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On the Energy Release Rate at the Crack Tips in a Finite Pre-Strained Strip

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Abstract: The influence of the initial finite stretching or compressing of the strip containing a single crack on the Energy Release Rate (ERR) and on the SIF of mode I at the crack tips is studied by the use of the Three-Dimensional Linearized Theory of Elasticity. It is assumed that the edges of the crack are parallel to the face planes of the strip and the ends of the strip are simply supported. The initial finite strain state arises by the uniformly distributed normal forces acting at the ends of the strip. The additional normal forces act on the edges of the crack. The elasticity relations for the strip material are given by the harmonic type potential. The corresponding boundary-value problem is solved by employing FEM. The numerical results on the influence of the initial finite strain state the values of the ERR and of the SIF of mode I are presented. In particular, it is established that the values of the ERR and of the SIF of mode I decrease (increase) monotonically with an increase (decrease) in the initial stretching (compression).

Keywords: Crack, Energy Release Rate, finite initial strain, harmonic type potential, Stress Intensity Factor of mode I, strip.

1 Introduction

A typical problem of fracture mechanics is the determination of the Energy Release Rate (ERR) or of the Stress Intensity Factors (SIF) at the tips of cracks occurring in structural members. A large number of investigations have been carried out in this field, and the corresponding results are tabulated in many reference books, such as

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Sih (1973). At present, numerical methods based on the domain-integral formulation for ERR and SIF computations are being developed intensively. A review of these investigations is given in Gosz, Dolbow and Moran (1998). It should be noted that the above-mentioned studies for brittle and quasi-brittle materials have been performed within the framework of the linear theory of elasticity, according to which the uniformly distributed normal stresses acting along the cracks do not influence the ERR and SIF. However, the experimental investigations listed in Guz (2008a, 2008b) show that these stresses act on the failure of the structural members containing cracks. In recent years in Kaminskii and Galatenko (2005), Kaminskii and Bogdanova (2009) and others within the framework of the linear and quasi-linear fracture mechanics a two-parameter model has been developed for computing the above-mentioned influence. Note that for this purpose in these investigations besides SIF the additional fracture parameter (T-stress) is introduced. Consequently, two-parameter models cannot take into account the influence of the uniformly distributed normal stress acting along the cracks on the SIF at the crack tips. This influence can be taken into account by the use of the Three-Dimensional Linearized Theory of Elasticity (TDLTE), which is detailed in Guz (1999). The TDLTE for problems of brittle fracture was developed in Guz (2008 a, 2008b). A review of investigations on the mechanics of brittle fracture of pre-stressed materials is given in Bogdanov, Guz and Nazarenko (2009), Guz (2009).

It follows from Guz (2008a, 2008b, 2009), Bogdanov, Guz and Nazarenko (2009) that, up to now, results on the influence of initial stresses on the SIF are obtained for an infinite body and studies on the effect of initial tension or compression along cracks located in finite regions on the SIF are absent. The importance and of such an inquiry is evident and it deserves to be a topic.

The first attempts in this field was made in the papers Akbarov, Yahnioglu and Turan (2004), Akbarov and Turan (2009a) and the investigations were carried out for a simply supported and initially stretched (compressed) strip containing a crack on whose edges some additional uniformly distributed normal forces operate. A plane-strain state is considered and the material of the strip was assumed orthotropic with normalized mechanical properties. In the paper Akbarov and Turan(2009b) the investigations Akbarov, Yahnioglu and Turan (2004), Akbarov and Turan (2009a) was developed for sandwich plate with interface cracks. However, it was assumed in Akbarov, Yahnioglu and Turan (2004), Akbarov and Turan (2009a, 2009b) that the initial strain state in the strip is small and is determined within the framework of the classical linear theory of elasticity. Consequently, the results obtained in Akbarov, Yahnioglu and Turan (2004), Akbarov and Turan (2009a, 2009b) regard the strip fabricated from the rigid or moderately rigid materials and are not applicable for the strip fabricated from the various elastomers which are suitable for the finite initial stretching or compression of the strip. Therefore, in the present paper the investigations carried out in Akbarov, Yahnioglu and Turan (2004), Akbarov and Turan (2009a, 2009b) are developed for a finite initially stretched (or compressed) simply supported strip containing the crack. The values of the ERR and the influence of the initial stretching (compressing) on these values are analysed. In particular cases, the SIF of mode I is also analysed within the foregoing contents.

