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A 3D Computational Model of RC Beam Using Lower Order Elements with Enhanced Strain Approach in the Elastic Range

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Abstract: A procedure has been described to carry out three-dimensional elastic analysis of reinforced concrete beam employing finite element technique, which uses lower order elements. The proposed procedure utilizes 8-noded isometric solid /hexahedral elements HCiS18 with enhanced assumed strain (EAS) formulation, recently developed in the literature, to predict load-deformation and internal stresses produced in case of a simply supported RC beams in the elastic regime. It models the composite behaviour of concrete and reinforcements in rigid /perfect bond situation and their mutual interaction in bond-slip condition considering continuous interface elements at the material level. Although, bond-slip relation are very much non-linear in behaviour even at the beginning of the loading condition, predictions from the proposed model /procedure are found to be very close to the experimental observations as far as accuracy is concerned in the elastic range. The sole purpose of this paper is to demonstrate the general applicability and to explore the potentiality of using lower order solid elements in the 3D finite element analysis with an aim of developing a general analytical method for the study of reinforced concrete beam in the elastic range.

Keyword: Lower order elements, Finite element approach, Three-dimensional, Enhanced assumed strain, Perfect bond, Bond-slip, linear elastic analysis, RC beam.

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1 Introduction

Recent developments in finite element techniques permits consideration of three-dimensional (3D) modeling of reinforced concrete structures with the availability of high speed computing facilities. A lot of works based on two-dimensional (2D) modeling of RCC structure without reinforcements and based on various integral methods has been reported in different reputed journals in last few decades. Attempts were made to improve performance of 2D isoparametric element based formulation using reduced and selective integration schemes, B-bar method, additional incompatible modes, but to a few specific problems and also under certain conditions of mixed formulation. Recently Cazzani et al. developed a four-node hybrid assumed-strain finite element derived within the framework of first order deformation theory, particularly for the analysis of laminated composite plates. All these attempts were made aiming at removing inherent difficulties (locking etc.) particularly in thin structures. Even these methods can only analyze certain specific problems where it is possible to study the behaviour of the structure with necessary simplification by adopting the assumptions of 2D analysis. On the other hand, one may opt for 3D modeling to avoid the shortcomings of 2D modeling in order to achieve most realistic analysis and to arrive at an optimal solution.

The standard quadratic 20-noded solid /hexahedral element has been used in many applications of 3D analysis, though it has high number of nodes involving a large number of degrees of freedom and necessitates large computational time and cost. Since comparatively lower order elements have the advantages for 3D analysis due to easy mesh generation, data interpretation and lower computational time, improvement of such type of element performance has drawn attention of the investigators. Among the lower order elements, the linear isoparametric elements are the simplest constant strain elements. However it has got some well-known deficiencies as far as the finite element analysis is concerned. It cannot represent the state of stress in pure bending accurately due to inherent volumetric and transverse shear locking phenomenon.

Several methods were described by Wilson et al. (1973), Taylor et al. (1976) in this regard to improve the performance of the standard linear quadrilateral and hexahedral elements with the introduction of additional imaginary incompatible degrees of freedom to represent different modes of deformation, of course that are condensed prior to the assembly of elements. The same is being used even nowadays by Li et al. (2005) for the three-dimensional finite element buckling analysis of honeycomb sandwich shells. This type of difficulty with the associated locking phenomena has also been overcome using a different concept of Enhanced Assumed Strain Approach (EAS) [Simo and Rifai (1990)] in near incompressible and bending situations, where the strain field is enhanced with inclusion of additional variables. A remarkable progress and accuracy has been obtained in this case by the element HCiS18 introduced by Sousa et al. (2002 & 2003), even with the coarser meshes. Kim et al. (2004) developed a simple triangular solid element using an assumed strain field to alleviate the locking effect for the analysis of plates and shells. In the present case, the element HCiS18 has been used to model the parent material i.e. concrete of the reinforced concrete structures.

