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Parameter Identification Method of Large Macro-Micro Coupled Constitutive Models Based on Identifiability Analysis

Jie Qu[1,2], Bingye Xu[3] and Quanlin Jin[4]

Abstract: Large and complex macro-micro coupled constitutive models, which describe metal flow and microstructure evolution during metal forming, are sometimes overparameterized with respect to given sets of experimental datum. This results in poorly identifiable or non-identifiable model parameters. In this paper, a systemic parameter identification method for the large macro-micro coupled constitutive models is proposed. This method is based on the global and local identifiability analysis, in which two identifiability measures are adopted. The first measure accounts for the sensitivity of model results with respect to single parameters, and the second measure accounts for the degree of near-linear dependence of sensitivity functions of parameter subsets. The global identifiability analysis adopts a sampling strategy with only a limited number of model evaluations, and the strategy is a combination of Latin-hypercube sampling, one-factor-at-a-time sampling and elitism preservation strategy. The global identifiability index is the integration of the corresponding local index. A hybrid global optimization method is designed to identify the parameter. Firstly, the genetic algorithm is adopted to identify the model parameter rudely, and then the obtained parameter is further refined through the improved Levenberg-Marquardt algorithm. The niching method is used to maintain the population diversity and to choose the initial value for the Levenberg-Marquardt algorithm. A transition criterion between the genetic algorithm and the Levenberg-Marquardt algorithm is proposed, through the improvement on the average objective function value of the chromosomes and the objective function value of the best chromosome. During optimization by the Levenberg-Marquardt algorithm, the local identifiability analysis is taken at the beginning stage of each iteration, and then the variable with poor identifiability remains unchanged in this iteration; the

[1] South China University of Technology, Guangzhou, P. R. China
[2] Guangdong Key Laboratory of Automotive Engineering, Guangzhou, P. R. China
[3] Department of Engineering Mechanics, Tsinghua University, Beijing, P. R. China
[4] Advanced Manufacture Technology Center, China Academy of Machinery Science & Technology, No.18, Xueqing Road, Haidian District, Beijing, 100083, P. R. China

problem of violation constraint for some solution is solved through adjusting the search step length. At last, taking Ti-6Al-4V as an example, a set of satisfactory material parameters is obtained. The calculated results agree with the experimental results well. The identified results show that some parameters involved in the model are poorly identifiable; at the same time, the identifiability analysis method can provide a guide to experiment design.

Keywords: Constitutive model, parameter identification, identifiability analysis, hybrid global optimization method.

1 Introduction

With the development of computer technology and numerical simulation technology, CAE technology has been widely applied to many fields, such as metal forming, polymer processing, chemical engineering, etc. A successful CAE analysis needs numerical simulation software, geometry model, constitutive model and related parameters, boundary condition and initial value etc. To fulfill the need of numerical simulation, many complex models have been developed. However, only a small fraction of models have been applied in practice. Due to that these models general involves in multiple physical processes and many parameters, then it is difficult to separate a physical process from the other physical processes. Similarly, it is also difficult to relate every parameter with the standard ideal experiments for mechanistic approach [Mahnken and Stein, (1996)]. At current, how to successfully identify the parameter of these complex and large models with high confidence degree has become one of the most important obstacles for the further application of CAE technology. Inverse analysis applies optimization techniques to identify the material parameters by minimizing a particular norm of the difference between the calculated and the experimental results [Gelin and Ghouati, (1996)], so it can identify all the parameters once involved in the model as long as the parameters are identifiable for the provided experimental data. Gupta, Sorooshian and Yapo(1998) note that parameter identification methodology has several important parts including: (1) the selection of the appropriate experimental data and the handling technology of the errors presented in the measured data, (2) the definition of the objective function that measures the error between the model predictions and the experimental datum and couples a priori information, and (3) the optimization algorithm that can used to optimize the selected objective function etc. Based on the assumption that the model residuals are uncorrelated and homoscedastic [Vicky, Vicente and Mariano,(1998)], the least squares criterion is introduced to defining the objective function. To consider the case that the output variables of the model are more than one, the weighted least squares criterion

[Qu, Jin and Xu, (2008)] is introduced. Based on the assumption that the criterion is the maximum likelihood, minimum variance, asymptotically unbiased estimator and that the variance of errors in the observed data is assumed to be related to the magnitude of the data, the hetereoscedastic maximum likelihood error[HMLE] criterion is developed [Sorooshian and Dracup, (1980)]. At the same time, many other evaluation criterions, such as mean absolute error [Hall, (2001)], coefficient of efficiency [Sorooshian and Dracup, (1980)], first serial correlation coefficient [Hall, (2001)] etc, have proposed to identify the model parameters. Optimization method is also one of the most important factors, which determine the results of the parameter identification. Local search method, such as projected Newton method [Mahnken, (2002)], Levenberg-Marquardt method [Schnur and Zabaras, (1992)], BFGS method [Kok, Beaudoin and Tortoelli, (2002)], multipoint approximation method [Yoshida, Urabe, Hino and Toropov, (2003)], Simplex method [Kajberg, Melin and Stahle, (2004)] are enough to obtain satisfactory parameter identification for a simple problem, because a good initial, which is located in the same valley with the optimum, can be obtained based on a priori information usually. But for some complex problems, it is difficult to obtain good initial values, thereby leading to difficulties in obtaining satisfactory parameters with any high degree of confidence by local search algorithms. To overcome the above difficulties, many global search methods, based on evolutionary strategies mainly, are developed. Conceptual rainfall-runoff model, whose objective function takes on structure of multiple optima on several scales, is calibrated successfully by shuffled complex evolution method. The method combines the controlled random search algorithms, the competitive evolution with the complex shuffling[Duan, Sorooshian and Gupta(1992); Duan, Gupta, Sorooshian and Gupta,(1993)]. Optimal lens design is obtained by real-coded genetic algorithm, in which a feasibility enforcement operator(FEO) is proposed to make an infeasible solution feasible [Ono, Kobayshi and Yoshida,(2000)]. A hybrid global optimization method, which combine the strength of genetic algorithm, the Levenberg-Marquardt algorithm with the variable error polyhedron algorithm and couples the concept of complex, is developed [Qu, Jin and Xu,(2005)] and the method is applied to the parameter identification of a viscoplastic model considering dynamic recrystallization [Qu, Jin and Xu, (2005)] and the identification of a superplastic model considering microstructure evolution [Qu, Jin and Xu, (2008)]. Genetic algorithm (GA) based multiple objective optimization method is applied to determine the viscoplastic constitutive equations for superplastic alloy and to identify the corresponding material parameter [Lin and Yang, (1999)]. However, no matter what algorithm and objective function are applied, the parameter identification is based on a assumption that the parameter set is well identifiable. However, the large and complex, especially the mechanistic model, are often poorly identifiable [Brun, Reichert and Künsch, (2001)]. A main

reason for the poor identifiability is that "what we would like to know about the internal description of the system ... is of a substantially higher order than what can be observed about the external description of the system " [Beck, (1987)]. Mechanistic model is characterized by an extensive use of causal hypothesis based on the current understanding of how process work and provides a rational basis for the prediction. For the parameter identifiability analysis, sensitivity analysis method is the basic method [Hornberger and Spear,(1980)]. These methods identify parameters that do or do not have a significant influence on model simulations of real world observations for specific catchments. However, the sensitivity analysis method can't deal with the interaction among parameters. Deeper graphical analysis of sensitivity functions has proven to be a valuable tool for relatively simple models [Walter and Pronzato, (1990)]. However, when the deeper graphical analysis method is applied to deal with large and complex models, this approach fails because it is no more possible to analyze efficiently the extensive graphical output that is produced. Recently, three alternative approaches which address the problem of parameter identifiability for large models with many parameters have been suggested. The first has been brought up by Weijers and Vanrolleghem (1997). It applied the sensitivity functions and the Fisher information matrix to analyses the parameter identifiability. It focuses on the problem of the selection of best identifiable parameter subsets for parameter estimation and has been to calibration of ASM1. The second approach has been suggested by Brun, Reichert and Kunsch (2001). It is based on linear regression diagnostics and applied the collinearity index to evaluate the identifiability [Belsley, (1991)]. In addition to the problem of the identifiable parameter subset selection, it focuses on the analysis of parameter interdependencies. A special attempt is made to incorporate prior knowledge about parameter values and uncertainties in a sound and transparent way. The approach has been applied to the identifiability analysis of Activated Sludge Model No. 2d [Brun, Martin, Kühni, Siegrista, Gujera and Reicherta,(2002)] and biogeochemical model of lake Zürich [Omlin, Brun and Reichert, (2001)]. The third approach is proposed by Doherty and Hunt(2009). In the method, the direction cosine between a parameter and its projection onto the calibration solution space is applied to define parameter identifiability, and it varies between zero and one, with zero indicating complete non-identifiability and one indicating complete identifiability. However, Hill states that the measure proposed by Doherty and Hunt is likely to do a poor job of parameter identifiability in common situations [Hill, (2010)]. However, the above criterion is local in the parameter space and the analysis result depends on the a priori assumption on the parameter value [Weijers and Vanrolleghem,(1997)]. At current, sampling-based methods are used to deal with the global sensitivity analysis and identifiability analysis [Helton, Johnson, Sallaberryc and Storlie, (2006)]. Morris (1991) proposed a random OAT(One-factor-At-a-Time) sampling plans for

preliminary computational experiments. The method consists of repetitions of a local method whereby the derivatives are calculated for each random sampled parameter by adding a small change to the parameter. The finial effect will then be calculated as the average of a set of partial effects. Then the local sensitivities get integrated to a global sensitivity measure. To enhancing the computation efficiency, Griensven et al proposed a novel sampling strategy that is a combination of latin-hypercube and one-factor-at-a-time sampling that allows a global sensitivity analysis for large model with only a limited number of model run [Van Griensven, Meixner, Grunwald, Bishop, Diluzio and Srinivasan, (2006)].

This paper propose a parameter identification method of large and complex macro-micro coupled constitutive models based on global and local identifiability analysis, taking the parameter identification of a superplastic model [Jin and Hai, (1997)] as example. In section 2 and 3, the model is given and the objective function is given respectively. Section 4 starts with the description of the global and local identifiability analysis method. Section 5 gives the result on the Global analysis. Section 6 gives a hybrid global optimization, which incorporate the strength of the genetic algorithm and the levenberg-marquardt method. The levenberg-marquardt method is corrected through the introduction of the identifiability analysis. Parameter identification result and the related discussion are given in section 7. At last, conclusions are drawn in Section 8.

2 Model description [Jin and Hai, (1997)]

To describe the flow behavior and grain growth during superplastic forming, a macro-micro coupled superplastic model is developed.

