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Micro-macro Approaches Coupled to An Iterative Process for Nonlinear Porous Media

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Abstract: An iterative homogenization approach is proposed in order to predict the nonlinear hydro-mechanical behaviour of porous media. This process is coupled to classical and modified secant extended methods and linear homogenization predictive schemes. At convergence of the iterative process, same equivalent behaviour is obtained for any secant method, any simplified homogenization used for the linear comparison material and for any initial porosity of the media. An application to the study of the nonlinear behaviour of clayey sediments is presented. The model parameters quantification is based on oedometric experimental results for different clays.

keyword: Nonlinear homogenization, Iterative process, Hydro-mechanical behaviour, Clayey sediments.

1 Introduction

Biot (1941) and then Coussy (1995) have proposed a rigourous thermodynamical framework for the modelling of hydro-mechanical behaviour of porous media. This theory assumes that the porous medium can be represented by two continuous media in interaction : a solid phase which constitutes the skeleton and a fluid phase which saturates the pores. Macroscopic constitutive laws can so be derived from this formalism for linear and nonlinear porous materials, Biot (1973), Coussy (1995).

In this paper we adopt a micromechanical approach which allows us to take into account the geometry of the microstructure and the mechanical properties of the solid phase constituting the skeleton in the overall behaviour of the porous medium. An elementary volume is chosen as to be representative of the porous medium and is submitted to an hydro-mechanical uniform loading characterized by a macroscopic strain and a fluid pressure. The resolution of the induced local problem posed on this volume gives the generated local fields. Then an adequate averaging scheme allows us to link the microscale to macroscale and to determine the overall response of the porous medium.

Various averaging schemes have been developped for saturated porous media with linear skeleton. They are mainly based on homogenization works devoted to the prediction of the effective properties of composites in which the phases have a linear behaviour. Auriault and Sanchez-Palencia (1977) have so used the framework of the homogenization theory of periodic media to predict the poro-elastic behaviour of materials with periodic patterns. In this approach the local problem must be solved by finite element method, Devries, Dumontet, Duvaut, and Léné (1989). This numerical homogenization makes it possible to simulate the global stress-strain responses of nonlinear materials, for example nonlinear viscoelastic composites or damaged materials, Zhang and Xia (2005). This approach is also essential to estimate the effective properties of materials which exhibit an ordered complex microstructure like textile reinforced materials, Haasemann, Kastner, and Ulbricht (2006). For disordered microstructure, various explicit approaches based on Eshelby's result, Eshelby (1957), for a review refer to Aboudi (1991), Bornert, Bretheau, and Gilormini (2001), and analytical solutions of the local problem, can also be applied to predict the linear poro-elastic behaviour of satured porous media. These linear homogenization schemes lead to different predictions following the assumed representation of the microstructure. They might not be sufficiently accurate, especially in the case of high contrast between the phase properties like in porous materials or in the case of significant porosities.

Micromechanical formulations for nonlinear porous materials have more recently received attention, Suquet (1997), Ponte-Castaneda and Suquet (1998), Bornert, Bretheau, and Gilormini (2001). These modelling are too often obtained as extensions of linear homogenization methods. In a first stage the nonlinear local equations are

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linearized by affine or secant formulations for example. The tangent or secant stiffness tensors depend to local strains in the skeleton which generally evolve the overall load. At each step of loading, complementary relations are then introduced in order to replace the local strains with averaged strains following the concept usually referred to as linear comparison material, Ponte-Castaneda (1991). Various linearization procedures have been proposed like the affine approach, Masson, Bornert, Suquet, and Zaoui (2000), second order affine procedures, Ponte-Castaneda and Suquet (1998), Bornert, Masson, Ponte-Castaneda, and Zaoui (2001), incremental, Hill (1965a), the classical secant method, Chu and Hashin (1971), Berveiller and Zaoui (1979), Weng (1990) or second order secant, called modified secant method, Suquet (1997), and some of their variants, Bardella (2003), Qiu and Weng (1992). The second stage consists in solving at each step of loading the linearized local problem by an usual linear homogenization method appropriate for the geometry of the microstructure. The relevance and the performance of the nonlinear effective properties of the porous media so obtained depend on both approximations induced by the linearization procedure and by the linear homogenization scheme. Comparisons of these different procedures have been made, see for example, Suquet (1997) and Rekik, Bornert, Auslender, and Zaoui (2005).

In this work we propose an iterative homogenization process which gives consistency to some secant homogenization procedures for random porous material by leading to a same prediction of the effective behaviour. In previous work this iterative process has been successfully proposed to predict the behaviour of linear reinforced composites, Benhamida and Dumontet (2003) and porous materials, Benhamida, Djeran-Maigre, Dumontet, and Smaoui (2005). Here it is extended in the nonlinear domain in order to predict the hydro-elastoplastic behaviour of porous materials.