RC structures are highly non-homogeneous medium due to discrete presence of the reinforcements. Till date hardly a few literatures are reported, where due attention has been paid to model concrete and the reinforcing steel with different physical and mechanical properties, which needs to be combined together through an interaction model to represent its composite behaviour. Hence when RCC structures are modeled based on continuum mechanics, contribution and distribution of stiffness of reinforcements should be given due importance. In general there are three methods available for modeling of reinforcement, e.g. the discrete, the smeared and the embedded approach. The first one represents reinforcements by truss elements those are connected to the mesh at the concrete/parent element nodes and hence finite element mesh generation becomes dependent on reinforcement layout. The second one (smeared) is more suitable for homogeneous or uniformly distributed reinforcements, such as wall panels. So they can't be generally applied to 3D structures. Within embedded approach, proposed by Elwi and Hrudey (1989), Barzegar and Madipuddi (1994), these restrictions were removed and even the reinforcements are superimposed as one dimensional uni-axial element with the same displacement field as parent/concrete element without any additional node /DOF. They are allowed to intersect the parent element at any location and hence mesh design becomes independent of reinforcement layout. Here the author has used the same method proposed by and Maddipudi (1994), Cheng and Fan (1993) due to it's simplicity to handle problems of 3D analysis of reinforced concrete structures in perfect bond situation, when linear behaviour is studied to model the reinforcement and as it's an appealing method for straight reinforcements in isoparametric configurations.

Accounting for interaction between parent material/concrete and the reinforcements are done to make RCC structure to behave in a more realistic way because concrete is a strong, relatively durable in compression and reinforcements are strong, ductile in tension. This composite action requires transfer of load between concrete and steel. This load transfer mechanism is referred as bond-slip, which is depicted as continuous stress field in the vicinity of steel-concrete interface. As the loading on RCC structures are gradually increased, this bond-slip increases, as result relaxation of steel stress takes place more and more and an equilibrium is set up in the domain. This phenomenon of interaction between the materials are modeled within the embedded approach where nodal D.O.F.s are increased by the slip D.O.F.s for each element and as a result global stiffness matrix size is increased dramatically. Another approach was initially introduced by G. Beer (1985) using isoparametric joint /interface element and was later on used by Hartl et al. (2000), where bond slip situations are being considered introducing supplementary interface elements of zero thickness. Within this approach, global displacement field is calculated at first considering perfect bond between reinforcements and concrete and then the slip is calculated by relaxing the perfect bond at the material level.

2 Motivation

In most of the earlier works of finite element analysis of reinforced concrete structures, emphasis has been given to predict load deformation characteristics either in terms of simplified 2D analysis in most of the cases or in terms of 3D analysis using higher order elements. In all these cases, hardly a few literatures are reported which includes the discrete presence of reinforcements in its exact position, which can evaluate the composite action of reinforced concrete behaviour and which includes the interaction between the components forming the RCC structures. With the developments of science and technology in different areas individually, this paper simulates the elastic response of reinforced concrete beam considering (1) lower order solid elements which reduces time and associated cost in terms of easy and simple mesh generation together with data interpretation, (2) reinforcements as 1D truss elements considering only the axial deformation in its exact spatial position without affecting the parent element mesh in perfect bond situation following embedded approach and (3) mutual interaction between concrete and reinforcements in terms of bond-slip phenomenon using continuous interface elements at the material level, that too without affecting size of global stiffness matrix.

Sufficient effort has been attributed to develop a number of subroutines for the purpose specific to this problem, which doesn't uses any block available commercially. It is shown that the accuracy of the formulation in interpreting the elastic response in this highlighted area is highly comparable to that of the existing analytical models. The present paper is an initial attempt on a continuing investigation of the finite element analysis of reinforced concrete members utilizing lower order solid hexahedral elements including assessment of the effect of reinforcement together with bond slip. Ultimate purpose of this research is to make feasible the detail analytical study of the behaviour of the reinforced concrete members through their entire elastic and inelastic ranges using non linear material properties as well as failure criteria of concrete, of course incorporating cracking phenomenon. Further, the utility of the analytical model may be verified from the extensive experimental investigations to establish its true potentiality.