The flow stress is expressed in terms of the volume fraction f_k of the k-th deformation mechanism, the grain size D, the effective strain ε_e and the effective strain rate $\dot{\varepsilon}_e$

$$\begin{aligned} \sigma_e &= \sum_{k=1}^{3} f_k A_k \dot{\varepsilon}_e^{m_k} D^{m_k p_k} = \phi(f_k, \dot{\varepsilon}_e, D), \\ S_{ij} &= \frac{2}{3}\phi \dot{\varepsilon}_{ij}/\dot{\varepsilon}_e. \end{aligned} \tag{1}$$

A particular form of Eq.1 can be given as

$$\begin{aligned} \varphi(\varepsilon_e, \dot{\varepsilon}_e, D, f_k) = \sum_{k=1}^{2} f_k A_k \left(\frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_0}\right)^{m_k} \left(\frac{D}{D_0'}\right)^{m_k p_k} \\ + f_3 A_3' \left(1 - \exp(-\beta \varepsilon_e)\right) \left(\frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_0}\right)^{m_3} \left(\frac{D}{D_0'}\right)^{m_3 p_3} \end{aligned} \tag{2}$$

where k=1,2,3 represents diffusion creep deformation mechanism, grain boundary sliding deformation mechanism and dislocation creep deformation mechanism respectively; A_1, A_2 and A_3' are the threshold stress for the corresponding deformation mechanism; m_k and $m_k p_k$ are strain rate sensitivity coefficient and the grain size sensitivity coefficient of the flow stress; β is strain sensitivity of flow stress for diffusion creep deformation mechanism. D_0' and $\dot{\varepsilon}_0$ are unit grain size and unit strain rate. f_k can be expressed by

$$f_1 = \frac{f_1^*}{f_1^* + f_2^* + f_3^*}, \quad f_2 = \frac{f_2^*}{f_1^* + f_2^* + f_3^*}, \quad f_3 = \frac{f_3^*}{f_1^* + f_2^* + f_3^*} \tag{3}$$

where f_1^*, f_2^* and f_3^*can be obtained from

$$\begin{aligned} f_1^* &= 0.5 - 0.5\text{th}\left(l_1 \ln\left(\frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_1}\right)\right), \\ f_2^* &= \text{sech}\left(l_2 \ln\left(\frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_2}\right)\right), \\ f_3^* &= 0.5 + 0.5\text{th}\left(l_3 \ln\left(\frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_3}\right)\right) \end{aligned} \tag{4}$$

l_k is the control parameter and $\dot{\varepsilon}_k$ is a characteristic strain rate, they can be obtained from empirical expressions as

$$\dot{\varepsilon}_1 = \dot{\varepsilon}_0 10^{R_1 - R_1' \frac{D}{D_0'}}, \quad \dot{\varepsilon}_2 = \dot{\varepsilon}_0 10^{R_2 - R_2' \frac{D}{D_0'}}, \quad \dot{\varepsilon}_3 = \dot{\varepsilon}_0 10^{R_3 - R_3' \frac{D}{D_0'}} \tag{5}$$

R_k and R_k' are the limit index and the grain size sensitivity index for the corresponding characteristic strain rate. The evolution equation of D can be expressed by

$$\dot{D} = \frac{2}{D}\left\{M\gamma_0 - \frac{CD}{4} + \sqrt{\left(M\gamma_0 - \frac{CD}{4}\right)^2 + \frac{M\alpha'}{a} f_2 A_2 \left(\frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_0}\right)^{m_2+1} \left(\frac{D}{D_0'}\right)^{m_2 p_2}}\right\} \tag{6}$$

$M\gamma_0$ is the driving force for grain growth , $M\alpha'/a$ is the driving force coefficient for grain growth due to grain boundary sliding and C is the drag coefficient for grain growth due to grain size. To be simple, $M\gamma_0$ and $M\alpha'/\alpha$ can be regarded as two independent parameters.

Strain rate sensitivity index m is one of the vital mechanical parameters for determining material superplasticity and can be obtained from

$$m = \frac{d\sigma}{d\dot{\varepsilon}} \tag{7}$$

Then there remain 22 parameters, viz. A_1, A_2, A_3', m_1, m_2, m_3, p_1, p_2, p, l_1, l_2, l_3, R_1, R_2, R_3, R_1', R_2', R_3', $M\alpha'/a$, C, $M\gamma_0$, β to be identified. From the perspective of physical significance, these parameter can be divided into some basic subsets, which is given in Table 1.

Table 1: Grouping of model parameter from the perspective of physical significance

subset name	element	physical significance
$\sim_1$	A_1, p_1, m_1	Flow stress for diffusion creep deformation mechanism
$\sim_2$	A_2, p_2, m_2	Flow stress for grain boundary sliding deformation mechanism
$\sim_3$	A_3', p_3, m_3, β	Flow stress for dislocation creep deformation mechanism
$\underset{\sim}{f}_1$	R_1, R_1', l_1	Mainly factor for volume fraction of diffusion creep deformation mechanism
$\underset{\sim}{f}_2$	R_2, R_2', l_2	Mainly factor for volume fraction of grain boundary sliding deformation mechanism
$\underset{\sim}{f}_3$	R_3, R_3', l_3	Mainly factor for volume fraction of dislocation creep deformation mechanism
$\underset{\sim}{D}_t$	$M\gamma_0, C$	Factors that attribute to the grain growth due to thermal
$\underset{\sim}{D}_s$	$M\alpha'/a$	Factors that attribute to the grain growth due to boundary sliding

3 Problem Description

3.1 Objective function

For the constitutive equations described in Eq.1-Eq.7, macroscopic stress-strain relation, macroscopic strain rate sensitivity index-strain relation and microscopic grain size-time relation can be obtained. The objective functions can be defined in terms of the quadratic sums of the difference between the experimental and computational data for the stress-strain relationship, train rate sensitivity index-strain relation and the grain growth– time relationship

$$f_1(\mathbf{x}) = \sum_{i=1}^{n_1}\sum_{j=1}^{m_i} r_{ij}^{\sigma}\cdot r_{ij}^{\sigma}, f_2(\mathbf{x}) = \sum_{p=1}^{n_3}\sum_{q=1}^{m_p} r_{pq}^{m}\cdot r_{pq}^{m}, f_3(\mathbf{x}) = \sum_{l=1}^{n_2}\sum_{k=1}^{m_l} r_{lk}^{d}\cdot r_{lk}^{d} \tag{8}$$

where $f_1(\mathbf{x})$, $f_2(\mathbf{x})$ and $f_3(\mathbf{x})$ are the quadratic sums of the weighted residuals for stress , rate sensitivity index and grain size respectively;$\mathbf{x}$ represent the material

parameter vector, and its upper bound and lower bound can be defined by $\mathbf{U}$ and $\mathbf{L}$ respectively; r_{ij}^{σ}, r_{kl}^{d} and r_{pq}^{m} represent the normalized difference between the calculated value and the experimental value for stress, grain size and strain rate sensitivity index respectively and can be obtained from

$$r_{ij}^{\sigma}=\frac{\sigma_{ij}^{c}-\sigma_{ij}^{e}}{\sigma_{i}^{*}}, r_{lk}^{d}=\frac{d_{lk}^{c}-d_{lk}^{e}}{d_{l}^{*}}, r_{pq}^{m}=\frac{m_{pq}^{c}-m_{pq}^{e}}{m_{p}^{*}} \tag{9}$$

where σ_{ij}^{c} and σ_{ij}^{e} are the calculated and experimental stress for the same time j and the same strain rate i respectively. m_i is the number of the recorded stress-strain data for the strain rate i , and n_1 is the number of the recorded strain rates considering the stress-strain relationship. d_{kl}^{c}, d_{kl}^{e}, m_{pq}^{c}, m_{pq}^{e}, k, l, p, q, m_l, m_p, n_2 and n_3 are similar to σ_{ij}^{c}, σ_{ij}^{e}, i, j, m_i and n_1. σ_{i}^{*} , d_{l}^{*} and m_{p}^{*} are the average experimental values of the stress, the grain size and strain rate sensitivity index respectively during a test, and can be obtained from

$$\begin{aligned}
\sigma_{i}^{*}&=\frac{\sum_{j=1}^{m_i}\sigma_{ij}^{e}\left(t_{i,j+\frac{1}{2}}-t_{i,j-\frac{1}{2}}\right)}{t_{i,m_i+\frac{1}{2}}-t_{i,\frac{1}{2}}},\\
d_{l}^{*}&=\frac{\sum_{k=1}^{m_l}d_{l,k}^{e}\left(t_{l,k+\frac{1}{2}}-t_{l,k-\frac{1}{2}}\right)}{\left(t_{l,m_l+\frac{1}{2}}-t_{l,\frac{1}{2}}\right)},\\
m_{p}^{*}&=\frac{\sum_{q=1}^{m_p}m_{pq}^{e}\left(t_{p,q+\frac{1}{2}}-t_{p,q-\frac{1}{2}}\right)}{\left(t_{p,m_p+\frac{1}{2}}-t_{p,\frac{1}{2}}\right)}
\end{aligned} \tag{10}$$

$t_{i,j}$ is the time when σ_{ij}^{e} is recorded and $t_{l,k}$ and $t_{m,p}$ is similar to $t_{i,j}$. Based on the physical consideration [Wu,(1997)], the following constraints is defined

$$\frac{A_1\dot{\varepsilon}_e^{m_1}D^{m_1p_1}}{A_2\dot{\varepsilon}_e^{m_2}D^{m_2p_2}}<1, \quad \frac{A_2\dot{\varepsilon}_e^{m_2}D^{m_2p_2}}{A_3\dot{\varepsilon}_e^{m_3}D^{m_3p_3}}<1 \tag{11}$$

From physical significance, volume fraction of every deformation mechanism must be larger than some value. At the same time, the parameter, which is associated with this deformation mechanism only, will be insensitive to the objective function, if the volume fraction is too small. Based on the above consideration, the following constraints should be fulfilled

$$\frac{f_1}{f_{1\min}}>1, \quad \frac{f_2}{f_{2\min}}>1, \quad \frac{f_3}{f_{3\min}}>1 \tag{12}$$

Based on Eq. 6, it can be concluded that $M\gamma_0$ must be large than $CD/4$, otherwise the grain can't grow under the static conditions. Then the following formulae should be fulfilled:

$$M\gamma_0 - \frac{CD}{4} > 0 \tag{13}$$

The parameter identification aims to find a set of appropriate parameters which make $f_1(\mathbf{x})$, $f_2(\mathbf{x})$ and $f_3(\mathbf{x})$ be minimal simultaneously, on condition that Eq. 11, Eq.12 and Eq.13 are fulfilled. This is a typical multiple objective optimization problem with inequality constraints. To be simple, we translate this multiple objective optimization problem into single objective problem through weight method and introduce the inequality constraints by the penalty function method [Hu,(1990)].

$$\begin{aligned}\varphi(\mathbf{x}) = {} & (w_1 f_1 + w_2 f_2 + w_3 f_3)(\mathbf{x}) \\ & + \sum_{i=1}^{n_1+n_2+n_3} \left(C_{1i}^2 + C_{2i}^2 + C_{3i}^2 + C_{4i}^2 + C_{5i}^2 + C_{6i}^2(\mathbf{x})\right)(\mathbf{x})\end{aligned} \tag{14}$$

where w_1, w_2 and w_3 are the weights for $f_1(\mathbf{x})$, $f_2(\mathbf{x})$ and $f_3(\mathbf{x})$respectively . $C_{1i}(\mathbf{x}) \sim C_{6i}(\mathbf{x})$ are the penalty functions for flow stress consideration, volume fraction and static grain growth, and are defined respectively as

$$\begin{aligned}
C_{1i}(\mathbf{x}) &= \max\left(0, \gamma_{1i}\left(\frac{A_1\dot{\varepsilon}_e^{m_1}D^{m_1p_1}}{A_2\dot{\varepsilon}_e^{m_2}D^{m_2p_2}} - 1\right)\right), \\
C_{2i}(\mathbf{x}) &= \max\left(0, \gamma_{2i}\left(\frac{A_1\dot{\varepsilon}_e^{m_1}D^{m_1p_1}}{A_2\dot{\varepsilon}_e^{m_2}D^{m_2p_2}} - 1\right)\right), \\
C_{3i}(\mathbf{x}) &= \max\left(0, \gamma_{3i}\left(\frac{f_{1\min}}{f_1} - 1\right)\right), \\
C_{4i}(\mathbf{x}) &= \max\left(0, \gamma_{4i}\left(\frac{f_{2\min}}{f_2} - 1\right)\right), \\
C_{5i}(\mathbf{x}) &= \max\left(0, \gamma_{5i}\left(\frac{f_{3\min}}{f_3} - 1\right)\right), \\
C_{6i}(\mathbf{x}) &= \min\left(0, \gamma_{6i}\left(M\gamma_0 - \frac{CD}{4}\right)\right)
\end{aligned} \tag{15}$$

γ_{1i}, γ_{2i}, γ_{3i}, γ_{4i}, γ_{5i} and γ_{6i} are the penalty factors and are set as the large positive number.