Throughout the paper, by repeated indices which are used only in the right side of the relations and equations are summed over their ranges.

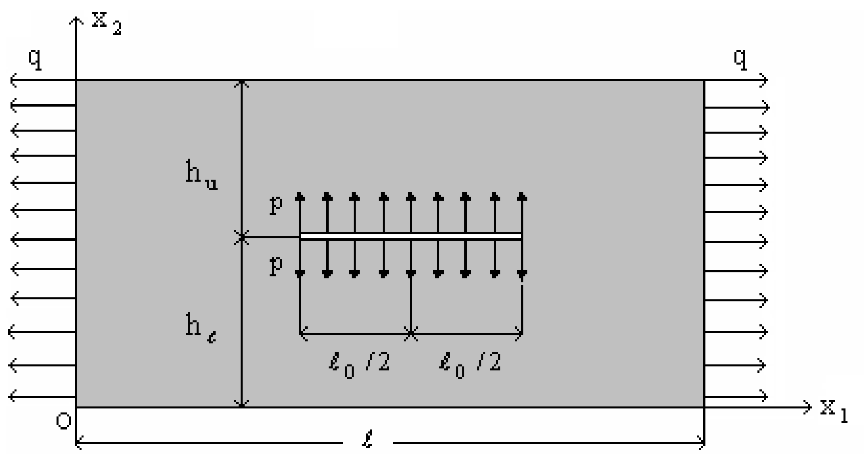

Figure 1: The geometry of the considered strip

2 Formulation of the problem

We consider a strip which in the natural state occupies the region $\Omega = \{0 \le x_1 \le \ell,\ 0 \le x_2 \le h\}$, in Ox_1x_2 plane and $-\infty < x_3 < +\infty$ for that, where x_1,x_2 and x_3 are the Lagrangian coordinates of the strip points in the Cartesian system of coordinates $Ox_1x_2x_3$. This system of coordinates is associated with the strip shown in Fig. 1. Assume that the strip contains a crack which is in a plane $x_2 = h_a$ in the natural state. The length of the crack is equal to ℓ_0 and the location of that is symmetric with respect to $x_1 = \ell/2$. It is supposed that the strip material is isotropic, homogeneous, compressible hyperelastic one with the elastic potential of the harmonic type.

Assume that uniformly distributed normal forces of intensity q operate at the strip ends and cause the initial strain state which is determined as follows:

$$u_m^0 = (\lambda_m - 1)x_m, \quad \lambda_m = const_m \tag{1}$$

where u_m^0 is a displacement and λ_m is the elongation along the Ox_m axis. As we consider the plane strain state, therefore we assume that

$$\lambda_3 = 1, \quad u_3^0 \equiv 0. \tag{2}$$

At the same time, the positions of the points of the strip are determined by the Lagrangian coordinates y_i in the Cartesian system of coordinates $Oy_1y_2y_3$ which is associated with the initial state of the strip. The relation between the coordinates y_i and x_i can be written as follows:

$$y_i = \lambda_i x_i. \tag{3}$$

Below we will use the equations and relations of TDLTE written in the system of coordinates $Oy_1y_2y_3$ and the values regarding this system will be denoted by upper prime.

Now we assume that after the appearance of the foregoing initial state additional normal forces of intensity p (p≪q) are applied to the edges of the crack. We investigate the influence of the initial tension (or compression) of the strip on the ERR at the crack tips caused by the action of additional normal forces p. For this purpose we write the equilibrium equations and corresponding boundary conditions of TDLTE for the considered case.

The equilibrium equations:

$$\frac{\partial Q'_{11}}{\partial y_1} + \frac{\partial Q'_{21}}{\partial y_2} = 0; \quad \frac{\partial Q'_{12}}{\partial y_1} + \frac{\partial Q'_{22}}{\partial y_2} = 0; \tag{4}$$

The mechanical relations:

$$Q'_{ij} = \omega'_{ij11}\frac{\partial' u_1}{\partial y_1} + \omega'_{ij12}\frac{\partial' u_1}{\partial y_2} + \omega'_{ij21}\frac{\partial' u_2}{\partial y_1} + \omega'_{ij22}\frac{\partial' u_2}{\partial y_2}. \tag{5}$$