3 FE Formulation

3.1 Concrete

Concrete is considered as the most important structural material in the last century in many areas of civil engineering beginning from buildings to bridges, offshore structures, storage structures etc. It consists of hardened cement paste with aggregates embedded in it. Hence it is highly heterogeneous medium and as a result behaviour is very complex. A considerable effort has been given by the various investigators to model the constitutive laws of concrete under different loading and unloading conditions at different stress levels. However in this paper, initial effort has been given only to model concrete at very low stress level. Hence concrete may assumed to behave linearly elastic and isotropic even in multi-axial stress states for all engineering purpose. From this standpoint only two material parameters are required viz. Young's modulus (E)and Poisson's ratio(μ) for finite element modeling of concrete /parent material of reinforced concrete structures. A classical displacement based isometric formulation is followed with three translational degrees of freedom at each node of 8-noded solid hexahedral elements to model the parent material (concrete) of the reinforced concrete. Using the standard elasticity matrix for the parent material D_P, strain displacement matrix B_P, 3D transformation matrix , volume considered V_P, P is the subscript to denote the parent material and their usual inter-relationships for the continuum in 3D stress state, the element stiffness is derived in a very straight-forward way using the above relationships as

$$K_P^e = \sum_P B_P^T \left[T_{\varepsilon,gl}^T\right] D_P \left[T_{\varepsilon,gl}\right] B_P dV_P. \tag{1}$$

The following is the shape or interpolation function $\left\{N_i^P\right\} = \frac{1}{8}(1+\xi.\xi_i)(1+\eta.\eta_i)(1+\varsigma\varsigma_i)$, $i = 1$ to 8 and ξ, η, ς being the intrinsic co-ordinates of the element for the 8-noded Serendipity (parent) element have been utilized for the purpose. However, the element stiffness matrix (size 24x24) formulated thus can not infer about the internal stresses set up due to it's inability to represent the state of pure bending strains and due to fictitious inclusion of large shear strains (parasitic shear). This effect of parasitic shear strain together with volumetric locking becomes significant with large aspect ratio of the element and hence structural response (deflection) is grossly underestimated as well as become dependent on mesh design. Several methods have been proposed to remove this deficiency of linear 8-noded isoparametric solid elements. Here an enhanced strain formulation proposed by Sousa et al. (2002) is incorporated based on extra compatible modes of deformation which don't have physical meaning and are eliminated at the element level by static condensation method. In particular element is designated as HCiS18, where 18 nos. of new extra variables are associated in addition to the usual strain field and the augmented strain matrix becomes

$$\left\{\varepsilon_P'\right\} = \left\{\varepsilon_P\right\} + \left\{\varepsilon_\alpha\right\} = \left[B_P \quad B_\alpha\right] \begin{Bmatrix} w_P \\ w_\alpha \end{Bmatrix} \tag{2}$$

With

$$N_\alpha = \frac{1}{2}(1-\xi^2)(1-\eta^2)(1-\varsigma^2) \tag{3}$$

is the bubble function, the enhanced part of the strain matrix becomes

$$B_\alpha = \frac{|J_0|}{|J|} T_0 B_\alpha^{18} \tag{4}$$

Where J_0 and J is the Jacobian determinant evaluated respectively at $\xi = \eta = \varsigma = 0$ and at each Gauss points, T is the transformation matrix and $\left[B_\alpha^{18}\right]$ is obtained from the Sousa et al. (2002).

With this enhanced strain components, the size of the element stiffness matrix becomes 42×42, which is reduced to 24×24 by static condensation. It is well established and has also proved its worth in evaluating the performance of the reinforced concrete structures in the present investigation. To demonstrate the accuracy and potentiality of the modified element, finite element solution of simply supported RC beams are compared in the case study and discussion.