3.2 Determination of parameter range

Eq. (14) is used to identify the parameters of Ti-6Al-4V at 927˚C[Ghosh and Hamilton, (1979)]. The grain growth-time data can be available for strain rates

$\dot{\varepsilon}$ of 2.0×10^{-4}, 5.0×10^{-5} and static with initial grain size D_0 of 6.4 μm; the obtained stress-strain data includes$\dot{\varepsilon}$ of 5.0×10^{-3} and 5.0×10^{-5} with $D_0 = 6.4\mu$m, of 1.0×10^{-3}, 2.0×10^{-4} with $D_0 = 9.0\mu$m and of 5.0×10^{-3}, 2.0×10^{-4} with $D_0 = 11.5\mu m$; Strain rate sensitivity index-time data includes $\dot{\varepsilon}$ of 2.0×10^{-4} with $D_0 = 9.0\mu$m and $\dot{\varepsilon}$ of 1.0×10^{-3} with $D_0 = 6.4\mu$m.

The domains of the parameters can be determined from their physical significance and sensitivity analysis result. For most parameters, such as A_1, A_2 and A_3' et al., whether they are positive or negative can be determined from physical significance, but it is difficult to determine their variation regions. So, the selected parameter domain is very large in order to assure that the optimum is within the parameter domain. For some parameters, whether it is positive or negative even can't be judged, such as R_1, R_2etc. In this paper, the elementary ranges of the parameter values first are given from their physical significance [Wu, (1997)] and other reference [McQueen and Ryan,(2002)], then the ranges are refined through nominal range sensitivity analysis method.

Based on physical significance, threshold stress A_1, A_3, A_3' must be positive and the corresponding domains are set to$\left(0.01, 10^9\right)$, $\left(0.1, 10^5\right)$, $\left(0.1, 10^5\right)$ respectively. The domains for the strain rate sensitivity index m_1, m_2, m_3 are defined to $(0.01, 16)$, $(0.01, 16)$, $(0.01, 10)$ respectively. The domains for grain size sensitivity factor p_1, p_2, p_3 are set to $\left(10^{-6}, 100\right)$, $\left(10^{-6}, 100\right)$, $(0.01, 10)$ respectively. With the strain increasing, the flow stress increases, then β must be positive and its region is set to $\left(10^{-3}, 10^3\right)$. The control parameter for l_1 must be positive from physical consideration. f_2^* and f_3^* are even functions for $\dot{\varepsilon}_e/\dot{\varepsilon}_2$ and $\dot{\varepsilon}_e/\dot{\varepsilon}_3$ respectively, so it is assumed that they are all positive. Their regions are set to $\left(10^{-6}, 10^3\right)$. It is difficult to judge whether R_k and R_k' are positive or negative, and their regions are set to $\left(-10^3, 10^3\right)$. The driving force $M\gamma_0$ for grain growth is positive and it can be obtained that its low limit is $3.78e^{-03}$ from experimental data, and then its domain is set to $\left(10^{-3}, 10^1\right)$ conservatively. Similarly, the driving force coefficient $M\alpha'/a$ for grain growth due to grain boundary sliding and the drag coefficient C for grain growth due to grain size are set to $\left(10^{-6}, 1\right)$ and $\left(10^{-8}, 1\right)$ respectively. On the basis of the above analysis, the domains of the material parameters determined from their physical significance are given in the third column of table 2.

The global search ability of the designed algorithm depends on the evolutionary algorithm. In order to increase the identification efficiency, the ranges of parameter values should be small as possible. Because the model is very complex and highly nonlinear, it is difficult to refine the parameter ranges by analytical method. We have to turn to numerical method for help and the nominal range sensitivity analysis method [Hu,(1990)] is selected to refine and shrink the ranges of the parameter values. It is assumed that the contributions due to the stress-strain data and the

grain growth- time data are equal for objective function, then ω_1 and ω_3 is set to 1 , ω_2 is set to 1.56. A set of parameters, which is given in the fourth column of Table 2, is selected; only a parameter is changed every time and other parameters remain unchanged. There exist three instances. One case, such as p_2, p_1 et al, the parameter ranges are such large that the objective function cannot be accurately calculated due to overflow, Not-a-Number values etc. [Frey and Patil, (2002)]; secondly, the recombination of some parameter value violates the constraint defined through Eq.11, Eq.12 and Eq.13, then causes that the objective function is too large; lastly, the optimal value may be within a narrow range for some parameters, such as A_2, A_3, l_1, l_2, R_1, R_1', R_3', C, $M\gamma_0$ et al, and the large ranges of parameter values cause to much ineffective computation . Based on the above analysis, the ranges of some parameter values are shrunken. The shrunken parameter value ranges are given in the fifth column of Table 2.

Table 2: Initial range of parameter values, selected parameter for shrinking range of parameter values, the corrected range of parameter values

Parameter	Dimension	Initial Range	Selected parameter $\mathbf{x}_b$	Corrected range
m_1		$(10^{-2},16)$	0.497	(0.2,2.82)
m_2		$(10^{-2},16)$	0.725	(0.1,0.85)
m_3		$(10^{-2},10)$	0.878	(0.1,0.85)
R_1		$(10^{-3},10^{3})$	-2.462	$(-4.51,-1.0)$
R_2		$(10^{-3},10^{3})$	-3.237	$(-100,4.37)$
R_3		$(10^{-3},10^{3})$	-0.100	$(-0.94,9.48)$
R_2'		$(10^{-3},10^{3})$	0.0160	$(-10,6.18)$
R_3'		$(10^{-3},10^{3})$	0.228	$(0.23,1)$
$M\alpha'/a$	$\mathrm{mm^2{\cdot}s^{-1}{\cdot}Mpa^{-1}}$	$(10^{-6},1)$	0.0264	$(3.4\times10^{-3},0.1)$
A_1	MPa	$(10^{-2},10^{9})$	27.152	$(4,10^{+3})$
A_2	MPa	$(10^{-1},10^{5})$	384.488	$(4,233)$
A_3'	MPa	$(10^{-1},10^{5})$	38.097	(15,500)
$M\gamma_0$	$\mathrm{mm^2{\cdot}s^{-1}}$	$(10^{-5},1)$	0.00439	$(1.14\times10^{-3}, 2.78\times10^{-2})$
β		$(10^{-3},10^{3})$	31.137	(0.13,100)
p_1		(0,100)	1.061	$\left(10^{-2},2.60\right)$
p_2		(0,100)	0.456	(0.01,3.60)
p_3		(0,10)	2.418	(-4.74,10)
R_1'		$(10^{-3},10^{3})$	0.179	(0.01,0.23)
l_1		(10-6,103)	1.529	(0.1,46.75)
l_2		$(-10^{3},10^{3})$	-0.06273	(-0.39,1)
l_3		(10-6,103)	0.254	(0.1,10)
C	$\mathrm{mm{\cdot}s^{-1}}$	$(10^{-8},1)$	0.00158	$(10^{-4},2\times10^{-3})$

3.3 characteristics of objective function

To develop a reasonable identificability analysis method and optimization method, the characteristics of the objective function firstly should be studied. The section aims to detect the location of the optima and to obtain the global information about sensitivity and feasible solution. A theoretical analysis to obtain the above information is impossible due to the high complexity of the objective function, then we have to turn to the numerical method. This section apply the sampling strategy and grid method to study the characteristics [McQueen and Ryan,(2002)].

To study the characteristics of the objective function, 100000 groups of parameters are obtained based on the Latin-hypercube sampling strategy. The cumulative frequency plot of $\sqrt{\varphi(x)}$ for 100000 model runs are given in Fig.1. It can be seen that objective function value for most combination of parameters are very large. The large objection function value includes three cases. One is that the solutions are feasible and the difference between the calculated value and the experimental value is large, then causing to relatively large objective function. However, the further study shows that only a small fraction of solutions are feasible, and the percent of the feasible solution is less than 1%. The second is that the solutions violate some constraints then causing to large function. The last is that the solution is numerical infeasible, and the objective function is given a huge value to make the computation continue . At the same time, the study shows that the objective function value nearly is independent of the sampled value for every parameter then it is hardly to further shrink the parameter domain and to obtain the effective information on the optima. Every parameter is within the feasible domain, however, most of combinations of parameter values violate one or more than one constraints , then causing to large objective function. The above study shows that only a small fraction of parameter space are feasible and it is difficult to distinguish feasible solution space and infeasible solution space further.

Grid method is used to study the non-convex state of the objective function. A set of parameters, which is given in the fourth column of Table 2, is selected. Every time, two parameter are selected and every parameter domain is equally divided into 100 sections. Every portion of a selected parameters is combined with every portion of another selected parameter, and other parameters are fixed. For each combination of two selected parameters, there are 10101 groups of parameter. The number of local optima for some combinations of parameter are given in Table 3.

The study shows that the number of local optima due to the combinations of the parameters within a subset as given in Table 1, is larger generally than the number of that caused by the combinations of the parameters belonging to different groups. At the same time, for arbitrary recombination of β with other parameters , the

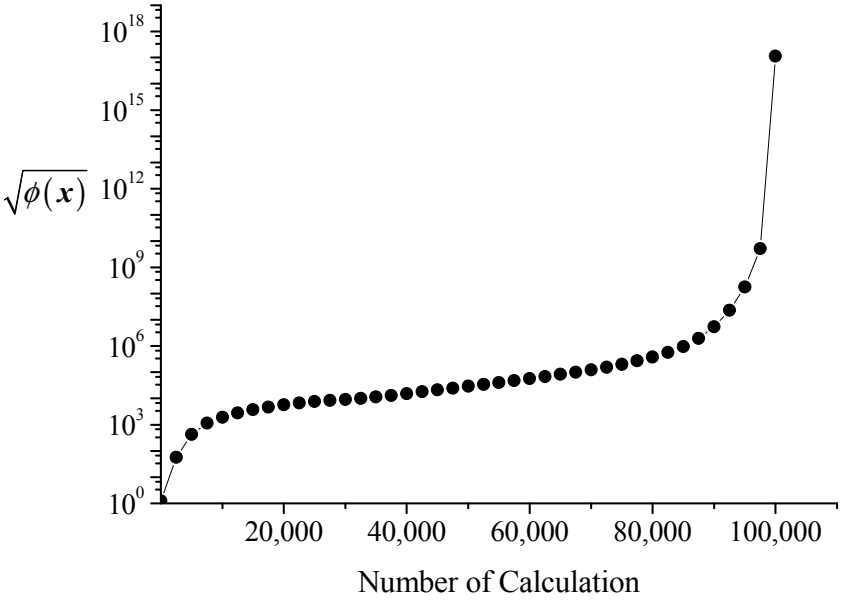

Figure 1: Cumulative frequency plot $\sqrt{\phi(\mathbf{x})}$ for100000 model runs.

number of local optima is relatively small. This is due to that β is only related to flow stress for dislocation creep deformation mechanism and independent of constraint. The study shows that the objective function is highly non-convex also.

4 Identifiability analysis

A schematic flowchart of the parameter identification based on parameter identifiability analysis is given in Fig. 2. The first two steps includes the definition of the model and the selection of the experimental data. Both steps have been carried out in section 2 and 3. Steps 3 is crucial for the parameter identification result. It consists of the following tasks. Firstly, the value range of every parameter must be determined, based on the physical significance , related references and experimental data; secondly, all of the model parameters have to been classified into some groups, mainly based on physical mechanism; thirdly, some constraint should be provided to ensure that the identified parameter are physically reasonable; fourthly, scale factors for all model output have to be ascertained, mainly according to the demands of the user. Steps 3 mainly is based on human expert knowledge and has recourse to numerical technology partially. Step 4 aims to find some good initial parameter in virtue of global identifiability analysis and global optimization method.

