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Multi-Disciplinary Optimization for Multi-Objective Uncertainty Design of Thin Walled Beams

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Abstract: The focus of this paper is concentrated on multi-disciplinary and multi-objective optimization for thin walled beam systems considering safety, normal mode, static loading-bearing and weight, in which the uncertainties of the parameters are described via intervals. The size and shape of the cross-section are treated as design parameters during optimization. Considering the lightweight and safety, the design problem is formulated with two individual objectives to measure structural weight and maximum energy absorption, respectively, constrained by the average force, normal mode and maximum stress. The optimization problem with uncertainties is further transformed into a deterministic optimization based on interval number programming. The approximation models, coupled with the design of experiment (DOE) technique, are employed to construct objective functions and constraints. The uncertain optimization problem characterized with these approximation models is performed and applied to a practical thin walled beam design problems.

Keywords: crashworthiness; multi-disciplinary multi-objective optimization; uncertainty; interval programming; approximation model

1 Introduction

Thin-walled beams are widely used in automotive industry and other engineering applications as energy-absorbing components to attenuate the initial kinetic energy so as to enhance the safety of occupants in the event of the crash accidents (Kurtaran et al. 2002). Therefore, it is of importance to investigate their energy absorption

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mechanisms and to optimize structural performance responses by considering both crashworthiness performance and manufacturing costs (e.g. component weight). However, most conventional design methods are limited to models and parameters under deterministic assumption, in which all the design variables and parameters involved are assumed to be certain and no variability in the simulation outputs. In practical engineering problems, it is noted that the uncertainty and randomness are most inherent and inevitably involved in stages of structural designs, such as uncertainties and randomness relevant to geometries, boundary and loading conditions and material properties. In such case, the final design achieved via the deterministic optimization may not well satisfy the desired goal or sometimes become unfeasible due to the variability of structural performance. Although the deterministic method has been successfully applied to a range of practical design problems, there is an increasing demand to consider non-deterministic factors in the procedure of real-world structural designs in order to ensure structural safety and to avoid occurrence of breakage and collapse in extreme events in the presence of uncertainties (Schueller and Jensen 2008).

In the past two decades, the finite element analysis oriented optimization has been developed as an effective tool to seek a desirable crashworthiness design for a full vehicle or its components (Jansson et al. 2003; Forsberg and Nilsson. 2006; Duddeck. 2008). As aforementioned, the dominant optimization techniques by far are deterministic methods which are used to improve thin-walled beam designs by incorporating some unknown variables of size and shape parameters. For instance, Zarei et al. (2006) studied multi-objective crashworthiness design optimization of circular aluminum tubes. Hou et al. (2008) proposed a multi-objective optimization of multi-cell sections for the crashworthiness design, by maximizing the energy absorption and at the same time minimizing the peak force. Lanzi et al. (2004) proposed multi-objective optimization of composite absorber shape under crashworthiness requirements. However, the uncertainties relevant to gauge thickness, geometry and material parameters of are not considered in the above mentioned studies. To obtain a reliable design, uncertainty should be considered in the design process of the crashworthiness of the thin-walled beams. Zhu et al. (2007) studied the front side rail lightweight design based on robust optimization method. Lönn et al. (2009) proposed an approach to robust optimization of impact problem using meta-models, which are created for both the mean value and the standard deviation of the response. The above mentioned works all used the probability method, in which system parameters of uncertainty were treated as stochastic numbers based on precise probability distributions. Unfortunately, for design problem of thin walled beam, it is generally difficult and computationally cost to specify a precise probability distribution.

Thus, to overcome the aforementioned difficulty of probability methods, it is gradually important to seek efficient optimization method for the design of thin walled beams with uncertainties. In recent years, it can be found that the interval method has been applied to model uncertainties in a wide range of structural design problems, in which only the bounds of the uncertain parameters are needed rather than their precise probabilistic distributions. In terms of the interval method, a new family of optimization methods for uncertain problems, interval number programming, has been attracted much attentions. Since most of the practical engineering problems are nonlinear, it is generally difficult to express interval numbers in an explicit form based on simulation analysis models. Hence there have been some research efforts which focus on nonlinear interval number programming (NINP). Ma (2002) used a deterministic optimization method to obtain the interval of the nonlinear objective function and then converted it into a three-objective optimization problem. Jiang et al. (2007a,b,c) recently proposed an NINP model used to transform a general NINP problem to a deterministic optimization, and further developed several efficient algorithms to solve the transformed two-layer optimization problem. Jiang et al. (2007d) has applied the interval uncertain optimization to thin-walled beams, in which the approximation models are managed by a sequence of sub-problems in the uncertain design space. Zhao et al. (2010) proposed a new method of nonlinear interval-based programming problem based on approximation models and a local-densifying method, which is applied to the pratical thin-walled beams. Li et al. (2010) studied the thin-walled beam problem using multi-objective uncertain optimization method, in which the uncertain parameters are modeled by interval number. These studies have shown the importance and the necessary to introduce interval programming to the design of thin-walled beams.