The iterative process is based on an iterative process of homogenization which consists in building Representative Elementary Volume of the porous material, by adding low porosities gradually to the skeleton, until reaching the final porosity of clay, according to a method close to differential scheme, Zimmerman (1991). At a given stage of the process, the behaviour of the porous intermediate media, if it is linear elastic, can be obtained by any simplified homogenization method and then becomes the skeleton of the following step. The equivalent homogeneous behaviour of the porous material is then obtained at the convergence of the process. The introduction of this process leads to the same equivalent behaviour whatever the used homogenization method is and this is valued even for significant porosities, Benhamida and Dumontet (2003). One can use for example the approach of the diluted distributions, Aboudi (1991), the self-consistent method, Hill (1965a), the concentric spheres, Christensen and Lo (1979), the Hashin's bounds, Hashin and Shtrikman (1963) or even the morphological representative pattern, Hervé and Zaoui (1993). This convergence can be explained by the use at each iteration of the predictive homogenization models with low porosities where the various estimates coincide. This iterative homogenization is coupled here with the secant methods of nonlinear homogenization. Two secant procedures have been considered, the classical secant method, Berveiller and Zaoui (1979), where the local strain in the skeleton is simply averaged on the phase or secant modified method, Suquet (1997), where a second-order moment of the averaged strain is used. We show that these classical or modified secant approaches coupled to iterative process lead to the same nonlinear behaviour for all rates of porosities whatever the selected simplified homogenization approach is.

This approach is here applied to the study of the nonlinear hydro-mechanical behaviour of compacted clays. The model parameters quantification is based on oedometric experimental results obtained with an original oedometric cell, Grunberger, Djeran-Maigre, Velde, and Tessier (1994), Djeran-Maigre, Tessier, Velde, and Vasseur (1998). This cell allows us to apprehend the three-dimensional phenomena by measuring simultaneously the lateral strain and the pore pressure. From these measurements of compaction, we identify the geometry and mechanical parameters of the micromechanical model. A nonlinear behaviour is chosen with power law dependency for the skeleton and the microstructure of clays is modelled by spherical pores.

After having recalled in section 2 the principle of nonlinear homogenization methods, we describe in section 3 the iterative process and its coupling with the secant approaches. We present finally in section 4 the application of the modelling to the prediction of the linear and nonlinear hydroelastic behaviour of clayes.

2 Recalls of nonlinear homogenization

The porous medium considered here is supposed to be made up of a porous phase embedded into a connected solid skeleton. The skeleton is supposed to have a nonlinear elastic behaviour. The porous phase is saturated with a fluid. Small deformations are assumed. Let Y be a Representative Elementary Volume of this porous medium. This volume consists of a solid part Y_S occupied by the skeleton and of a part Y_V which represents the pores of the medium. The interface between the skeleton and the pores is noted by Γ_v and the external boundary of Y by ∂Y. In a strain approach of homogenization, the representative volume is subjected to an hydro-mechanical macroscopic loading in terms of a mechanical strain $\boldsymbol{E}$ and a uniform pressure p. This loading generates at the microscale local fields of displacement $\boldsymbol{u}(y)$, strain $\boldsymbol{\varepsilon}(y)$ and stress $\boldsymbol{\sigma}(y)$ solutions of a cellular problem, which are written after secant linearization, in the following way :

$$\begin{aligned}
&\nabla.\boldsymbol{\sigma}(y) = 0, \forall y \in Y,\\
&\boldsymbol{\sigma}(y) = \boldsymbol{C}_s^{sct}(\boldsymbol{\varepsilon}(y)) : \boldsymbol{\varepsilon}(y), \forall y \in Y_s,\\
&\boldsymbol{\varepsilon}(y) = \frac{1}{2}(\nabla \boldsymbol{u}(y) + \nabla^T \boldsymbol{u}(y)), \forall y \in Y_s,\\
&\boldsymbol{\sigma}(y).\boldsymbol{n} = -p\,\boldsymbol{n}, \forall y \in \Gamma_v,\\
&<< \boldsymbol{\varepsilon}(y) >>= \boldsymbol{E},
\end{aligned} \tag{1}$$

where $\boldsymbol{C}_s^{sct}$ denotes the secant tensor of the skeleton which depends on the local deformation, $\boldsymbol{n}$ the unit normal at the considered boundary, the double dot products : is the product of a fourth-order tensor with a second-order tensor, the single dot . is the product of a second-order tensor with a vector and the notation $(\boldsymbol{u} \otimes \boldsymbol{n})_S$ denotes the symmetrized tensorial product of the displacement defined by :

$$(\boldsymbol{u}(y) \otimes \boldsymbol{n})_S)_{ij} = \frac{1}{2}(u_i n_j + u_j n_i). \tag{2}$$

Due to the difficulty of defining the strain in the pores, it was here necessary to introduce a specific strain average defined by :

$$<< \boldsymbol{\varepsilon}(y) >>= \frac{1}{|Y|}\int_{\partial Y} (\boldsymbol{u}(y) \otimes \boldsymbol{n})_S dS = \boldsymbol{E}. \tag{3}$$

