

## A Novel SINS/IUSBL Integration Navigation Strategy for Underwater Vehicles

Jian Wang<sup>1</sup>, Tao Zhang<sup>1,\*</sup>, Bonan Jin<sup>1</sup> and Shaoen Wu<sup>2</sup>

**Abstract:** This paper presents a novel SINS/IUSBL integration navigation strategy for underwater vehicles. Based on the principle of inverted USBL (IUSBL), a SINS/IUSBL integration navigation system is established, where the USBL device and the SINS are both rigidly mounted onboard the underwater vehicle, and fully developed in-house, the integration navigation system will be able to provide the absolute position of the underwater vehicle with a transponder deployed at a known position beforehand. Furthermore, the state error equation and the measurement equation of SINS/IUSBL integration navigation system are derived, the difference between the position calculated by SINS and the absolute position obtained by IUSBL positioning technology is used as the measurement information. The observability of the integration system is analyzed based on the singular value decomposition (SVD) method. Finally, a mathematical simulation is performed to demonstrate the effectiveness of the proposed SINS/IUSBL integration approach, and the observable degrees of the state variables are also analyzed.

**Keywords:** SINS/IUSBL, integration navigation, singular value decomposition.

### 1 Introduction

Recently, underwater vehicles have been widely used to survey the ocean resources, such as environmental monitoring, underwater inspection of estuaries, surveillance, harbors and pipelines, and geological and biological surveys, et al. In fact, the implementation of the underwater navigation system is the key of the successful operation of the underwater vehicles [Luke, Liu and Charl (2008); Miller, Farrell and Zhao (2010)].

For the merits of high accuracy in short time, complete independence, strong anti-interference ability, comprehensive and high update frequency of navigation message, strap-down inertial navigation system (SINS) has been the backbone navigation in the underwater navigation system, however, unbounded positioning errors induced by the uncompensated rate gyro and accelerometer degrade the performance of SINS over time, the positioning errors should be corrected by other aided navigation devices. The electromagnetic wave signal is severely attenuated in the water, which limits the application of the global positioning system (GPS) [He, Wang and Zhou (2018); Shi, Zhang and Li (2018)]. At present, the aiding methods of the underwater navigation

---

<sup>1</sup> Department of instrument science and engineering, Southeast university, Nanjing, 210096, China.

<sup>2</sup> Department of computer science, Ball State University, Muncie, USA.

\* Corresponding Author: Tao Zhang. Email: 101011356@seu.edu.cn.

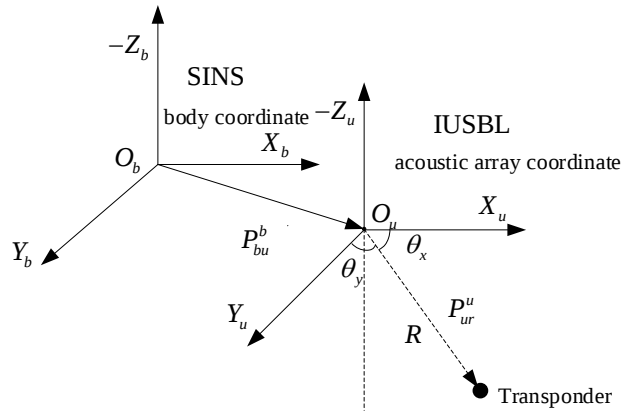
include underwater acoustic positioning, geomagnetic matching, gravity matching, terrain matching, etc. In fact, terrain matching, geomagnetic matching and gravity matching are all based on the geophysical field information, the matching databases of the mission area are needed in advance, in addition, the testing equipment is expensive and the complexity of matching calculation increases with the increase of the dimension. Due to the long distance transmission and the small attenuation loss of the underwater acoustic wave, the underwater acoustic positioning technology has been fully developed [Wolbrecht, Gill, Borth et al. (2015); Wu, Zapevalova, Chen et al. (2018)]. Available underwater acoustic positioning systems include long base line (LBL), short baseline line (SBL) and ultra-short base line (USBL), the positioning accuracy of LBL is high, however, which needs cumbersome and time-consuming installation and calibration procedures; SBL is rigidly installed onboard a vessel hull, however, which is affected by the noise and the natural bending of the hull. USBL does not need to lay the acoustic array on the seabed, which provide factory-calibrated and the fast deployable system with the small size and the portability, therefore, USBL has been the primary choice for underwater positioning.

