

## Nonlinear Finite Element Analysis of RC Structures Incorporating Corrosion Effects

Smitha Gopinath<sup>1,2</sup>, A. Ramachandra Murthy<sup>1</sup> and Nagesh R. Iyer<sup>1</sup>

**Abstract:** This paper presents the mathematical modeling techniques for nonlinear finite element analysis of RC structure to incorporate uniform corrosion effects. Effect of corrosion has been simulated as reduction in effective cross-sectional area of reinforcing bar, reduction in bonding phenomena and as reduction in material properties of reinforcing bar such as yield strength and elastic modulus. Appropriate constitutive laws for (i) corroded rebar elements and (ii) bond slip with corroded bar have been described. Procedure has been outlined to determine the global damage indicator by secant stiffness based approach. A corroded RC beam has been analysed to validate the proposed model and results have been compared with experimental response. A RC chimney has been analysed by considering the uniform corrosion effects. The result of corroded chimney shows the growth of damage with respect to increase in age of the structure. The results will give an insight for the maintenance and repair measures to be taken during the service life.

**Keywords:** Damage Indicator; Reinforced Concrete Structures; Corrosion; Non-linear Finite Element Analysis, Chimney.

### 1 Introduction

Analysis and design of reinforced concrete (RC) structures require better understanding of the behaviour of the constituent materials, their combinations, and material degradation procedures to ensure safety and serviceability. Reinforcement corrosion is the major cause of damage and early failure of RC structures worldwide with subsequent enormous costs for maintenance and repair. Further, increased demand for controlling the air pollution in our country demand for tall chimneys. This stack like structure experiences material deterioration mainly due to corrosion with increase in age. Severe corrosion occurring to the component part require repair which is very costly and needs specialized manpower. Also it is very

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<sup>1</sup> CSIR-Structural Engineering Research Centre, CSIR Campus, Taramani, Chennai, India, 600113

<sup>2</sup> smithag@sercm.org, smithagopinath13@yahoo.com, Fax: 91-44-22541508

difficult to inspect and find out the areas affected by corrosion. In such situation, the importance of an analytical model which is capable of simulating the actual structural behaviour comes into role. If the model is capable of predicting the cracking details and damage state of a structure as age progresses, it will be beneficial for the maintenance of the structure. The aim of any analytical model is to obviate the need for the experiments, at the same time it should be capable of simulating the response behaviour more realistically. Nonlinear finite element analysis (FEA) is the best option to predict the response behaviour of such RC structures, wherein material modelling plays very important role.

A constitutive law for simulating behaviour of corroded bars was developed and bond was modeled by using 4-noded isoparametric elements [Lee et al. (2000)]. Methodologies were proposed [Kratzig et al. (2000)] for modelling of local damage and deterioration phenomena on material point level as well as its mapping onto structural level. The applicability of this mapping was found to be very useful in the health monitoring of industrial structures such as cooling tower.

Research was carried out to simulate localized and uniform corrosion effects [Dekoster et al. (2003), Lundgren (2001)]. It was found that localized corrosion is sensitive to size of 'rust elements' while predicting the response behaviour. The corrosion effects were modeled by reducing the cross-sectional area of the reinforcing bar by modifying the constitutive laws of steel-concrete interface to simulate the bond behaviour [Coronelli and Gambarova (2004)]. Experiments were conducted [Shin et al. (2007)] to find out the effect of rate of corrosion on the mechanical properties of reinforcing bar.

This paper focuses on the development of methodologies to predict damage due to corrosion in terms of effective material properties and geometry of the member. The analytical models used are capable of simulating the deformability and load carrying capacity of the corroded structural components. The mechanisms such as reduction in cross-sectional area and reduction in yield stress and elastic modulus of reinforcing bars have been simulated with the existing analytical models. A model has been proposed to simulate the bond-slip of corroded bar. The numerical validation is carried out by analyzing an experimentally tested RC beam. Further, a RC chimney is analysed for uniform corrosion effects. A secant stiffness approach is proposed to find the damage indicator, which will give an indication about the damage state of the structure for a particular load level.