The equations (4) and (5) are satisfied in

$$\Omega'_1 = \Omega' - (L'^{+} \cup L'^{-}) \tag{6}$$

where

$$\Omega' = \left\{0 \le y_1 \le \ell', 0 \le y_2 \le h'\right\},$$

$$L'^{\pm} = \left\{y_2 = h'_{a\pm 0}, \ell'/2 - \ell'_0/2 < y_1 < \ell'/2 + \ell'_0/2\right\}. \tag{7}$$

According to (3), in (7) the following notation is used

$$h' = \lambda_2 h, \quad \ell'_0 = \lambda_1 \ell_0, \quad h'_u = \lambda_2 h_u, \quad h'_a = \lambda_2 h_a. \tag{8}$$

In (4) and (5) through Q'_{ij} the perturbations of the components of Kirchoff stress tensor are denoted. The notation u'_{α} $(\alpha = 1,2)$ shows the perturbations of the components of the displacement vector. The values of $\omega'_{ij\alpha\beta}$ $(i; j; \alpha; \beta = 1,2)$ in (5) are determined through the mechanical constant of the strip material and through the initial state (1). According to Guz (2008a, 2008b), consider the determination$\omega'_{ij\alpha\beta}$. As it has been noted above, we assume that the elasticity relations of the strip material are given by harmonic type potential. This potential is given as follows:

$$\Phi = \frac{1}{2}\lambda s_1^2 + \mu s_2 \tag{9}$$

where λ and μ are the mechanical constants, s_1 and s_2 are invariants of the Green's strain tensor and are determined by the following formulae:

$$s_1 = (\lambda_1 - 1) + (\lambda_2 - 1) + (\lambda_3 - 1),$$
$$s_2 = (\lambda_1 - 1)^2 + (\lambda_2 - 1)^2 + (\lambda_3 - 1)^2 \tag{10}$$

where

$$\lambda_i = \sqrt{1 + 2\varepsilon_i}. \tag{11}$$

In (11), ε_i are the principal values of the Green's strain tensor ε_{ij} which are determined through the components of the displacement vector by the following expressions:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_\alpha}{\partial x_i}\frac{\partial u_\alpha}{\partial x_j}\right). \tag{12}$$

In this case, the determination of the components of the Lagrange stress tensor S_{ij} through the potential Φ and the expression of the Kirchoff stress tensor Q_{ij} through the S_{ij} are given as follows:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial}{\partial \varepsilon_{ij}} + \frac{\partial}{\partial \varepsilon_{ji}}\right)\Phi, \quad Q_{ij} = S_{in}\left(\delta_n^j + \frac{\partial u_j}{\partial x_n}\right). \tag{13}$$

Note that the expressions (9)-(12) are written in the arbitrary system of Cartesian coordinate system without any restriction related to the association of this system to the natural or initial state of the considered strip.

According to (13) for the considered case the relations between the perturbation of the Kirchoff stress tensor and the perturbation of the components of the Lagrange stress tensor can be written as follows:

$$Q'_{nm} = \frac{\lambda_n}{\lambda_1\lambda_2}\left(\lambda_m S_{nm} + S_{ni}^0 \frac{\partial u'_m}{\partial y_i}\right) \tag{14}$$

According to (13) the following notation is used.

$$S_{ni}^{0}=\frac{1}{2}\left(\frac{\partial}{\partial\varepsilon_{ni}^{0}}+\frac{\partial}{\partial\varepsilon_{in}^{0}}\right)\Phi^{0}, \tag{15}$$

$$S_{nm}=\lambda_{nmij}\frac{\partial u_i}{\partial x_j} \tag{16}$$

where

$$\lambda_{nmij}=\frac{1}{4}\left(\delta_k^i+\frac{\partial u_i^0}{\partial x_k}\right)\left(\frac{\partial}{\partial\varepsilon_{kj}^{0}}+\frac{\partial}{\partial\varepsilon_{jk}^{0}}\right)\left(\frac{\partial}{\partial\varepsilon_{nm}^{0}}+\frac{\partial}{\partial\varepsilon_{mn}^{0}}\right)\Phi^{0}. \tag{17}$$

Taking into account the expressions (9)-(11), from (15) we obtain that

$$S_{11}^{0}=\left[\lambda(\lambda_1+\lambda_2-2)+2\mu(\lambda_1-1)\right];\quad S_{12}^{0}=0,$$

$$S_{22}^{0}=\left[\lambda(\lambda_1+\lambda_2-2)+2\mu(\lambda_2-1)\right];\quad S_{33}^{0}=2\mu(1-\lambda_2) \tag{18}$$

According to the problem statement, we can write $S_{11}^{0}=q$ and $S_{22}^{0}=0$ from which we obtain

$$\lambda_2=\frac{2(\lambda+\mu)-\lambda\lambda_1}{\lambda+2\mu}. \tag{19}$$

So, for fixed values of the material constants λ and μ the initial stretching or compression of the strip can be estimated through the parameter λ_1 only.