A single SINS and the USBL acoustic positioning system can't meet the positioning requirements of underwater vehicles. The research of SINS aided by USBL acoustic positioning system shows important theoretical significance and practical value for high precision and long-time navigation of underwater vehicles. Traditional USBL is mainly used to locate and track underwater targets. By analyzing the characteristics of USBL positioning, a new USBL positioning mode based on inverted acoustic array was proposed in Vickery [Vickery (1998)]. With a transponder deployed at a known position, a SINS/IUSBL integration navigation system used as the main sensor suite installed onboard the underwater vehicle will be able to provide the absolute position of the underwater vehicle. Now we can consider a typical positioning scenario, where an underwater vehicle equipped with an INS and an IUSBL array sails in the water, a nearby transponder's relative position with respect to the acoustic array coordinate can be obtained by interrogating the transponder and listening for its acoustic responses, and the considerable distances range typically from a few meters to several kilometers, so we can obtain the absolute position of the underwater vehicle depend on the known position of the transponder and the transponder's relative position. In recent years, the SINS/USBL integration technique has gained more and more attention [Zhang, Mong and Ma (2016); Morgado, Oliveira, Silvestre et al (2013); Morgado, Oliveira, Silvestre et al (2010); Morgado, Oliveira, Silvestre et al (2013)]. In previous research, the SINS and USBL device work separately, the calibration of the installation error angle is required before each experiment. In this paper, all the research is based on the SINS/IUSBL integration navigation system, which can achieve the free calibration and high-precision navigation and positioning.

The rest of the work is given by as follows: the principle of the SINS/IUSBL integration strategy is presented in Section 2. Section 3 reports the SINS/IUSBL integration navigation model and filtering equations. The observability analysis of the integration strategy is given in Section 4. Section 5 presents the simulation test and discussion. Conclusion is summarized in Section 6.

## 2 Structure

The integration prototype system is mainly composed of the USBL device and the SINS, they are fully developed in-house, the installation error matrix  $C_u^b$  from the acoustic array coordinate to the body coordinate and the position lever arm  $P_{bu}^b$  can be calibrated beforehand. Once calibrated successfully, the prototype system can achieve the free calibration. When the underwater vehicle sails in the water, we can obtain the range measurement  $R$  by measuring the round-trip time between the acoustic array center and the transponder, the direction information  $\theta_{mx}$  and  $\theta_{my}$  can be computed by the measured the delay difference or the phase difference, the scheme of the SINS/IUSBL integration strategy is shown in Fig. 1.



**Figure 1:** The scheme of SINS/IUSBL positioning theory

In this paper, the positioning error of the SINS is fixed by the USBL. As shown in Fig. 1, the USBL device and SINS are both rigidly mounted on the carrier, and fully developed in-house. A transponder is firstly deployed at a known position  $P_r^e$ , which can be calibrated through the conventional LBL positioning method. Furthermore, based on the calibration cosine matrix  $C_u^b$  and the heading and attitude cosine matrix  $C_b^n$  provided by SINS, we can get the absolute position of USBL in the navigation coordinate system. Finally, the absolute position of the SINS  $P^e$  can be obtained by compensating the position lever arm  $P_{bu}^b$  between the USBL and the SINS. The principle can be given by as follow:

$$P^e = P_r^e - C_n^e C_b^n C_u^b P_{ur}^u - C_n^e C_b^n P_{bu}^b \quad (1)$$

## 3 Model of SINS/IUSBL integration

The error model of SINS can be given by Savage [Savage (1998)].