However, it is noted that most of the above mentioned methods focus only on single disciplinary application of thin-walled structures rather than several different disciplines. As a matter of fact, there has been a increasingly demand to include multiple disciplinary measures, such as safety, NVH, durability, and other attribute performances, into the analysis of the thin walled beam structures. In contrast to a number of optimization designs of single disciplinary, there are few reported studies concentrated on the multidisciplinary optimization of the thin-walled structures, although such structures are mostly characterized with multidisciplinary design requirements. In this study, we only limit the discussion to multidisciplinary design optimization (MDO) which in most cases can be equivalently represented as a multi-objective programming scheme. The general approach for a multi-objective design problem is to convert the multiple individual objectives into an aggregated single objective function via direct weighting method and other compromising programming schemes (Luo et al., 2005), so as to find the desirable Pareto solutions.

However, it is not easy even for an experienced design engineer to identify a predominant objective from a long list of design requirements of MDO problem (Wang et al. 2007). Furthermore, there often exist uncontrollable variations or uncertainties in parameters of MDO problems. Hence, it is necessary to develop a multidisciplinary optimization method including uncertainties for multi-objective design of the thin walled beam structures.

2 Statement of the problem

The optimization problem can be specifically stated as a multidisciplinary problem involving the "disciplines" of safety, static load-bearing and the first order vibration mode. In safety discipline, the thinned walled beam is a key part of energy absorption in the frontal crash and its structural crashworthiness performance is commonly represented by the deformation mode and the absorbed energy. This can affect greatly the crash performance of full vehicle. For the human safety issues, the mean crushing force F_a that occurs during the crash should be under certain criteria, which is very important in the automotive design and manufacturing. Therefore the structural crashworthiness performance of the front side rail should be guaranteed primarily in the design process. In static bending discipline, the thin walled is subjective to the load to test the structure maximum stress. In normal mode discipline, the first order vibration mode is selected.

Design variables are composed by structural considerations (e.g, sheet metal thickness, geometric shape, etc.). The lightweight design has been treated as an optimization problem in the previous study [zhang *et al.* (2007); Pan *et al.* (2009)], where structure weight is the objective function subject to the structural performance constraints. The maximum energy absorption is chosen as objective function. The average rigid force, normal mode and maximum stress are regarded as constraints.

The uncertainty widely exists in material property, component structures, impact speed, etc. As a result, in thin-walled beam design, an uncertain multi-objective multi-disciplinary optimization for crashworthiness problem can be given in the following form:

Minmize $\{W(\mathbf{x},\mathbf{a}), E(\mathbf{x},\mathbf{a})\}$

Subject to:
Safety:

$$F_a(\mathbf{x},\mathbf{a}) \le [v_1^L, v_1^R]$$

Normal mode:

$$[b_1^L, b_1^R] \le f_q(\mathbf{x}, \mathbf{a}) \le [v_2^L, v_2^R]$$

Static load bearing:

$$S_m(\mathbf{x}, \mathbf{a}) \le \left[v_3^L, v_3^R\right]$$

$$\mathbf{a} \in \mathbf{a}^I = [\mathbf{a}^L, \mathbf{a}^R], \tag{1}$$

$$a_i \in a_i^I = [a_i^L, a_i^R],\ i = 1, 2, \cdots, q,$$

$$x_{il} \le x_i \le x_{iu},\ i = 1, 2, \cdots, n$$

Where $\mathbf{x}$ denotes an n-dimensional vector. $\mathbf{a}$ is a q-dimensional uncertain vector which collects all of the uncertain parameters in the thinned wall beam model, and its uncertainty is modeled by an interval vector $\mathbf{a}^I$. The superscripts I represent an interval, and L and R denote lower and upper bounds of the interval. v^I denotes the allowable interval of the constraint.

For a specific design vector $\mathbf{x}$, the objection functions and constrains will form intervals, as the uncertain parameters are all intervals, and are nonlinear function of $\mathbf{x}$ and $\mathbf{a}$. In the following sections, an interval programming will be introduced to solve above complex uncertain optimization problem.

3 Uncertain multi-objective multi-disciplinary optimization based on interval programming method for design of thin walled beam

3.1 The treatment of the objective function

An order relation implies that an interval number is better than another but not that one is larger than another. In reference [Han *et al.*(2008); Jiang et al. (2008a,b)], an order relation $\le_{mw}$ was adopted to treat objective function. Similarly, the uncertain objective function in Equation (1) can be transformed into a deterministic multi-objective optimization problem using the order relation $\le_{mw}$:

$$\min_{\mathbf{x}} \left[m(W^I(\mathbf{x}, \mathbf{a})), w(W^I(\mathbf{x}, \mathbf{a}))\right]$$

$$\min_{\mathbf{x}} \left[m(E^I(\mathbf{x}, \mathbf{a})), w(E^I(\mathbf{x}, \mathbf{a}))\right]$$

$$\begin{matrix} m(W(\mathbf{x}, \mathbf{a})) = \frac{1}{2}(W^L(\mathbf{x}) + W^R(\mathbf{x})) & m(E(\mathbf{x}, \mathbf{a})) = \frac{1}{2}(E^L(\mathbf{x}) + E^R(\mathbf{x})) \\ w(W(\mathbf{x}, \mathbf{a})) = \frac{1}{2}(W^R(\mathbf{x}) - W^L(\mathbf{x})) & w(E(\mathbf{x}, \mathbf{a})) = \frac{1}{2}(E^R(\mathbf{x}) - E^L(\mathbf{x})) \end{matrix} \tag{2}$$