Thus taking into account (16)-(18), from (14) and (5) we obtain the following expressions for the componentsω'_{ijnm}:

$$\omega'_{1111}=\frac{\lambda_1}{\lambda_2}(\lambda+2\mu),\quad \omega'_{2222}=\frac{\lambda_2}{\lambda_3}(\lambda+2\mu)$$

$$\omega'_{1122}=\lambda,\quad \omega'_{2112}=\omega'_{1212}=\frac{2\mu\lambda_2}{\lambda_1+\lambda_2},$$

$$\omega'_{1221}=\frac{2\mu\lambda_1^2}{\lambda_2(\lambda_1+\lambda_2)}, \tag{20}$$

$$\omega'_{1112}=\omega'_{1121}=\omega'_{1211}=\omega'_{1222}=\omega'_{2111}=\omega'_{2121}=\omega'_{2122}=0,$$

$$\omega'_{2211}=\lambda,\quad \omega'_{2121}=\frac{2\mu\lambda_2}{\lambda_1+\lambda_2}$$

Consider the boundary conditions for the perturbation state. We assume that the following conditions are satisfied.

$$u_2'\big|_{y_1=0;\ell'}=0; \quad Q_{11}'\big|_{y_1=0;\ell'}=0; \quad Q_{22}'\big|_{y_2=0;h'}=0$$

$$Q_{21}'\big|_{y_2=0;h'}=0; \; Q_{22}'\big|_{L'^{\pm}}=-p; \quad Q_{21}'\big|_{L'^{\pm}}=0 \tag{21}$$

Thus, the investigation of the stress-strain state in the considered strip is reduced to the solution to boundary value problem (4)-(21). After determination of the stress-strain state we can calculate ERR (denoted by γ) by the use of the expression

$$\gamma=-\frac{1}{2}\frac{\delta U(\ell_0)}{\delta \ell_0} \tag{22}$$

where

$$U(\ell_0)=\frac{1}{2}\iint\limits_{\Omega_1'}\left(Q_{11}'\frac{\partial u_1'}{\partial y_1}+Q_{12}'\frac{\partial u_2'}{\partial y_1}+Q_{21}'\frac{\partial u_1'}{\partial y_2}+Q_{22}'\frac{\partial u_2'}{\partial y_2}\right)dy_1dy_2. \tag{23}$$

3 Method of Solution: FEM modelling

As an analytical solution to problem (4)-(21) cannot be obtained, we will investigate this problem by employing the FEM. For this purpose, we introduce the functional

$$\Pi=\frac{1}{2}\iint\limits_{\Omega_1'}\left(Q_{11}'\frac{\partial u_1'}{\partial y_1}+Q_{12}'\frac{\partial u_2'}{\partial y_1}+Q_{21}'\frac{\partial u_1'}{\partial y_2}+Q_{22}'\frac{\partial u_2'}{\partial y_2}\right)dy_1dy_2 - \int\limits_{L'^{+}} pu_2'^{+}dy_1+\int\limits_{L'^{-}} pu_2'^{-}dy_1 \tag{24}$$

Taking into account the relations (5) the functional (24) can be written as follows:

$$\Pi=\frac{1}{2}\iint\limits_{\Omega_1'}\omega_{ij\alpha\beta}'\frac{\partial' u_\alpha}{\partial y_\beta}\frac{\partial u_j'}{\partial y_i}dy_1dy_2-\int\limits_{L'^{+}} pu_2'^{+}dy_1+\int\limits_{L'^{-}} pu_2'^{-}dy_1 \tag{25}$$

According to expressions (5), (14), (16), (17) and (20), it is easy to verify that $\omega_{ij\alpha\beta}'=\omega_{\beta\alpha ji}'$. According to Guz (2009), by using these relations, it is proven that the equations (4) and the boundary conditions (21) (except the conditions written for u_2') are Euler equations of the functional (25). Consequently from the relations $\delta\Pi=0$ we obtain the equations (4) and the conditions (21). Thus, in this way the validity of the functional (25) for FEM modelling of the considered problem is proven.

