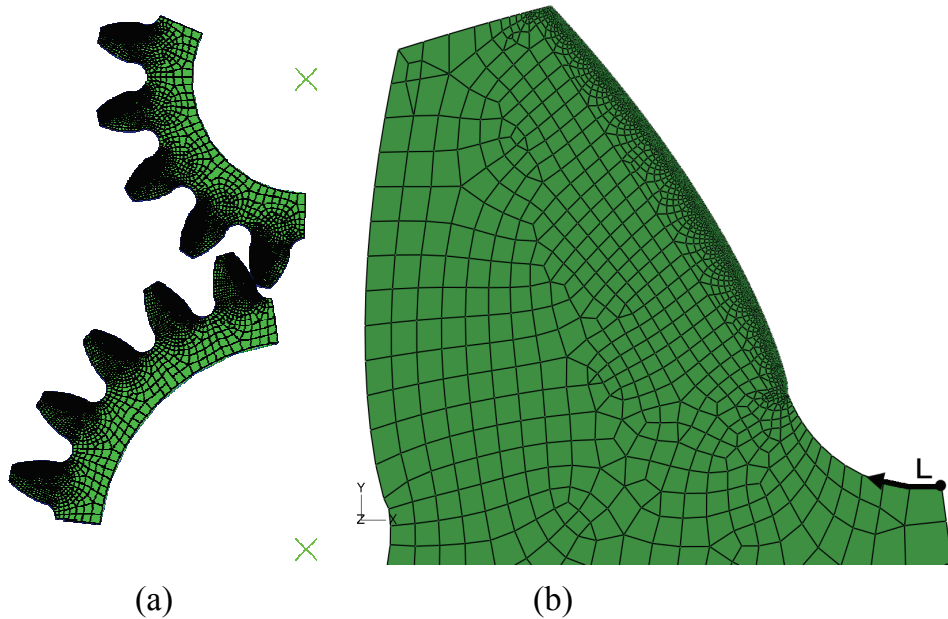


Figure 1: (a) Model of spur gear teeth; (b) detailed mesh of one sun gear tooth and the definition of L for each tooth

modelled by linear segments with sufficient accuracy. However, the onset of transfer from one pair to the next depends on the deformation of the teeth induced by the applied torque. Furthermore, it can be assumed that the observed transfer behaviour may change with progressing wear, underlining the need of finite element simulations.

3.3.2 Slip rate

Figure 3 shows the slip rate of the sun tooth and planet tooth. The slip rate decreases and increases linearly before and after the pitch point is reached, respectively, because it is defined by the geometry and kinematics of the spur gear. There is only rotation, but no relative movement of the mating contact gear tooth at the pitch point occurs. Thus, the slip rate decreases towards and increases after passing the pitch point by the contact. Because the local deformations in the contact, particularly, tangential to the surface, are negligibly small, determining the slip rate does not require an FE model. The slip rate can be obtained from tracking the contact point during the rigid body rotation of the two gears, where their geometry slowly changes due to wear.

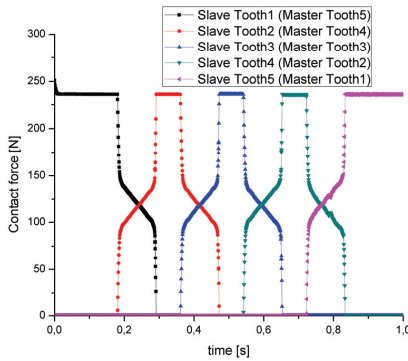


Figure 2: Evolution of the total contact force

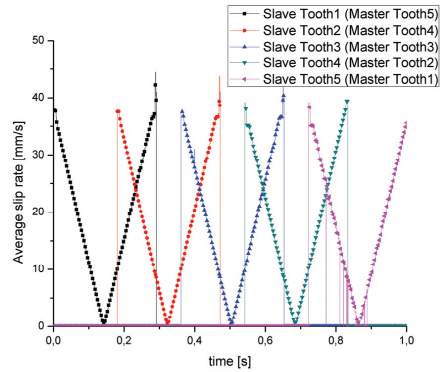


Figure 3: Evolution of the slip rate

For the following wear simulation, the average wear is based on the individual results of teeth 2, 3 and 4 only. Teeth 1 and 5, which are located at the boundary of the model, exhibit an unrealistic high compliance and can be ignored.