Where m and w denote the midpoint and radius of interval, respectively. For each specific $\mathbf{x}$, the bounds of the objective functions caused by uncertainty can be obtained:

$$\begin{aligned}
&E^L(\mathbf{x}) = \min_{a\in\Gamma} E(\mathbf{x},\mathbf{a}), \quad E^R(\mathbf{x}) = \max_{a\in\Gamma} E(\mathbf{x},\mathbf{a}),\\
&W^L(\mathbf{x}) = \min_{a\in\Gamma} W(\mathbf{x},\mathbf{a}), \quad W^R(\mathbf{x}) = \max_{a\in\Gamma} W(\mathbf{x},\mathbf{a}),\\
&\Gamma = \left\{\mathbf{a}\left| a_i^L \le a_i \le a_i^R,\ i=1,2,...,q\right.\right\}
\end{aligned} \tag{3}$$

Through Equation (3), the uncertain vector $\mathbf{a}$ is eliminated and the deterministic objective functions are obtained.

The midpoint of objective function interval in Equation (3) analogously minimizes the average value of the uncertain objective function, and the radius analogously minimizes the deviation. Through minimizing the deviation, the design robustness can be ensured.

Using linear combination method to deal with the multiple objectives and each objective can be transformed as following assessment function:

$$\begin{aligned}
&\min_{\mathbf{x}} f_{d1}(\mathbf{x},\mathbf{a}) = (1-\beta)(m(W(\mathbf{x},\mathbf{a}))+\xi)/\varphi + \beta(w(W(\mathbf{x},\mathbf{a}))+\xi)/\psi\\
&f_{d2}(\mathbf{x},\mathbf{a}) = (1-\beta)(m(E(\mathbf{x},\mathbf{a}))+\xi)/\varphi + \beta(w(E(\mathbf{x},\mathbf{a}))+\xi)/\psi
\end{aligned} \tag{4}$$

Where $0 \le \beta \le 1$ is a weight factor, and its different values will lead to different optimization. ξ is a number making m and w non-negative. ϕ and ψ are the normalization factors of objectives.

3.2 The treatment of the constraint function

The possibility degree of interval number represents certain degree that one interval number is larger or smaller than another. The reference [Jiang *et al.* (2008c,d)] gives a definition of the satisfactory degree, which was adopted to deal with constraints in this paper. The uncertain constraints in Equation (1) can be transformed into the following deterministic constraints:

$$P(C_i^I \ge v_i^I) \ge \lambda_i, i=1,2,3$$

$$P(b_1^I \ge C_3^I) \ge \lambda_3, \tag{5}$$

where

$$\begin{aligned}
C_1^I &= \left[F_a^L(\mathbf{x}), F_a^R(\mathbf{x})\right] = \left[\min_{a\in\Gamma} F_a^L(\mathbf{x},\mathbf{a}), \max_{a\in\Gamma} F_a^R(\mathbf{x},\mathbf{a})\right] \\
C_2^I &= \left[f_q^L(\mathbf{x}), f_q^R(\mathbf{x})\right] = \left[\min_{a\in\Gamma} f_q^L(\mathbf{x},\mathbf{a}), \max_{a\in\Gamma} f_q^R(\mathbf{x},\mathbf{a})\right] \\
C_3^I &= [S_m^L(\mathbf{x}), S_m^R(\mathbf{x})] = \left[\min_{a\in\Gamma} S_m^L(\mathbf{x},\mathbf{a}), \max_{a\in\Gamma} S_m^R(\mathbf{x},\mathbf{a})\right]
\end{aligned} \tag{6}$$

$$\Gamma = \left\{\mathbf{a} \,\middle|\, a_i^L \le a_i \le a_i^R,\; i = 1,2,...,q\right\}$$

Where λ_i, $i = 1,2,3$ is predetermined satisfactory degree level of the constraint. C_i^I, $i = 1,2,3$ of constraint at $\mathbf{x}$ which is caused by the uncertainty. λ_i, $i = 1,2,3$ can be control the feasible field of $\mathbf{x}$. A larger λ_i, $i = 1,2,3$ means a stricter restriction to the constraint and where by a smaller feasible design space.