3.2 Embedded modeling of Reinforcements :

The straight reinforcement bars are modeled utilizing classical embedded approach proposed by Elwi and Hrudey (1989), Cheng and Fan (1993) and Hartl et al. (2000), where the same displacement field of the parent element is assigned. Hence in the structural domain, the reinforcement layout remains independent of element mesh. The only requirement is to identify the elements with reinforcement(s) and their sectional properties together with its orientation, which may be taken care of by a preprocessing subroutine. Once it is identified it becomes very simple to handle problems of three-dimensional RC structures in perfect bond situations.

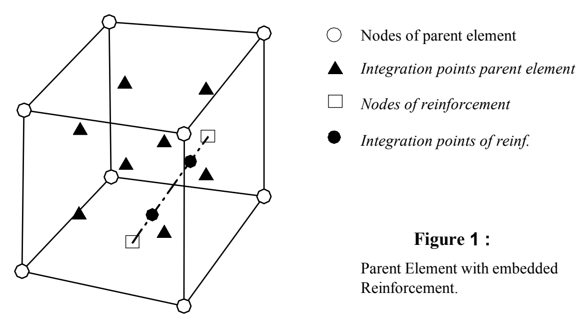


Figure 1 :
Parent Element with embedded Reinforcement.

The stiffness of the reinforcements is calculated as one-dimensional elements embedded in the space of parent element and is then super-imposed on the stiffness of the parent element. The reinforcement end nodes are generated independently within the respective parent element. The same strain displacement matrix B_P (used for the parent element) is utilized to evaluate the stiffness of the reinforcements. Since the reinforcement is considered as one-dimensional, the stiffness (integration) is to be evaluated along the path of the reinforcement(s). In order to integrate the stiffness contribution of the reinforcement(s) the strain displacement matrix has been computed at the respective gauss point(s) of the reinforcements expressed in terms of the intrinsic coordinates of the parent element. A Newton root finding algorithm in 3D is used for this purpose, where the known integration points of reinforcement in global coordinates are computed in local coordinates using an inverse mapping procedure based on iterative method by Barzegar and Madipuddi (1994). Thus the stiffness contribution of reinforcement towards the element becomes

$$K_R^e = \sum_{RB} B_P^T T_{\varepsilon,gl}^T D_R T_{\varepsilon,gl} B_P dV_R, \quad (5)$$

Where D_R is the elasticity matrix for the reinforcement in local coordinates, RB is the number of reinforcement elements within the parent element and R is the subscript used to denote reinforcement.

When the integration points within the local coordinates are known, the element stiffness matrix may be computed simply by adding equation (1) and (5).

3.3 Modeling of Bond-Slip :

In the previous stiffness formulation, a rigid interconnection between the contact surfaces of the concrete and the reinforcement is assumed. Most interestingly this perfect bonding is not true throughout the loading history of a reinforced concrete structures. And hence a need for consideration of relative movement between the mating surfaces becomes important for such structures. This relative movement is known as bond slip. The perfect bonding of the reinforcements with the concrete over-predicts the shear transfer and this lead to an over or under estimation of the response of the structure depending on specific situation.

In finite element method, bond-slip can be modeled in a conventional way by means of interface elements. In this respect, bond-link element was introduced by Ngo & Scordelis (1967) at first, later on bond-zone element was introduced Groot(1981) and subsequently contact elements by Mehlhorn(1987). As per the literature review, recommendations are given to modify the constitutive law of either concrete or reinforcements, which can't be implemented within the embedded approach. G. Beer (1985) introduced an elegant way of continuous interface element and subsequently by H. Hartl et al. (2002), where bond-slip is accounted for at the material level by introducing interface elements supplementary between reinforcement and concrete after the displacement field has been computed based on rigid bond condition. Then the steel stress is relaxed due to bond-slip.