## **2 Material Models for Stiffness Degradation**

In the present study, load induced deterioration is interpreted as irreversible material damage effects presumably incorporated into the material models of FEA

software called FINEART [2006]. In nutshell these models include [Smitha et al. (2009)]:

- Stress-strain behaviour of uniaxial state of concrete in compression
- Hyperbolic tension softening for tensile cracking
- Bilinear stress-strain behaviour of reinforcing steel
- Inelastic bond behaviour.

Schematic representation of the uniaxial stress-strain behaviour of constituent materials of reinforced concrete along with the corresponding changes of the material state and their influence on the stiffness parameters have been illustrated in Figure 1. For instance, concrete tension cracking along with corrosion leads to replacement of the effective elasticity modulus to  $E_{cs}$  which is the elastic modulus of corroded bar. Yielding of reinforcement leads to replacement of the yield stress  $F_y$  with yield strength of corroded bar.

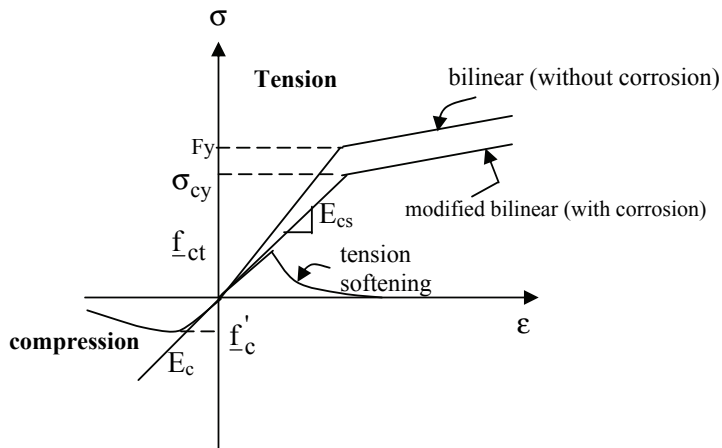


Figure 1: Uniaxial stress-strain diagram for reinforced concrete

Further, among the time- dependent deterioration phenomena, corrosion is the most significant one. The mathematical modeling techniques to incorporate the corrosion effects in nonlinear FEA are described below.

### 3 Mathematical Modelling of RC Member with Corroded Reinforcing Bar

The following mechanisms are considered for simulating the corrosion effects.

- Reduction of effective cross sectional area of bar
- Reduction of tensile stress carried by concrete around the bar by bonding phenomena
- Slip between bar and surrounding concrete
- Reduction in yield stress and elastic modulus of reinforcing bar

All the above mechanisms are quantitatively considered by means of constitutive modeling. The theoretical aspects applied for incorporating these mechanisms are described in detail in the subsequent sections.

#### 3.1 Degradation Model for Cross-sectional Area Reduction

Possible damages due to corrosion are attributed to the loss of cross-sectional areas of reinforcing bars. A degradation model was proposed as shown in Figure 2 [Sarja and Vesikari (1996)] to simulate the parameters such as age in years ' $t'$ ' and the corrosion rate ' $k'_s$ '. The area of N number of corroded reinforcing bars at a particular age is given by

$$A_s(t) = N \cdot \frac{\pi(D_0 - k_s t)^2}{4} \quad (1)$$

where N= number of reinforcing bars,  $D_0$  = Initial diameter of reinforcing bar before corrosion.

The reduced amount of area will cause reduction in structural stiffness especially in the cracked elements.

#### 3.2 Constitutive Law for Corroded Rebar Elements

An analytical model was developed [Shin et al. (2007)] to determine the relationship of the rate of corrosion on the mechanical behaviour of reinforcing bar. In the present study, the same model as given in Table 1 for modelling stress-strain relationship of corroded bar has been used. According to this model, by knowing the uniform rate of corrosion, the material properties of reinforcing bar such as yield strength and elastic modulus can be computed.