The equivalent homogeneous behaviour is then defined by the following relation between strain-stress averages, Dormieux, Molinari, and Kondo (2002) :

$$\boldsymbol{\Sigma} = < \boldsymbol{\sigma}(y) >= \boldsymbol{C}^{hom}(\boldsymbol{E}) : (\boldsymbol{E} - p\boldsymbol{B}(\boldsymbol{E})), \tag{4}$$

where $\boldsymbol{C}^{\mathrm{hom}}$ denotes the equivalent stiffness tensor, $\boldsymbol{B}$ the Biot tensor where both depend on the macroscopic loading and the simple brackets $< . >$ denote the classical average on Y defined by :

$$< \boldsymbol{\sigma}(y) >= \frac{1}{|Y|}\int_Y \boldsymbol{\sigma}(y)\, dy, \tag{5}$$

with $\boldsymbol{\sigma}(y)$ the stress solution of the local problem (1).
In a similar way, the stress homogenization approach consists in imposing to the Representative Elementary Volume a macroscopic stress loading $\boldsymbol{\Sigma}$ instead of the macroscopic strain $\boldsymbol{E}$. The effective behaviour is then defined by the following relation :

$$<< \boldsymbol{\varepsilon}(y) >>= \boldsymbol{S}^{hom}(\boldsymbol{\Sigma}) : \boldsymbol{\Sigma} + p\,\boldsymbol{S}^{hom}(\boldsymbol{\Sigma}) : \boldsymbol{B}(\boldsymbol{\Sigma}), \tag{6}$$

where $\boldsymbol{\varepsilon}$ is the solution of the local problem with stress loading and $\boldsymbol{S}^{hom}$ the equivalent compliance tensor. Generally, even in linear case, the two approaches don't lead to the same behaviour, $\boldsymbol{S}^{hom}$ and $\boldsymbol{C}^{\mathrm{hom}}$ don't be inverse and consequently the two Biot tensors differ.
Approximations of these effective properties can be explicitly obtained by introducing the concept of linear comparison composite, Ponte-Castaneda (1991), and by exploiting the classical homogenization methods of linear elasticity. We assume in order to simplify the developpements that the skeleton has a linear behaviour for purely hydrostatic loading and a nonlinear behaviour in shear. The skeleton behaviour is also considered isotropic, so the secant tensor of the skeleton is written as :

$$\boldsymbol{C}_s^{sct}(\boldsymbol{\varepsilon}(y)) = 3k_s\boldsymbol{J} + 2\mu_s^{sct}(\varepsilon_{eq}(y))\,\boldsymbol{K}, \tag{7}$$

where k_s denotes the bulk modulus supposed to be constant, μ_s^{sct} the secant shear coefficient of the skeleton which evols with the local equivalent strain ε_{eq} defined by $\varepsilon_{eq}(y) = \sqrt{\frac{2}{3}e_{ij}(y)\,e_{ij}(y)}$ with $\boldsymbol{e}(y)$ the deviatoric part of the strain tensor given by $\boldsymbol{e}(y) = \boldsymbol{\varepsilon}(y) - \frac{1}{3}(tr\boldsymbol{\varepsilon})\mathbf{I}$, with $\mathbf{I}$ the second order identity tensor. $\boldsymbol{J}$ and $\boldsymbol{K}$ are the fourth-order tensors defined by :

$$J_{ijkh} = \frac{1}{3}\delta_{ij}\delta_{kh},\; K_{ijkh} = \frac{1}{2}(\delta_{ik}\delta_{jh} + \delta_{ih}\delta_{jk}) - J_{ijkh}, \tag{8}$$

with δ_{ij} the components of $\mathbf{I}$.
The strain classical secant approach, Berveiller and Zaoui (1979), consists in approximating the nonlinear behaviour of the cellular problem (1) with (7) by the linear

comparison material with the stiffness tensor :

$$\boldsymbol{C}_s = \boldsymbol{C}_s^{sct}((<\boldsymbol{\varepsilon}(y)>_{Y_S})_{eq}), \tag{9}$$

where the brackets $< . >_{Y_S}$ indicate here the classical average of the strains taken on the skeleton part Y_S.

In the strain modified secant method, Suquet (1997), the behaviour law uses the second moment order of the equivalent local strain :

$$\boldsymbol{C}_s = \boldsymbol{C}_s^{sct}(\sqrt{<\varepsilon_{eq}^2(y)>_{Y_s}}). \tag{10}$$