3.3 Deterministic optimization

Through above treatments, the uncertain optimization problem Equation (1) can be transformed into a following deterministic multi-objective optimization problem:

$$\begin{aligned}
&\min\nolimits_{\mathbf{x}} f_{d1}(\mathbf{x},\mathbf{a}) = (1-\beta)(m(W(\mathbf{x},\mathbf{a}))+\xi)/\varphi + \beta(w(W(\mathbf{x},\mathbf{a}))+\xi)/\psi \\
&f_{d2}(\mathbf{x},\mathbf{a}) = (1-\beta)(m(E(\mathbf{x},\mathbf{a}))+\xi)/\varphi + \beta(w(E(\mathbf{x},\mathbf{a}))+\xi)/\psi
\end{aligned} \tag{7}$$

$$s.t. \quad P(C_i^I \ge v_i^I) \ge \lambda_i, i = 1,2,3$$

$$P(b_1^I \ge C_3^I) \ge \lambda_3,$$

$$x_{il} \le x_i \le x_{iu}£\neg i = 1,2,\cdots,n$$

Where

$$\begin{aligned}
C_1^I &= \left[F_a^L(\mathbf{x}), F_a^R(\mathbf{x})\right] = \left[\min_{a\in\Gamma} F_a^L(\mathbf{x},\mathbf{a}), \max_{a\in\Gamma} F_a^R(\mathbf{x},\mathbf{a})\right] \\
C_2^I &= \left[f_q^L(\mathbf{x}), f_q^R(\mathbf{x})\right] = \left[\min_{a\in\Gamma} f_q^L(\mathbf{x},\mathbf{a}), \max_{a\in\Gamma} f_q^R(\mathbf{x},\mathbf{a})\right] \\
C_3^I &= [S_m^L(\mathbf{x}), S_m^R(\mathbf{x})] = \left[\min_{a\in\Gamma} S_m^L(\mathbf{x},\mathbf{a}), \max_{a\in\Gamma} S_m^R(\mathbf{x},\mathbf{a})\right]
\end{aligned} \tag{8}$$

$$\Gamma = \left\{\mathbf{a} \,\middle|\, a_i^L \le a_i \le a_i^R,\; i = 1,2,...,q\right\}$$

4 Uncertain multi-objective multi-disciplinary optimization based on approximation models

Finite simulations with acceptable accuracy are computationally very expensive. To improve the optimization efficiency, the approximation models have been applied instead of actual simulation models. Equation (6) can be formulated as a following approximation optimization problem:

$$\begin{aligned}&\min_{\mathbf{x}} \tilde{f}_{d1}(\mathbf{x},\mathbf{a}) = (1-\beta)(m(\tilde{W}(\mathbf{x},\mathbf{a}))+\xi)/\varphi+\beta(w(\tilde{W}(\mathbf{x},\mathbf{a}))+\xi)/\psi \\ &\tilde{f}_{d2}(\mathbf{x},\mathbf{a}) = (1-\beta)(m(\tilde{E}(\mathbf{x},\mathbf{a}))+\xi)/\varphi+\beta(w(\tilde{E}(\mathbf{x},\mathbf{a}))+\xi)/\psi\end{aligned} \tag{9}$$

$s.t.\ P(\tilde{C}_i^I \geq v_i^I) \geq \lambda_i,\ i=1,2,3$

$P(b_1^I \geq \tilde{C}_3^I) \geq \lambda_3,$

$x_{il} \leq x_i \leq x_{iu},\ i=1,2,\cdots,n$

where

$$\begin{aligned}&C_1^I = \left[\tilde{F}_a^L(\mathbf{x}),\tilde{F}_a^R(\mathbf{x})\right] \\ &= \left[\min_{a\in\Gamma}\tilde{F}_a^L(\mathbf{x},\mathbf{a}),\max_{a\in\Gamma}\tilde{F}_a^R(\mathbf{x},\mathbf{a})\right],\quad \begin{aligned}&C_2^I = \left[\tilde{f}_q^L(\mathbf{x}),\tilde{f}_q^R(\mathbf{x})\right] \\ &= \left[\min_{a\in\Gamma}\tilde{f}_q^L(\mathbf{x},\mathbf{a}),\max_{a\in\Gamma}\tilde{f}_q^R(\mathbf{x},\mathbf{a})\right],\end{aligned} \\ &C_3^I = [\tilde{S}_m^L(\mathbf{x}),\tilde{S}_m^R(\mathbf{x})] \\ &= \left[\min_{a\in\Gamma}\tilde{S}_m^L(\mathbf{x},\mathbf{a}),\max_{a\in\Gamma}\tilde{S}_m^R(\mathbf{x},\mathbf{a})\right]\end{aligned} \tag{10}$$

$$\Gamma = \left\{\mathbf{a}\,\middle|\,a_i^L \leq a_i \leq a_i^R,\ i=1,2,...,q\right\}$$