3.3.3 Average contact velocity

The average contact velocity is defined by the distance that the contact point P moves in the local coordinate system of the gear tooth surface under consideration, $L_2 - L_1$, and the corresponding time increment, $t_2 - t_1$, by

$$v_c(P) = \frac{|L_2 - L_1|}{t_2 - t_1}.$$

In this study, the average contact velocity of the gear teeth model is computed as follows. Firstly, the profiles of one mating sun gear tooth and planet gear tooth are obtained from coordinates of the nodes in the FE model in its undeformed configuration (see Figure 4). A clockwise angular velocity is applied to the sun gear for the given time increment. The rotation of the planet gear follows this rotation in such a way that it geometrically remains in contact with the sun gear, i.e., the overlap is minimised. In this way, the movement of the contact, $L_2 - L_1$, can be obtained, and the average contact velocities of sun gear and planet gear can be determined. The result is a smooth curve with a very low level of numerical noise, as shown in Figure 5. It should be noted that corresponding data obtained directly from the deformed FE model are very noisy and lead to extreme scatter in the predicted wear profile.

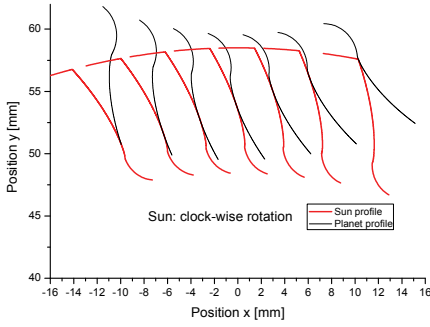


Figure 4: Rotation of one mating gear contact pair driven by the rotation of the sun tooth (only selected increments are shown)

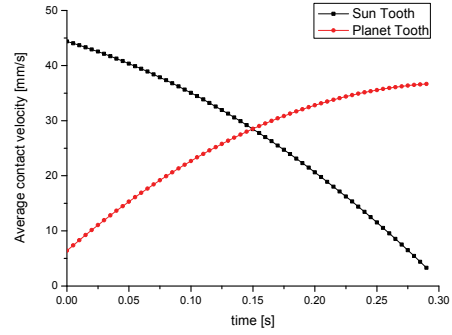


Figure 5: Average contact velocities for the sun and planet tooth as determined from their rigid body motion and contact tracking

3.3.4 Wear distribution

Once the contact force, slip rate and average contact velocity are determined, the wear distribution on the surfaces of the sun gear and the planet gear can be obtained by the CForce method. The resulting wear distribution for 1 turn of the sun gear is shown in Figure 6, and Figure 7 shows the wear distribution for the planet gear. Only the wear results for the inner gear teeth are shown. It should be noted that when the wear of multiple rotations are desired, the ratio of turns for the sun and planet gear (24/16) should be used to calculate the wear on the smaller planet gear. Because the wear of the planet gear is 25% greater than the wear on the sun gear (cf. Figures 6 and 7 at small L), the wear for multiple turns will be approximately 88% greater on the planet gear than the wear on the sun gear.

It can be seen from Figure 6 and Figure 7 that the wear distributions for the sun gear and the planet gear agree well using the CForce method (Eq. (3)) and the conventional Ipdstdt method (Eq. (2)). The agreement with the Ipdstdt method validates the CForce method and the computation of the average contact velocity proposed in the previous section. Furthermore, the CForce method achieves a much higher accuracy and robustness compared with that of the Ipdstdt method. The stability of the CForce method originates from the integral nature of the global force $F_N = \int p dX$, which, in contrast to the Ipdstdt method, does not require a spatial resolution of the contact pressure profile.

The observed oscillations in the Ipdstdt method result from integrating the contact pressure over the slip distance based on the nodal FE results. Because the contact is

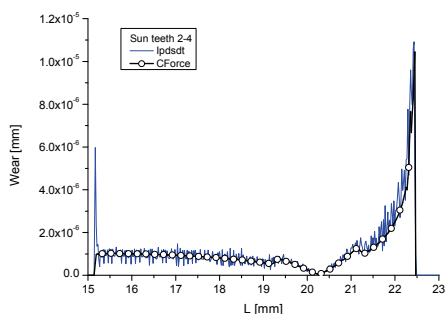


Figure 6: Wear distribution on the surfaces of the sun gear

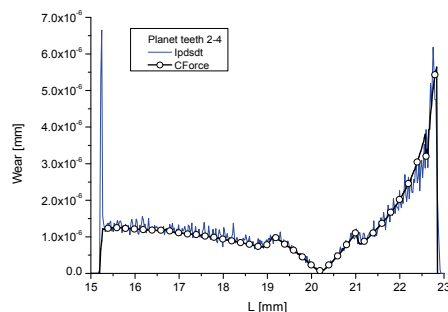


Figure 7: Wear distribution on the surfaces of the planet gear

very small but moves over the entire tooth surface, it is difficult to mesh the entire part to achieve both acceptable computation time and an accurate resolution of the pressure distribution. Thus, the wear integration by the Ipdst method shows large variations depending on how much is known of the contact pressure profile and if slip information is available for a specific surface node.