The homogenized stiffness tensor $\boldsymbol{C}^{\text{hom}}$ can be then defined by the relation :

$$\boldsymbol{C}^{hom}(\boldsymbol{E}) = \boldsymbol{C}_s - \phi\, \boldsymbol{C}_s : << \boldsymbol{A}(y) >>_{Y_v}, \tag{11}$$

where $\boldsymbol{C}_s$ denotes the stiffness tensor of the linear comparison porous media defined by (9) or (10) according to the adopted secant approach, ϕ is the porosity of material and $\boldsymbol{A}(y)$ is the localization tensor defined by :

$$<< \boldsymbol{A}(y) >>_{Y_v} : \boldsymbol{E} = \frac{1}{|Y_v|} \int_{\Gamma_v} (\boldsymbol{u}(y) \otimes \boldsymbol{n})_S\, dS, \tag{12}$$

where $\boldsymbol{u}(y)$ is the displacement solution of the linearized cellular problem (1) with the laws (7)-(9) or (7)-(10) without pore pressure ($p = 0$). The Biot tensor is obtained then by the following relation, Dormieux, Molinari, and Kondo (2002) :

$$\boldsymbol{B}(\boldsymbol{E}) = \boldsymbol{I} - \boldsymbol{C}_s^{-1} : \boldsymbol{C}^{hom} : \boldsymbol{I}. \tag{13}$$

In a similar way, the linearization of the stress homogenization approach leads with classical secant method to the compliance tensor :

$$\boldsymbol{S}_s = \boldsymbol{S}_s^{sct}((<\boldsymbol{\sigma}(y)>_{Y_S})_{eq}), \tag{14}$$

and with modified secant method :

$$\boldsymbol{S}_s = \boldsymbol{S}_s^{sct}(\sqrt{<\sigma_{eq}^2(y)>_{Y_s}}), \tag{15}$$

where $\boldsymbol{S}_s^{sct}$ denotes the secant compliance tensor of the skeleton, σ_{eq} is the Von-Mises equivalent stress defined by $\sigma_{eq}(y) = \sqrt{\frac{2}{3} s_{ij}(y)\, s_{ij}(y)}$ and $\boldsymbol{s}(y)$ is the deviatoric part of the stress tensor. The homogenized compliance tensor $\boldsymbol{S}^{\text{hom}}$ can be then defined by the relation:

$$\boldsymbol{S}^{hom}(\boldsymbol{\Sigma}) = \boldsymbol{S}_s - \phi\, \boldsymbol{S}_s : << \boldsymbol{D}(y) >>_{Y_v}, \tag{16}$$

where $\boldsymbol{D}(y)$ is the concentration tensor defined by :

$$<< \boldsymbol{D}(y) >>_{Y_v} : \boldsymbol{\Sigma} = \boldsymbol{S}_s^{-1} : \frac{1}{|Y_v|} \int_{\Gamma_v} (\boldsymbol{u}(y) \otimes \boldsymbol{n})_S\, dS, \tag{17}$$

where $\boldsymbol{u}(y)$ is the displacement solution of the linearized cellular problem (1) in stress approach with the laws (7)-(14) or (7)-(15) without pore pressure ($p = 0$). The Biot tensor in stress approach is obtained then by the following relation :

$$\boldsymbol{B}(\boldsymbol{\Sigma}) = \boldsymbol{I} - (\boldsymbol{S}^{hom})^{-1} : \boldsymbol{S}_s : \boldsymbol{I}. \tag{18}$$

For simplified geometry of the REV with spherical or ellipsoidal pores, Ehselby's results, Eshelby (1957), can be used to solve the linearized local problems at each step of loading with explicit developpements. The classical linear approximations of the homogenized coefficients for the elastic comparison porous media, such as the diluted approximations in strain or stress approaches, the self-coherent scheme, the Mori Tanaka's method, or the Hashin-Shtrikman bounds, allow then to estimate the nonlinear homogenized behaviour of the porous media, Aboudi (1991), Bornert, Bretheau, and Gilormini (2001). However, these estimates have same inherent limitations of the simplified homogenization methods in linear elasticity, which coincide for low porosities, but quickly diverge and can become not exploitable for significant porosities. Moreover, these approaches often give a very imprecise estimate of local fields and remain not easily usable in local criteria of damage. The classical and modified secant methods lead in addition to different predictions for the nonlinear behaviour, Suquet (1997). To remedy these various limitations, we propose to couple the nonlinear homogenization presented above with an iterative process.

3 Iterative process

This iterative process is inspired in its principle by the homogenization method known as differential scheme, Norris (1985), McLaughlin (1977), proposed for the linear elastic porous media by Zimmerman (1991). The porous media is built by adding low porosities $\Delta\phi_j$ gradually to the skeleton until reaching the final porosity of the medium $\phi = \sum_{j=1}^{n} \Delta\phi_j$. At a stage (i) of the process, the porous material is then composed of a skeleton whose behaviour is that of the equivalent homogeneous medium of the preceding stage and whose porosity is given by :

$$\phi^{(i)} = \Delta\phi_i\, (\phi_s + \textstyle\sum_{j=1}^{i} \Delta\phi_j)^{-1}. \tag{19}$$