Table 1: Equations for Mechanical properties of corroded rebar

Properties	Corrosion Rate (%)	Equations
<b>Yield strength</b>	uniform corrosion	$\sigma_{cy} = \left(1 - 1.24 \frac{\Delta W}{100}\right) F_y$
<b>Elastic modulus</b>	uniform corrosion	$E_{cs} = \left(1 - 0.75 \frac{\Delta W}{100}\right) E_s$
<b>Nominal values</b>	$F_y$ = Yield strength of original bar $E_s$ = Elastic modulus of original bar $\Delta W$ = percentage weight loss due to corrosion	$\sigma_{cy}$ = yield strength of corroded bar $E_{cs}$ = Elastic modulus of corroded bar

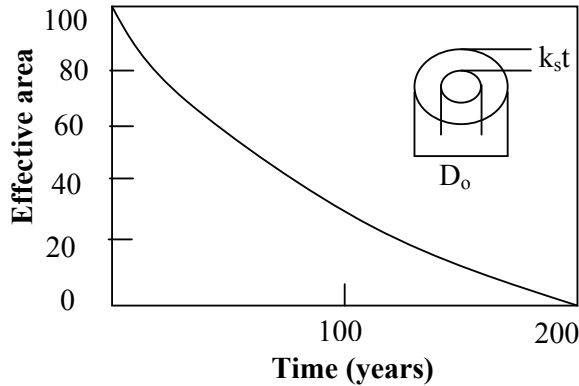


Figure 2: Numerical model for corrosion

### 3.3 Constitutive law for bond-slip with corroded bar

In the present study, a mathematical model has been proposed to incorporate the effect of corroded reinforcement due to bond-slip. For this, once cracking is observed in a finite element, bond-slip action is investigated. Since bond stresses in RC members arise from the change in force along the corroded reinforcing bar, the effect of bond becomes more pronounced in the vicinity of cracks. After cracking, the stresses in concrete is taken over by the corroded reinforcement to some extent. Due to this, a difference in strain between corroded reinforcement and concrete arise, leading to a relative displacement between them. This difference causes further transfer of the tensile stresses from the concrete to corroded reinforcement. The basic relations of this one-dimensional stress transfer process are elucidated using differential elements in Figure 3.

Based on equilibrium check,

$$d\sigma_s(x) = \tau_b(x, s) \frac{U_s}{A_s} dx \quad (2)$$

$$d\sigma_c(x) = -d\sigma_s(x) \frac{A_s}{A_{c,eff}} \quad (3)$$

and the continuity condition yields for bond slip,  $s(x)$

$$s(x) = v_s(x) - v_c(x) \quad (4)$$

where slip  $s(x)$  is defined as the difference between displacement of corroded reinforcement,  $v_s(x)$ , and displacement of concrete  $v_c(x)$ ,  $\tau_b$  is the bond stress,  $A_s$  is

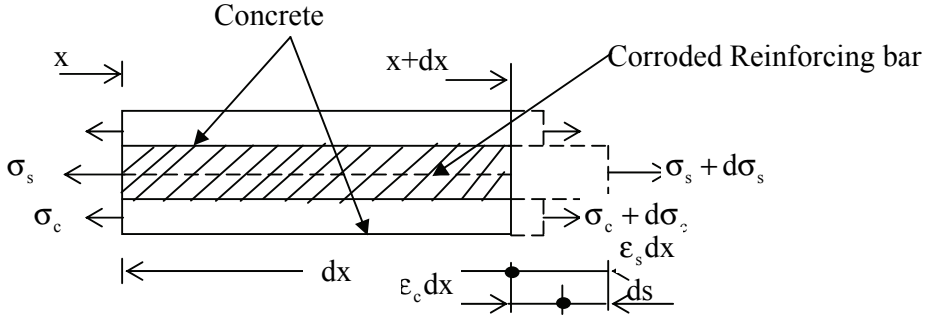


Figure 3: Equilibrium conditions for bond-slip

the cross sectional area of corroded reinforcement,  $A_{c,eff}$  is the effective sectional area of concrete region and,  $U_s$  is the perimeter of corroded reinforcement.

Eqn. (4) can be expressed in differential form as

$$\frac{ds(x)}{dx} = \frac{dv_s(x)}{dx} - \frac{dv_c(x)}{dx} = (\epsilon_s - \epsilon_c) \tag{5}$$

The relation suggested by CEB [1990] is used to model the bond-slip phenomenon in the nonlinear response analysis. By introduction of suitable bond laws and integration over the crack spacing  $s_{rm}$  as shown in Figure 4, functions for the constitutive steel stress, and bond slip are derived as follows.