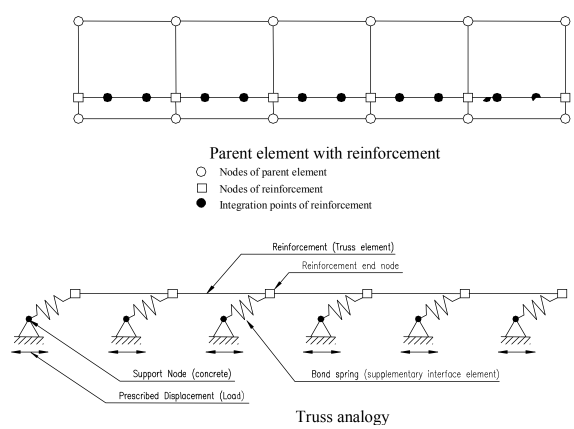


Figure 2 : Supplementary Interface Model

The basic concept of supplementary slip algorithm is similar to that of a truss analogy (refer Fig. 2), when the reinforcements are embedded in a classical way in the parent element without slip D.O.F. The truss members are the reinforcements and the supports are the concrete. The end points of the reinforcements are connected to the psudo node on the concrete treated as support by bond spring, which are considered as continuous interface elements. Once the global displacement field is known, the strains along the reinforcement may be integrated and the same are referred as prescribed displacement of the supports. These support displacements get transferred to the end points of the reinforcements depending on the characteristic property of the bond spring.

Thus the relative displacement of the reinforcement support node and the adjacent reinforcement end node is referred as bond-slip. The difference of the reinforcement force computed thus with respect to the same considering perfect bond are mapped back as residual nodal forces to the parent element.

In order to calculate slip, truss model is analyzed considering the stiffness of the reinforcement as

$$K_R = A_R \sum_l B_R^T E_S B_R dl_R, \quad (6)$$

Where E_S is the elasticity modulus of reinforcements (for elastic analysis), B_R is the strain displacement matrix for 1D reinforcement and A_R is the cross sectional area of each reinforcements.

With B_j is the strain displacement matrix for the joint element, the stiffness of the continuous interface element as

$$K_j = \sum_S B_j^T k B_j dS \quad (7)$$

Where k is the interface stiffness depending on slip, calculated from bond-slip relationship (Fig. 3) as per Modelcode-90 [MC90] given by the following equation.

$$k = \frac{\tau_{\max}}{s_1^{\alpha}} \alpha s^{\alpha-1} \quad (8)$$

α, s, $\tau_{\max}$ are reinforcement-concrete interface properties depending on bond condition and confined/unconfined concrete.

It is to be noted that here an iteration is a must to obtain a convergent value of tangent stiffness as reinforcement-concrete interface behaviour is non-linear from the beginning of loading even when both concrete and steel remains in the elastic range. The end conditions are specified in terms of prescribed displacements as Dirichlet boundary condition. Once these displacements at the free nodes are calculated, the same set of equations are again solved for the revised slip until a good convergence is obtained with sufficient accuracy and then the relative displacements of the nodes along with the steel stress due to bond slip are calculated. Finally this stress is mapped back as residual nodal forces of the respective parent element.

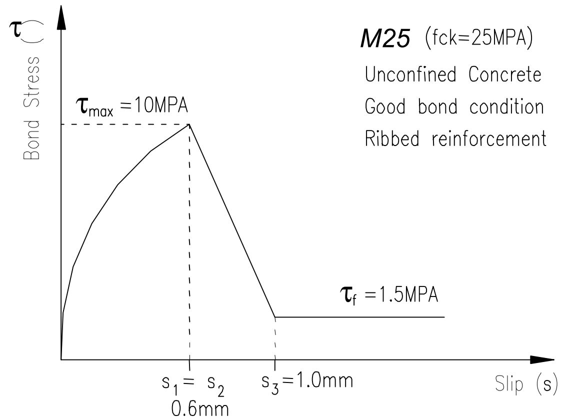


Figure 3 : Bond-Slip Relation (MC90)

