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A Simplified Finite Element Method for Modelling Functionally Graded Materials Using an Equivalent Thermal Field

Amira Ziane¹, Loubna Chikh¹, Noureddine Ziane^{1,2} and Giuseppe Ruta^{3,*}

¹Faculty of Technology, Djilali Liabes University of Sidi Bel Abbès, Sidi Bel Abbès, Algeria

²Laboratoire des Structures et Matériaux Avancés dans le Génie Civil et Travaux Publics, Université de Sidi Bel Abbès, Sidi Bel Abbès, Algeria

³Department of Structural and Geotechnical Engineering, University “La Sapienza”, Rome, Italy

*Corresponding Author: Giuseppe Ruta. Email: giuseppe.ruta@uniroma1.it

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ABSTRACT: This work presents a simplified Abaqus-based approach for the finite element numerical modelling of functionally graded materials (FGMs) using an equivalent thermal field. Indeed, in the quoted commercial code there already is the possibility to describe the variation of material properties by suitably tailoring and arranging discrete material layers, thus relying on the users' artwork and experience. On the other hand, the novelty of the proposed approach lies in exploiting the robust and reliable temperature-dependent material constitutive law available in Abaqus to represent the continuous spatial variation of FGM properties through a fictitious temperature field. In this way, one builds a robust and self-contained procedure that eliminates the need for layer-wise partitioning and external user-defined material subroutines (UMAT). Numerical studies on 3D plates and 2D beams show that quadratic elements provide results in excellent agreement with the UMAT method, while linear elements converge more slowly and exhibit notable discrepancies with the same reference. For the linear elastic static cases investigated, the proposed approach is efficient, reliable, and accessible, offering a practical tool for FGM modelling in Abaqus.

KEYWORDS: FGM; Abaqus; equivalent thermal field; UMAT

1 Introduction

Functionally graded materials (FGMs) constitute an advanced class of materials characterised by a continuous variation of mechanical, thermal, and/or other physical properties along one or more given direction(s). In many engineering applications, metal–ceramic combinations are considered because they allow the mechanical and thermal characteristics of the material to be tailored gradually along the structure. In detail, ceramic constituents are attractive for thermal protection because of their low thermal conductivity, which enables them to provide effective thermal insulation in thermal-barrier systems [1,2]. Lanthanum-based ceramic materials, for example, have been investigated for thermal-barrier applications owing to their low thermal conductivity and suitable thermal and chemical stability [1]. Similarly, recent developments in ZrO₂-based coatings have focused on improving phase stability while reducing thermal conductivity at high temperatures [2]. The thermal and environmental durability of ceramic-based barrier systems is another important consideration in high-temperature applications. Advanced ceramic materials and thermal-barrier coatings must withstand oxidation, corrosion, thermal cycling, and other degradation mechanisms encountered in severe environments [3,4]. Recent reviews of thermal and environmental barrier coatings for aerospace applications further emphasize the importance of low thermal conductivity, phase

stability, and environmental resistance in protecting hot-section components [4,5]. In complete thermal-barrier systems, the oxidation protection of the underlying metallic substrate is also closely associated with the bond coat and the thermally grown oxide, rather than with the ceramic topcoat alone [6]. These complementary thermal and protective functions make ceramic-metal systems attractive for applications in which thermal protection and mechanical performance must be achieved simultaneously.

Numerous studies have been devoted to the analysis of the static and dynamic behaviour of FGM structures using analytical and numerical approaches. For example, Sobhy and Zenkour [7] investigated the buckling and vibration behaviour of double-FGM nanoplates while accounting for porosity and material inhomogeneity. Al-Furjan et al. [8] analysed the stress and strain response of an FG-GPLRC disk using a three-dimensional refined higher-order theory. Attention was also devoted to the dynamic behaviour of rotating FGM structures. Du et al. [9–11] investigated different rotating FGM configurations, including a rectangular Mindlin plate, an FGM beam, and a hub-FGM micro-beam, using rigid-flexible coupling, point interpolation, and meshless approaches, respectively. The latter study also considered the effect of a thermal environment. In addition, Alnujaie et al. [12] investigated the buckling and free-vibration behaviour of multidirectional functionally graded sandwich plates. On the other hand, the heterogeneous nature and spatially varying properties of FGMs pose significant challenges for their experimental testing and characterization, while their manufacturing can also involve costly and time-consuming processes [13,14]. These challenges have contributed to the widespread use of numerical methods, particularly the finite element method (FEM), for investigating their thermo-mechanical behaviour.