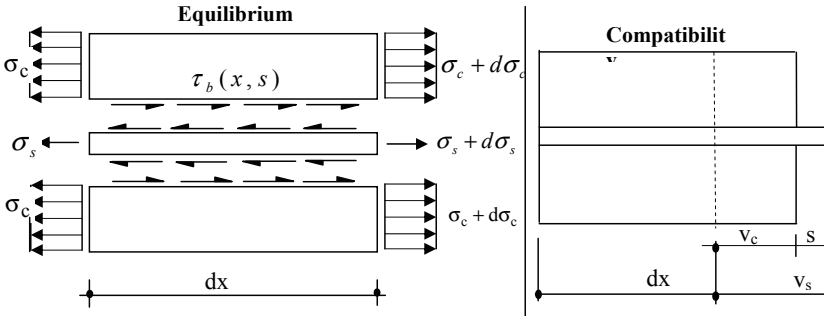


Figure 4: Basic conditions for stresses in steel and concrete and for bond slip

$$\sigma_s(x) = \sigma_s(x = 0) + \frac{U_s}{A_s} \int_{x=0}^{s_{rm}/2} \tau_b(x, s) dx, \tag{6}$$

$$\sigma_c(x) = \sigma_c(x=0) - (\sigma_s(x) - \sigma_s(x=0)) \frac{A_s}{A_{c,eff}} \quad (7)$$

$$s(x) = \int_{x=0}^{s_{rm}/2} \varepsilon_s(x) dx - \int_{x=0}^{s_{rm}/2} \varepsilon_c(x) dx \quad (8)$$

### 3.3.1 Bond stress- slip model for corroded bar

According to CEB-FIP [2000], loss of bond strength is potentially more severe than loss of bar cross-section. Experimental results demonstrate that under general corrosion bond strength can be reduced by 50% when loss of bar cross section is only 10-12%. Magnusson [2000] developed a local bond stress-slip relation as shown in Figure 5 for a bar with good bond condition and other bond conditions. The same model is used in the present study for corroded bar.

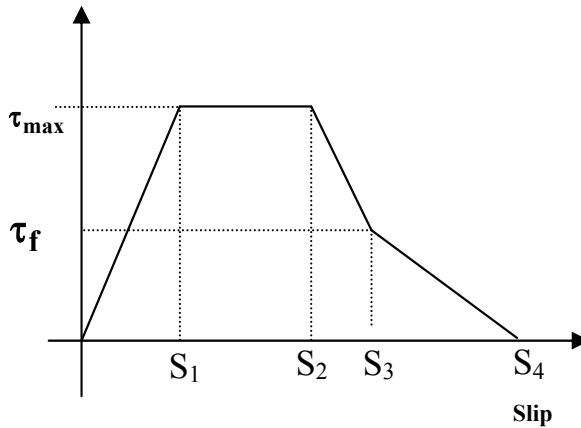


Figure 5: Constitutive law for bond-slip

The parameters used in the above relationships are given below.

For good bond condition,  $\tau_{max} = 0.45f'_c$  and for corroded bar,  $\tau_{max} = 0.225f'_c$

$S_1=1.0$  mm,  $S_2=3.0$ mm,  $S_3=5.8$  mm (clear rib spacing)  $S_4=27.6$  mm (3 x rib spacing),  $\beta=0.4$ ,  $\tau_f=0.4 \tau_{max}$ ,

$$\tau = \tau_{max} \left( \frac{S}{S_1} \right)^\beta \quad \text{for } S \leq S_1 \quad (9)$$



#### 4 Structural Damage Simulation

Structural damage is generally understood as a stiffness reduction phenomena due to material degradation. Softening of nonlinear response is reflected in the corresponding load-displacement diagram by change of its slope in the successive load increments. Driven by the deterioration process, the structural response behaviour will change over life time generally in a nonlinear manner. Thus, the damage accumulation will be nonlinear and any response simulation will also be nonlinear.