The behaviour of this intermediate porous medium is obtained by a classical homogenization method in linear elasticity and becomes that of the skeleton of the following stage $(i+1)$. At each step of loading, the equivalent homogeneous behaviour of the linear comparison porous medium is obtained with convergence of the succession of these intermediate homogenizations, the process being initialized with the secant mechanical properties of the solid part of a material. Thus, the stiffness tensor at an intermediate stage (i) is written :

$$\boldsymbol{C}_s^{(i)} = \boldsymbol{C}_s^{(i-1)} - \phi^{(i)} \boldsymbol{C}_s^{(i-1)} : << \boldsymbol{A}(y, \boldsymbol{C}_s^{(i-1)}, \phi^{(i)}) >>_{Y_V}, \tag{20}$$

where the localization tensor $\boldsymbol{A}$ differs following the linear schemes. For the diluted approximations, it is given by :

$$<< \boldsymbol{A}(y, \boldsymbol{C}_s^{(i-1)}, \phi^{(i)}) >>_{Y_V} = (\boldsymbol{I} - \boldsymbol{P}_s^{(i-1)} : \boldsymbol{C}_s^{(i-1)})^{-1} \tag{21}$$

with the polarization tensor $\boldsymbol{P}_s$ given by :

$$\boldsymbol{P}_s^{(i-1)} = \boldsymbol{S}_E^{(i-1)} : (\boldsymbol{C}_s^{(i-1)})^{-1}, \tag{22}$$

where $\boldsymbol{S}_E^{(i-1)}$ is the Esheby tensor of the skeleton phase given in the case of spherical pores by Mura (1987):

$$\boldsymbol{S}_E^{(i-1)} = p_E^{(i-1)} \boldsymbol{J} + q_E^{(i-1)} \boldsymbol{K}, \tag{23}$$

with the coefficients p_E and q_E :

$$p_E^{(i-1)} = \frac{1}{3} \frac{1+\nu^{(i-1)}}{1-\nu^{(i-1)}}, \qquad q_E^{(i-1)} = \frac{2}{15} \frac{4-\nu^{(i-1)}}{1-\nu^{(i-1)}}, \tag{24}$$

$\nu^{(i-1)}$ being the Poisson's ratio of the homogenized medium at the step $(i-1)$.
The localization tensor becomes with the upper Hashin-Shtrikman bound, Hashin and Shtrikman (1963) :

$$<< \boldsymbol{A}(y, \boldsymbol{C}_s^{(i-1)}, \phi^{(i)}) >>_{Y_V} = (\boldsymbol{I} - \phi^{(i)} \boldsymbol{P}_s^{(i-1)} : \boldsymbol{C}_s^{(i-1)})^{-1} \tag{25}$$

With a stress approach, the compliance tensor at the step (i) is written :

$$\boldsymbol{S}_s^{(i)} = \boldsymbol{S}_s^{(i-1)} - \phi^{(i)} \boldsymbol{S}_s^{(i-1)} : << \boldsymbol{D}(y, \boldsymbol{S}_s^{(i-1)}, \phi^{(i)}) >>_{Y_V} \tag{26}$$

with the concentration tensor given for the diluted approximations by :

$$<< \boldsymbol{D}(y, \boldsymbol{S}_s^{(i-1)}, \phi^{(i)}) >>_{Y_V} = (\boldsymbol{I} - \boldsymbol{Q}_s^{(i-1)} : \boldsymbol{S}_s^{(i-1)})^{-1} \tag{27}$$

and

$$\boldsymbol{Q}_s^{(i-1)} = (\boldsymbol{S}_s^{(i-1)})^{-1} : (\boldsymbol{I} - \boldsymbol{S}_E^{(i-1)}). \tag{28}$$

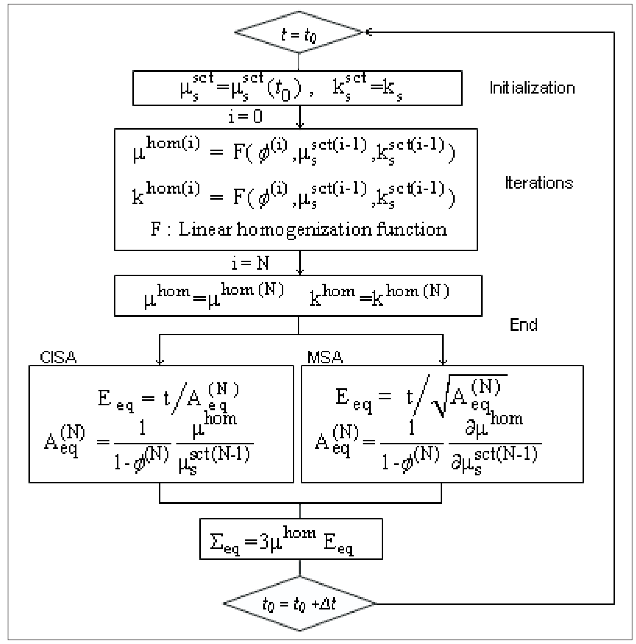

Figure 1 : Algorithm of the iterative process.