Table 3: Some combinations of parameter which number of local optima is larger than 10

Combination of parameter		Number of local optima	Combination of parameter		Number of local optima
m_1	p_1	48	m_2	A_2	27
m_1	A_1	34	m_2	p_2	44
p_1	A_1	49	p_2	A_2	61
m_3	p_3	67	R_1	R_1'	35
m_3	A_3	41	R_1	l_1	38
p_3	A_3	60	R_1'	l_1	33
R_2'	l_2	15	R_3	R_3'	40
R_2	R_2'	106	R_3'	l_3	15
R_2	l_2	15	R_3	l_3	34
$M\gamma_0$	C	48	l_2	C	11
R_2	C	17	R_2	R_3	16
R_1'	R_3'	12	A_2	l_1	11
p_2	A_3	16	p_2	A_1	12
R_3	R_2'	17	R_2	C	17

At last, the initial parameter is improved by the local optimization method, which include local identifiability analysis, largest identifiable subsets selection, parameter update, convergence judgment etc. Global identifiability analysis mainly includes computation of sensitivity matrix and the integration of the local measure to the global measure. In our method, the integration method apply the Latin-Hypercube sampling strategy, and the global optimization mainly applies genetic algorithm. The local identifiability analysis includes the computation of sensitivity matrix, parameter sort according to importance ranking, subset selection based on collinearity index and computation of determinant measure for selected subset etc. In our algorithm, the local optimization method applies Levenberg-Marquatdt method, which incorporate the local identifiability analysis.

4.1 parameter identifiability measures

Firstly, the applied notation in identifiability analysis is introduced; secondly, the measure used for parameter identifiability diagnosis is given; at last, the integration method from local measure to global measure is provided.

The experimental data are assumed to be described by

$$\mathbf{Y} = \boldsymbol{\eta}(\mathbf{x}) + \mathbf{r} \tag{16}$$

The observation vector $\mathbf{Y} = (Y_1, \cdots, Y_n)^T$ includes stress-strain relation, grain-size time series data, strain rate sensitivity index time series data and constraint value from the experimentation. $\boldsymbol{\eta}(\mathbf{x}) = (\boldsymbol{\eta}_1(\mathbf{x}), \cdots, \boldsymbol{\eta}_n(\mathbf{x}))^T$ denotes the simulation result of the superplastic model for stress, grain-size, strain rate sensitivity index and constraint value at the same strain and the same rate as there are observations in $\mathbf{Y}$. $\mathbf{r} = (r_1, \cdots, r_n)^T$ is residual vector.

To be identifiable, a parameter subset $\mathbf{K}$ has to fulfill three conditions. Firstly, each parameter in $\mathbf{K}$ must be sufficiently sensitive to $\boldsymbol{\eta}(\mathbf{x})$; Secondly, changes in the computational result due to changes in single parameters may not be approximately canceled by appropriate changes in other parameters in $\mathbf{K}$. The first condition is addressed by the sensitivity measure δ_j^{msqr} which is calculated for every parameter $x_j$separately; the second by the collinearity index γ_k, which is calculated for arbitrary parameter subsets $\mathbf{K}$; the third by the determinant measure ρ_k, which is calculated for the according parameter subsets $\mathbf{K}$. The sensitivity measure δ_j^{msqr} assesses individual parameter importance; the collinearity index γ_k analyses the parameter interdependencies. ρ_k takes into account the above identifiability conditions simultaneously and is suited for the assessment of relative identifiability of different parameter subsets.

The three measures δ_j^{msqr}, γ_k and ρ_k are all based on sensitivities, either the non -dimensional sensitivity matrix $\mathbf{s} = \{s_{ij}\}$ or the normalized matrix $\bar{\mathbf{s}} = \{\bar{s}_{ij}\}$ with elements

$$s_{ij} = \frac{w_i \partial \eta_i}{\eta_i^* \partial x_j}, \quad \bar{s}_{ij} = \frac{s_{ij}}{\|\mathbf{s}_j\|} \tag{17}$$

Respectively, here η_i^* is the average experimental values, which equal to σ_i^* , d_l^* or m_p^*, w_i is the weight coefficient, $\|\mathbf{s}_j\|$ is the Euclidean norm of the $j$thcolumn of $\mathbf{S}$. The sensitivity measure of δ_j^{msqr} is defined as [Brun, Reichert and Kunsch,(2001)]

$$\delta_j^{\text{msqr}} = \sqrt{\frac{1}{n}\sum_i^n s_{ij}^2} \tag{18}$$

It measures that the mean sensitivity of the simulation result to a change in the parameter x_j (in the mean square sense). A high δ_j^{msqr} means that the value of the parameter x_j has an important influence on the simulation result, a value of zero means that the simulation result is independent of parameter x_j. The collinearity index γ_k is defined as [Brun, Reichert and Kunsch,(2001)]

$$\gamma_k = \frac{1}{\min_{\|\beta\|=1} \|\bar{\mathbf{S}}_{\mathbf{K}}\beta\|} = \frac{1}{\sqrt{\tilde{\lambda}_{\mathbf{K}}}} \tag{19}$$

with $\bar{\mathbf{S}}_{\mathbf{K}}$ being an $n \times k$ submatrix of $\bar{\mathbf{S}}$ containing those columns that correspond to the parameters in $\mathbf{K}$; β being a vector of coefficients of length k and $\tilde{\lambda}_{\mathbf{K}}$ being the smallest eigenvalue of $\bar{\mathbf{S}}_{\mathbf{K}}^T \bar{\mathbf{S}}_{\mathbf{K}}$. γ_k measures the degree of near-linear dependence of the columns of $\bar{\mathbf{S}}_{\mathbf{K}}$. It equals unity if the columns are orthogonal and it reaches infinity if the columns are linearly dependent. If the columns are nearly linearly dependent, changes in the simulation result due to small changes in a parameter x_j can be compensated to a large extent by appropriate changes in other parameters in $\mathbf{K}$. This is indicated by a high collinearity index γ_k. If γ_k exceeds an empirically found threshold of approximately 10–15, then the corresponding parameter subset is poorly identifiable.

Brun et al proposed that the determinant measure $\rho_{\mathbf{K}}$ is defined as[Weijers and Vanrolleghem (1997);Brun, Reichert and Kunsch,(2001)]

$$\rho_{\mathbf{K}} = \det\left(\mathbf{S}_{\mathbf{K}}^T \mathbf{S}_{\mathbf{K}}\right) \tag{20}$$

with det( ) being the determinant function and λ_j being the eigenvalues of $\mathbf{S}_{\mathbf{K}}^T \mathbf{S}_{\mathbf{K}}$. $\rho_{\mathbf{K}}$ is equal to the determinant of $\mathbf{S}_{\mathbf{K}}^T \mathbf{S}_{\mathbf{K}}$, and is corresponding to the volume of confidence regions under the assumption of neglecting measurement error. ρ_k is rather a relative measure suited for comparison of parameter identifiability of different parameter subsets.

4.2 Integration of a local measure to a global measure

The above parameter identifiability measures are local measures and cannot provide any global measure of identifiability measure for the entire parameter space. Therefore, these above measures do not provide robust and reliable approach for the evaluation of parameter identifiability since the objective functions are highly non-linearly related to the parameters. At current, the integration methods from local measure to global measure are based mainly on the sampling strategy. Till now, the applied sampling strategy mainly includes Monte Carlo sampling and Latin-Hypercube sampling [Fishman and Monte Carlo,(1996)].The Monte Carlo sampling produces samples drawn from a specified distribution (typically a uniform distribution) [Fishman and Monte Carlo,(1996)]. Monte Carlo sampling is robust, but may require a large number of simulations and consequently large computational resources. For large model, the computation is hardly to endured to obtain a good evaluation. Latin-Hypercube sampling [Iman, RL, Conover and WJ,(1980)] is based on Monte Carlo sampling and uses a stratified sampling of the parameter space which is preferable over Monte Carlo sampling. For uniform probability distributions on the parameters, the sampling proceeds as follows. A number of samples N_{LHS} is defined. Then the parameter range is subdivided into N_{LHS} equal intervals. Random values of the parameters are generated such that

for each of the N parameters, each interval is sampled only once. This approach results in N_{LHS} non-overlapping realizations and the model is run N_{LHS} times. In our method, Latin-Hypercube sampling is used to evaluate the global identifiability analysis. However, based on the above analysis, most of sampled points violates constraints, causing to large objective function value. However, the actual parameters does not violate the constraints. Inspired by the elitism preservation strategy [Michalewicz,(1992)], we only consider the feasible and nearly feasible solutions, when the global measures are evaluated. The global sensitivity matrix and the Fisher information matrix can be defined by respectively

$$\begin{cases} S_{ij} = \frac{1}{n*} \sum_{n*} \left| \frac{w_i \partial \eta_i}{\eta_i^* \partial x_j} \right| \\ \tilde{S}_{ij} = \frac{1}{n*} \sum_{n*} \left| \frac{s_{ij}}{\|\mathbf{s}_j\|} \frac{s_{jm}}{\|\mathbf{s}_m\|} \right| \end{cases} \tag{21}$$

Subject to:

$$\varphi(\mathbf{x}) \leq \varphi^* \tag{22}$$

$\mathbf{x}$ are sampled by Latin-Hypercube sampling method.

Where $n*$ is the number of $\mathbf{x}$, whose objective function value is less than φ^*, when N_{LHS} samples are defined. The global sensitivity measure of δ_j^{msqr}can be defined as

$$\delta_{Gj}^{\text{msqr}} = \sqrt{\frac{1}{n} \sum_{i}^{n} S_{ij}^2} \tag{23}$$

5 Global analysis

Under the different selection of $\phi*$, the global sensitivity rank of all the parameters are given in Table 4.

From Table 4 , it can be seen that the parameters (m_2, R_2, p_2, R_2', l_2, A_2)associated with grain boundary sliding deformation are relatively sensitive and nearly are all in the subsets of the top 10 parameters of the parameter importance ranking. At the same time, the parameters (R_1, p_1, R_1', l_1, A_1)associated with diffusion creep deformation mechanism are relatively insensitive and nearly are all in the subsets of the lowest 10 parameters of the parameter importance ranking. Based on the superplastic theory, the grain boundary sliding deformation is the important deformation mechanism, however, diffusion creep deformation mechanism is the least important deformation mechanism, so the analysis result is consistent with the superplastic theory. β and $M\alpha'/a$ are the lowest 2 parameter of the parameter importance ranking. $M\alpha'/a$ show the grain size growth kinetics due to grain sliding,

Table 4: Global Sensitivity results for superplastic constitutive model under the different selection of ϕ^*

φ* / Parameter	10^6	10^5	10^4	10^3	φ* / Parameter	10^6	10^5	10^4	10^3
m_1	5	1	5	6	R_1'	7	12	4	1
m_2	6	2	2	10	R_2'	2	6	7	3
m_3	11	4	6	9	R_3'	4	10	13	15
R_1	17	19	16	14	$M\alpha' \cdot a^{-1}$ $/\mathrm{mm}^2 \cdot \mathrm{s}^{-1} \cdot \mathrm{MPa}$	21	21	21	21
R_2	3	8	9	5	A_1 / MPa	19	13	14	11
R_3	10	15	18	17	A_2 / MPa	9	5	3	8
$M\gamma_0 / \mathrm{mm}^2 \cdot \mathrm{s}^{-1}$	12	16	11	12	A_3' / MPa	18	11	15	19
β	22	22	22	22	l_1	14	17	10	4
p_1	20	14	17	13	l_2	1	7	8	2
p_2	8	3	1	7	l_3	13	18	20	20
p_3	15	9	12	18	$C / \mathrm{mm} \cdot \mathrm{s}^{-1}$	16	20	19	16

however, the main contribution of the grain growth is due to heat, so the sensitivity of $M\alpha'/a$ is relatively low. β show the strain hardening for dislocation diffusion. However, the dislocation diffusion isn't the dominant deformation mechanism. At the same time, β and $M\alpha'/a$ are the only two parameters that cannot contribute to constraints.