Since the problem is symmetric with respect to $y_1' = \ell'/2$ under FEM modelling we consider only half the region Ω_1'. In this case, the part around the crack tip is modelled by singular finite elements (Tan and Gao (1990), Zienkiewicz and Taylor (1989)). For the remaining part of the region, the standard quadratic Lagrange-family rectangular finite elements are used (Fig. 2).

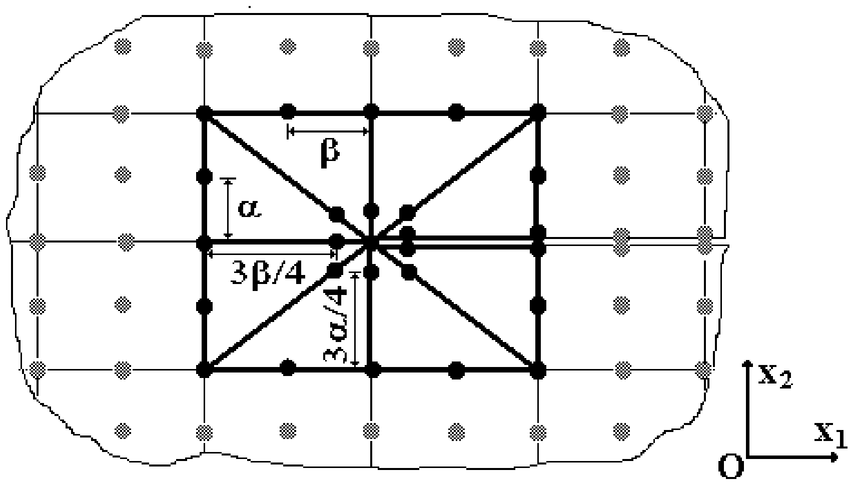

Figure 2: Finite element modelling at the crack tip.

According to (22), under calculation of the ERR we use the approximate expression

$$\gamma \approx -\frac{1}{2}\frac{U(\ell_0'+\Delta\ell_0')-U(\ell_0')}{\Delta\ell_0'}. \tag{26}$$

the values of the increment $\Delta\ell_0'$ are determined from the convergence requirement of the values γ for each combination of the problem parameters h_a, $\ell_0/2\ell$, h/ℓ, λ/μ and λ_1. The influence of the initial stretching (or compressing) of the strip the values of ERR (i. e. γ) will be estimated through the parameter λ_1.

Under $h_a = h/2$ (Fig. 1) for the considered problem mode I takes place and in this case we can calculate the values of SIF (K_I) through γ. But in the cases for which $h_a \neq h/2$ the mixed mode arises and in such cases we will analyse the influence of the initial tension or compression of the strip on the values of ERR, (i.e. γ).

According to Guz (2008b, 2009) and the expressions (20) for calculating K_I we use the following relations.

$$\left[K_I^2(\gamma_2'^{(1)}+\gamma_2'^{(2)})\mu_1'\gamma_{21}'^{(1)}-\gamma_2'^{(1)}\gamma_{21}'^{(2)}\right]=4\gamma i^{-1}(\mu_1'\gamma_{21}'^{(1)}\gamma_{22}'^{(2)}-\gamma_{21}'^{(2)}),\quad i=\sqrt{-1} \quad (27)$$

where

$$\mu_{1,2}'=i\sqrt{A'\pm\sqrt{A'^2-A_1'}},$$

$$2\gamma_{21}^{(1)}=-\left(\omega_{2112}'\omega_{1122}'\mu_1'^2-\omega_{1111}'\omega_{1212}'\right)B_1'^{-1}$$
$$-\left(\omega_{2222}'\omega_{2112}'\mu_1'^2+\omega_{2112}'\omega_{1221}'-\omega_{1212}'\left(\omega_{1122}'+\omega_{1212}'\right)\right)B_2'^{-1},$$

$$\gamma_{22}'^{(2)}=-2\left(\omega_{2222}'\omega_{2112}'B_1^{-1}\mu_1'^4+\omega_{1122}'\omega_{1221}'B_2'^{-1}\right),$$