Where $\tilde{W}(\mathbf{x},\mathbf{a})$ and $\tilde{E}(\mathbf{x},\mathbf{a})$ are approximation models of weight and maximum energy absorption of thin-walled beam, respectively. $\tilde{F}_a(\mathbf{x},\mathbf{a})$,$\tilde{f}_q(\mathbf{x},\mathbf{a})$ and $\tilde{S}_m(\mathbf{x},\mathbf{a})$ are approximation models of constraint, respectively. $\tilde{f}_{d1}(\mathbf{x},\mathbf{a})$ and $\tilde{f}_{d2}(\mathbf{x},\mathbf{a})$ are the assessment functions based on the approximation models of the objective functions (termed as "approximation assessment functions"). $\tilde{C}_i^I$,$i=1,2,3$ are intervals of approximation constraint functions. Here, the design vector $\mathbf{x}$ and uncertain vector $\mathbf{a}$ are both used as input variables in the construction process. Hence $\tilde{W}(\mathbf{x},\mathbf{a})$, $\tilde{E}(\mathbf{x},\mathbf{a})$,$\tilde{F}_a(\mathbf{x},\mathbf{a})$,$\tilde{f}_q(\mathbf{x},\mathbf{a})$ and $\tilde{S}_m(\mathbf{x},\mathbf{a})$ are all explicit functions with respect to $\mathbf{x}$ and $\mathbf{a}$, instead of only $\mathbf{x}$ as we usually do for deterministic optimization problems.

Design of experiment (DOE) provides a means to selection of the sampling points in the space of input variables in a more efficient way when creating approximation models for objective functions and constraints. There are many different experimental design methods available, such as the factorial, Koshal, composite, Latin Hypercube and D-optimal design, etc. Here, we made use of the Latin Hypercube Design (LHD) [Morris and Mitchell (1995)] for its uniformity of sampling in an unknown design space and uncertain space.

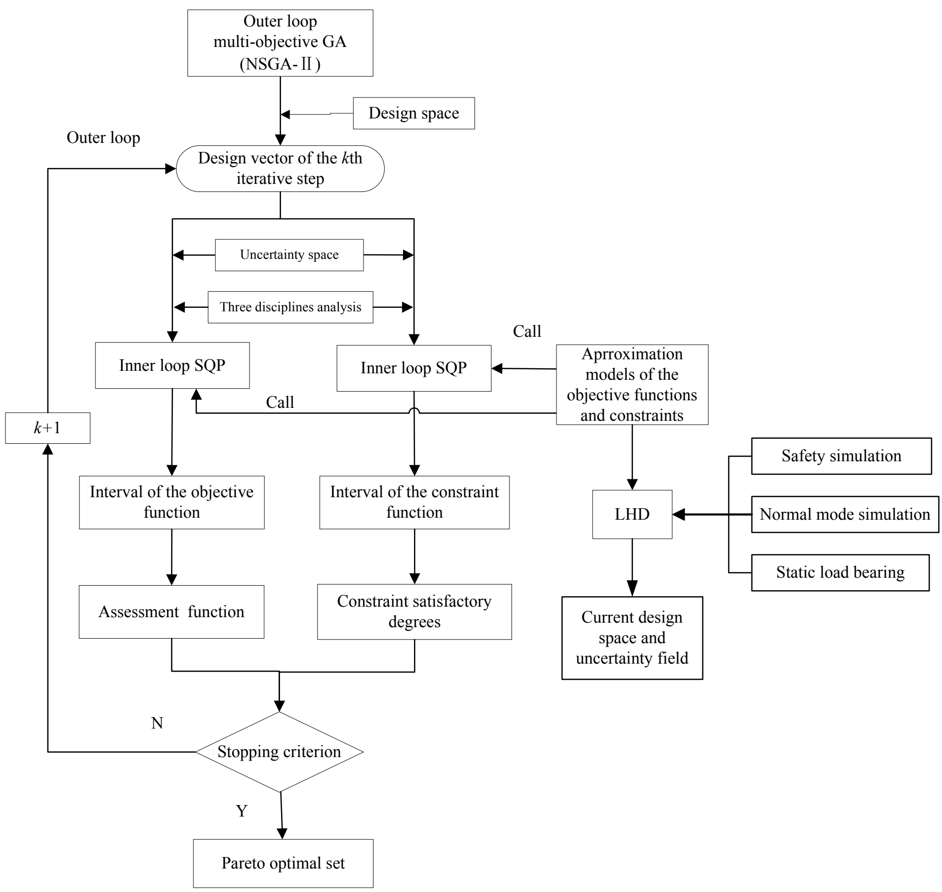

Figure 1: The optimization chart based on approximation models

Fig.1 shows the flowchart of the optimization. **x** and **a** are both used as the input variables, and sampling points using Latin Hypercube procedure centered at the design space and uncertain field. After inputting the sampling points into the three disciplines simulation analysis, then the samples used to construct the approximation models of the objective functions and constraints. Obviously, it is two-loop nesting optimization problem. Here, the Non-dominated Sorting Genetic Algorithm II [Deb (2001); Deb, Pratap, Agarwal and Meyarivan (2002)] and sequential quadratic programming (SQP) [Boggs and Tolle (1995)] are used as the outer layer and inner layer optimization solver, respectively. In the outer layer, an amount of individuals of the design vector **x** are generated by multi-objective genetic algorithm named NSGA-II, which is employed to optimize the design vector. In the inner layer, the SQP method for each individual will be called two times to obtain the intervals of objective functions and constraints based on these approximation models. Then the approximate assessment function values can be calculated based on these intervals. As a result, the Pareto set can be obtained.

