Radial Basis Function and Genetic Algorithms for Parameter Identification to Some Groundwater Flow Problems

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Abstract: In this paper, a meshless method based on Radial Basis Functions (RBF) is coupled with genetic algorithms for parameter identification to some selected groundwater flow applications. The treated examples are generated by the diffusion equation with some specific boundary conditions describing the groundwater fluctuation in a leaky confined aquifer system near open tidal water. To select the best radial function interpolation and show the powerful of the method in comparison to domain based discretization methods, Multiquadric (MQ), Thin-Plate Spline (TPS) and Conical type functions are investigated and compared to finite difference results or analytical one. Through two sample problems in groundwater flow, we demonstrate the computational capacity of RBF in simulating time dependent problems and the possibility of simultaneous estimation of multiple groundwater parameters computational feasibility when it is coupled to simple Genetic Algorithms (GAs). Performance of variously based RBF is compared.

keyword: meshless methods, radial basis functions, multiquadric, conical, genetic algorithm, groundwater flow.

1 Introduction

The application of numerical methods to solve groundwater governing partial differential equations (PDEs) is common since the advent of computers. Many techniques such as finite difference, finite element, boundary element and finite volume have been used extensively for modelling and simulation issues. All these domain discretization based methods rely on local interpolation by using special appropriate mesh structure. Grid genera-

tion is very often carried out through third party software that might not be affordable by the end users and often require special training for users for working on such software. These methods offer certainly the best alternative solution of PDEs as they are soundly based on good mathematical and physical background and concepts. However, they require intensive data preparation and are time consuming even with the use of highly advanced grid generators, especially for 3D analysis, where constant remesh of the domain are required.

Element free/meshless methods have been developed recently to overcome these drawbacks. The literature abounds on these topics and many variants of meshless methods do exist [Lin and Atluri (2001); Belytschko, Lu, and Gu (1994); Cheng, Golberg, Kansa, and Zammito (2003)]. Here we will restrict ourselves to a specific class of RBF based meshless technique.

Duing the past decade radial basis function, which is one of the meshless schemes, is growing and gaining popularity in science and engineering communities [Chen, Brebbia, and Power (1999); Cheng, Golberg, Kansa, and Zammito (2003); Cho, Golberg, Muleshkov, and Li (2004); Wong and Hon (2000); Kansa (1990a,b); (Kansa and Hon(2000); Li, Cheng, and Chen (2003)]. The technique is based on a set of points and selected radial functions. Its implementation is straightforward compared to other methods (domain discretization methods, boundary element method), and simplify tremendously input data preparation and avoids any meshing.

In this paper, the radial basis function technique is investigated to solve some selected time dependent groundwater flow problems:

- predicting drawdown based on known aquifer parameters (1D example), [Wang and Anderson (1982)],

- solving two-dimensional groundwater fluctuation in a leaky confined aquifer system near open tidal wa-

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ters (2D example), [Zhongua and Jiao (2001)].

The technique is first developed for direct simulation and three type of radial functions have been used and their results are compared to the finite difference method. Then it is coupled with Simple Genetic Algorithm (GAs) for parameter identification of storativity, transmissivity of the aquifer and leakance for 2D analysis.

The paper is organized as follows. In the next section a brief description of groundwater flow problem types and definition of physical statement of the problem are given. Then a brief introduction of Kansa's method is given in section 3. GAs with the formulation of the objective function are presented in section 4. Finally, we present two numerical examples to validate our proposed approach.

2 Groundwater flow problem types

The major purposes of computer modelling for groundwater flow are to analyze the aquifer system and assess the impacts of a selection of possible strategies. First, a suitable conceptual model is constructed based on the information gained from site and observation survey, experimental measurements, topographic and climatic data that will deliver a good hydrogeological understanding of the key flow processes of the system. Unfortunately however, the modelling is very often based on limited data and a large number of assumptions and the results should be used with great caution. The ability of providing an increasingly accurate representation of the groundwater system increases with time, resources, and the technical expertise available and applied.

A mathematical model is a set of governing equations, which, subject to certain assumptions, quantifies the physical processes active in the aquifer system being modelled.

Parameter identification is of fundamental issue in the modelling of groundwater systems as many of these parameters are difficult to measure directly, and that most of the subsurface is inaccessible, [Carrera and Neuman (1986); Sun (1997)].

Methods that have been previously used on the inverse groundwater problem typically range from guesswork, which is referred to as trial and error calibration, to various attempt of different levels of sophistication at automatic calibration.

The computational difficulties associated with the estimation of these parameters are related to ill-posedness of this inverse problem, spatial distribution sparseness of observations and inherent measurement errors.

In this paper, we will restrict ourselves to some time-dependent groundwater flow models [Bear and Verruijt (1987)] for material survey [Wang and Anderson (1982); Zhongua and Jiao (2001)], and direct as well as inverse problems assessment.

The main governing equation is summarized below [Bear and Verruijt (1987); Wang and Anderson (1982)]:

$$S\frac{\partial h}{\partial t} = T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) + L(h_z - h) \tag{1}$$

subject to some initial and boundary conditions. Herein h denotes the hydraulic head, S the storage coefficient, T the transmissivity and L the leakance as discussed in one section of the paper.