$$2\mu_1'^{-1}\gamma_{21}'^{(2)}=\left(\omega_{2112}'\omega_{1122}'\mu_1'^2+3\omega_{1111}'\omega_{1212}'\right)B_1'^{-1}, \quad (28)$$

$$2\mu_1'^{-1}\gamma_2'^{(1)}=\left(\omega_{1111}'+\omega_{2112}'\mu_1'^2\right)B_1'^{-1}+\left(\omega_{1122}'+\omega_{1212}'\right)B_2'^{-1}+$$
$$\left(3\omega_{2222}'\omega_{2112}'\mu_1'^2-\omega_{2112}'\omega_{1221}'+\omega_{1212}'(\omega_{1122}'+\omega_{1212}'\right)B_2'^{-1},$$

$$\gamma_2'^{(2)}=\mu_1'\left(\omega_{1111}'-\omega_{2112}'\mu_1'^2\right)B_1'^{-1}$$

In (28) the constants A', A_1', B'_1 and B'_2 are calculated by the use of the following formulae:

$$A'=\frac{1}{2\omega_{2222}'\omega_{2112}'}\left[\omega_{1111}'\omega_{2222}'+\omega_{2112}'\omega_{1221}'-(\omega_{2211}'+\omega_{1212}')^2\right], \quad (29)$$

$$A_1'=\omega_{1111}'\omega_{1221}'(\omega_{2222}'\omega_{2112}')^{-1},$$

$$B'_1=\mu_1^2\left(\omega'^2_{1212}+\omega_{1212}'\omega_{1122}'+\omega_{1221}'\omega_{2112}'\right)-\omega_{1111}'\omega_{1221}',$$

$$B'_2=\omega_{2222}'\omega_{1212}'\mu_1^2-\omega_{1122}'\omega_{1221}'.$$

4 Numerical Results and Discussions

First we consider the validity of the algorithm and programmes which are composed by the authors and are used in the present numerical investigations. For this purpose consider the case where the initial strains (or stresses) are absent in the strip, i.e. $\lambda_1 = 1.0$ and $\lambda/\mu = 1.0$. It is evident that in this case the obtained results must coincide with the corresponding ones obtained within the framework of the classical linear theory of elasticity. Assume that $h_a = h/2$ and analyse the values of K_I obtained for various problem parameters. Tab. 1 shows the values of $K_I^{(s)}/K_{I\infty}$, $K_I^{(f)}/K_{I\infty}$ and $K_I^{(E)}/K_{I\infty}$ where $K_{I\infty} = p\sqrt{\pi\ell_0}$, $K_I^{(s)}$ and $K_I^{(f)}$ are the values of the SIF for mode I calculated by using the exact solution for an infinite plane, the approximate series given in Sih (1973), and the present approach, from the values of the nodal displacements of the singular triangular finite elements shown in Fig. 2 respectively. Moreover, in Tab. 1, $K_I^{(E)}$ shows the values of SIF for mode I calculated from the values $\partial U/\partial \ell_0$. The agreements of the corresponding results given in Tab. 1 provide support for the numerical approach used and proposed. Moreover, the results given in Tab. 1 agree with the mechanical consideration, according to which, the values of $K_I^{(s)}/K_{I\infty}$, $K_I^{(f)}/K_{I\infty}$ and $K_I^{(E)}/K_{I\infty}$ must simultaneously tend to unity with decreasing ℓ_0/ℓ and ℓ_0/h.

Table 1: The values of SIF for $\lambda_1 = 1.0$, $h/\ell = 0.20$, $h_u/\ell = h_A/\ell = h/2\ell$.

ℓ_0/ℓ	ℓ_0/h	$K_I^{(s)}/K_{I\infty}$	$K_I^{(f)}/K_{I\infty}$	$K_I^{(E)}/K_{I\infty}$
0.080	0.80	1.2406	1.2406	1.1930
0.075	0.75	1.2009	1.2009	1.1716
0.060	0.60	1.1444	1.1444	1.1108
0.050	0.50	1.0936	1.0931	1.0729
0.040	0.40	1.0473	1.0473	1.0409

Now we turn to consideration of the results which characterize the influence of the initial stretching and compression of the strip on the values of ERR and, in particular cases, on the values of K_I. Assume that $\lambda/\mu = 1.0$ and consider the graphs given in Figs. 3 and 4. Note that the graphs given in Fig. 3 show the dependencies between $K_I/K_{I\infty}$ and λ_1 in the case where $h_u = h_a = h/2$, for various values of h/ℓ and $\ell_0/2\ell$, but the graphs given in Fig. 4 show the dependencies between $\gamma\mu/K_{I\infty}$ and λ_1 in the case where $\ell_0/2\ell = 0.15$ for various values of h_u/ℓ and h/ℓ.