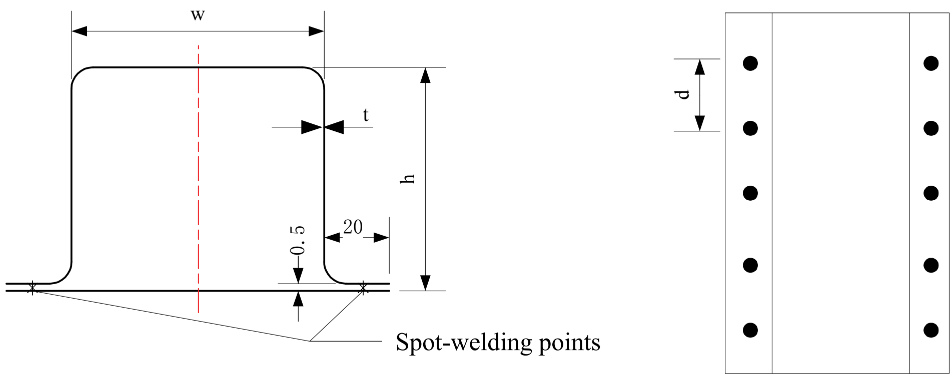

Figure 2: A closed-hat beam impacting the rigid wall and its cross section (mm)

Table 1: Details of NSGA-II specific parameters used

GA parameter name	Value
Population size	50
Number of generation	200
Probability of crossover	0.9
Probability of mutation	0.1
Distribution index for crossover	2.0
Distribution index for mutation	2.0

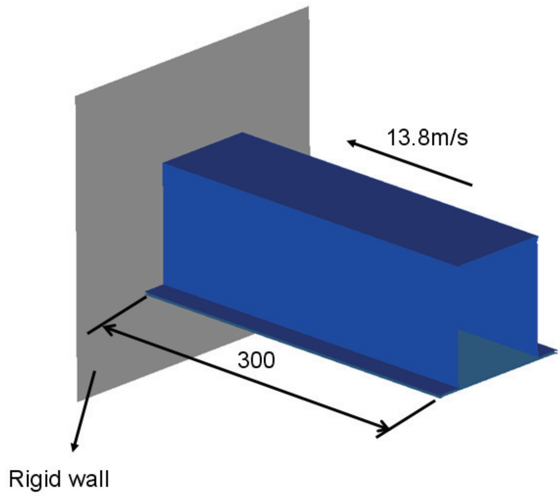

Figure 3: The longitudinal impact model

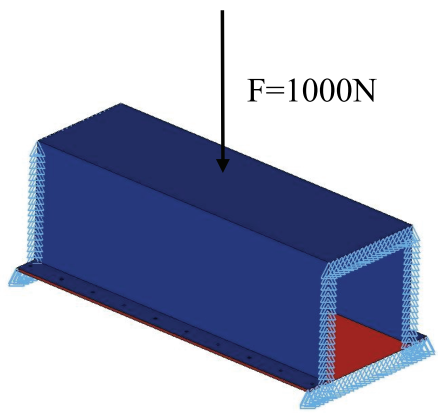

Figure 4: The model with vertical loading condition

5 Application

5.1 Optimization of the thin walled beam

This thin-walled beam problem is modified from the numerical example in reference [Zhang, Li and Zhong (2009)]. The cross-section geometry and spot-welds along the length of the section are shown in Fig. 2. In safety discipline, the compu-

Table 2: Material properties of thin walled beam

Modulus of elasticity E	2.0×10^5Mpa
Poisson's ratio ν	0.27
Density ρ	7.85×10^{-3}Kg / mm^{-3}
Yield stress σ_s	525 Mpa

Table 3: Accuracy of RS models for the thin walled beam

Approximation model	R^2_{adj}
$\tilde{f}_1$	0.998
$\tilde{f}_2$	0.985
$\tilde{F}_a$	0.954
$\tilde{f}_q$	0.998
$\tilde{S}_m$	0.995

Table 4: Show the typical Pareto fronts of the optimization result

	w(mm)	h(mm)	d(mm)	t(mm)	W(kg)	$-E_d$
1	80.00	80.00	31.481	1.742	1.649	-13510
2	80.84	80.23	25.767	1.787	1.700	-13783
3	85.92	80.03	22.368	1.839	1.792	-13849
4	89.54	80.00	19.347	1.912	1.896	-13909
5	93.22	80.30	15.655	1.973	1.993	-13956
6	95.44	80.00	13.207	2.052	2.092	-13991
7	96.5	80.00	12.093	2.145	2.198	-14012
8	96.94	85.30	10.000	2.178	2.290	-14022
9	94.86	104.08	10.000	2.125	2.403	-14036
10	95.24	104.05	10.102	2.198	2.488	-14039

Table 5: The optimization result of the thin walled beam

	$-E_d$	w(mm)	h(mm)	d(mm)	t(mm)
Point 10 applied on FEA for true assessment function	-13889	95.24	104.05	10.102	2.198
Point 10 on the Pareto front (optimization)	-14039	95.24	104.05	10.102	2.198
Error percentage	1.07				

tational model is shown in Fig. 3. The initial velocity is 13.8 m/s with the impact duration time of 40ms. In Fig.4, the model is for the disciplines of normal mode and static load bearing.