3 Radial basis functions formulation

After the success of the radial basis function in the approximation of surface and scattered data, the technique have been further extended to solving partial differential equations in the early 1990 by Kansa [Kansa (1990a,b)]. Then, many developments and applications of such technique have been followed (see, e.g., [Buhmann (2000); Chen, Brebbia, and Power (1999); Cheng, Golberg, Kansa, and Zammito (2003); Cho, Golberg, Muleshkov, and Li (2004); Wong and Hon (2000); (Kansa and Hon(2000); Schaback and Wendland (2000)] and the references therein.)

The following radial basis functions

$$\begin{aligned}
\varphi(r) &= (r^2 + c^2)^{(2n+1)/2} \text{ multiquadric,} \\
\varphi(r) &= (r^2 + c^2)^{-(2n+1)/2} \text{ inverse multiquadric,} \\
\varphi(r) &= e^{-\beta^2 r} \text{ Gaussian} \\
\varphi(r) &= r^{2n} \log(r) \text{ polyharmonic splines}
\end{aligned}$$

are widely used in the RBF formulation and also in the integral equation.

The governing partial differential equations are of the type:

$$Ah = f \text{ in } \Omega, \quad \text{and} \quad Bh = g \text{ on } \partial\Omega \tag{2}$$

where A is an arbitrary differential operator (which is a Laplace operator for our case) and B is an operator imposed for boundary conditions, that can be Dirichlet, Neumann, or Robin type.

Following Kansa's RBF collocation scheme [Kansa (1990b)], the application of collocation radial basis functions to boundary value problem 2 start by first selecting a set of boundary collocation points $\{(x_1, y_1), ..., (x_b, y_b)\}$ and interior collocation points $\{(x_{b+1}, y_{b+1}), ..., (x_{d+b}, y_{d+b})\}$. The unknown solution h of the problem can be expressed as

$$h(x,y) = \sum_{j=1}^{d+b} \alpha_j \varphi_j(x,y) + \sum_{j=d+b+1}^{N} \alpha_j P_j(x,y) \qquad (3)$$

where $\{\alpha_j\}_1^{d+b}$ are unknown coefficients to be determined. $\varphi_j(x,y) = \varphi(r)$ is the radial basis function and r denotes the distance between the two points (x,y) and (x_j, y_j). The polynomials P_j are the basis of the space of 2-variant polynomials of order not exceeding q and $N - b - d = (q+1)!/(2!(q-1)!)$.

After selecting used radial basis functions, the equation (3) is substituted in equation (2) to yield the following system:

$$\sum_{j=1}^{d+b} \alpha_j A\varphi_j(x_i, y_i) + \sum_{j=b+d+1}^{N} \alpha_j A P_j(x_i, y_i)$$

$$= f(x_i, y_i) \quad \text{for} \quad i = b+1, ..., b+d \qquad (4)$$

$$\sum_{j=1}^{d+b} \alpha_j B\varphi_j(x_i, y_i) + \sum_{j=d+b+1}^{N} \alpha_j B P_j(x_i, y_i)$$

$$= g(x_i, y_i) \quad \text{for} \quad i = 1, ..., b \qquad (5)$$

$$\sum_{j=1}^{d+b} \alpha_j P_i(x_j, y_j) = 0 \quad \text{for} \quad i = 1, ..., N-b-d \qquad (6)$$

Then the coefficients $\{\alpha_j\}_1^N$ constitute the solution of the system (4), (5) and (6).

Although the matrix resulting from this formulation is not symmetric, the technique can be applied successfully to a problem with Dirichlet condition. In case of a mixed Neumann condition the method is much less accurate at nodes on and adjacent to the Neumann boundary. This is one of the main numerical disadvantages of the Kansa's method [Kansa (1990b)]. To overcome this drawback, improved schemes were proposed in [Fedoseyev, Friedman, and Kansa (2002)], [Fornberg, Driscoll, Wright, and Charles (2002)], [Larsson and Fornberg (2002)]. Their strategy is based on an additional arbitrary set of nodes adjacent to the boundary and located inside or outside the domain.

In the case of steady state problem, as it was investigated in this paper, the technique can be applied after the use of the finite difference scheme, specially the θ-weighted method, for approximating the time derivative. So in each time step n the treated problem is as follows:

$$h^n - h^{n-1}$$
$$= \Delta t \theta(\frac{T}{S} \nabla^2 h^n - \frac{L}{S} h^n)$$
$$+ \Delta t (1-\theta)(\frac{T}{S} \nabla^2 h^{n-1} - \frac{L}{S} h^{n-1}) + \Delta t \frac{L}{S} h_z \qquad (7)$$

where $0 \leq \theta \leq 1$, and Δt is the time step. The used notation h^n and h^{n-1} are $h(x,y,t^n)$ and $h(x,y,t^{n-1})$ computed at t^n and t^{n-1} respectively.