$$\begin{cases} \dot{\boldsymbol{\phi}} = \boldsymbol{\phi} \times \boldsymbol{\omega}_{in}^n + \delta \boldsymbol{\omega}_{in}^n - \mathbf{C}_b^n (\boldsymbol{\varepsilon}^b + \mathbf{w}_g^b) \\ \delta \dot{\mathbf{v}}^n = \mathbf{f}_{sf}^n \times \boldsymbol{\phi} + \mathbf{v}^n \times (2\delta \boldsymbol{\omega}_{ie}^n + \delta \boldsymbol{\omega}_{en}^n) - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \delta \mathbf{v}^n + \mathbf{C}_b^n (\nabla^b + \mathbf{w}_a^b) \\ \delta \dot{\mathbf{p}} = \mathbf{M}_{pv} \delta \mathbf{v} + \mathbf{M}_{pp} \delta \mathbf{p} \\ \dot{\boldsymbol{\varepsilon}}_i^b = 0 \quad (i = x, y, z) \\ \nabla_i^b = 0 \quad (i = x, y, z) \end{cases} \quad (2)$$

where  $\boldsymbol{\phi} = [\phi_E \ \phi_N \ \phi_U]^T$  denotes the attitude error;  $\delta \mathbf{v}^n = [\delta v_E \ \delta v_N \ \delta v_U]^T$  is the velocity error;  $\delta \mathbf{p} = [\delta \varphi \ \delta \lambda \ \delta h]^T$  denotes the position error;  $\boldsymbol{\varepsilon}^b = [\varepsilon_x^b \ \varepsilon_y^b \ \varepsilon_z^b]^T$  is the constant drift of gyroscope,  $\boldsymbol{\omega}_g^b = [\omega_{gx}^b \ \omega_{gy}^b \ \omega_{gz}^b]^T$  is a random Gaussian white noise vector of the gyroscope,  $\nabla^b = [\nabla_x^b \ \nabla_y^b \ \nabla_z^b]^T$  is constant bias of the specific force,  $\mathbf{w}_a^b = [\omega_{ax}^b \ \omega_{ay}^b \ \omega_{az}^b]^T$  is the random errors in the body frame, the subscripts x, y and z denote the right, the front and the upward directions of the body frame, and the subscripts E, N, U denote the east, north and upward direction, respectively.

$$\mathbf{M}_{pv} = \begin{bmatrix} 0 & 1/R & 0 \\ \sec L / R & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{M}_{pp} = \begin{bmatrix} 0 & 0 & -v_N / R^2 \\ v_E \sec L \tan L / R & 0 & -v_E \sec L / R^2 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The state vector can be chosen as:

$$\mathbf{X} = \mathbf{X}_{SINS} = [\phi_U \ \phi_E \ \phi_N \ \delta v_E \ \delta v_N \ \delta v_U \ \delta \varphi \ \delta \lambda \ \delta h \ \varepsilon_x^b \ \varepsilon_y^b \ \varepsilon_z^b \ \nabla_x^b \ \nabla_y^b \ \nabla_z^b]^T \quad (4)$$

The state error equation of the integrated navigation system can be described:

$$\dot{\mathbf{X}} = \mathbf{F}_{SINS} \mathbf{X}_{SINS} + \mathbf{G}_{SINS} \mathbf{W}_{SINS} \quad (5)$$

where  $\mathbf{F}_{SINS}$  and  $\mathbf{G}_{SINS}$  can be obtained according to the Eq. (2), and the difference between the position calculated by SINS and the absolute position obtained by IUSBL positioning technology is used as the measurement, the measurement model of the SINS/IUSBL integration navigation system can be established as follow:

$$\mathbf{Z} = \mathbf{P}_{SINS}^e - \mathbf{P}^e = \mathbf{H} \mathbf{X} + \mathbf{V} \quad (6)$$

where  $\mathbf{H} = [\mathbf{0}_{3 \times 6} \ \mathbf{I}_{3 \times 3} \ \mathbf{0}_{3 \times 6}]$ ,  $\mathbf{V}$  is the Gaussian random measurement noise with zero mean and the covariance  $\mathbf{R}$ .