Early FEM approaches to FGM modelling were mainly based on discretising the structure into several homogeneous layers (see Fig. 1 for a benchmark simple geometry), each characterised by uniform mechanical properties deduced from a given variation law [15–17]. Although this approach is widely used in the literature, it presents several limitations, including a long time for model preparation, the need for many layers, and the loss of accuracy in representing the continuous material variation; we will show these limitations in the next section, where we present this approach.

To overcome these limitations, FGM modelling in commercial software such as Abaqus [18] has increasingly relied on user subroutines such as UMAT [19–22], which allows for the definition of a continuous variation of mechanical properties at the integration points. This approach provides high accuracy; however, it requires advanced programming skills, the installation of several software tools, as well as significant computational resources. These requirements limit its use in many studies, especially for engineers and designers working with limited hardware capabilities.

Therefore, there is a clear need to develop a simpler and more efficient method that does not require complex partitioning, advanced programming skills, or high-performance hardware.

It should be emphasised that the individual Abaqus functionalities employed in our newly proposed strategy, namely, the predefined analytical fields and the temperature-dependent material properties, are already available in the software. The novelty of the present work does not therefore lie in introducing these functionalities themselves, but rather in their suitable combination through an equivalent fictitious temperature mapping. In our benchmark investigation, which is limited, for the sake of simplicity, to grading in one direction only, such a mapping is denoted $T_f(y)$, varies in the direction of the functional grading, and is suitably designed for a continuous FGM modelling. As a matter of fact, the proposed procedure transforms the prescribed spatial variation of the FGM properties into an equivalent temperature-dependent representation, such that the fictitious temperature field $T_f(y)$ reproduces the required material variation at the integration points. The fictitious temperature has no direct physical thermal meaning and serves solely as an equivalent computational variable for mapping the spatial material gradation onto the temperature-dependent material capability of Abaqus, while not preventing to superpose the investigation of the effect of

a real temperature gradient onto the previous study. This provides a systematic and simple FGM modelling procedure that avoids layer-by-layer discretization and user-defined material subroutines such as UMAT or USDFLD.

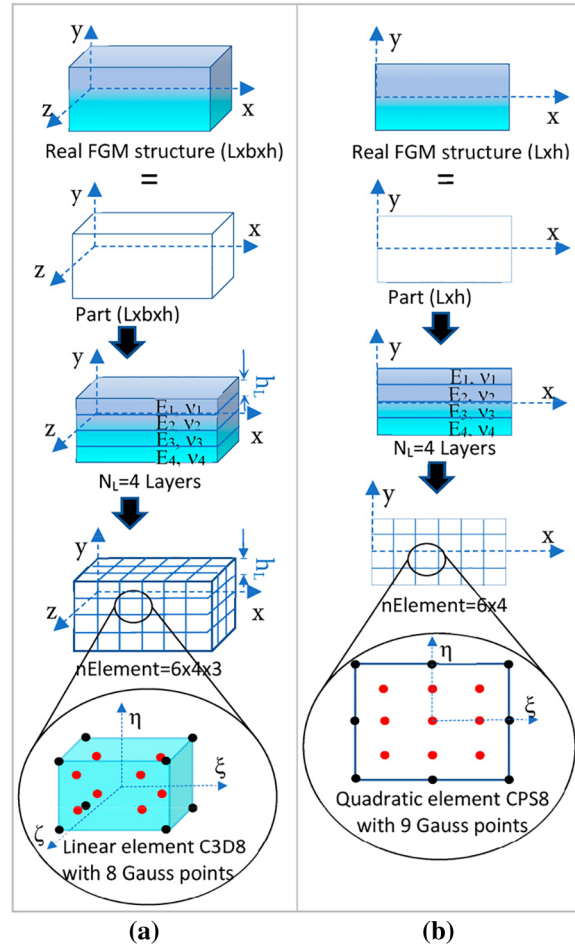


Figure 1: Partition method for bodies in: (a) 3D ($L \times b \times h$), divided into 4 layers, meshed with $6 \times 4 \times 3$ C3D8 elements (8 Gauss points); (b) 2D ($L \times h$) divided into 4 layers, meshed with 6×4 CPS8 elements (9 Gauss points).