The parameter combination with high correlation coefficient based on global analysis are given in table 5. From table, it can be seen that the correlation coefficient of the parameter combination general is high, if the two parameters, such as (m_1, p_1), (m_2, p_2) etc, are all within a subset as given in Table 1. The high correlation coefficient between some parameters shows that there are strong interdependence between these parameters. However, at the same time, the correlation coefficient is low, if the two parameters , such as (m_1, p_2), (m_2, A_3') etc, are selected from different basic subsets as given in Table 1. The above results show that there may be seriously identifiable problem for the superplastic model, due to the existence of many parameter combinations with high correlation coefficient.

Table 5: Parameter combination with high correlation coefficient based on global analysis

φ* / Subset name	10^6	10^5	10^4	10^3
$\sim^1$	(m_1,p_1), 0.98	(m_1,p_1), 0.97	(m_1,p_1), 0.97	(m_1,p_1), 0.96
	(m_1,A_1), 0.98	(m_1,A_1), 0.97	(m_1,A_1), 0.97	(m_1,A_1), 0.96
	(p_1,A_1), 0.99	(p_1,A_1), 0.99	(p_1,A_1), 0.99	(p_1,A_1), 0.99
$\sim^2$	(m_2,p_2),0.96	(m_2,p_2),0.96	(m_2,p_2),0.96	(m_2,p_2),0.95
	(m_2,A_2),0.96	(m_2,A_2),0.96	(m_2,A_2),0.96	(m_2,A_2),0.96
	(p_2,A_2),0.99	(p_2,A_2),0.99	(p_2,A_2),0.99	(p_2,A_2),0.99
$\sim^3$	(m_3,p_3),0.97	(m_3,p_3),0.97	(m_3,p_3),0.97	(m_3,p_3),0.97
	(m_3,A_3'),0.97	(m_3,A_3'),0.98	(m_3,A_3'),0.97	(m_3,A_3'),0.96
	(p_3,A_3'),0.99	(p_3,A_3'),0.99	(p_3,A_3'),0.99	(p_3,A_3'),0.99
$\underset{\sim}{f}_1$	(R_1,R_1'),0.95	(R_1,R_1'),0.95	(R_1,R_1'),0.94	(R_1,R_1'),0.94
	(R_1,l_1),0.92	(R_1,l_1),0.92	(R_1,l_1),0.92	(R_1,l_1),0.88
	(R_1',l_1),0.94	(R_1',l_1),0.94	(R_1',l_1),0.94	(R_1',l_1),0.91
$\underset{\sim}{f}_2$	(R_2,R_2'),0.96	(R_2,R_2'),0.96	(R_2,R_2'),0.96	(R_2,R_2'),0.95
	(R_2,l_2),0.95	(p_3,A_3'),0.96	(p_3,A_3'),0.95	(p_3,A_3'),0.94
	(R_2',l_2),0.95	(R_2',l_2),0.95	(R_2',l_2),0.94	(R_2',l_2),0.93
$\underset{\sim}{f}_3$	(R_3,R_3'),0.97	(R_3,R_3'),0.98	(R_3,R_3'),0.97	(R_3,R_3'),0.98
	(R_3,l_3),0.91	(R_3,l_3),0.91	(R_3,l_3),0.91	(R_3,l_3),0.90
	(R_3',l_3),0.89	(R_3',l_3),0.89	(R_3',l_3),0.90	(R_3',l_3),0.90
$\underset{\sim}{D}_t$	$(M\gamma_0,C)$,0.97	$(M\gamma_0,C)$,0.97	$(M\gamma_0,C)$,0.97	$(M\gamma_0,C)$,0.97

6 Optimization method

Based on the numerical results, it is known that the objective function has the following characteristics:

1) Objective function is non-convex and there exist thousands of local optima, then the main framework of the proposed optimization method must be based on some global search optimization method.

2) Due to numerical problem, the objective function can't be calculated for some parameter combinations, and then the optimization method must be able to deal with discontinuous problem.

3) The model may be poorly identifiable, then the identifiability analysis is required to obtain a identifiable parameter subsets during optimization.

4) In parameter space, only a small fraction of parameter combinations are feasible.

Based on the above demand, a global search-based hybrid optimization method is designed. The method absorbs the strengths of the GA [Michalewicz,(1992); Dumitrescu, Lazzerini, Jain and Dumitrescu,(2000); Herrera, Lozano and Verdegay,(1996)] and the L-M algorithm [Mor,(1978); Yuan and Sun, (1999)]. The global search ability of GA makes GA find a global optimum in probability 1; however, its convergent speed is very slow. The L-M algorithm is an excellent algorithm which is designed to solve nonlinear least squares problem, however, it is local optimization algorithm and its solution depends on initial value of parameter. In this method, some candidates for a good initial value for the local optimization method are obtained through GA. Some initial values are selected based on the niching technology [Hua, Wu and Tian,(2008)] and are refined by the L-M algorithm. The best obtained solution is regarded as the material parameter. The flowchart of the global optimization method is given in Figure 2.

Among them, G_m is the set maximum generation number, ε_a is the control parameter for the maximum value of the relative mean correction of the objective function correction value of the 80% leading chromosomes in the population during successive 5 generations, ε_b is the control parameter for the maximum value of the relative objective function correction value of the best chromosome during successive 5 generations, n_{niche} is the number of niches which are necessary for local optimization method, h_g is the current generation number, f_i is fitness function value which considering objective function value and crowding mechanism, rat_a and rat_b are the maximum value of the relative objective function correction value of the best chromosome and the maximum value of the relative mean correction of the objective function correction value of the 80% leading chromosomes in the population during successive 5 generations. $\mathbf{PN}_i$ represents a niche.

6.1 Real-coded genetic algorithm (RGA)

To increase the precision and computational efficiency, my method applies the real-coded genetic algorithm (RGA) . RGA is a global optimization technique inspired by the evolutionary process of the natural life and It mainly includes the production operator, the crossover operator and the mutation operator. A real number vector (parameter vector) is regarded as a representation of the problem. The initial population is generated as follows [Qu, Jin and Xu,(2008); Qu, Jin and Xu, (2005)]

$$\begin{cases} K_{\mathrm{i}} = L_{\mathrm{i}} + \delta\left(U_{\mathrm{i}} - L_{\mathrm{i}}\right) & \beta < \max\left(\left(\frac{U_{\mathrm{i}}}{L_{\mathrm{i}}}\right), \left(\frac{L_{\mathrm{i}}}{U_{\mathrm{i}}}\right)\right)^{\pounds\alpha} \\ K_{\mathrm{i}} = \operatorname{sgn}\left(L_{\mathrm{i}}\right) 10^{(\lg|L_{\mathrm{i}}| + \delta(\lg|U_{\mathrm{i}}| - \lg|L_{\mathrm{i}}|))} & \text{else} \end{cases} \tag{24}$$

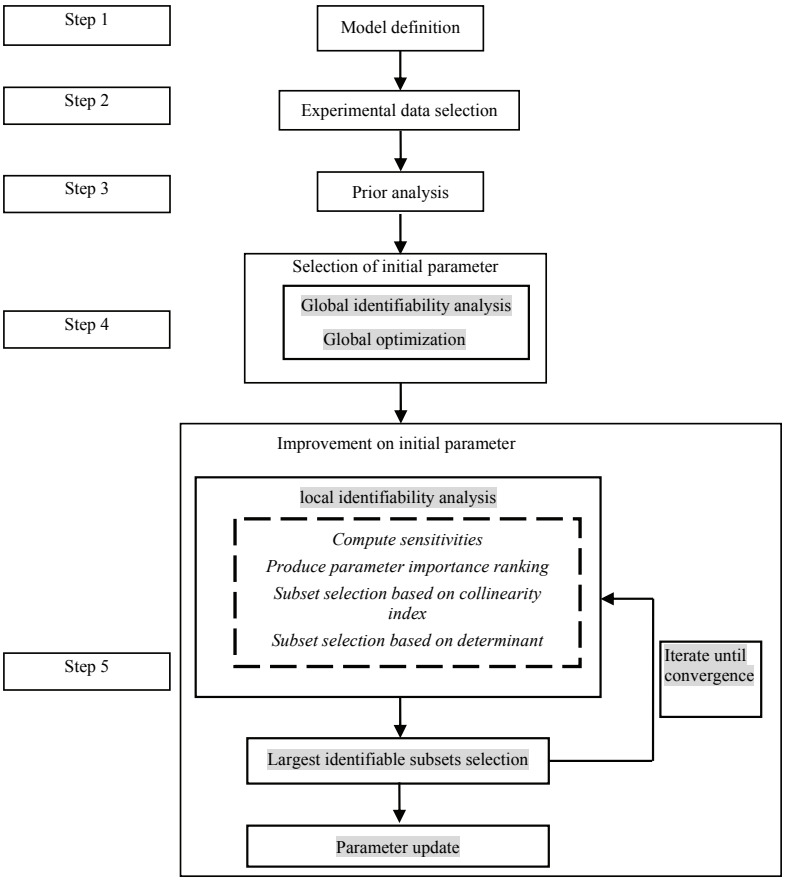

Figure 2: Schematic flowchart of parameter identification for large complex model based on identifiability analysis

sgn is the symbolic function. If $L_i < 0$, then $\text{sgn}(L_i) = -1$; if $L_i > 0$, $\text{sgn}(L_i) = 1$. When U_i is positive and L_i is negative, the actual value range is mapped into a positive value range through a linear transformations, then the sampling is taken in the transitional value zone , the obtained parameter value is re-mapped into the actual value range through the accordingly inverse transformation lastly. So it is assumed that L_i and U_i are simultaneously positive or negative in the operator.

Non-uniform mutation operator [Dumitrescu, Lazzerini, Jain and Dumitrescu,(2000)] can make a uniform search in the population at the initial stage of the optimization

process and a very local search at a later stage. The applied mutation operator is

$$\begin{cases} \begin{cases} K_i = sgn(L_i)\,10^{\left(\lg(|K_{0i}|)+(\lg(|U_i|)-\lg(|K_{0i}|))\left(1-\alpha^{\left(1-\frac{t}{g\max}\right)^5}\right)\right)} & \text{if } \delta \geq 0.5 \\ K_i = sgn(L_i)\,10^{\left(\lg(|K_{0i}|)-(\lg(|K_{0i}|)-\lg(|L_i|))\left(1-\alpha^{\left(1-\frac{t}{g\max}\right)^5}\right)\right)} & \text{if } \delta < 0.5 \end{cases} \\ \qquad \text{if } \beta > \max\left(\left(\frac{U_i}{L_i}\right), \left(\frac{L_i}{U_i}\right)\right)^{\pounds\alpha} \\ \begin{cases} K_i = K_{0i} + (U_i - K_{0i})\left(1-\alpha^{\left(1-\frac{t}{g\max}\right)^5}\right) & \text{if } \delta \geq 0.5 \\ K_i = K_{0i} - (K_{0i} - L_i)\left(1-\alpha^{\left(1-\frac{t}{g\max}\right)^5}\right) & \text{if } \delta < 0.5 \end{cases} \\ \qquad \text{else} \end{cases} \tag{25}$$

where K_{0i} is the gene of parent, t and $g_{\max}$ is the current generation number and the set maximum generation number respectively. To improve the global search ability, it is recommended that$g_{\max}$ is larger than the actual maximum evolution generation G_m.