Rearranging the system (7) we get the following system

$$(1 + \Delta t \theta \frac{L}{S}) h^n - \Delta t \theta \frac{T}{S} \nabla^2 h^n$$
$$= (1 - \Delta t(1-\theta)\frac{L}{S})h^{n-1} + \Delta t(1-\theta)\frac{T}{S} \nabla^2 h^{n-1} + \Delta t \frac{L}{S} h_z \qquad (8)$$

After time discretisation, the RBF method can then be applied each time when the head is needed. So, at each time n, the head is approximated by:

$$h^n(x,y) = \sum_{j=1}^{m} \alpha_j^n \varphi(r_j) + \alpha_{m+1}^n x + \alpha_{m+2}^n y + \alpha_{m+3}^n \qquad (9)$$

where m is the number of used collocation nodes and r_j is the Euclidean distance between the points (x,y) and (x_j, y_j). To know the value of h^n, the coefficients $(\alpha_1^n, \alpha_2^n, ..., \alpha_{m+3}^n)$ have to be determined. Substituting (9) into (8), and applying the resulting equation to each node and using the three extra equations

$$\sum_{j=1}^{m} \alpha_j^n = \sum_{j=1}^{m} \alpha_j^n x_j = \sum_{j=1}^{m} \alpha_j^n y_j = 0,$$

we obtain an algebraic system $A\alpha^n = b^n$ to be solved at each time step t_n.

4 Genetic algorithms and problem formulation

4.1 Genetic algorithms

The genetic algorithm method is becoming one of the most popular technique in the optimization domain. It is based on the process of biological evolution [Holland (1979); Michalewicz (1994)], and has been widely applied for parameter identification in many engineering fields, especially in water resources, see [Gorelick (1983); Ouazar and Cheng (1999)]. More sophisticated GAs such as messy [Halhal, Walters, Ouazar, and Savic (1997)] are not discussed in this paper.

The simple genetic algorithms works with a design family representing a population of individuals. Each element of the population, called chromosome, is given as a number of binary strings (0110...01) placed end to end and representing the design variable in its binary form. Within an evolution iteration three basic genetic operators are applied to produce stronger individuals for simple genetic process which are: selection, crossover and mutation. The length of the string is given by the precision required to evaluate the corresponding design variable.

Starting by a random selection of an initial population, the objective function value (fitness) of each solution (individual) must be computed before the application of any genetic operators. The solution of higher values are considered as the best parent who can give birth to a best child in maximization problem and vise versa in the minimization one. The next generation is that of reproduction and the new generation is again taken as parents for other new generations. The process is considered as terminated when convergence is detected or when the specified maximum number of generations is reached. As the maximum value of the objective function value is not known only a maximum number of iterations is specified to stop the process and the best chromosome within these iterations can be selected as the optimum solution of the problem.

The main steps required to build up a GA for an optimization process are summarized in seven steps, i.e. Coding, String formation, Initial population, Fitness evaluation, Reproduction, Crossover, Iterative process and termination rule. For most real-world problems, these pseudo-optimal solutions are still much better than those that could be obtained using less robust methods. The GA must have some control parameters such as population size, (n)-usually 4 to 100, and probabilities for applying genetic operators, e.g. crossover probability (Pcross)- usually 0.5 to 1, and mutation probability (Pmut)-usually 0.01 to 0.1.

Herein, this strategy is applied for identification of aquifer parameters which are storativity, transmissivity and leakance using RBF method for solving direct problems.

4.2 Objective function minimization

The objective of the parameter identification problem in the field of groundwater is to determine some aquifer parameters such as the transmissivity S, the storativity T and leakance or specific leakage L [Carrera and Neuman (1986)], [Gorelick (1983)]. The determination of these parameters can be done by formulating an objective function based on some knowledge of measurement data value of the head h at a specific points p inside the domain. These engineering measurements can be done at any specific time t^* and since it is assumed that the parameters do not vary with time and they just depend on space, their values at any point of the domain are still the same during the time process. So, the genetic algorithm scheme can be coupled with radial basis function only at time step t^*. Then, the objective function for the parameter identification inverse problem, can be defined in the following way:

$$\min \frac{1}{2} \sum_{j=1}^{p} (h_{com}(x_j, y_j, t^*) - h_{mes}(x_j, y_j, t^*))^2 \qquad (10)$$

where $h_{mes}(x_j, y_j, t^*)$ and $h_{com}(x_j, y_j, t^*)$ are the given engineering measurements and computed head values at time t^*. The parameter S, T and L are considered as the design variables for such problem.