The manuscript is organised as follows: [Section 2](#) sketches the main methods used in standard FEM codes (we take Abaqus as a paradigm) to analyse FGMs; [Section 3](#) will present the proposed new strategy in detail, showing how our approach allows the continuous variation of mechanical properties to be represented accurately, while avoiding classical layer discretisation and the use of user subroutines. [Section 4](#) presents the numerical results for standard benchmark static problems by the proposed technique and the comparison with those obtained by the libraries at ease at date. [Section 5](#) presents some conclusive remarks and perspectives relative to other techniques and the applicability of the proposed technique to design. An Appendix, divided into two parts, presents a standard UMAT code to investigate the simple problems tested and, by contrast, the very few lines that are proposed here to obtain the same results by means of the fictitious temperature field.

The key contributions of this work can be summarised as follows: 1) Development of a simplified FGM modelling method that does not require complex layer partitioning or advanced programming skills; 2) Reduction of model preparation time and computational requirements, enabling the use of standard

hardware; 3) Accurate representation of the continuous material variation through the equivalent thermal field; 4) Validation of the proposed method through numerical examples.

2 Main Methods Used in Standard FEM Codes

Here we outline the usual main strategies to model FGM structures in Abaqus [18], since these materials are not directly dealt with by the software: indeed, the module that considers the material properties does not provide any variation in function of a Cartesian coordinate frame x, y, z .

In the present study, the mechanical properties of the FGM in a very simple element like that in Fig. 1 are assumed to follow a power-law combined with a mixture-law distribution, which is customary in the literature and technologically sound:

$$\begin{aligned} E(y) &= E_m + (E_c - E_m) \left(\frac{y}{h} + \frac{1}{2} \right)^p \\ \nu(y) &= \nu_m + (\nu_c - \nu_m) \left(\frac{y}{h} + \frac{1}{2} \right)^p \end{aligned} \quad (1)$$

with $E(y), \nu(y)$ Young's modulus and Poisson's ratio of the FGM in function of the coordinate y . These physical properties of the FGM obviously depend on the thickness h , the power index p , and on the separated properties E_c, ν_c, E_m, ν_m of the ceramic and of the metal, respectively.

2.1 Partition Method

We start modelling FGMs by a simplified multi-layer discretisation approach, as in Fig. 1. The numerical framework to tackle the considered problem is based on the classical Newton–Raphson iterative algorithm as in Fig. 2.

For the partition method, the values of $E(y), \nu(y)$ are evaluated from Eq. (1) at the mid-surface of each layer using an external calculation tool. These values are then assigned to the corresponding layers, effectively converting the continuous material variation into a stratified heterogeneous configuration suitable for a subsequent numerical analysis.

However, this method exhibits significant limitations: the major, which is present at once, is that the method requires a strict correspondence between the mesh and the stratification, such that the number of elements through the thickness must exactly match the number of physical layers. This restriction considerably reduces modelling efficiency, particularly in terms of local accuracy. Indeed, the validity of this approach is primarily limited to reduced integration elements, as C3D8R or CPS4R, with a single Gauss point. For fully integrated elements, as C3D8 in Fig. 1, the method reaches its limits: the presence of integration points at different heights ($\eta = \pm 1/\sqrt{3}$) would require distinct material properties within a single element. Assigning a single Young's modulus E_l and Poisson's ratio ν_l per layer introduces an approximation error, compromising the reliability of the results for these more complex elements. Moreover, the method is hindered by the labour-intensive nature of the partitioning process: dividing the body into multiple layers constitutes a time-consuming pre-processing step, making the implementation unnecessarily complex.

Remark that the flowchart in Fig. 2 represents a simplified case, restricted to a linear elastic static analysis under a single load increment. Although the problem is linear, the procedure is systematically governed by the Newton-Raphson iterative loop. In this configuration, the algorithm performs two iterations: the first initialises the system and computes the residual, the second validates the system convergence after correcting the displacement vector $\{\Delta U_{It}\}$.