In the present investigation, material deterioration phenomena at local level is mapped into the structural level by means of introducing damage indicators. A damage indicator (DI) for a certain state of deformation takes a value of zero in the undamaged virgin state and increases with growing damage. Further, in case of structural disintegration DI arrives at a value of '1'. Global damage indicator is determined using a secant stiffness approach, which is expressed as

$$DI = \left( 1 - \frac{K_{ult}}{K_{initial}} \right) \text{ for } K_{initial} < K < K_{ult} \quad (10)$$

where  $K_{initial}$  is the initial secant stiffness and  $K_{ult}$  is the secant stiffness at ultimate load.

#### 5 Validation Studies

##### 5.1 RC Beam

The performance of the proposed model is verified by comparison between analytical and experimental response behaviour [Shayanfar and Safiey (2008)] of a RC beam. The beam is 250mm x 250mm in cross-section and supported over a clear span of 2000mm. The details of the reinforcement and geometry of the beam is shown in Figure 6. The beam is subjected to 2-point loading. The properties of the constituent materials are given in Table 2. The beam has been analysed for 7.9% of corrosion.

Table 2: Material Properties of RC beams

Concrete				Corroded Reinforcement	
$f_c$ (MPa)	$E_c$ (MPa)	$f_{ct}$ (MPa)	$\epsilon_{cu}$	$f_y$ (MPa)	$E_s$ (MPa)
70.1	38500	3.67	0.004	374.35	$1.88 \times 10^5$

Due to symmetry, half span of the beam has been modeled employing 10 number of quadrilateral shell element with 5 degrees of freedom per node. Each element

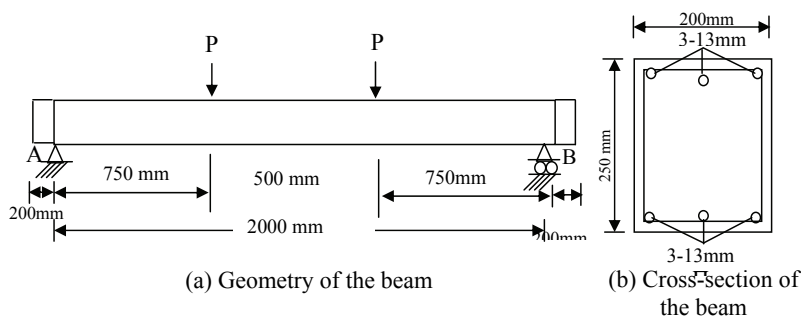


Figure 6: Details of RC beam

is divided into 5 layers. Reinforcement is smeared into the concrete layer in the corresponding locations. The analysis has been carried out through displacement imposed loading on the beam. The load-deflection curve obtained by using the proposed model is shown in Figure 7 along with the corresponding experimental results [Shayanfar and Safey (2008)]. From Figure 7, it can be observed that the response obtained from the proposed model is in very good agreement with the experimental results. From the load-deformation response, it can be observed that there is flexible behaviour in the elastic range compared to the corresponding experimental behaviour. This may be due to the fact that effect of shear is not accounted for in the proposed model.

## 5.2 Analysis of a Chimney with Corrosion Effects

A RC chimney of 275 m height as shown in Figure 8 is used for the nonlinear analysis. Outer diameters of the shell are 16.5 m at top and 28.0 m at the base. Shell thickness at top and base are 0.7 m and 1.6 m respectively. The chimney has been designed for wind loads specified by IS:4998 [1992] and IS:875 [1987]. Material properties are given in Table 3. Nonlinear FEA has been carried out for uncorroded as well as for corroded structure with 8% uniform corrosion.