Different crossover operators have different advantages. For example, random simplex crossover operator [Herrera, Lozano and Verdegay,(1996)] makes use of the information of multiple-parent and isn't easy to be trapped by minor optimum; SBX crossover operator [Dumitrescu, Lazzerini, Jain and Dumitrescu,(2000)] can get descendants similar to their parents. In order to apply the advantages of the SBX crossover operator, the random simplex crossover operator and Arithmetical crossover operator [Michalewicz,(1992)], a hybrid crossover operator is designed, enlightened by the hybrid combinatorial optimization[Preux and Talbi,(1999)] and of simultaneously applying multiple mutation operators [Hong, Wang and Chen, (2000)]. To enhance the global search ability, these crossover are improved by the same method with Non-uniform mutation operator. Inspired by the shuffled complex evolution method [Duan, Sorooshian and Gupta,(1992)], several parents are firstly selected from population **P** as a complex **A**; then the crossover operator and mutation operator are applied in a complex; the generated children from *A* are shuffled in *P* at last. The detail is given in [Qu, Jin and Xu, (2008)].

Generation-alteration Operator (GAO)determines how to choose pairs of parents for generating the children by the crossover operators and the mutation operators, and determines how to select parents to survive in the next generation. The method applies simply Roulette rule. The selection probability f_i of chromosome i is codetermined by the objective function value and the crowding state of the chromosome.

N chromosomes are first sorted in order of decreasing objective function. Due to the objective function value, the selection probability f_i' of chromosome i is calculated

as

$$f_i' = \frac{i}{(N+1)N} \; i = 1, \cdots, N \tag{26}$$

where i represents the position of chromosomes in the population after sorting. However, if the selection probability f_i is determined completely by the objective function value, the chromosomes tend to concentrate on the domain near to some local optima at the later stage of genetic algorithm based on the Roulette rule, and then the genetic algorithm is easy to premature. To overcome the problem, this paper applies the niching technology to maintain the population diversity [Miller and Shaw,(1996); Sareni and Krähenbühl,(1998)]. In nature, a niche can be viewed as a subspace in the environment that can support different types of life. For each niche, the physical resources are finite and must be shared among the population of that niche. The sharing method is the best known and applied among the niche technology. Fitness sharing modifies the search landscape by reducing the payoff in densely populated regions. Typically, the shared fitness f_i of an individual i with fitness f_i' is simply

$$f_i = \frac{f_i'}{m_i'} \tag{27}$$

Where m_i' is the niche count which measures the approximate number of individuals with whom the fitness f_i' is shared. The niche count is calculated as follows

$$m_i = \sum_{i=1}^{N} sh(d_{ij}) \tag{28}$$

Where d_{ij}represents some distance between the chromosome i and j. It returns 1 if the elements are identical, zero if their distance is higher than a threshold of dissimilarity, and an intermediate value at intermediate level of dissimilarity. In this method, we applied triangular sharing function as follows [Sareni and Krähenbühl, (1998)]

$$sh\left(d_{ij}\right) = \begin{cases} 1 - \frac{d_{ij}}{d_s} & \text{if } d_{ij} < \sigma_s \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

d_{ij} is characterized by a similarity metric and is defined by

$$d_{ij} = \sqrt{\sum_{m=1}^{n} \left(\frac{x_{im} - x_{jm}}{U_m - L_m} \right)^2} \tag{30}$$

$\mathbf{x}_i$ and $\mathbf{x}_j$ are two parameter vectors,$d_s$denotes the threshold of dissimilarity and is recommended as follows

$$d_s \in \left(\frac{\sqrt{n}}{100}, \frac{\sqrt{n}}{20} \right) \tag{31}$$

6.2 Levenberg-marquardt Algorithm

Classic optimization method is based on Taylor expansion of the objective function. A quadratic Taylor expansion of the square of the objective function $\varphi(\mathbf{x})$ around $\mathbf{x}_t$ can be expressed as [Yuan and Sun,(1999)]

$$\begin{aligned} \varphi'(\mathbf{x}) = \varphi'(\mathbf{x}_t) + (\mathbf{x}-\mathbf{x}_t)^T \mathbf{J}(\mathbf{x}_t)^T \mathbf{r}(\mathbf{x}_t) \\ + \frac{1}{2}(\mathbf{x}-\mathbf{x}_t)^T \left[\mathbf{J}(\mathbf{x}_t)^T \mathbf{J}(\mathbf{x}_t) + \sum \mathbf{r}_i(\mathbf{x}_t) \nabla^2 \mathbf{r}_i(\mathbf{x}_t) \right] (\mathbf{x}-\mathbf{x}_t) \end{aligned} \tag{32}$$

where $\mathbf{J}(\mathbf{x}_t) = \mathbf{r}'(\mathbf{x}_t)$, $\mathbf{r}$ is the residual vector associated with stress-strain relationship, grain growth-time relationship , strain rate sensitivity index-strain relationship and constraint violation. The L-M algorithm, based on the Gauss-Newton model $q_t^{\mathrm{G}}(\mathbf{x}_{t+1})$ and the trust region technology, is designed to solve nonlinear least square problems. The iteration formula is as follows

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \left(\mathbf{J}(\mathbf{x}_k)^T \mathbf{J}(\mathbf{x}_k) + \mu_k \mathbf{I} \right)^{-1} \mathbf{J}(\mathbf{x}_k)^T \mathbf{r}(\mathbf{x}_k) \tag{33}$$

μ_k is Levenberg-Marquardt parameter. To make the identification result more reliable, some corrections are made and are given as follows

1. The parameter identifiability analysis technology is taken to find the largest identifiable subset. The procedure of finding the identifiable subset is as follows. Firstly, the sensitivity matrix and sensitivity measure δ_j^{msqr} is calculated ,and sort parameters in order of decreasing sensitivity measure δ_j^{msqr}, if $\delta_j^{\mathrm{msqr}} / \delta_{\max}^{\mathrm{msqr}} < 0.01$, then x_j is regarded as insensitive. Secondly, select the most sensitive parameter in each subset as given in table 1 and group them into a initial subset, and iteratively add or remove the element from the subset, until a largest subset identifiable $\mathbf{K}_{id}$ is obtained and other parameters are regarded as un-identifiable. Thirdly, the un- identifiable parameters are frozen and only the identifiable parameters are updated during this iterative step. At last, the frozen variable is activated after the convergence of this iteration.

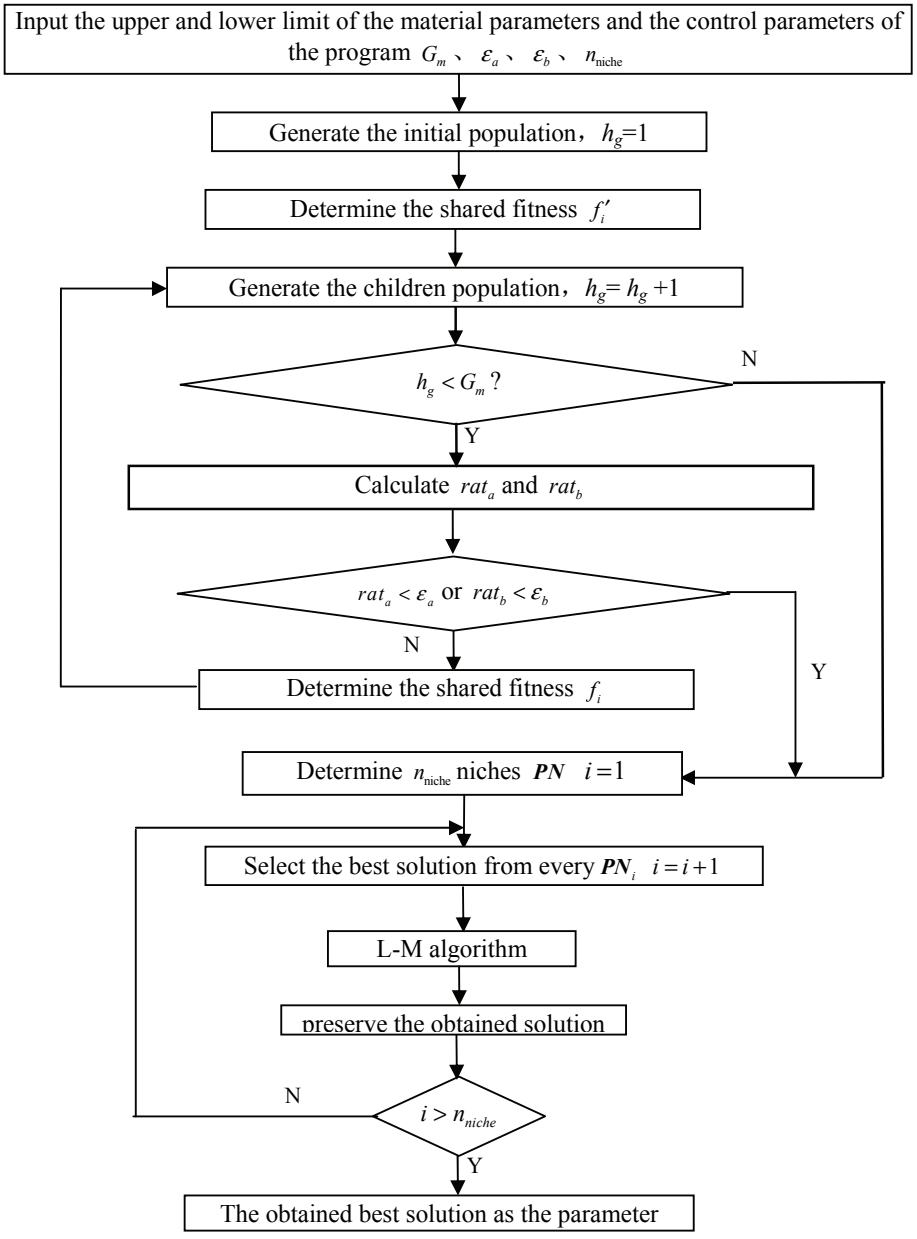

Figure 3: Main frame of designed global optimization method

2. Sometimes, the parameters may be out of upper and lower limit of the according parameter during the optimization. To solve the problem, the iterative step length h_t is reduced and the iteration is performed again.

3. The objective function for some parameter recombination may not be accurately evaluated due to numerical problem. In such a case, the search step length is halved and the search is taken again.

The rest of the algorithm remains unchanged. The flowchart of the corrected L-M algorithm is given in figure 3. Where $\mathbf{x}_0$ is the initial value, σ is the control parameter for $\|\mathbf{D}_t\mathbf{s}_t\|$; ε_s is the convergent control parameter for the ratio ρ_t between the predicted reduction in the objective function and the actual reduction; ε_ρ is the control parameter for ρ_k to adjust the iteration step length h_k;ε_x is the convergent control parameter for the iteration increment $\mathbf{s}_t$; ε_h is the control parameter for determining how to deal with $\mathbf{x}_{t+1}$; $\mathbf{D}_t$ is the scaling matrix. More details regarding the L-M algorithm can be found in paper [Moré,(1978);Yuan and Sun,(1999)] .

6.3 Transition criterion and selection of initial value for L-M algorithm

The transition criterion between the genetic algorithm and the L-M algorithm decide when to stop the genetic algorithm and trigger the L-M algorithm. Enlightened by the convergent criterion of the local optimization algorithm based on gradient [Yuan and Sun,(1999)] and stop criterion of genetic algorithm [Michalewicz,(1992)], the maximum value rat_b of the ratio between the correction value of the objective function of the best chromosome in the nth generation population relative to the best chromosome in the (n-1)th generation population and the objective function value of the best chromosome in the (n-1)th generation population during successive 5 generations is introduced; at the same time, the maximum value rat_a of the ratio between the correction value of the mean of the 80% leading chromosomes in the nth generation population and that of the 80% leading chromosomes in the (n-1)th generation population during successive 5 generations is introduced also. When rat_a or rat_b is less than ε_a or ε_b, then the genetic algorithm is stopped and the L-M algorithm is triggered. In the method, ε_a and ε_b are all recommended to be set to 10^{-5}.