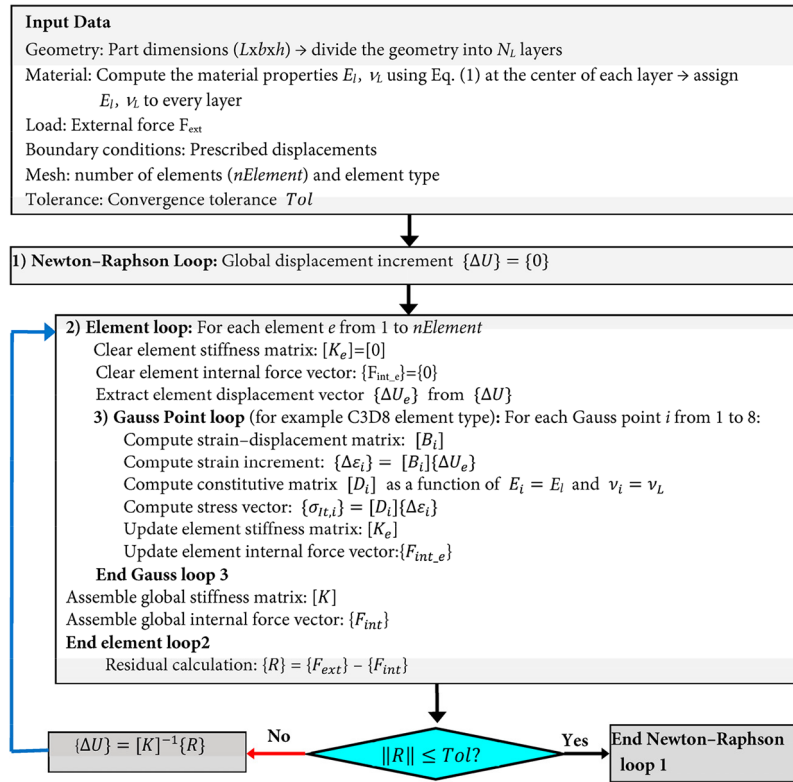


Figure 2: Flowchart of the partition method for linear elastic analysis.

2.2 UMAT Subroutine Method

The UMAT (User Material) is a subroutine designed to provide engineers and researchers with full flexibility in defining mechanical constitutive laws beyond the standard models available in the software material library. Here, the UMAT is used to control the material behaviour at each integration point, thereby allowing a rigorous customisation of the structural response.

In a linear elastic static analysis, the role of the UMAT is to define the direct relationship between stress and strain according to Hooke's law. Although the problem is linear, the algorithm is embedded within a Newton-Raphson iterative procedure, see Fig. 3.

During the computation, the subroutine must perform two essential tasks: compute the material stiffness matrix $[D_i]$, referred to as constitutive matrix, and update the stress tensor $\{\sigma_{It,i}\}$ at the end of each increment. This procedure ensures that, from the second iteration onward, the residual of the internal forces is minimised, thus assuring the convergence of the system.

The use of UMAT, even in a linear case, presents several important methodological advantages: it allows the introduction of complex anisotropy or material properties that depend on external parameters, not always efficiently handled by native material models; further, it provides full traceability of internal variables through state ones, facilitating detailed post-processing of results at the material level. Finally, this approach constitutes a robust foundation for subsequent extensions.

However, this method also presents several significant drawbacks. From a numerical standpoint, the systematic use of an external subroutine written in Fortran increases the computational time. From an implementation perspective, it requires a high level of mathematical rigour, since even a minor error in the definition of the constitutive matrix $[D_i]$ may prevent convergence, even for a simple linear elastic problem.

Moreover, this method introduces a major practical limitation: the installation of the compilation environment (Intel Fortran and Visual Studio) is complex and requires precise configuration of the environment variables under operative systems as Windows.