For uncorroded chimney, the maximum drift obtained is 1.75% corresponding to an ultimate load capacity of 1.62 times the applied load. With 8% uniform corrosion, the chimney has been analysed for different ages by making use of the model shown in Figure 2. It is observed that as the age of the structure progress, there is considerable reduction in load and deformation capacity of the structure due to corrosion. The response behaviour obtained from the analysis for various cases is shown in Figure 9. Table 4 gives details on the reduction in load carrying capacity

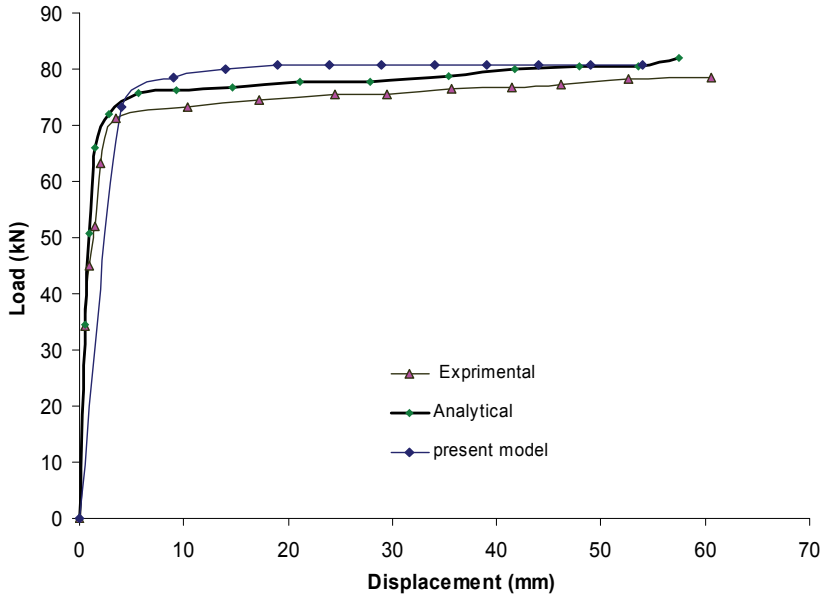


Figure 7: Load vs displacement of RC beam

Table 3: Material properties for RC chimney

Parameters	Concrete	Steel
Young's modulus $E_c, E_s$ , (kN/mm <sup>2</sup> )	29.58	205
Poisson's ratio $\nu_c, \nu_s$	0.15	0.3
Yield stress $f_y$ (N/mm <sup>2</sup> )	-	415
Compressive strength $f_c$ (N/mm <sup>2</sup> )	35	-
Tensile strength $f_t$ (N/mm <sup>2</sup> )	3.5	-
Maximum strain, $\epsilon_{cu}$	0.0035	0.15

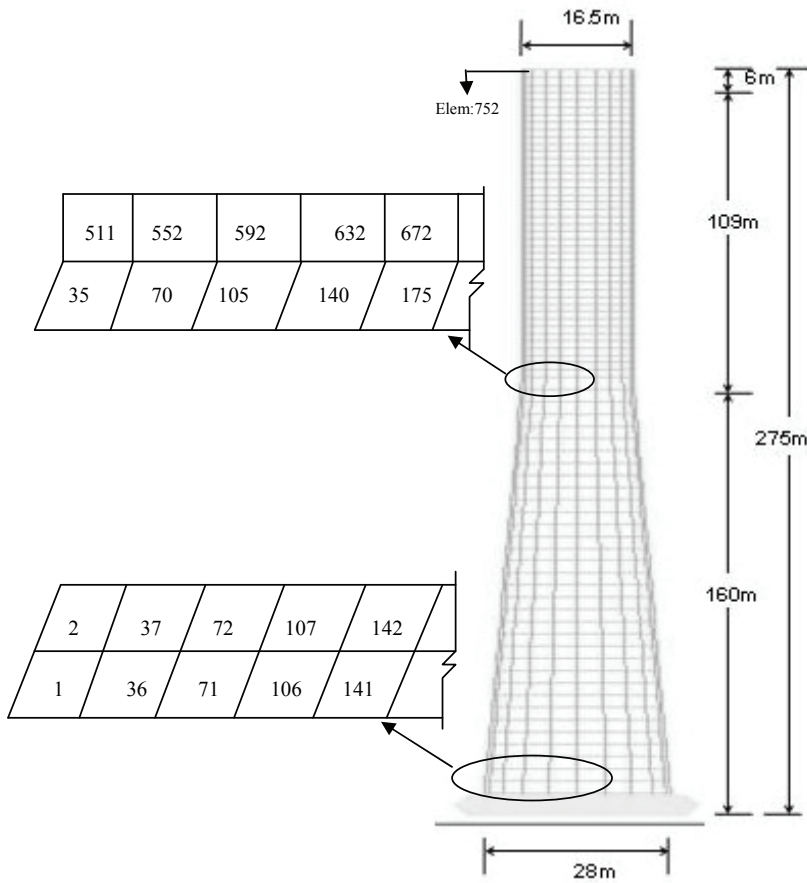


Figure 8: Finite element idealization of RC Chimney

and drift for various cases.