4 Investigated Case Studies :

4.1 Analytical Application :

The formulation followed has been implemented with FORTRAN computer program [12, 13, 14]. A single span simply supported RC beam with the geometrical properties as shown Fig. 6 is investigated, which is subjected to only uniformly distributed load (=w) of 1.6 t/m-run over the entire span. The beam is 5.0m long (=L) and the cross section 200(b) $\times$ 400(D) with 2 nos. 20dia ordinary ribbed reinforcing steel(= A_{st}) placed at 40mm(=d') above the bottom. The concrete has the characteristic strength of 25Mpa, elastic modulus E_c = 25000Mpa, Poisson's ratio μ = 0.15 and the reinforcement bar has the elastic modulus E_s = 200000 MPa. It is assumed that concrete is unconfined as per the considered configuration and the bond condition is good. Accordingly the bond-slip parameters are assumed as $s_1 = 0.6$mm, $\alpha = 0.4$ and $\tau_{\max} = 2.0\sqrt{f_{ck}} = 10.0MPa$.

Both the parent material /concrete and the reinforcement are assumed to be linear elastic within the specified load range. The mesh of 50 elements of size 200 cube is generated with a preprocessing subroutine for the parent material, where 8-noded solid isoparametric elements with EAS formulation are implemented. With the supplied end points /profile of the reinforcements in the beam, the reinforcement mesh is also generated within each element for which stiffness contribution is

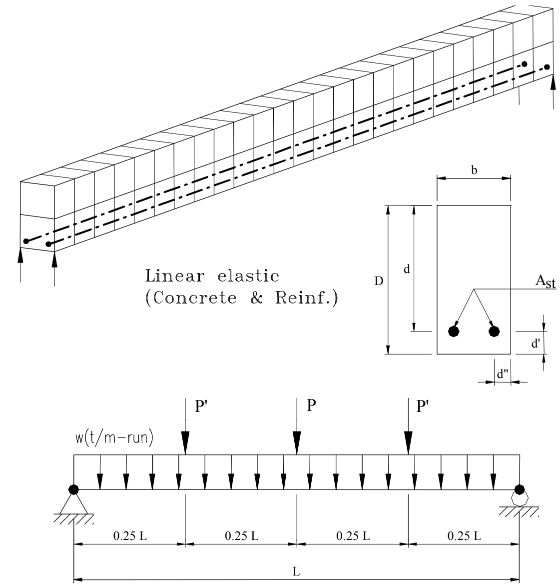
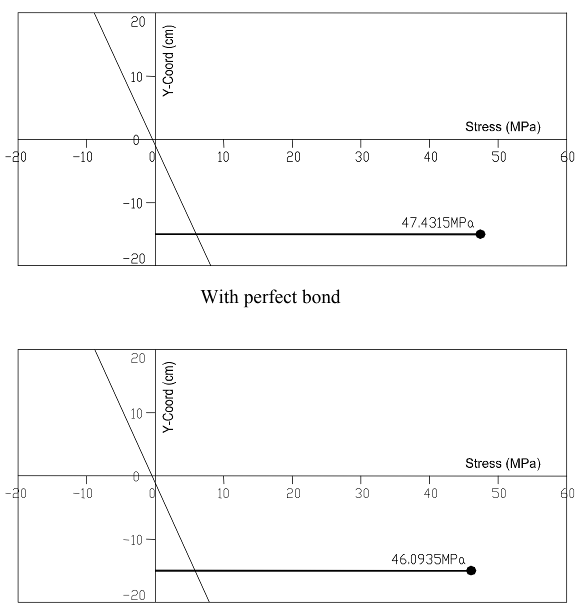