In practice, the construction of the nonlinear homogenized response involves two nested loops, one internal for the loading steps and one external for the iterative process, Figure 1. The local equivalent strain in the skeleton or equivalent stress in strain approach is here considered as a parameter t following the suggested method, Bornert, Bretheau, and Gilormini (2001), with $t = (< \varepsilon(y) >_{Y_S})_{eq}$ for the classical secant method (CLSA) and $t = < \varepsilon_{eq}^2(y) >_{Y_s}$ for the modified secant method (MSA).

4 Application to hydro-mechanical behaviour of clays

The application of the previous described micromechanical approach requires the knowledge of mechanical be-

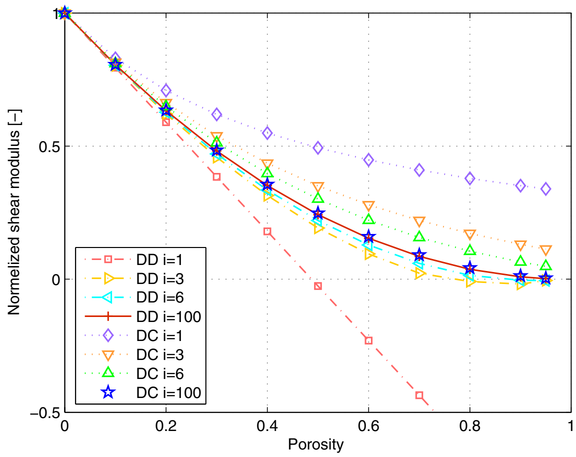

Figure 2 : Equivalent shear modulus of the clay normalized by the shear modulus of the skeleton versus porosity for diluted approximation in strain and stress approaches coupled to the iterative process after 1, 3, 6 and 100 iterations.

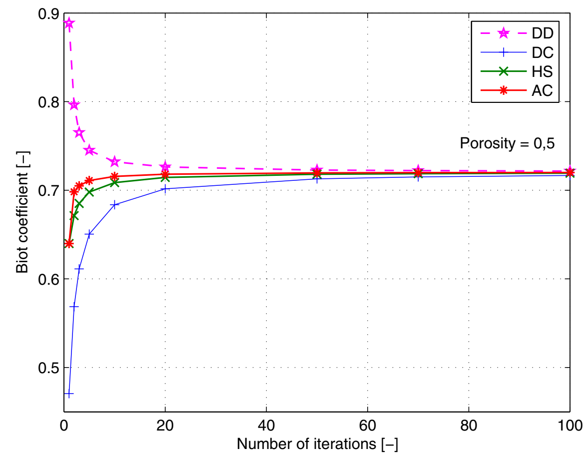

Figure 3 : Biot coefficient of the Bouzule clay versus iteration numbers for various homogenization methods.

haviour of the skeleton, as well as porosity of the media.These parameters were identified from compaction tests carried out on various clays by Djeran-Maigre, Tessier, Grunberger, Velde, and Vasseur (1998) at G3S Laboratory of the Ecole Polytechnique on an original oedometric cell.

4.1 Identification of the skeleton's elastic properties

In this experimental tests the axial stress is applied and with the axial strain, the radial stresses and the pore pressure were measured thanks to this specially designed cell. The experimental data allowed us to identify clays behaviour on a macroscopic scale with an elastoplastic law of modified Cam-Clay type, Pouya, Djeran-Maigre, Lamoureux-Var, and Grunberger (1998), Djeran-Maigre and Gasc-Barbier (2000). The skeleton's elastic properties of clays were identified by inverse homogenization from beginning of the unloading compaction curve measurements. The elastic behaviour of three clays, the illite-Salins-14, the Bouzule and the Marais Poitevin, was then characterized, Benhamida, Djeran-Maigre, Dumontet, and Smaoui (2005). In this study, we present only the results concerning the Bouzule whose bulk modulus of the skeleton k_s was identified to be 461 MPa, the elastic shear modulus μ_s^e to be 445 MPa and initial porosity to be 20.5%. The nonlinear evolution of the shear modulus is put as a power law :

$$\mu_s^{sct}(\varepsilon_{eq}(y)) = \frac{\sigma_0}{3\,\varepsilon_0}\left(\frac{\varepsilon_{eq}(y)}{\varepsilon_0}\right)^{m-1}, \tag{29}$$

with $\sigma_0 = 3\mu_s^e\varepsilon_0$ and m a parameter which varies between 0 and 1. In the following applications m is taken equal to 0.3.

4.2 Hydro-elastic behaviour

First, we study the elastic phase of the equivalent homogeneous hydro-mechanical behaviour of the clay. For that, the iterative process of homogenization presented at section 3 is coupled with various simplified homogenization methods. The evolutions of the equivalent homogeneous shear modulus versus porosity are presented in Figure 2 for diluted approximations in strain (DD) or stress (DC) approaches coupled with the iterative process after 3, 6 and 100 iterations and without coupling (1 iteration). We show that the stress and strain approaches, which lead to different predictions without iterative process, converge to a same prediction with the increasing number of iterations even for significant porosities. Initially the strain approach gives nonphysical predictions for porosities beyond 30% to 40% but after convergence of the iterative process they tend to zero as expected. The predictions obtained by the two approaches become in particular inverse of each other.