5 Numerical experiments

In order to demonstrate the applicability and efficiency of the adopted coupling technique of radial basis function and genetic algorithms, two examples (1D and 2D) of known analytical solution are used. The one-dimensional example deals with the prediction of drawdown based on known aquifer parameters. The two-dimensional example concerns groundwater flow in a leaky confined aquifer system near open tidal water. The results are obtained by using three kind of radial functions: Multiquadric, Thin-Plate Spline and Conical functions and
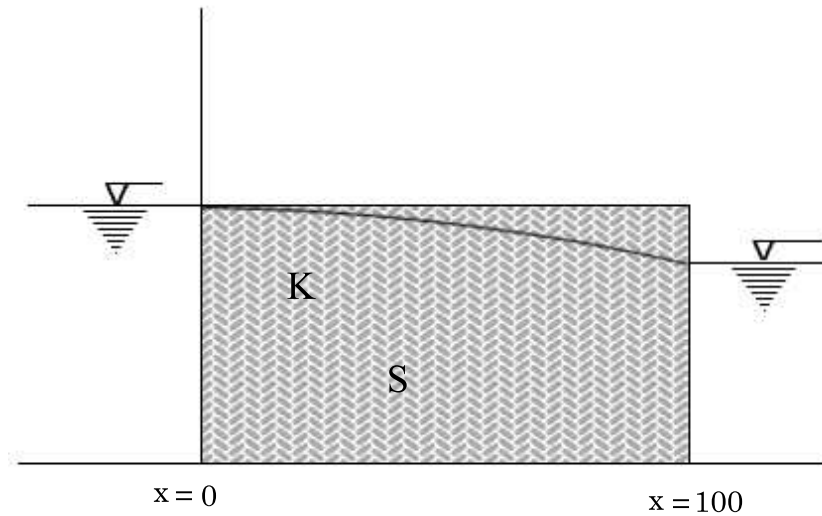
Figure 1 : Problem statement for drawdown prediction example.

then compared with the finite difference and analytical results [Zhongua and Jiao (2001)].

The modelling of these problems has been carried out in two steps: In the first one, the examples are solved as a direct problem by assuming that all involved parameters are known. The efficiency of RBF algorithm is illustrated and compared with the finite difference and/or analytical solution, for one and two dimensional cases. In the second step, some parameters arising in the flow equation such as storativity and transmissivity are considered as unknowns and only values of the head at some specific points are given by engineering measurements at some given times. For the one dimensional case, the rearrangement of the governing equation allows us to consider the problem as one parameter determination only. While for two dimensional case, one, two or three parameters are to be determined. In this case the RBF technique is coupled with GAs at the time of measurement to find out the optimum values of the parameters.

5.1 One dimensional case

5.1.1 Direct simulation

Consider first a hypothetical problem as illustrated in Figure 1 [Wang and Anderson (1982)]. The example is used to illustrate the solution of one-dimensional example by predicting drawdown based on known aquifer parameters. The aquifer head is controlled at the downstream boundary. The initial head in the aquifer at time

$t = 0$ is H1 $= 16$ m and at time $t = 0$, the head at the downstream boundary is suddenly lowered to level H2 $= 11$ m. The change in boundary conditions implies that the head in the aquifer immediately starts to fall and the drawdown should be calculated.

The parameters T and S are the measures of the overall ability of the aquifer to store and transmit water, respectively. This one dimensional problem is governed by the following PDEs:

$$\begin{cases} S\dfrac{\partial h}{\partial t}(x,t) = T\dfrac{\partial^2 h}{\partial x^2}(x,t) \text{ for } 0 < x < 100;\ t > 0 \\ \quad h(0,t) = 16 \text{ for } t \geq 0 \\ \quad h(100,t) = 11 \text{ for } t > 0 \\ \quad h(x,0) = 16 \text{ for } 0 \leq x \leq 100 \end{cases} \quad (11)$$

The steady state analytical solution of the problem is available. It is simply a linear variation of the head which constitutes an asymptote to time dependant solution. Using MQ-radial basis function, the obtained results are presented in Figure 2. The behavior of the head at times $t = 5mn, 15mn$ and $t = 25mn$ is illustrated and the overall trend at infinity is converging to steady state solution. The used values for time step $\Delta t, \theta, T$ and S are $5mn, 1, 2 \times 10^{-2}m^2$ and 2×10^{-3} respectively. The overall solution behavior is in good agreement with the finite difference method.
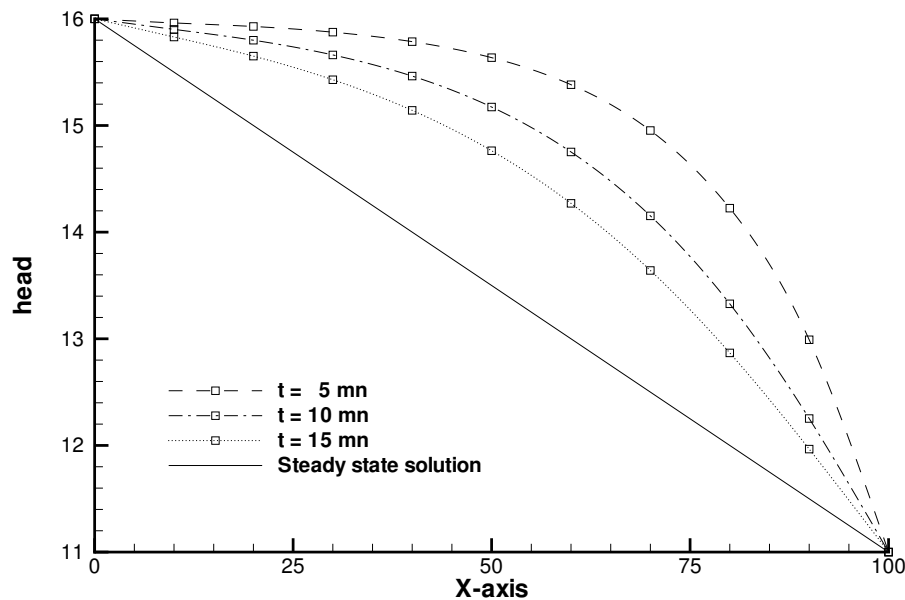
Figure 2 : MQ-numerical solution of the hydraulic head for 1D test problem.