Like Hua, Wu and Tian (2008), the chromosomes are first sorted in the last generation population, with the order of decreasing objective function; then n_{niche}niches are sequentially constructed, beginning with the best chromosome; at last, the best chromosome of each niche is regarded as the initial value respectively for L-M algorithm.

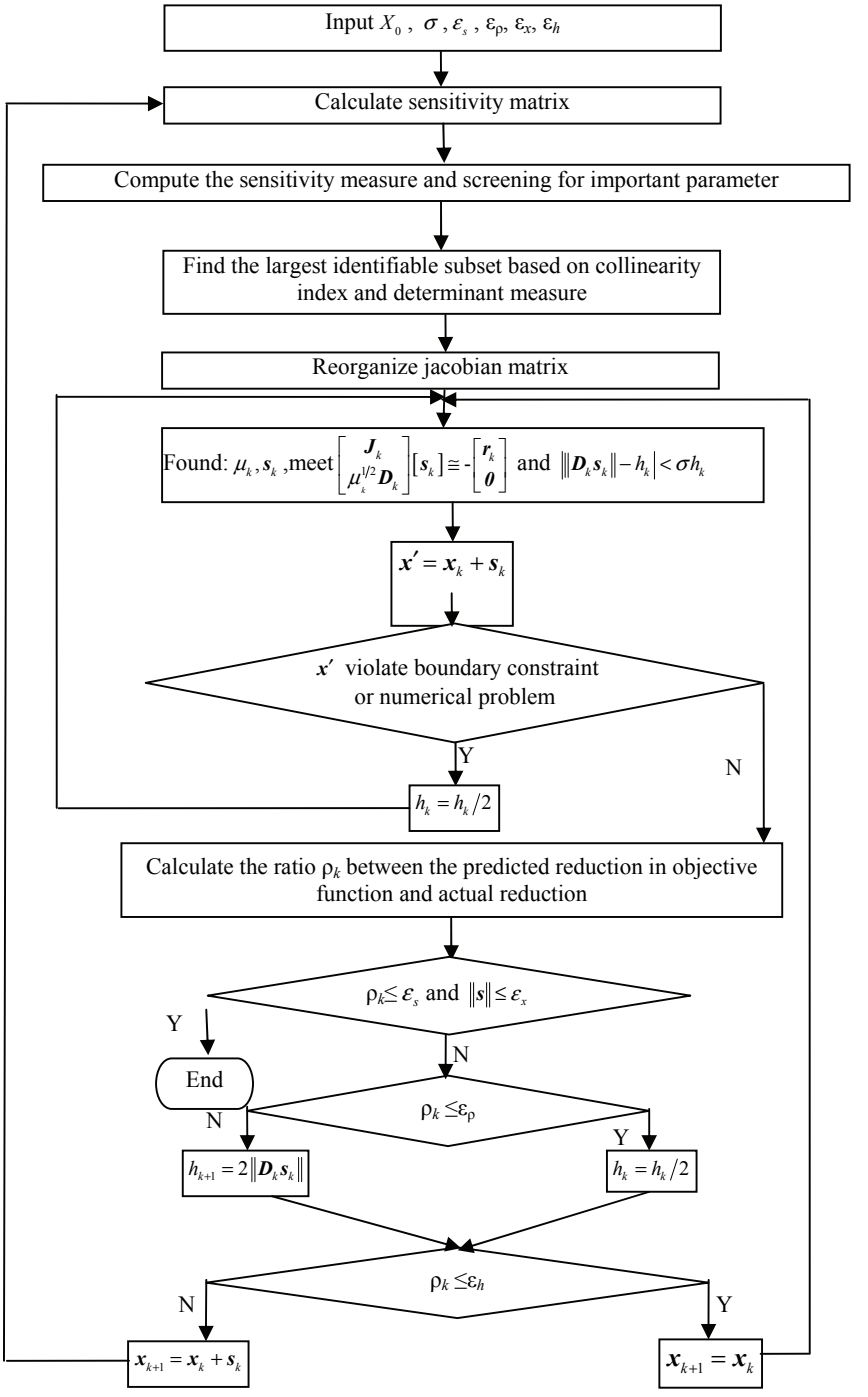

Figure 4: Flowchart of improved Levenberg-Marquardt method

7 Parameter identification result and discussion

The comparisons between the calculated and experimental data are given in figure 3, figure 4 and figure 5. The lines with solid symbols represent the experimental data due to Ghosh and Hamilton (1979), and the lines with empty symbols are obtained from the computations. From figure 3a-c, it can be seen that very close agreements have been obtained for the stress-strain data and the maximum relative error do not exceed 3%. From figure 4, it can be seen that relative to stress-strain relation, the discrepancy is more serious a little for grain growth-time relation. This may be due to that the experimental error for grain size is larger than that for stress. Relative to the grain growth-time relation and the stress-strain relation, the gap between the predicted strain rate sensitivity index and the experimental ones is large. This may be due to the larger experimental error and that the information, which is contained in the objective function, due to strain rate sensitivity index is relative less. To verify the reliability of the parameter identification result, the identified parameter is used to predict the stress-strain relation for $\dot{\varepsilon} = 2.0 \times 10^{-4}$ with D_0=6.4μm, the grain growth –time relation for $\dot{\varepsilon} = 1.0 \times 10^{-3}$ and $\dot{\varepsilon} = 5.0 \times 10^{-3}$ with D_0=6.4μm, the strain rate sensitivity index –strain relation for $\dot{\varepsilon}$ of 2.0×10^{-4} with D_0=6.4μm. For the stress-strain relation, the maximum relative error doesn't exceed 5%, for the grain growth-time relation, the maximum relative error doesn't exceed 15% and for the strain rate sensitivity index –strain relation, the maximum relative error doesn't exceed 10%. Sum up the above comparative result, it can be seen that the maximum relative errors between the predicted and the experimental results doesn't exceed 15%.

The above results show that the identified parameters are a set of satisfactory parameters for describing the experimental data. However, the identifiability analysis shows that the model is poorly identifiable and some parameters are hardly to be identified well. one factor is that the objective function is insensitive nearly to some parameters, and the other is that there exists strong interdependence between some parameters, then causing large collinearity index.

100 sets of different parameters are selected by niching technology, then sensitivity analysis is taken at the first iteration for every set of initial value. The analysis result is given in Table 6, where 'Number' means the times that the rank of the parameter is the according serial number. It can be seen that m_3 and R_3' are the top 2 parameters of the parameter importance ranking at every initial value. At the same time, p_1 and A_1 are the lowest 2 parameters always; At the same time, the times is 87 that the rank of parameter m_1 is 20, the times is 12 that the rank of parameter m_1 is 19; the times is 1 that the rank of parameter m_1 is 18. and that, $\delta_{p_1}^{\mathrm{msqr}}/\delta_{m_3}^{\mathrm{msqr}} < 0.01$, $\delta_{A_1}^{\mathrm{msqr}}/\delta_{m_3}^{\mathrm{msqr}} < 0.01$ and $\delta_{m_1}^{\mathrm{msqr}}/\delta_{m_3}^{\mathrm{msqr}} < 0.01$ are always satisfied for every initial value. It shows it is difficult to identify these parameters,

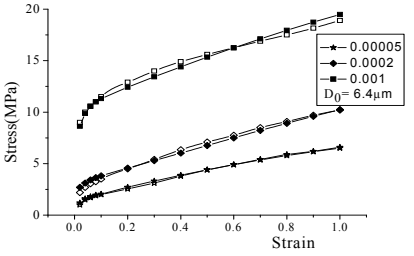

Figure 5a. Comparison of calculated results (solid symbol) and experimental results (empty symbol) on stress vs. strain curves for Ti-6Al-4V at 927°C

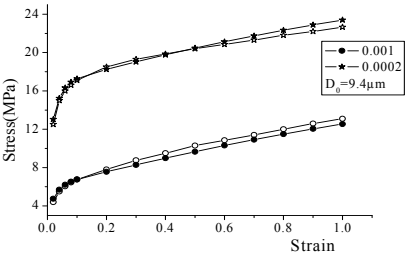

Figure 5b. Comparison of calculated results (solid line) and experimental results (broken line) on grain growth curves for Ti-6Al-4V at 927°C

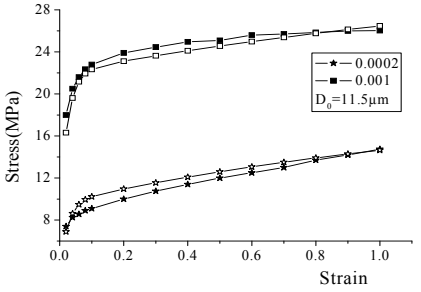

Figure 5c. Comparison of calculated results (solid line) and experimental results (broken line) on grain growth curves for Ti-6Al-4Vat 927°C

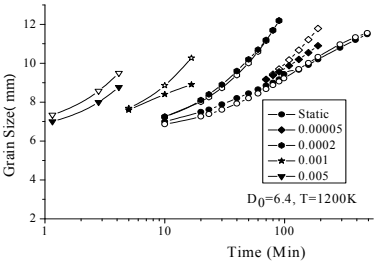

Figure 6. Comparison of calculated results (solid line) and experimental results (broken line) on grain growth curves for Ti-6Al-4Vat 927°C

which are associated with the flow stress on the diffusion creep deformation mechanism. Parameters within subsets $\underset{\sim}{\overrightarrow{\sigma}}_2$, $\underset{\sim}{\overrightarrow{\sigma}}_3$ and $\underset{\sim}{\overrightarrow{\sigma}}_1$, strain rate sensitivity coefficient are the most sensitive parameters always, and p_k are the least sensitive parameters except $\underset{\sim}{\overrightarrow{\sigma}}_3$, within which the strain sensitivity coefficient is the least sensitive parameter. Comparing Table 6 with Table 4, it can seen that there exists huge difference. This mainly is due to that parameters in Table 6 are all feasible and don't violate any constraint, however, most of parameters are infeasible and most of them violate constraints in Table 4.

The average of the absolute value of the correlation coefficient between the sensitive parameter are given in Table 7. "Number" is the times that the parameter is sensitive for the 100 initial value. It can be seen that parameters m_2, m_3, R_1, $M\gamma_0$, p_2, p_3, R'_1, R'_3, $M\alpha/\alpha'$, A_2, A'_3, l_1, l_3 and C are β are all sensitive for every initial value. β is sensitive nearly for every initial vale except one. It can be seen that

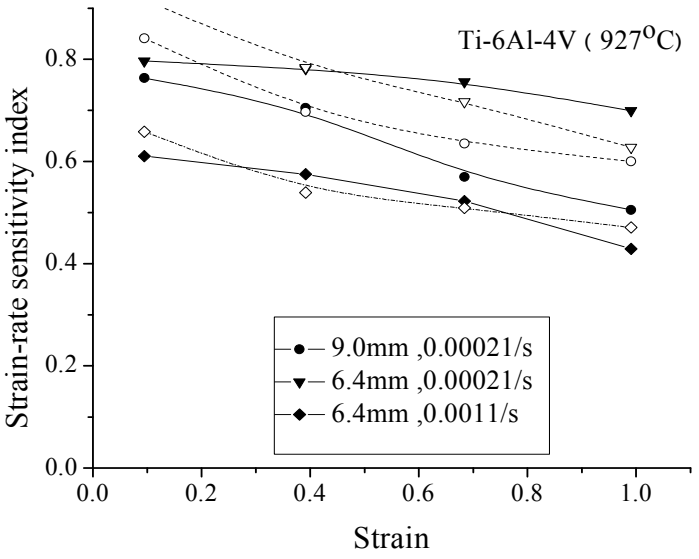

Figure 7: Comparison between the predicted instantaneous values of strain rate sensitivity during tensile test and the experimental results for Ti-6Al-4V at 927°C

the correlation coefficient of the parameter combination generally is high, if the two parameters, such as (m_2, p_2), (m_3, p_3) etc, are all within a subset as given in Table 1. Especially for the combinations of (m_2, p_2), (m_3, p_3), (m_3,A_3'), (R_2,R_2'), ($M\gamma_0$,C), (p_2,A_2), (p_3,A_3') etc, the correlation coefficient even exceed 0.99. However, at the same time, the correlation coefficient is low, if the two parameters , such as (m_1,p_2), (m_2,A_3') etc, are selected from different subsets Table 1. Especially, the correlation coefficient between β and $M\gamma_0$ is nearly to 0.