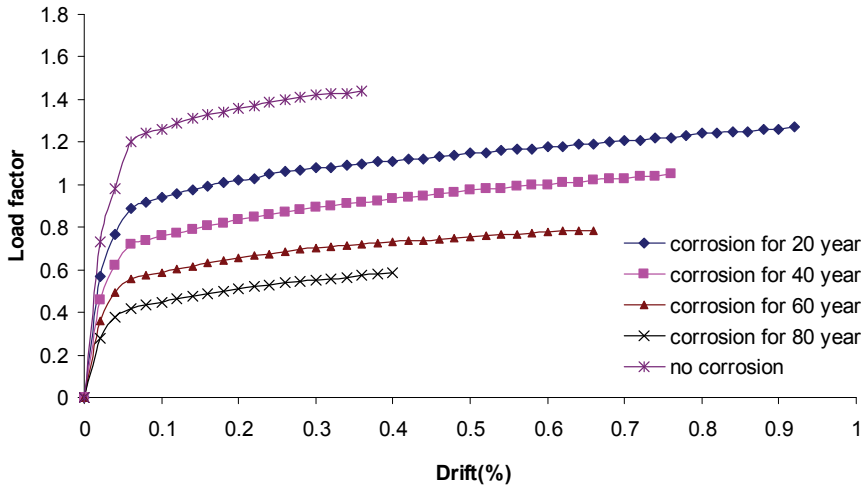


Figure 9: Load vs Drift for different age of the chimney

Table 4: Reduction in load and drift due to corrosion

Age(years)	20	40	60	80
Percentage reduction in load carrying capacity	25	37.5	59.4	65.5
Percentage reduction in drift	4	25	29.2	58.3

Based on the secant stiffness approach, damage indicators have been evaluated for all the cases. Figure 10 shows the growth of damage at global level and it can be observed that when the structure is above 40 years, it may need regular inspection and maintenance due to stiffness reduction. The structure with 80 years old is susceptible to 90% damage and the load coming on the structure is about 50% of the applied load.

Further, to get an understanding on the local behaviour of chimney, number of finite elements cracked and yielded has been traced. Figure 11 shows the details of the number of cracked finite elements for different displacement levels. If chimney is 80 years old, out of 770 elements, 710 elements are cracked for a displacement of 2m, whereas only 410 elements are cracked if there is no corrosion. Similar interpretations can be made for other ages.

Due to corrosion, reinforcement properties such as yield stress and elastic modulus gets reduced. The proposed model is also capable of giving information about the