5.1.2 Inverse simulation

For the inverse simulation, the T and S parameters are replaced by T/S, so we can have just one design variable for this equation (11) to be determined. It is assumed that the optimal value of T and S are the same given in direct simulation. The genetic algorithm is then coupled to radial basis function to find out this value. The applied GAs parameters are given in Table 1, and the search domain of the ratio T/S, which is dimensionless variable, is set to $[1, 20]$. At each genetic algorithm iteration, the problem (11) is solved until the time iteration of given measurement heads, and then the objective function is evaluated. The GAs process run until it exceeds the maximum number of iterations, which is set to 100 for this example. The obtained optimum value for T/S is 9.9935 with a relative error 6.5×10^{-3}.

Table 1 : Parameters for 1D and 2D inverse problems.

GAs parameter	Value
Crossover probability P_c	0.5
Mutation probability P_m	0.02
Number of populations	5
Number of generations	100

5.2 Two dimensional case

In this example, the collocation RBF technique is extended to the solution of two-dimensional groundwater fluctuation in a leaky confined aquifer system near open tidal waters [Zhongua and Jiao (2001)]. The problem is solved as a direct one with known aquifer parameters and as an inverse, for parameter identification of aquifer coupling RBF and GAs via known head values at some specific position.

5.2.1 Direct simulation

Groundwater in coastal areas is commonly disturbed by tidal fluctuations. A two-dimensional analytical solution has been developed [Zhongua and Jiao (2001)] to describe the groundwater fluctuation in a leaky confined aquifer system near open tidal water under the assumption that the groundwater head in the confined aquifer fluctuates in response to sea tide whereas that of the overlaying unconfined aquifer remains constant. It is important to understand the response of groundwater to tidal fluctuation of coastal water.

The configuration of the aquifer system is shown in Figure 3. It consists of three layers: leaky confined aquifer, semi-permeable layer and unconfined aquifer. It is assumed that the shallow unconfined aquifer has a large specific yield, which can effectively damp the tidal effect
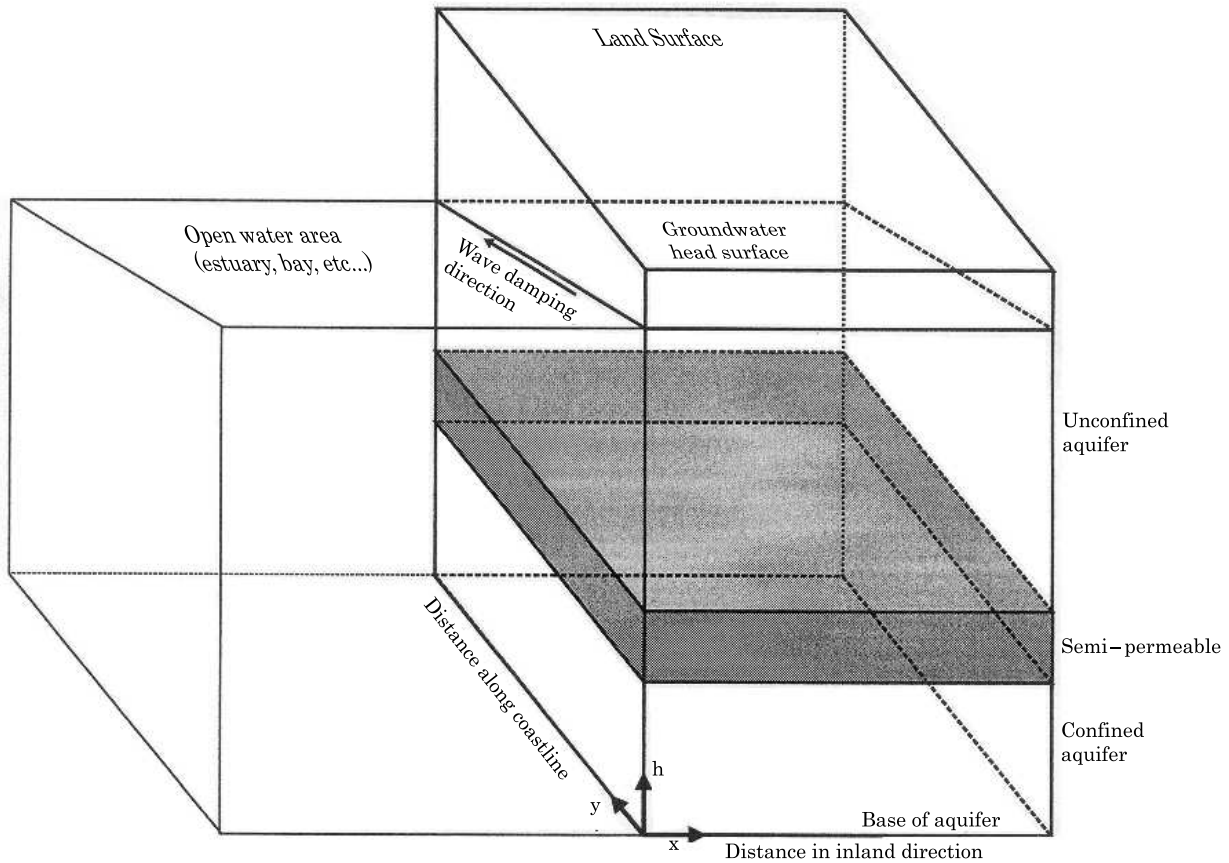
Figure 3 : Conceptual model of 2D groundwater flow in leaky confined aquifer system near open tidal water [Zhongua and Jiao (2001)].

so that the tidal fluctuation in the unconfined aquifer is negligible compared with that in the confined aquifer.