#### 4 Analysis on observability based on SVD method

The observable degree of the state variable has been analyzed based on the singular value decomposition (SVD) method [Tang, Wu and Wu (2009); Zhou and Cai (2013)], the initial state vector can be expressed as a function of the observation vector during filtering periods.

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_0 \\ \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \vdots \\ \mathbf{Z}_k \end{bmatrix} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}\mathbf{F}_0 \\ \mathbf{H}\mathbf{F}_1 \\ \vdots \\ \mathbf{H} \sum_{j=1}^{k-1} \mathbf{F}_j \end{bmatrix} \mathbf{X}_0 = \mathbf{R}_k \mathbf{X}_0 \quad (7)$$

where  $\mathbf{R}_k$  is the observability matrix of the dynamic system, the observability of the initial state vector  $\mathbf{X}_0$  depends on the characteristics of the stepped observability matrix (SOM), the singular value decomposition of  $\mathbf{R}_k$  is given by as follow:

$$\mathbf{R}_k = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad (8)$$

where  $\mathbf{U} = [\mathbf{u}_1 \mathbf{u}_2 \dots \mathbf{u}_{(k+1)m}]$ ,  $\mathbf{V} = [\mathbf{v}_1 \mathbf{v}_2 \dots \mathbf{v}_n]$ ,  $\mathbf{\Sigma} = \text{diag}(\mathbf{S}, \mathbf{0})$ ,  $\mathbf{S} = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_r)$ ,

$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r$ ,  $\sigma_i$  is the singular value of  $\mathbf{R}_k$ , we can get

$$\mathbf{Z} = \mathbf{H}\mathbf{X}_0 = \sum_{i=1}^r \sigma_i \mathbf{u}_i \mathbf{v}_i^T \mathbf{X}_0 = \sum_{i=1}^r \sigma_i (\mathbf{v}_i^T \mathbf{X}_0) \mathbf{u}_i \quad (9)$$

$$\mathbf{X}_0 = (\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T)^{-1} \mathbf{Z} = \sum_{i=1}^r \frac{\mathbf{u}_i^T \mathbf{v}_i}{\sigma_i} \mathbf{Z} \quad (10)$$

The observability of the state variable is defined as follow:

$$\eta_k = \frac{\sigma_i}{\sigma_0}, \quad i = 1, 2, \dots, n \quad (11)$$

where  $\eta_k$  denotes the observability of the  $k$ th state variable,  $\sigma_0$  is the singular value corresponding to the external measurement,  $\sigma_i$  is the singular value that maximizes the expression  $[\frac{\mathbf{u}_i^T \mathbf{v}_i}{\sigma_i} \mathbf{Z}]_k$ .

## 5 Simulation Test and Discussion

In order to verify the performance of the proposed SINS/IUSBL integration algorithm, a mathematical simulation test is performed in this section, the main parameters of IMU and USBL are listed in Tab. 1 and Tab. 2, respectively.

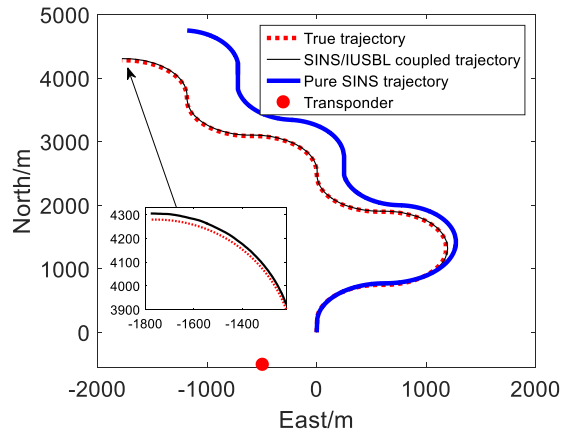
**Table 1:** Parameters of IMU

|        | Gyroscope ( $^{\circ}/\text{h}$ ) |              | Accelerometer ( $\mu g$ ) |              |
|--------|-----------------------------------|--------------|---------------------------|--------------|
|        | Constant                          | Random drift | Constant                  | Random drift |
| x-axis | 0.02                              | 0.005        | 100                       | 50           |
| y-axis | 0.02                              | 0.005        | 100                       | 50           |
| z-axis | 0.02                              | 0.005        | 100                       | 50           |