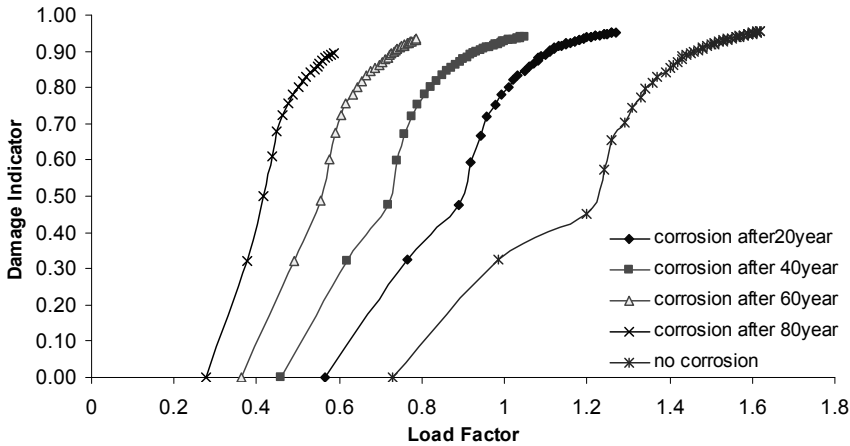


Figure 10: Damage indicator vs load factor for different ages of the chimney

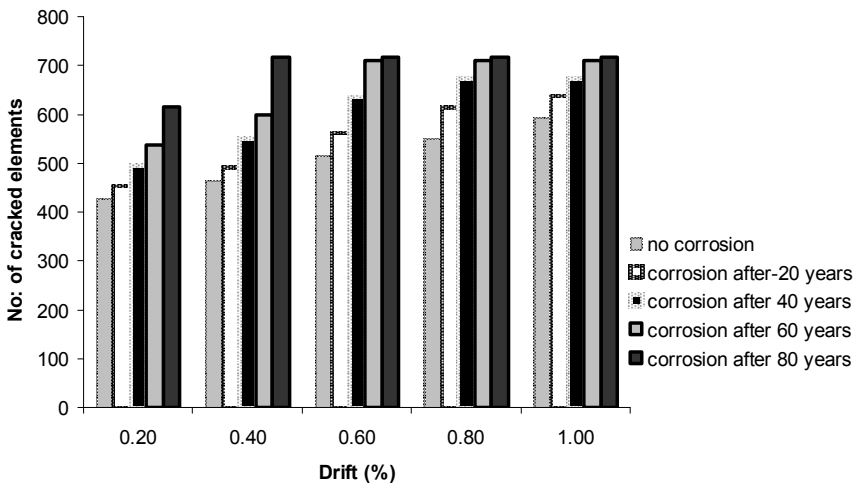


Figure 11: Details of number of cracked elements

number of yielded elements. For example, if the chimney is 80 years of age, out of 770 elements, 570 elements are yielded whereas if the structure is without any corrosion effect only 490 elements would have been yielded. Similar interpretations can be made for different age of the structure. The details are shown in Figure 12.

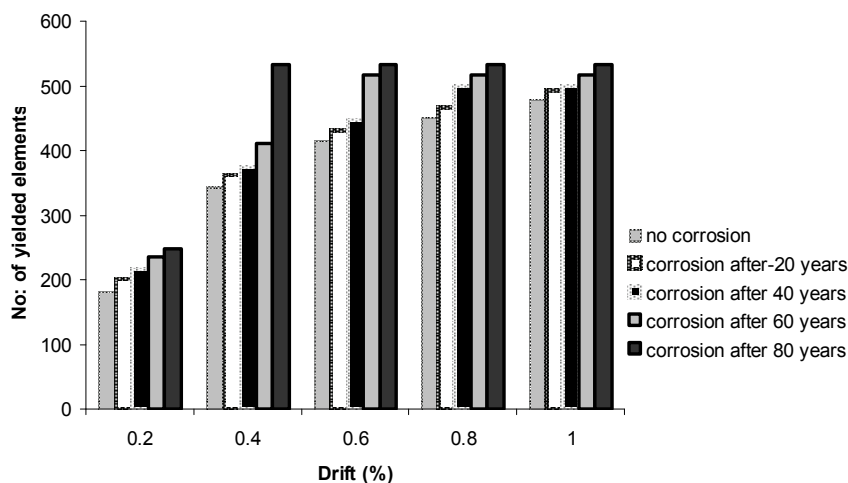


Figure 12: Details of number of yielded elements

## 6 Summary & Conclusions

Nonlinear FEA of RC structural components has been carried out accounting for corrosion effect. Mathematical model has been developed by considering the effect of corrosion as reduction in effective cross-sectional area of bar, reduction in bonding phenomena, reduction in material properties of reinforcing bar such as yield strength and elastic modulus. A procedure has been outlined to determine the global damage indicator by secant stiffness based approach. A corroded RC beam has been analysed to validate the proposed model and results have been compared with experimental response behaviour. RC chimney has been analysed by considering the uniform corrosion effects. The result of corroded chimney shows that the growth of damage increases with the age of the structure replicating the real behaviour. Based on the present investigations, it can be concluded that due to corrosion, both load carrying capacity and deformability decreases and there is a need to ensure safety and serviceability at any point of time as the age of the structure progresses.

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