In this study, the thickness t, height h, width w, and space $d$of each two neighboring spot-welding are taken as design variables. The weight W and maximum energy absorption E_d are used as objective functions. Material properties of thin walled beam are listed in Table 2. Because of the manufacturing and measurement errors, E, υ and σ_s are treated as uncertain parameters, and the uncertainty level is $\pm 10\%$ off from their nominal values, namely $E \in [189000\text{MPa}, 231000\text{MPa}]$,$\upsilon \in [0.27, 0.33]$,$\sigma_s \in [472.5\text{MPa}, 577.5\text{MPa}]$.As a result, the uncertain multi-objective multi-disciplinary optimization problem is formulated as:

$$\begin{aligned}
&\underset{t,d,w,h}{\text{Min}}\, f_1(t,d,w,h,) = W\\
&f_2(t,d,w,h,E,\nu,\sigma_s) = -E_d
\end{aligned}$$

$$\begin{aligned}
&\text{s.t}\quad F_a(t,d,w,h,E,\nu,\sigma_s) \le [115\text{KN}, 125\text{KN}]\\
&[450\text{Hz}, 500\text{Hz}] \le f_q(t,d,w,h,E,\nu,\sigma_s) \le [650\text{Hz}, 750\text{Hz}]\\
&E \in [189000\text{MPa}, 231000\text{MPa}]\\
&\nu \in [0.27, 0.33]\\
&\sigma_s \in [472.5\text{MPa}, 577.5\text{MPa}]\\
&0.8\text{mm} \le t \le 2.5\text{mm}\\
&10\text{mm} \le d \le 50\text{mm}\\
&80\text{mm} \le w \le 120\text{mm}\\
&80\text{mm} \le h \le 120\text{mm}
\end{aligned} \tag{11}$$

In Equation (11), the normal modes were calculated under the free-free condition. In the multidisciplinary design optimization problem, there are 4 global (system) thickness design variables including t,d,w,h. The material parameters E,ν,σ_s are treated as shared uncertain parameters. The quadratic polynomial response surface is used to construct the approximation models for the objective function and constraints. The weight W is related to the component geometry and thickness. In the construction progress,t,d,w,h,E,ν and σ_s are both used as the input variables of the polynomial response surface. Therefore, the objective functions and constraints are explicit functions with respect to the design vector (t,d,w) and the uncertain vector (E,ν,σ_s), instead of only design vector (t,d,w) as we usually do for deterministic optimization. Initial 60 sampling points are selected through LHD to construct the quadratic response surface approximate models for the objective functions and constraints within design space and uncertainty space. According to the classical response surface method (RSM) theory, the larger the values of R^2_{adj}, the better the model fits [Fang (2005)]. The regression analysis results are given in Table 3. The model fits are very good.

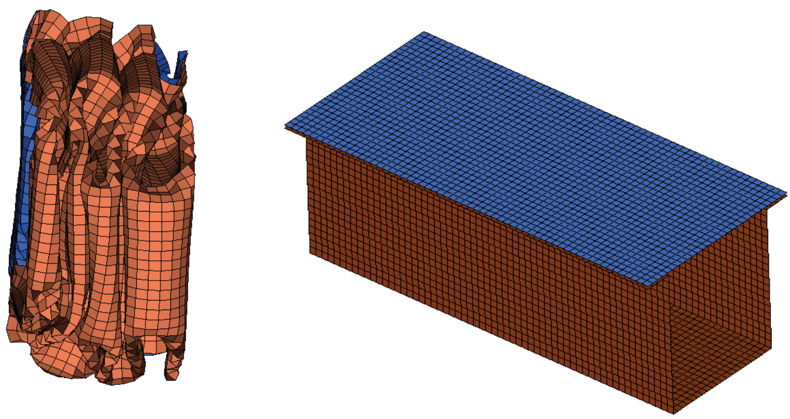

Figure 5: The Finite element model and a possible deformation of the close-hat beam impacting the rigid wall

In safety disciplines, the Finite Element Method (FEM) simulation is carried out in the explicit non-linear finite element code LS-DYNA. This finite element model consists of 5760 shell elements. A 300 kg mass is attached to the free end of these beams during the crash analysis to supply enough crushing energy. The finite mesh model and a typical deformation behaviour of the finite element model can be seen in Fig.5. Fig.6 shows the component of the test thin walled beam and deformation. In normal mode and static load bearing disciplines, the finite element model as shown in Fig.7 contains approximately 5940 elements and 6138 nodes. The thin walled beam is subjected to the vertical force 1000 N. The FEM simulation of this problem is solved to the first order vibration mode and maximum stress by Optistruct Software.

The parameters in NSGA-II are specified as the Table 1. The possibility degree levels λ of the constraint and β are set to 0.90 and 0.5, respectively. ξ, φ and ψ are all specified as 0 for f_1 and f_2 .