The wear distributions in Figures 6 and 7 clearly show visible features approximately 1 mm from the pitch point (indicated by minimum wear). This phenomenon is due to the redistribution of the contact force during the transition from one tooth to the next, as reported in (Flodin and Andersson 1997; Flodin and Andersson 2001; Dhanasekaran and Gnanamoorthy 2008).

3.3.5 Effect of torque

With a relatively general and robust tool for wear prediction, parametric studies can be performed, and the effects on the expected wear distribution can be easily investigated. To demonstrate this potential, a second case was computed at a reduced torque of 4000 Nmm. The predicted wear profiles are shown in Figure 8 and 9 for the sun gear and planet gear, respectively.

First, we obtained the expected result: approximately, the amount of wear is simply half of the wear for a torque of 8000 Nmm. This result is not surprising because all the assumptions are based on the fact: the contact is small compared with the dimension of the tooth and thus, Hertzian (linear) contact theory is applicable. Therefore, the kinematical variables, namely slip rate and average contact velocity, remain nearly unchanged. However, we can observe a shift of the point when force transition begins or ends (observe features approximately 1 mm from both sides of the pitch point). At a lower torque, the gears are under maximum load (single

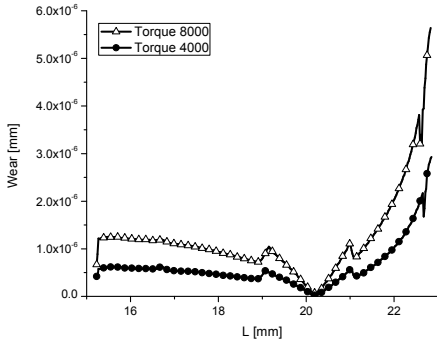


Figure 8: Wear distribution on the surfaces of the sun gear for two torques

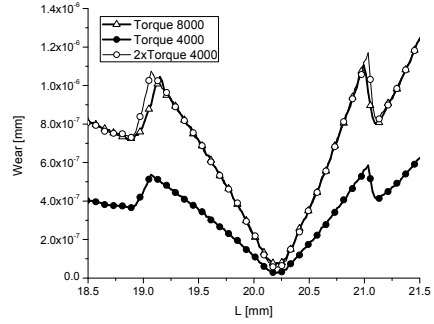


Figure 9: Wear distribution on the surfaces of the planet gear – investigation of linear scaling

gear contact) for a longer period of time due to the reduced bending of the gear teeth. By scaling the wear profile by a factor of two (see Fig. 9, open circles), the maximum wear would actually be slightly higher when a gear is operated at half torque but for twice the time. Or, the other way around, the elasticity of the gear favours distributing the force on two gears with increasing applied torque, resulting in a slightly reduced wear at the same energy being transmitted.

4 Conclusions

The recently proposed method to efficiently compute wear, the CForce method, has only been applied to problems with only a single contact. In the present paper, it is shown how the post-processing software, Wear Processor, can be extended so that it can be applied to multi-contact problems using the CForce method. An emphasis was placed on the robustness and simplicity of the approach to ensure reliable predictions at reduced cost of the meshing of the contacting bodies and finite element computation.

As a prominent example, the case of a gear train was considered, where the torque is transmitted, which alternates between two pairs of teeth in contact. To obtain robust data for the average contact velocity, the kinematics of the contact point was determined for undeformed gears that undergo a rigid body rotation only. A validation was performed by comparing the simulation results with those from the conventional Ipdstdt method, which computes the wear by integrating the product of the slip rate and local contact pressure. In this way, it was shown that the agreement between both methods is excellent, and furthermore, the CForce method is much

more robust because only the total contact force is used instead of a highly resolved pressure profile, which is difficult to be obtained.

The present work has opened up new possibilities to solve complex systems, such as gears, with sufficient accuracy and robustness. In a simple parametric study, it was shown that the results generally follow the expected linear scaling. However, when considering the wear distribution and maximum amount of wear in more detail, it becomes clear that coupling effects between the applied load and the elasticity of the system can make a difference in the resulting distribution of the contact force on more than one contact and thus, in the predicted wear profile.

In this sense, using the proposed implementation of the CForce method, valuable insights can be gained with respect to understanding of complex phenomena in mechanical systems whose performance or lifetime is limited by wear. Many examples could be given here, but examples of those with particular relevance that should be investigated are (i) the effect of wear on wear, i.e., the question of how wear changes the kinematics of the gear with consequences on history-dependent wear evolution, (ii) the influence of production tolerances, e.g., micro gears, on wear, and (iii) the effect of spectrum loading, whose existence is indicated by the results from the present work.

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