Same commentaries can be made on the prediction of

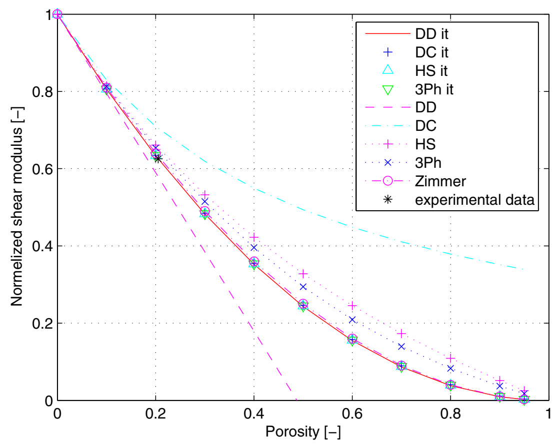

Figure 4 : Equivalent homogenous shear modulus of the clay normalized by the shear modulus of the skeleton versus porosity for various homogenization methods coupled or not with the iterative process. Here (it) means method coupled with iterative process

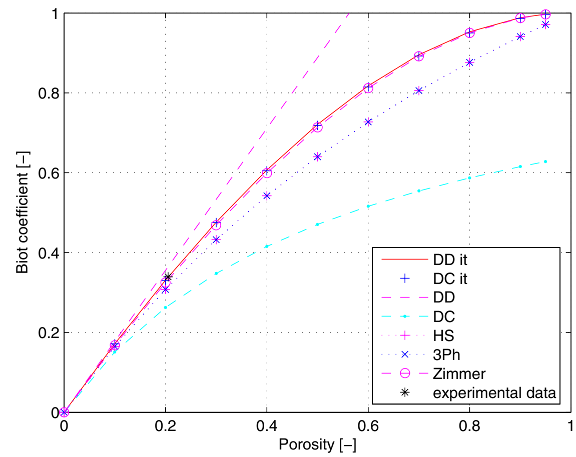

Figure 5 : Biot coefficient of the Bouzule clay versus porosity for various homogenization methods coupled or not with the iterative process. Here (it) means method coupled with iterative process

the Biot coefficient according to the iteration numbers plotted in Figure 3 for a porous media with an initial porosity of 50% . The diluted distributions method (DD) and (DC), the Hashin's upper bound (HS) and the self-consistent model (AC) give the same prediction after 100 iterations.

This convergence to the same equivalent behaviour for all homogenization methods and porosities, including now significant rates, can also be observed in Figure 4 and Figure 5 where the direct application of the diluted distributions method with a strain approach (DD) and a stress approach (DC), the Hashin's upper bound (HS), the three phases method (3Ph) and the differential scheme (Zimmer) are compared with those obtained by iterative process at convergence coupled to these methods and symbolized by (it). The experimental point used for the identification of elastic skeleton properties is also located on the curves. This convergence can be explained from the use at each iteration of the predictive homogenization models with low porosities where the various estimates coincide. The case of reinforced or unidirectional composites was also successfully studied, Benhamida and Dumontet (2003). Lastly, it has been shown that the localization or concentration tensors are also corrected by the introduction of the iterative process so that the strain and stress local fields become exploitable for the prediction of the microstructure damage, Brini, Pradel, Benhamida, and Dumontet (2003).

4.3 Nonlinear elastic behaviour

The same analysis is carried out now on the nonlinear homogeneous equivalent behaviour. The coupling of the iterative process is used, at each step of the loading, following the method presented at section 3. We present at Figures 6 the evolution of the deviatoric macroscopic stress depending on the deviatoric macroscopic strain, for an initial porosity of 50%, obtained by the classical secant approach coupled with the diluted distributions, strain approach (DD) and stress approach (DC), after 3, 6 and 100 iterations and without coupling (1 iteration). As previously in linear case, we show that the stress and strain approaches, which lead to different nonlinear predictions without iterative process, converge here to a same prediction with the increasing number of iterations. The comparison with the prediction given by the Hashin's upper bound (HS) coupled to classical secant approach is presented in Figure 7. At convergence of the iterative process, the two linear homogenization schemes give the same evolution of the deviatoric macroscopic stress versus the deviatoric macroscopic strain, for an initial porosity of 20,5%. The same result of convergence can be found in Figure 8 with the modified secant

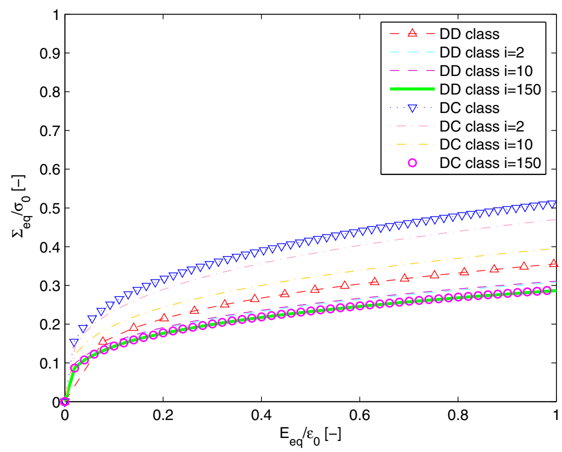

Figure 6 : Evolution of deviatoric equivalent stress according to equivalent strain by classical secant approach with diluted approximations coupled to the iterative process after 1, 3, 6 and 100 iterations. Initial porosity 50%.