Figure 4 : Single Span RC Beam System



With perfect bond

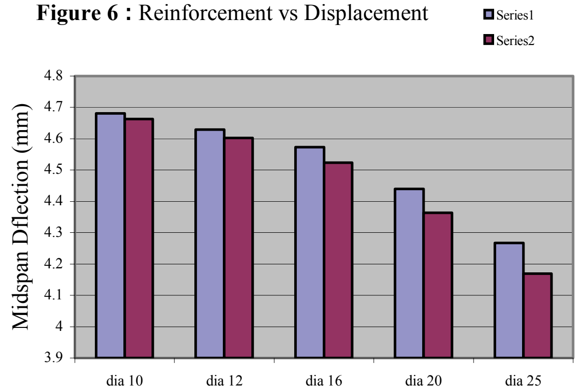
With bond-slip

Figure-5 : Stress Distribution Diagram (linear elastic)

added to the stiffness of the parent element as per equation (1) and (5).

Considering the material perfectly elastic and linear within the load range, the above sample problem was solved and the following stress diagram

Figure 6 : Reinforcement vs Displacement



(Fig. 5) has been obtained. One of the main objectives was to assess the performance of the element HciS18 in evaluating the bending situations considering incompressibility. With the following illustration of single span RC beam, it has been seen from the tab. 1 and Fig. 6 that the element shows results with 1.11% error only with element size 200x200x200.

Table 1: Comparison of Deflection & Stress

Description of Items		ϕ25-2Nos. (A)	ϕ25-2Nos. (B)	ϕ20-2Nos. (A)	ϕ20-2Nos. (B)	ϕ16-2Nos. (A)	ϕ16-2Nos. (B)	ϕ12-2Nos. (A)	ϕ12-2Nos. (B)	ϕ10-2Nos. (A)	ϕ10-2Nos. (B)
Mid-span deflection (mm)		4.2673	4.1686	4.4394	4.3640	4.5725	4.5238	4.6288	4.6023	4.6802	4.6625
Maxm. Stress at mid-span (Mpa)	Top-most layer	8.6892	8.5804	8.8639	8.7860	8.9895	8.9357	9.0346	9.0053	9.1299	9.1098
	Bottom-most layer	7.6446	7.3662	8.1195	7.9161	8.5299	8.3891	8.6980	8.6205	8.8567	8.8043
	Reinforce - ment	44.0173	42.4437	47.4315	46.0935	50.1655	49.2387	51.2908	50.7806	52.2638	51.9191

(A) : Without considering bond-slip
(B) : With considering bond-slip

The accuracy of the prediction in deflection using the same element has also been checked by considering a cross section 200mm × 400mm for various span, with and without a consideration for bond slip. As far as the bond slip is concerned, the scheme of the supplementary interface algorithm starts from the perfect bond analysis of reinforcement strains, which is obtained from global solution. Thus the stress is overestimated at the nodes which experiences highest strain increment. With the present iteration of the supplementary interface algorithm, due to continuous unloading no problem is encountered as long as reinforcement stress does not exceed its elastic limit.

It may be inferred that the element type shows good results even with coarser meshes, being almost accurate as best as the analytical case. So the same may be used for other engineering purposes. As the present study provides a basic step towards the better understanding of a very complex behaviour of reinforced concrete structure in its entire range of stress history before it collapses, these analysis results in its linear elastic regime may be considered as milestone towards the same. The following model has also been verified with some of the experimental works.

4.2 *Experimental verification :*

Three simply supported RC beams have been investigated from various references in the literature to validate the present model in the elastic regime. The first example has been taken from the test series of Shegg and Decanini, 1971 (mkd as RC-75-1) referred by Gomesand Awruch (2001), second one from the experiment reported by Burns, et al. 1966 (mkd as Burn-Siess beam) referred by Cho and Hotta (2002) and the third one by Bresler and Scordelis, 1963 (mkd as beam A-1) referred by Kwak and Filippou (1997). All these experiments were performed to obtain better understanding of load-deflection behaviour of RC beams loaded to failure level primarily. The maximum load in the linear range has been considered as equal to 25% of the failure load as reported by the literature. As shown in Fig. 4, these beams have only two numbers tensile reinforcement, but no longitudinal compressive or transverse shear reinforcements. The geometry, reinforcement details, finite element mesh and material properties are noted in the tab. 2 which shall be read in conjunction with Fig. 4.