The Dirichlet boundary condition for equation (1) on the inland side where x approaches infinity is $h = h_z$. It states that the tide has no effect far inland as x approaches infinity.

This two dimensional example has the following analytical solution in an infinite domain [Zhongua and Jiao (2001)].

$$h(x,y,t)$$
$$= h_z + A_1 \exp(-px - my)$$
$$\left[cos(a_1 t + b_1 y - \frac{a_1 S + b_1 mT}{2pT} x + c_1) \right] \quad (12)$$

The problem has been solved using RBF and implicit finite-difference numerical method in a truncated do-

main where the dimension is shown in Table 2. The time step is $1h$ and the number of used scattered data is 64 nodes on the boundary and 209 nodes inside the domain. This is much less than the number of nodes used in the FDM (30000). An unconfined aquifer with constant groundwater head is supposed to exist on the above of the treated confined one which causes the leakage between the two aquifers, with specific leakage being $L = 0.001$. For other aquifer parameters, Table 2 shows those based on the studies in Apalachicola Bay, Florida, USA SUN. Concerning the boundary conditions and following [Zhongua and Jiao (2001)] a no-flow condition are adopted at the boundaries with $y = 0$ and $y = 50$ km and $h = h_z$, $h = h_z + A_1 \exp(-my) \cos(a_1 t + b_1 y + c_1)$ for outlet and inlet boundaries respectively. The initial condition $h = 0$ is used everywhere. The obtained results for the direct numerical mode (without optimization) are presented in Table 3 for point $x = 0.3$ km and $y = 10$ km. Note that from Table 3 the solution matches closely with

Table 2 : Aquifer parameters used for 2D case [Zhongua and Jiao (2001)].

Tidal	Parameter	Value
Diurnal	Transmissivity T	700m^2
	Storativity S	0.002
	Length in inland direction	3km
	Length of coastal line	50km
	Amplitude at $x = 0$, A_1	0.342m
	Damping coefficient m_1	$5.48 \times 10^{-6}(1/\text{m})$
	Tidal speed a_1	-0.2618(1/h)
	Separation constant b_1	$1.67 \times 10^{-6}(1/\text{m})$
	Phase shift c_1	0.0
	Constant head h_z	0.0001

Table 3 : Groundwater heads (m) at $y = 10$km and $x = 0.3$km.

Time	Analytical solution	RBF solution (r^5 function)	RBF solution (MQ-function)	RBF solution (TPS-function)	FDM solution
1	0.2210	0.1696	0.1730	0.1737	0.2155
2	0.2046	0.1870	0.1899	0.1892	0.2017
3	0.1742	0.1659	0.1680	0.1672	0.1730
4	0.1320	0.1272	0.1284	0.1278	0.1319
5	0.0808	0.0777	0.0782	0.0777	0.0816
6	0.0240	0.0222	0.0218	0.0215	0.0257
7	-0.0343	-0.0352	-0.0363	-0.0364	-0.0318
8	-0.0904	-0.0944	-0.0922	-0.0921	-0.0870
9	-0.1402	-0.1395	-0.1419	-0.1416	-0.1361
10	-0.1805	-0.1792	-0.1819	-0.1815	-0.1758
11	-0.2086	-0.2066	-0.2095	-0.2090	-0.2032
12	-0.2224	-0.2200	-0.2229	-0.2223	-0.2165

the analytical solution for all radial basis functions we used. The results become accurate after the second time iteration, which is more or less logical due to a truncated domain, and a few nodes compared to FD are used. It can also be explained by the use of zero initial condition at the first iteration. We also noticed that the RBF solution (for all used type of functions) are more accurate than FD solutions. Comparing the applied radial basis functions, we numerically notice that during the simulations, Multiquadrics are somehow not easy to run since a suitable shape parameter must be selected and the shape free parameter of Conical and Thin-Plate Spline made them suitable for application. It was also tested, see Figures 4 and 5, that if the sources points are the same as the used nodes points, the values of the head at the boundaries