The existences of many parameter combinations with high correlation coefficient and with low sensitivity show that the identifiability of the superplastic model is poor, when only the experimental data in section 3.2 is available. To improve the identifiability of the model, the two approach can be applied. one approach is to simplify the model further, such as neglecting the terms associated with diffusion creep deformation mechanism; another approach is to increase the experimental data, such as increasing the number of grain size and strain rate.

The statistical result on the identifiability analysis result is given in Table 8. It can be seen that the parameters m_1, R_3, p_1, p_2, A_1, A_3', C are identified with probability 0, and this result shows that it is impossible to identify well these parameter , and only $M\gamma_0$, $M\alpha'/\alpha$ and $m_3$can be identified with probability 1, when the identifiability analysis is taken only at the first iteration; however, only parameter p_1,

Table 6: Sensitivity analysis result at the first iteration for 100 sets of different initial parameters

Rank	Parameter	Number	Parameter	Number	Parameter	Number	Parameter	Number	Parameter	Number
1	m_3	100								
2	R_3'	100								
3	m_2	99	A_2	1						
4	A_2	99	m_2	1						
5	R_1'	6	$M\gamma_0$	94						
6	A_3'	76	R_1'	18	$M\gamma_0$	3	R_1	3		
7	R_1'	62	A_3'	10	R_1	25	$M\gamma_0$	3		
8	R_1	72	A_3'	14	R_1'	14				
9	l_3	100								
10	R_3	50	C	41	p_2	8	$M\alpha/\alpha'$	1		
11	C	51	R_3	42	p_2	7				
12	p_2	83	C	8	R_3	7	$M\alpha/\alpha'$	2		
13	$M\alpha/\alpha'$	69	p_3	29	p_2	2				
14	p_3	71	$M\alpha/\alpha'$	28	l_2	1				
15	l_1	64	l_2	36						
16	β	49	l_1	34	R_2'	6	l_2	11		
17	R_2'	76	β	21	l_1	2	m_1	1		
18	R_2	60	β	19	R_2'	18	l_2	3	R_3	1
19	R_2	39	l_2	37	m_1	12	β	11		
20	m_1	87	l_2	12	R_2	1				
21	A_1	100								
22	p_1	100								

A_1, A_3' are identified with probability 0, and m_2, $M\gamma_0$, β, R_3', $M\alpha'/\alpha$ and $m_3$can be identified with probability 1. At the same time, the identifiable probability of other parameters is much higher, when the identifiability analysis is taken at every iteration. The above results show that it is necessary to take identifiability analysis duing every iteration to increase the identifiable probability.

The identified best parameter, the corresponding initial value and the relative stan-

Table 7: Correlation coefficient between the sensitive parameter at the first iteration for 100 sets of different initial parameters

	Correlation Matrix																		number
	m_2	m_3	R_1	R_2	R_3	$M\gamma_0$	β	p_2	p_3	R_1'	R_2'	R_3'	$M\alpha'/\alpha$	A_2	A_3	l_1	l_2	l_3	C	
m_2	1.00																		100
m_3	0.91	1.00																	100
R_1	0.95	0.84	1.00																100
R_2	0.79	0.95	0.64	1.00															74
R_3	0.96	0.95	0.93	0.85	1.00														99
$M\gamma_0$	0.62	0.51	0.56	0.47	0.54	1.00													100
β	0.26	0.22	0.27	0.16	0.25	0.0019	1.00												99
p_2	0.99	0.95	0.92	0.85	0.98	0.58	0.27	1.00											100
p_3	0.86	0.99	0.75	0.96	0.92	0.46	0.21	0.92	1.00										100
R_1'	0.96	0.84	0.98	0.72	0.95	0.64	0.22	0.94	0.79	1.00									100
R_2'	0.76	0.93	0.60	0.99	0.81	0.49	0.13	0.81	0.94	0.68	1.00								80
R_3'	0.94	0.97	0.88	0.91	0.98	0.60	0.21	0.96	0.94	0.93	0.89	1.00							100
$M\alpha'/\alpha$	0.87	0.79	0.78	0.73	0.80	0.70	0.32	0.87	0.75	0.84	0.73	0.84	1.00						100
A_2	0.98	0.93	0.93	0.81	0.97	0.55	0.29	0.996	0.90	0.94	0.77	0.94	0.85	1.00					100
A_3	0.88	0.99	0.77	0.95	0.94	0.44	0.24	0.93	0.996	0.80	0.91	0.94	0.74	0.92	1.00				100
l_1	0.85	0.90	0.81	0.84	0.91	0.50	0.20	0.90	0.89	0.86	0.79	0.91	0.81	0.89	0.90	1.00			100
l_2	0.74	0.91	0.58	0.98	0.79	0.49	0.12	0.79	0.91	0.66	0.99	0.87	0.71	0.74	0.87	0.76	1.00		49
l_3	0.87	0.67	0.94	0.51	0.84	0.54	0.24	0.81	0.59	0.92	0.47	0.78	0.65	0.82	0.61	0.63	0.46	1.00	100
C	0.62	0.52	0,54	0.49	0.54	0.99	0.17	0.57	0.47	0.63	0.52	0.61	0.70	0.54	0.45	0.50	0.52	0.52	1.00	100

dard error are given in fifth column, the fourth column and the sixth column respectively. Relative standard errors of some parameters are relatively large, especially for parameter R_3, its Relative standard errors even is larger than 1. Generally, the relative standard error is relative low, when the parameter must be identified or can't be identified.

During optimization procedure by L-M algorithm, the identifiable state of some parameters can change frequently. When $\mathbf{x}^0$, which is given in the fourth column in table 9, is regarded as initial value for the L-M algorithm, the optimization procedure is completed after 44 iterations due to convergence. The identifiable state of the variable during optimization procedure is given in Table 9. At the first iteration, the largest identifiable subsets is $\{m_3, M\gamma_0, R_1, M\alpha'/\alpha, l_1, \beta, R_3', m_2, R_2\}$. At the 2th iteration, R_1 and R_2become un-identifiable, however p_3, l_3 and R_1' become identifiable, the largest identifiable subsets become $\{m_3, M\gamma_0, R_1', l_3, M\alpha'/\alpha, l_1, \beta, R_3', m_2, p_3\}$.

Table 8: Statistical result on parameter identifiabilty analysis, the best identified parameter and corresponding initial value, and relative standard error

Parameter	Identifiable probability[1]	Identifiable probability[2]	Initial value	Identified value	Relative standard error
m_1	0.00	0.02	1.4483	1.4483	0.047
m_2	0.92	1.00	0.4757	0.4343	0.037
m_3	1.00	1.00	0.2215	0.2591	0.067
R_1	0.13	0.87	-1.6701	-1.0767	0.20
R_2	0.66	0.84	-19.6872	-94.9974	0.64
R_3	0.00	0.47	-0.2459	0.2919	2.26
$M\gamma_0$	1.00	1.00	0.002476	0.002576	0.024
β	0.99	1.00	60.9427	77.7040	0.092
p_1	0.00	0.00	0.4154	0.4154	0.15
p_2	0.00	0.11	0.3075	0.3075	0.22
p_3	0.04	0.59	-0.7849	-0.3589	0.44
R_1'	0.85	0.99	0.2888	0.3183	0.050
R_2'	0.19	0.75	-3.3831	-6.8963	0.38
R_3'	0.97	1.00	0.2238	0.2644	0.053
$M\alpha/\alpha'$	1.00	1.00	0.03705	0.03489	0.15
A_1	0.00	0.00	185.5861	185.5861	0.19
A_2	0.03	0.61	137.4271	137.4271	0.09
A_3'	0.00	0.00	409.4376	409.4376	0.01
l_1	0.37	0.95	0.8331	0.8208	0.18
l_2	0.40	0.61	0.01167	0.0200	0.44
l_3	0.18	0.84	0.6710	0.6064	0.034
C	0.00	0.04	0.0008323	0.0008323	0.033

Identifiable probability[1] means the identifiable probability when the identifiability analysis is taken only at the first iteration.
Identifiable probability[2] means the identifiable probability when the identifiability analysis is taken at each iteration.

At the first 8 iterations, the largest identifiable subset alters frequently. However, after the first 8th iteration, the largest identifiable subsets become steady and don't alter no more. This is due to that the variation ranges of the parameters are small. During the parameter identification procedure of the time, the un-identifiable parameter subsets is $\{m_1, p_1, p_2, A_1, A_2, A_3', C\}$.

8 Conclusion

In this paper, a systemic parameter identification method for the large and complex macro-micro coupled constitutive models is proposed and is used to identify the material parameters of the superplastic alloy Ti-6Al-4V, involved in a macro-micro coupling superplastic model, at 927°C.

The proposed method is based on the global and local identifiability analysis, in which two identifiability measures are adopted. The first measure accounts for the sensitivity of model results to single parameters, and the second measure accounts for the degree of near-linear dependence of sensitivity functions of parameter subsets. The global identifiability analysis adopts a sampling strategy that is a combination of Latin-hypercube sampling, one-factor-at-a-time sampling and elitism preservation strategy. The global identifiability index is the integration of the corresponding local index.

A hybrid global optimization method is designed. The designed optimization method incorporates the strengths of GA and the L-M algorithm. The difficulty of choosing appropriate initial values from GA for the L-M algorithm is overcome through the introduction of the niching technology. A transition criterion between GA and L-M algorithm is proposed, through the improvement on the average objective function value of the chromosomes and on the objective function value of the best chromosome in the population. The niching technology and the hybrid-type genetic operator are used to maintain the population diversity. During the L-M optimization procedure, identifiability analysis is taken at each iteration, and only the identifiable parameters are updated and the un-identifiable are frozen.

The identified results show that the superplastic model is poorly identifiable. When the above available experimental data is used, p_1, A_1, A_3' are identified only with the probability 0, and the identifiable probability of parameters m_1, R_3, p_2 and C are very low. The next goal of our research is how to design the experiment to insure that every parameter involved in the model can be identified well.

Acknowledgement: Finical support for this research is provided by Major National Science and Technology program of Ministry of Industry and Information Technology of the People's Republic of China (Grants2009ZX04014-082) and the

Table 9: Identifiable state of the variable during optimization procedure when L-M algorithm is applied

Iteration number	Identifiable variable	Iteration number	Identifiable variable
1	$m_3, M\gamma_0, R_1, M\alpha'/\alpha, l_1, \beta, R'_3, m_2, R_2$	6	$m_3, M\gamma_0, l_3, M\alpha'/\alpha, R_1, l_1, \beta, R'_3, R_3$
2	$m_3, M\gamma_0, R'_1, l_3, M\alpha'/\alpha, l_1, \beta, R'_3, m_2, p_3$	7	$m_3, M\gamma_0, R_1, M\alpha'/\alpha, l_1, \beta, l_2, R'_3, m_2, R_3$
3	$m_3, M\gamma_0, m_2, l_3, M\alpha'/\alpha, l_1, \beta, R'_3, p_3$	8	$m_3, M\gamma_0, R_1, M\alpha'/\alpha, R_2, l_2, l_1, \beta, R'_3, m_2, p_3$
4-5	$m_3, M\gamma_0, R_1, M\alpha'/\alpha, l_1, \beta, R'_3, m_2, p_3$	9-44	$m_3, M\gamma_0, R_1, M\alpha'/\alpha, R_2, l_2, l_1, \beta, R'_3, m_2, R'_2$

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