Note that in these figures the graphs separated by the letters a, b and c corresponding to the cases where $h/\ell = 0.10$; 0.15; 0.20, respectively. Moreover note that points

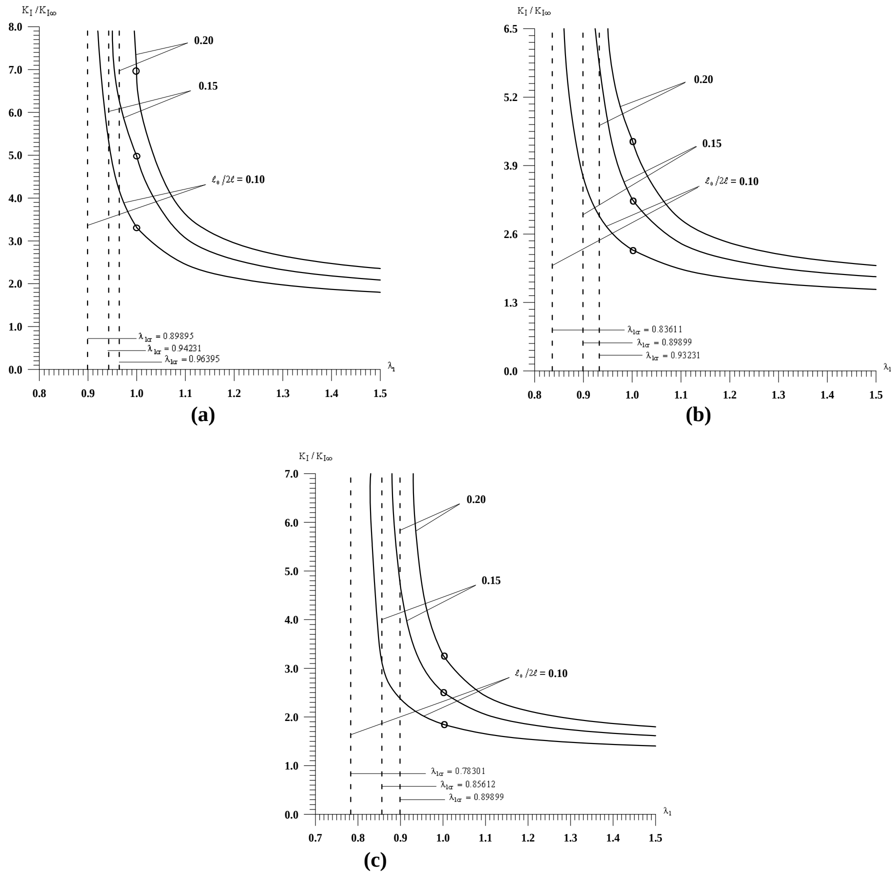

Figure 3: The graphs of the dependencies between $K_I/K_{I\infty}$ and λ_1 for the values of $\ell_0/2\ell$: (a) $h/\ell = 0.10$; (b) $h/\ell = 0.15$; (c) $h/\ell = 0.20$.

on these graphs which are indicated by circles show the values of $K_I/K_{I\infty}$ (in Fig. 3) and $\gamma\mu/K_{I\infty}$ (in Fig. 4) for the case where $\lambda_1 = 1.0$, i.e. for the case where the initial strain in the strip is absent.

The analyses of the graphs given in Figs. 3 and 4 show that the values of $\gamma\mu/K_{I\infty}$ and $K_I/K_{I\infty}$ decrease (increase) monotonically with increasing (decreasing) of the values of λ_1. Moreover, it follows from these graphs that $\gamma\mu/K_{I\infty}$ and $K_I/K_{I\infty} \to \infty$ as $\lambda_1 \to \lambda_{1cr.}$, where $\lambda_{1cr.}$ is the critical values of λ_1 which correspond to the loss of the stability of the considered strip with the crack. Consequently, for the critical values of λ_1 the uniqueness of the considered boundary-value problem (4)-(21) is violated. Therefore the results regarding the ERR have the physical meaning in