where a Neumann condition is imposed are not accurate. To alleviate this difficulty, the sources points at the boundaries with Neumann conditions are moved to outside the domain, as shown in [Fornberg, Driscoll, Wright, and Charles (2002); Larsson and Fornberg (2002)], and in a symmetric way of the inside nodes of domain near that boundaries. But this position is an arbitrary choice and it was remarked that any position outside the domain not far from the boundaries does not affect the solutions very much. The values of the head inside the domain were also improved. In Figure 6, we present the groundwater head distributions in leaky confined aquifer near open water with one diurnal tide for three selected time steps: $t = 4, 6$ and 8 hours. The 3D plotted results are obtained using Conical-type radial basis functions. In
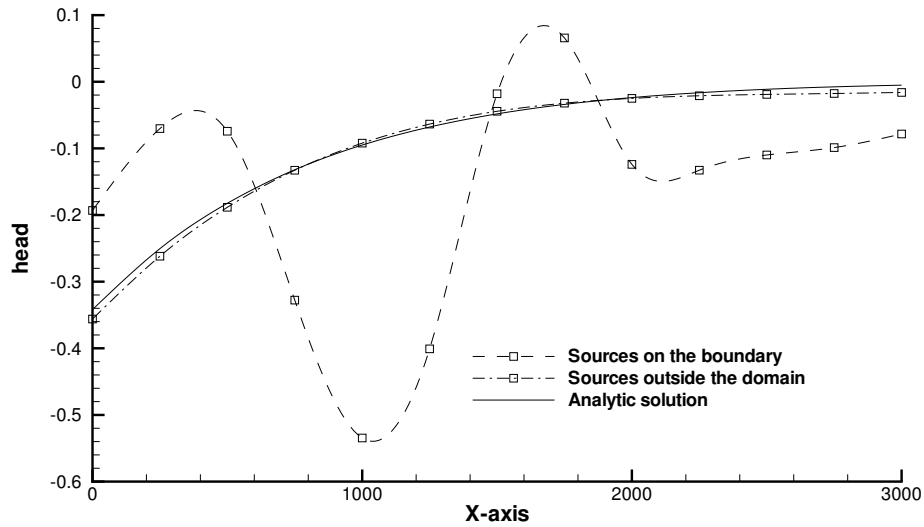
Figure 4 : Groundwater heads at $y = 0$ for $t = 12h$ using Conical RBF for different source points.
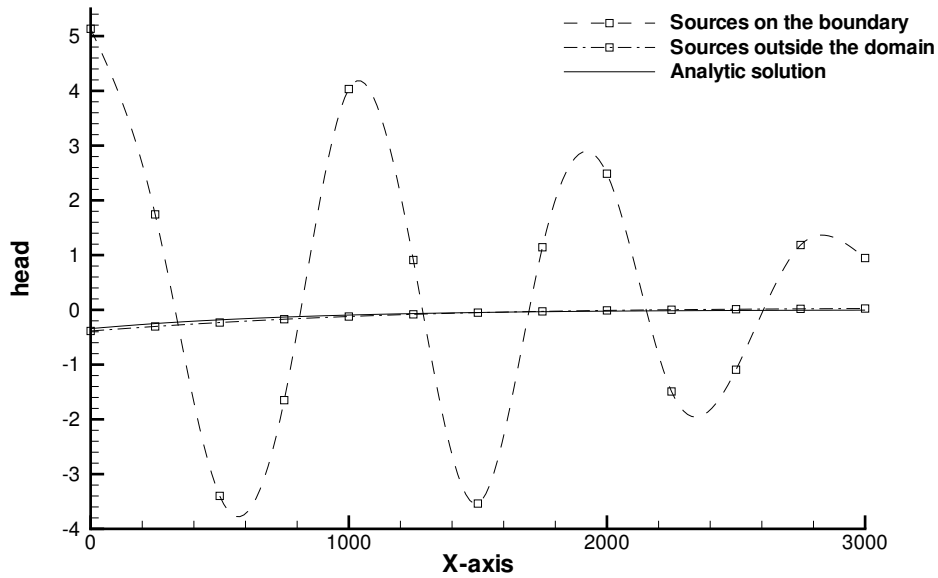

Figure 5 : Groundwater heads at $y = 0$ for $t = 12h$ using MQ-RBF for different source points.

Figure 6, the groundwater head in the aquifer is represented in three time steps to illustrate changes with space and time. They are in close agreement with the analytical solution provided in [Zhongua and Jiao (2001)].

5.2.2 Inverse simulation

In the next numerical experiment, we consider an inverse problem. The storativity S and/or transmissivity T and/or leakance can be considered as unknown parameters, whereas the design variables and the values of