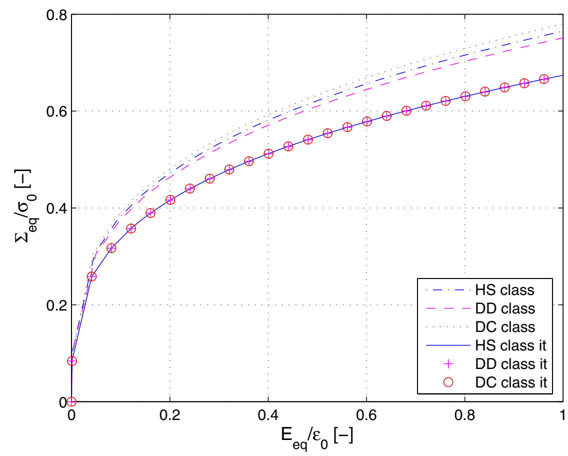

Figure 7 : Evolution of deviatoric equivalent stress according to equivalent strain by the classical secant approach for various homogenization methods with (it) and without coupling of the iterative process. Initial porosity of 20,5%.

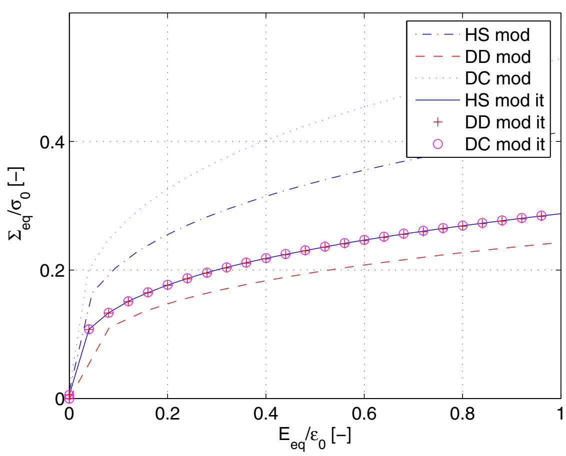

Figure 8 : Evolution of deviatoric equivalent stress according to equivalent strain by the modified secant approach for various homogenization methods with (i) or without coupling of the iterative process. Initial porosity 50%.

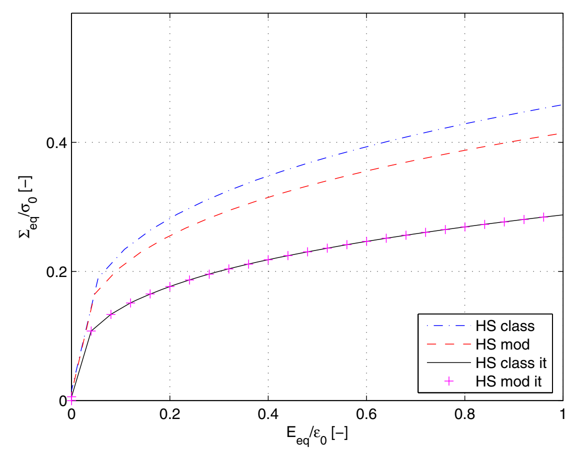

Figure 9 : Evolution of equivalent deviatoric stress depending on the equivalent strain by the classical and modified secant approaches and the upper limit of Hashin with or without iterative process. Initial porosity 50%.

method for a material with higher porosity. Finally the two secant methods can be compared in the Figure 9. Without coupling with the iterative process, the classical secant method leads to a stiffer prediction of the equivalent behaviour than that provided by the modified secant extension, which is in accordance with Suquet's results, Suquet (1997). With the iterative process, the two secant methods coincide. This result remains valid for any porosity.

The introduction of the iterative process leads then to the same nonlinear equivalent behaviour whatever the simplified method that is used for the homogenization of the linear comparison porous media, whatever the secant approaches and rates of porosity.

5 Conclusion

The iterative process of homogenization proposed in this work enables to simulate the nonlinear behaviour of porous material by unifying the predictions resulting from the classical and modified secant approaches, as well as the linear simplified homogenization methods. Applied to the hydro-mechanical behaviour of a clay, it allows us to simulate the nonlinear response of the material in a purely deviatoric loading. Currently, we continue with the simulation of a compaction test, which will allow us to compare the model with the experimental results. A more realistic microstructural geometry can be considered. The iterative method is particularly adapted to taking into account for example ellipsoids of the same sizes in order to represent, as well as possible, the distributed homogeneous clay particules or even different sized particules in order to take into account granulometric dispersion.

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