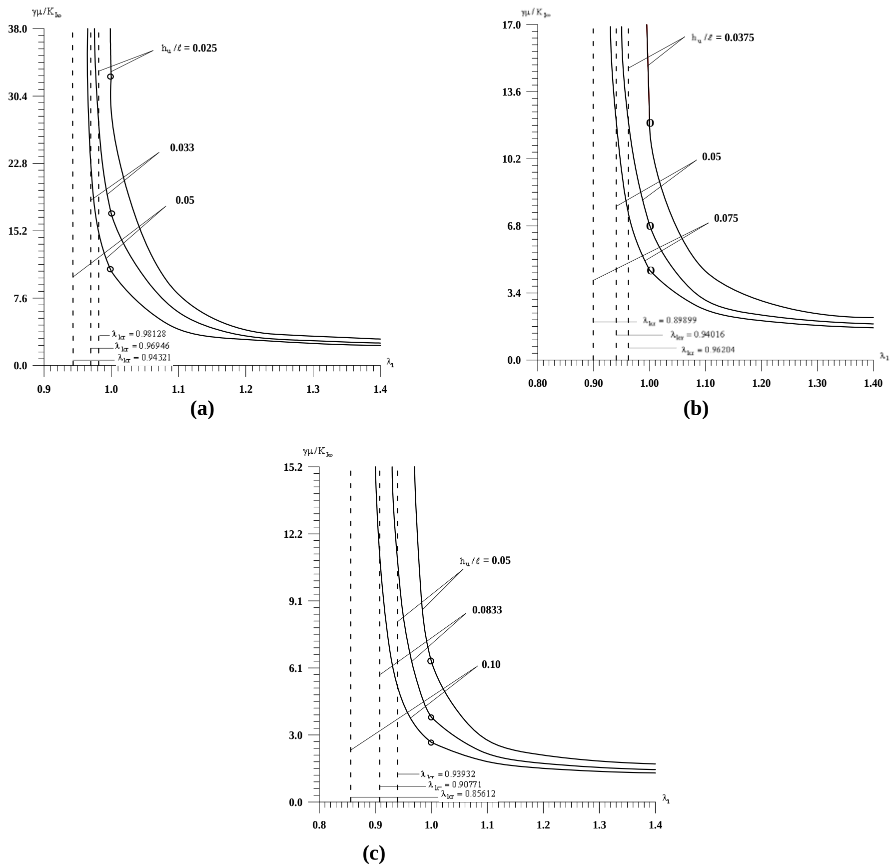

Figure 4: The graphs of the dependencies between $\gamma\mu/K_{I\infty}$ and λ_1 for the values of h_u/ℓ: (a) $h/\ell = 0.10$; (b) $h/\ell = 0.15$; (c) $h/\ell = 0.20$.

the cases where the initial compression of the strip is limited by the above-noted critical values of λ_1. Note firstly that, a similar result for infinite plane containing a crack was obtained in Guz (2008b) and it was established that $\lambda_{1cr.}$ for the infinite plane corresponds to the surface stability loss of the plane material.

The analyses of the obtained numerical results also show that in the quantitative sense the influence of the initial strain on the ERR depends significantly on the problem parameters h_u/ℓ and $\ell_0/2\ell$. The character of this dependence is formulated in the conclusions part of the paper.

5 Conclusions

In the present paper, within the framework of the TDLTE the influence of the finite initial tension and compression of the simply-supported strip containing a crack on the values of the Energy Release Rate (ERR) and on the SIF of mode I at the crack tips has been investigated. It was assumed that the initial stresses operated along the crack whose edges are parallel to the free face planes of the strip. The investigations were carried out by employing the FEM and the elasticity relations of the strip material are described by the harmonic potential.

The following concrete conclusions follow from the numerical analyses:

1. The values of the ERR and of the SIF of mode I decrease (increase) monotonically with finite initial tension (compression);

2. The influence of initial strains on the ERR and on the SIF of mode I increase monotonically with crack length; i.e. this influence becomes more significant with $\ell_0/2\ell$;

3. The influence of the finite initial strains on the ERR and on the SIF of mode I increase monotonically with crack location approaching the free face plane of the strip;

4. There exist such values of the initial compression strain under which a "resonance" type phenomenon takes place. These values of the initial compression strain correspond to the stability loss of the strip which contains the crack.

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