the head at some specific point in the domain are given as known engineering measurement data. These values are assumed to be known at time $t = 10h$. The storativity and/or transmissivity and/or leakance are then computed via GAs coupled with RBF algorithms, by minimizing the objective function given by the equation (10). Using the known value given in Table 2, the domain of search of the parameter design are $[500m^2/h, 800m^2/h]$ for transmissivity and $[10^{-3}, 3 \times 10^{-3}]$ for storativity, and $[0, 3 \times 10^{-3}]$ for the leakance. The GAs parameters
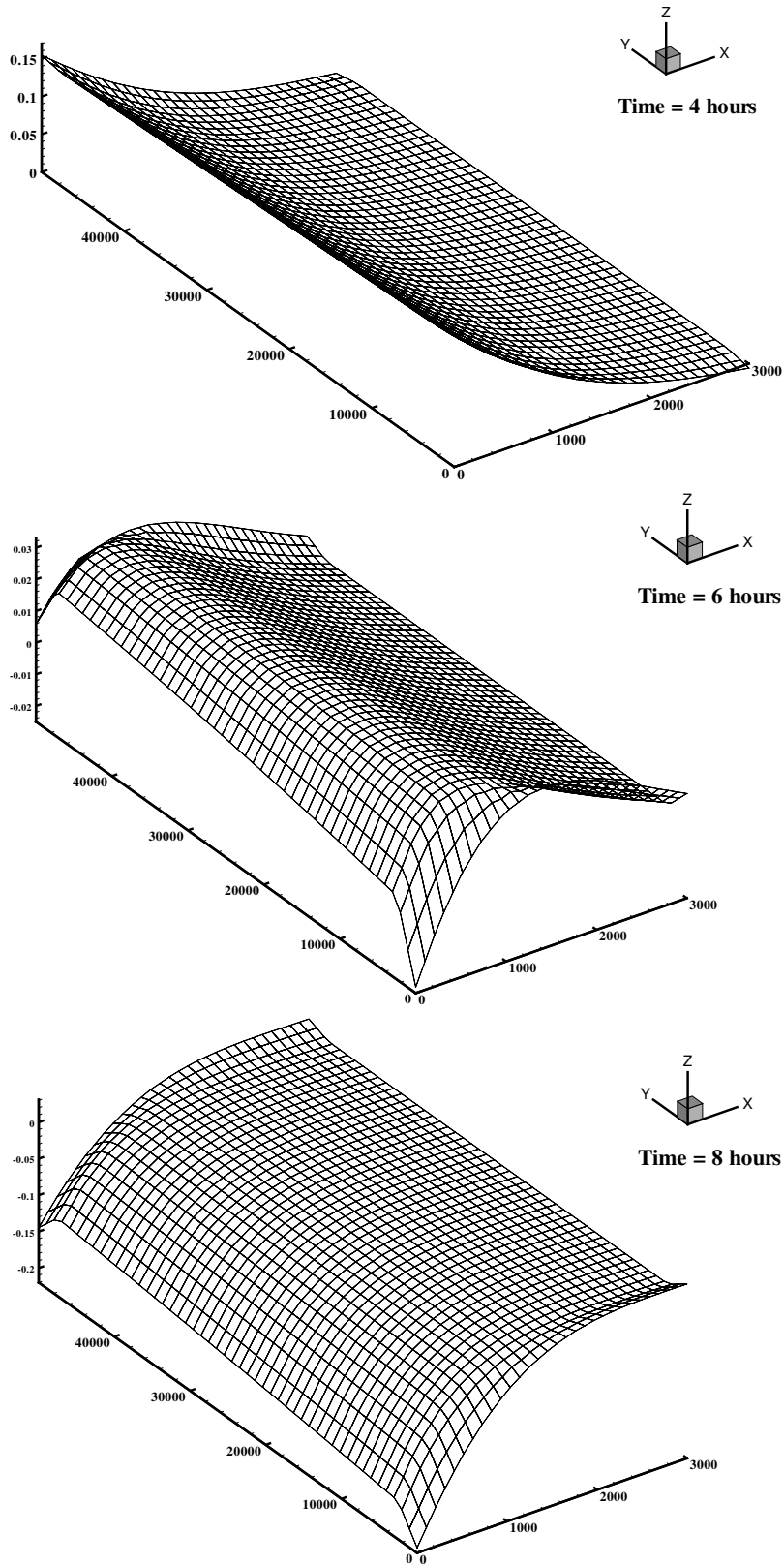
Figure 6 : Spatial groundwater heads distribution for $t = 4$h, $t = 6$h and $t = 8$h.

needed for the use of algorithm as probability of selection, mutation and crossover and also the number of population are given in Table 1. Ideal values of transmissivity, storativity, and/or leakance are found, they are:

- for simulation number 1 where leakance is kept fixed to its true value 0.001, we obtain 722.5 and 2.005×10^{-3} for transmissivity and storativity respectively,

- for simulation number 2, we took the three parameters as design variables and found $774.7, 2.3 \times 10^{-3}$, and 7.35×10^{-4} for transmissivity, storativity, and leakance respectively.

Notice that the differences observed between computed and true values can be explained by the fact that simulation runs stand for a finite domain approximating an infinite one. Let us mention also that for simplicity, we have considered GAs for parameter identification for problems with constant coefficients but the same technique could be applied to variable coefficient problems.

6 Concluding remarks

Through the two sample problems in groundwater flow, we have demonstrated the computational capacity of RBF in simulating time dependent problems and the possibility of simultaneous estimation of multiple groundwater parameters computational feasibility when it is coupled to simple GAs. The main advantage of the technique resides in the fact that it is straightforward to assess and implement. Of course, one pays the price of possible ill-conditioning of the fully populated matrix, and the trial and error in choosing constants involved in some functions and the lack of universality of the method. More numerical experiments and theoretical insight are needed for better assessment of the method: investigating especially domain decomposition technique (see, e.g., [Li and Hon (2004)]) for solving the ill-conditioning of the matrix, the optimal choice of the involved shape parameter and the selection of better radial basis function to deal with the accuracy of the solution for Neumann boundary condition type, to name a few improvements.

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