Figure 6: The component of the test thin walled beam and deformation

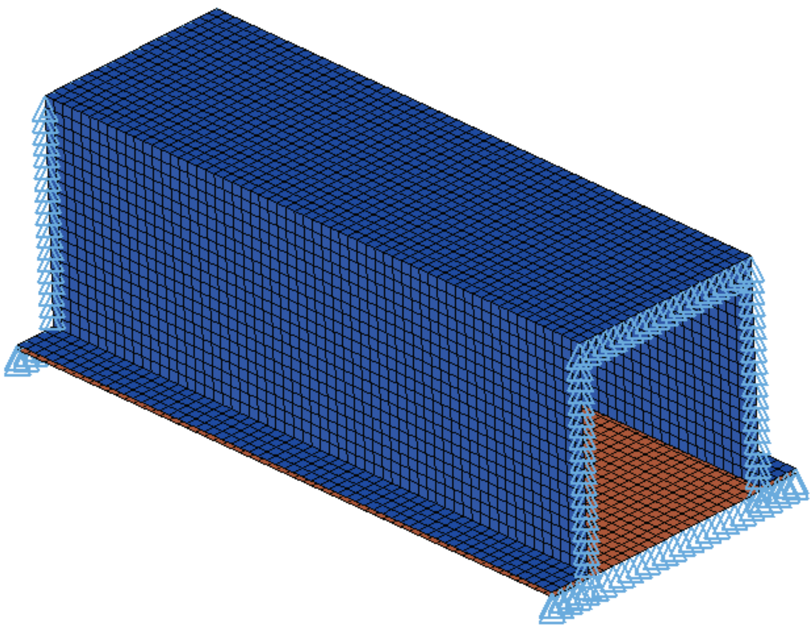

Figure 7: The Finite element model for the first order vibration mode and maximum stress

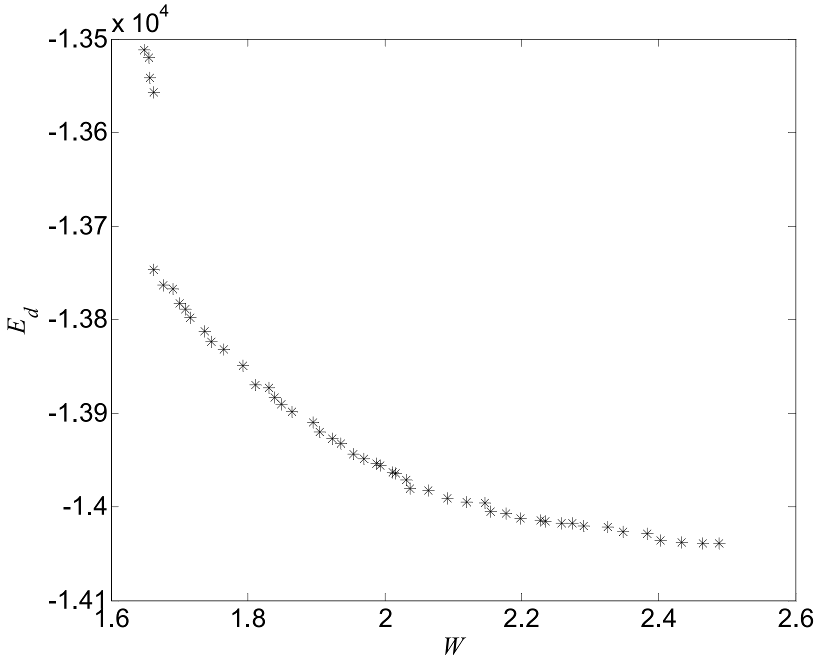

Figure 8: The Pareto set obtained

5.2 Optimization results and analysis

The outcome of the optimization search is shown in the Pareto front, displayed in Fig. 8, and the front is composed of 50 points. The weight function value is from 1.649 to 2.488 and the max energy absorbed assessment function value is from -1.351×10^4 to -1.404×10^4. Table 4 shows the typical Pareto fronts of the optimization result. According the decision making, considering the lightweight, the designer may choose the $1th$, the $2th$. When maximum energy absorption is considered, the designer can select the $9th$, the $10th$. In this paper, the maximum energy absorption is considered. Table 5 shows the optimization result of the thin walled beam by using finite element analysis (true assessment function value) and the surrogate function (approximation assessment function value). The error percentage between them is 1.07. It can be found that a fine result is obtained. True assessment function value is calculated referring to the reference [Jiang et al. (2008d)].

6 Conclusion

In this paper, the multi-objective multi-disciplinary optimization with interval number programming is used to design the thin walled beam problem. Three disciplines are studied, they are: safety discipline, normal mode and static load bearing, respectively. Uncertainties of parameters of material are described by intervals. The lightweight and maximum energy absorption are treated as objective functions. The system optimization problem is computationally intensive, involving high finite element models and analyses for safety, normal mode and static load bearing subsystems. The objective functions and constraints are constructed by the response surface approximation models based on LHD and then uncertain optimization is performed. The result provides the design engineering with a set of solution on the Pareto front to help their decision making. The method works well for the examples presented and the main advantage of the method is that it is well suited for problems with a small uncertainty.

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