Tab. 2 also includes the values of midspan deflections from the present model to compare the same with experimental observations for specific values of the loads from the literature. In order to study the effect of finite element mesh on present model with enhanced strain approach, two different categories of mesh configurations were considered. The comment in this regard is exactly same as done in case of pure analytical cases. Coarser meshes are producing better result, in fact more close the experimental values.

Table 2: Geometrical and Material properties used

Beam mkd		RC-75-1		Burn-Siess beam		Beam : A - 1	
b (mm)		153		152.4		305	
D (mm)		246		304.8		553	
L (mm)		3,000.0		2,743.2		3,677.0	
d' (mm)		25		50.8		63	
d" (mm)		25		25		40	
"w" - self wt (ton/m)		0.094		0.116		0.422	
P (ton)		0.00		1.00		11.25	
P' (ton)		0.78		0		0	
A_{st} (cm^2)		2.35		2.65		10.20	
F_{ck} (MPa)		31.1		18.2		24.5	
E_c (MPa)		30,653		21,000		23,674	
μ		0.15		0.19		0.17	
F_y (MPa)		550		310		566	
E_s (MPa)		200,000		155,000		222,180	
Midspan deflection (mm)	Experimental	1.250		0.690		1.275	
	Mesh size (b x d x L)	1 x 2 x 20	2 x 3 x 40	1 x 2 x 18	2 x 4 x 36	1 x 2 x 12	2 x 4 x 24
	Present study	1.139	1.077	0.661	0.649	1.213	1.177
Error (%)		8.89	13.88	4.14	5.93	4.83	7.66

5 Conclusion

The finite element formulation for the elastic analysis of simply supported RC beam based on standard linear hexahedral element has been presented, due to its simplicity in terms of easy mesh generation and data interpretation. Since this particular category of element exhibits some well-known deficiencies, it has been modified with the inclusion of enhanced strain modes. The performance of this new enhanced strain element is very similar to the higher order element. It has been found that the lower order elements modified thus, is extremely efficient and effective in the analysis of three-dimensional problems.

This model also includes the discrete presence of the reinforcement in arbitrary direction without affecting the parent element mesh. It has been shown that the concept of this FEM model (for incompressible situations) which includes the presence of the reinforcement in perfect bond condition and relaxed stress condition using bond-slip relation as per modelcode90 and its mathematical derivation is very simple and economical in terms of time consumption in terms of analysis efforts compared to other generalized methods. The validity of the formulation is verified by analyzing a few examples. It could be said that this model may work well for such reinforced concrete systems, where stiffness contribution of reinforcements are taken into account.

It has also been shown that the iterative scheme of the supplementary slip algorithm starts with the perfect bond predictions for reinforcement strains, which is obtained from the global solutions. Thus stresses are overestimated at the regions of the parent element domain, which experiences the highest strain. The approach causes no problem as long as stress in the reinforcement does not exceed the elastic limit within the iterative scheme. Since within the embedded approach, reinforcements are not restricted to the parent element nodes, the computational effort is reasonable. But when bond-slip is taken into account it requires higher time consumption. However this is becoming of irrelevant with the availability of high-speed computers. At the same time this model provides accurate representation of deformation and internal stress distribution.

Further to this effort of linear elastic 3D analysis reinforced concrete structures, this model may be upgraded to solve prestress concrete structures and may be extended to the non-linear regime too, which can include specific phenomenon such as cracking, shrinkage, creep and material anisotropy in higher load ranges. On the other hand, a verification of the results of the proposed analytical model in the non-linear range is important and will be done relating the output of the program to the experimental data. Finally, different parametric studies may also be included.

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