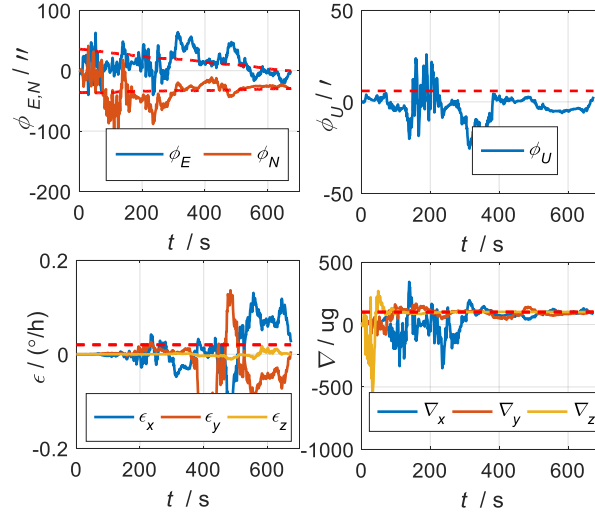
**Table 2:** Parameters of SINS/USBL integrated prototype system

| Range (%D) | Accuracy of $\theta$ ( $^{\circ}$ ) | $\phi$ ( $^{\circ}$ ) | $C_u^b$ ( $^{\circ}$ ) | $P_{bu}^b$ (m) |
|------------|-------------------------------------|-----------------------|------------------------|----------------|
| 0.5        | 0.1                                 | [0.01,0.01,0.1]       | [0.01,0.01,0.2]        | 0              |

The kinematic trajectory used in the simulation is the serpentine maneuver, as it contains the information of the angular and linear motion, the coordinate of the transponder relative to the starting position of the true trajectory is  $[-500, -500, -100]$  m. Based on the known kinematic trajectory and the relative coordinate of the transponder, we can get the physical parameters of the ideal model. In order to simulate the actual situation, the mathematical simulation test is performed with the error parameters in Tab. 1 and Tab. 2, simulation time lasts about 674 seconds, and the simulation results are shown in Fig. 2, Fig. 4 and Tab. 3.

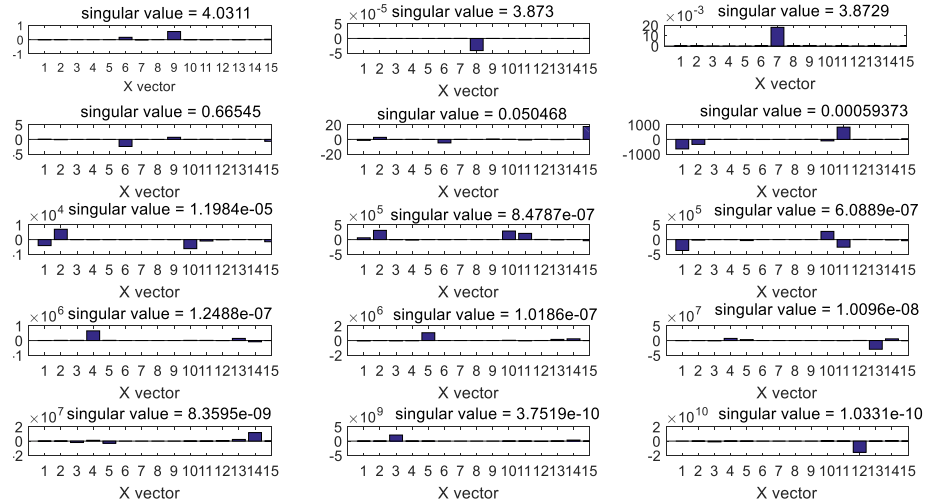
**Figure 2:** Comparison of different trajectories

As shown in Fig. 2, the red dotted line is the true trajectory, the proposed SINS/IUSBL integration algorithm can achieve a comparable performance to the true trajectory, and can obviously distinguish from the curve of pure SINS trajectory. The positioning accuracy in this simulation is 16.13 m, which demonstrates the effectiveness of SINS/IUSBL integration prototype system, and the positioning error of pure SINS can be fixed by USBL.



**Figure 3:** Curves of errors estimation

As shown in Fig. 3, the red dotted lines are the reference value. The attitude errors estimation and bias estimation of inertial sensors are also shown in the figure, however, they cannot be estimated correctly by introducing the aiding USBL in the KF, so the attitude errors and bias of inertial sensors are not compensated in the integration system. The only estimated position errors are compensated by the direct-feedback. In order to analyze the estimated results of the SINS/IUSBL filtering model, the observable degrees of the state variables have been analyzed based on SVD method.



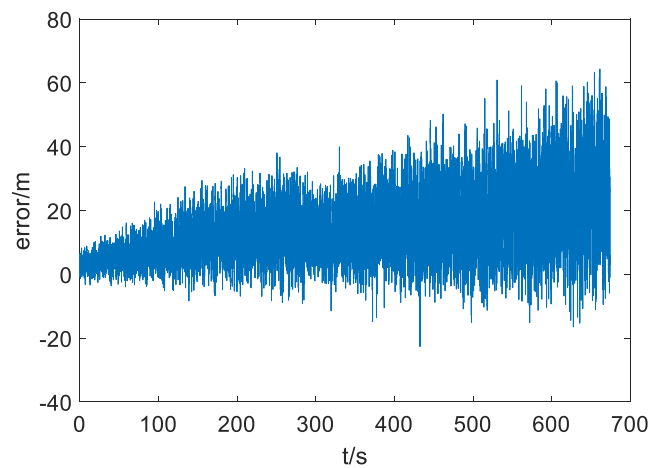
**Figure 4:** The observable degree of the state variable

**Table 3:** The observable degree of the state variable

| $X$              | observable degree | $X$               | observable degree |
|------------------|-------------------|-------------------|-------------------|
| $\phi_E$         | 1.57e-7           | $\delta h$        | 1                 |
| $\phi_N$         | 3.09e-6           | $\varepsilon_x^b$ | 2.19e-7           |
| $\phi_U$         | 9.69e-11          | $\varepsilon_y^b$ | 1.53e-4           |
| $\delta v_E$     | 3.22e-8           | $\varepsilon_z^b$ | 2.67e-11          |
| $\delta v_N$     | 2.63e-8           | $\nabla_x^b$      | 2.61e-9           |
| $\delta v_U$     | 1.72e-1           | $\nabla_y^b$      | 2.16e-9           |
| $\delta \varphi$ | 1                 | $\nabla_z^b$      | 1.3e-2            |
| $\delta \lambda$ | 1                 |                   |                   |

We can see from Fig. 4 and Tab. 3 that the observability of the position errors is best, as the position errors are set as the external measurement information, the observable degrees of  $\phi_U$ ,  $\varepsilon_z^b$ ,  $\nabla_x^b$ ,  $\nabla_y^b$  are relatively poor, and the observability of other state variables is also not ideal, they cannot be estimated correctly by introducing the aiding USBL in the KF, so the only estimated position errors are compensated in the system. The important reasons of the above phenomenon are caused by the positioning errors of USBL and the calibration errors of SINS/IUSBL integration prototype system. For example, the measurement error of the range increases with the distance increases in the actual underwater environment, resulting in the poor positioning accuracy of USBL, and the measurement error of the range over the time in the simulation is shown in Fig. 5.

The observability of the state variables of the SINS/IUSBL integration navigation system can be improved if we can obtain higher positioning accuracy of USBL and the smaller calibration errors of SINS/IUSBL integration prototype system. Due to the length limit of the paper, the simulation results in this case are not presented in the paper.

**Figure 5:** The measurement error of the range

## 6 Conclusion

In this paper, a novel SINS/IUSBL integration navigation strategy for underwater vehicles is studied, the mathematical simulation is designed to verify the effectiveness of the proposed SINS/IUSBL integration algorithm. In addition, the observability of system is affected by the positioning accuracy of USBL and the calibration errors of SINS/IUSBL integrated prototype system.

**Acknowledgement:** The author would like to thank the support in part by the National Natural Science Foundation of China (Grant No. 51375088), Inertial Technology Key Lab Fund, the Fundamental Research Funds for the Central Universities (2242015R30031, 2242018K40065, 2242018K40066), the Foundation of Shanghai Key Laboratory of Navigation and Location Based Services.

## References

- He, M.; Wang, H. B.; Zhou, L. K.** (2018): Symmetric learning data augmentation model for underwater target noise data expansion. *Computers, Materials & Continua*, vol. 57, no. 3, pp. 521-532.
- Luke, S.; Liu, H. H.; Charl, T.** (2008): Navigation technologies for autonomous underwater vehicles. *IEEE Transactions on Systems, Man and Cybernetics*, vol. 38, no. 4, pp. 581-589.
- Miller, P.; Farrell, J.; Zhao, Y.** (2010): Autonomous underwater vehicle navigation. *IEEE Journal of Oceanic Engineering*. vol. 35, no. 3, pp. 663-678.
- Morgado, M.; Oliveira, P.; Silvestre, C.** (2013): Embedded vehicle dynamics aiding for USBL/INS underwater navigation system. *IEEE Transactions on Control Systems Technology*, vol. 22, no. 1, pp. 322-330.
- Morgado, M.; Oliveira, P.; Silvestre, C.** (2010): Design and experimental evaluation of an integrated USBL/INS system for AUVs. *Proceedings of the 2010 IEEE International Conference on Robotics and Automation*, pp. 4264-4269.
- Morgado, M.; Oliveira, P.; Silvestre, C.** (2013): Tightly coupled ultrashort baseline and inertial navigation system for underwater vehicles: an experimental validation. *Journal of Field Robotics*, vol. 30, no. 1, pp. 142-170.
- Savage, P.** (1998): Strap-down inertial navigation integration algorithm design part 1: Attitude algorithms. *Journal of Guidance, Control, and Dynamics*, vol. 21, no. 1, pp. 19-28.
- Savage, P.** (1998): Strap-down inertial navigation integration algorithm design part 2: velocity and position algorithms. *Journal of Guidance, Control, and Dynamics*, vol. 21, no. 2, pp. 208-221.
- Shi, J.; Zhang, Z. J.; Li, Y. Y.** (2018): New method for computer identification through electromagnetic radiation. *Computers, Materials & Continua*, vol. 57 no. 1, pp. 69-80.
- Tang, Y. G.; Wu, Y. X.; Wu, M. P.** (2009): INS /GPS integration: global observability analysis. *IEEE Transactions on Vehicular Technology*, vol. 58, no. 3, pp. 1129-1142.
- Vickery, K.** (1998): Acoustic positioning systems new concepts-the future. *Proceedings*

*of the 1998 Workshop on Autonomous Underwater Vehicles*, pp. 103-110.

**Wolbrecht, E.; Gill, B.; Borth, R.** (2015): Hybrid baseline localization for autonomous underwater vehicles. *Journal of Intelligent & Robotic Systems*, vol. 78, no. 3, pp. 593-611.

**Zhang, Y. W.; Mong, M. G.; Ma, X. Y.** (2016): An algorithm used in underwater SINS /USBL integrated navigation: an experimental validation. *Navigation Positioning & Timing*, vol. 3, no. 2, pp. 7-13.

**Zhou, W. D.; Cai, J. N.** (2013): Observability analysis of GPS/SINS ultra-tightly coupled system. *Journal of Beijing University of Aeronautics and Astronautics*, vol. 39, no. 9, pp. 1157-1162.