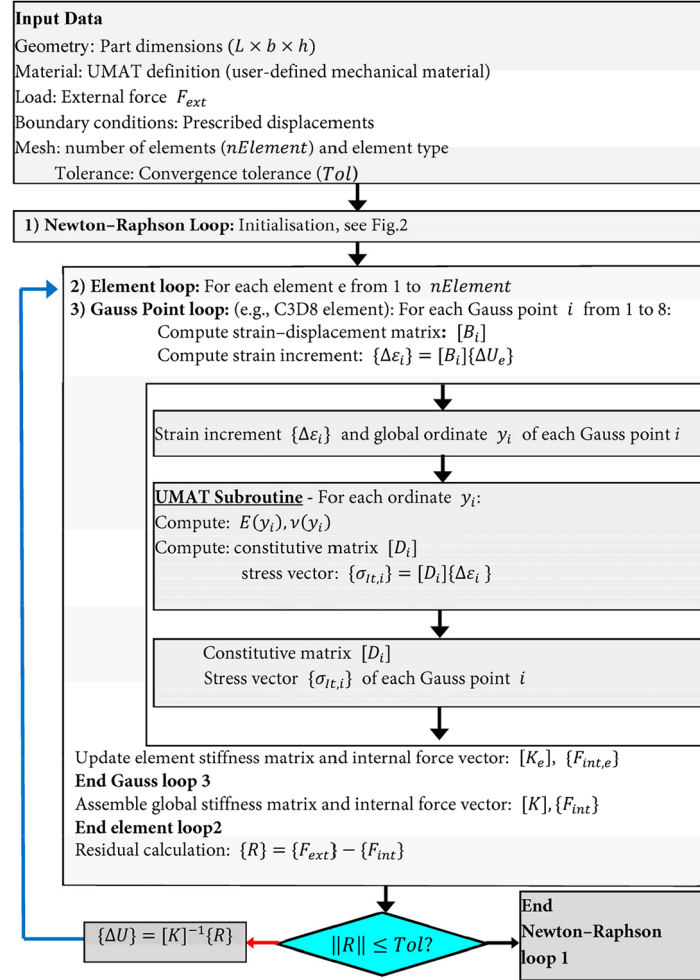


Figure 3: Flowchart of the UMAT subroutine method for linear elastic analysis.

As shown in Fig. 3, the multi-layer partitioning is no longer required when this method is used. However, the use of the UMAT subroutine in Fortran (file.for) requires that the material be defined using the *User Material* option in the material properties.

3 Equivalent Thermal Field Method

To overcome the limitations of the usual Partition and Subroutine methods, the present investigation introduces a new optimised modelling strategy. While the Partition approach suffers from a noticeable lack of accuracy and the Subroutine involves significant technical complexity due to the installation of several software tools, the method proposed here is marked by simplicity and efficiency: by removing these technical constraints while ensuring accurate results, it provides a reliable and accessible alternative for users seeking both performance and ease of implementation.

The proposed approach is based on the observation that in Abaqus/Standard Young's modulus E and Poisson's ratio ν can be defined as functions of the temperature ($Temp$), which itself can be defined as a function of the spatial coordinate y , the only direction of functional grading here:

$$\left\{ \begin{matrix} E(Temp) \\ \nu(Temp) \end{matrix} \right\} \text{ and } \{Temp(y)\} \Rightarrow \left\{ \begin{matrix} E(y) \\ \nu(y) \end{matrix} \right\} \quad (2)$$

The transitive relationship in Eq. (2) directly links E and ν to the coordinate y and provides a new modelling strategy that avoids the approximation associated with layer-wise partitioning while maintaining accurate evaluation of the material properties at the integration points, without requiring the technical complexity of a UMAT.

The implementation of the proposed approach requires knowledge of the element formulation and of the locations of the integration points along the material-gradient direction, defined here by the y -coordinate in the global reference system. Since Abaqus/Standard does not directly allow the variation of mechanical properties as a function of Cartesian coordinates, but allows this variation as a function of temperature, the values of the coordinate y are replaced by fictitious temperatures that can provide the required variation of E and ν , which are calculated according to the power-law distribution in Eq. (1) and introduced in the form of a three-column table. The first column contains the values of Young's modulus, the second those of Poisson's ratio, and the third column corresponds to the fictitious temperatures. The number of rows must match the number of distinct y -coordinates of all the integration points.

The proposed approach is therefore linked to the selected mesh through the integration-point locations in the material-gradient direction. In the present work, the mesh is generated using local seeding with the *By size* method and an *Approximate element size* sizing control. The number of elements through the thickness is selected to adequately capture the FGM gradation, while the number of elements in the other directions results from the prescribed approximate element size and the corresponding geometric dimensions. Consequently, a change in the element size or element formulation may modify the integration-point locations and, when the set of distinct y -coordinates changes, the temperature-property table must be regenerated. This regeneration is a preprocessing operation and does not modify the underlying power-law material distribution. For mesh refinements in the in-plane directions that leave the set of integration-point coordinates along the y -direction unchanged, the same material-property table can be retained. Thus, the mesh dependence of the proposed approach concerns the discrete representation of the material law at the integration points rather than the material law itself, which remains defined by Eq. (1). Although the power-law distribution in Eq. (1) is adopted as the reference gradation, the procedure is not intrinsically restricted to this distribution. Other prescribed continuous material-property functions can, in principle, be represented in the same manner, provided that their values can be defined at the required integration-point coordinates.

After determining the values of E and ν as functions of temperature at each integration point, a thermal field is created in which the temperature is set equal to the coordinate y ($Temp = y$). Once the Newton-Raphson iterations in Fig. 4 are performed, the coordinates of the integration points are automatically assigned to the fictitious temperatures, which leads to the definition of E and ν as functions of the spatial coordinate.

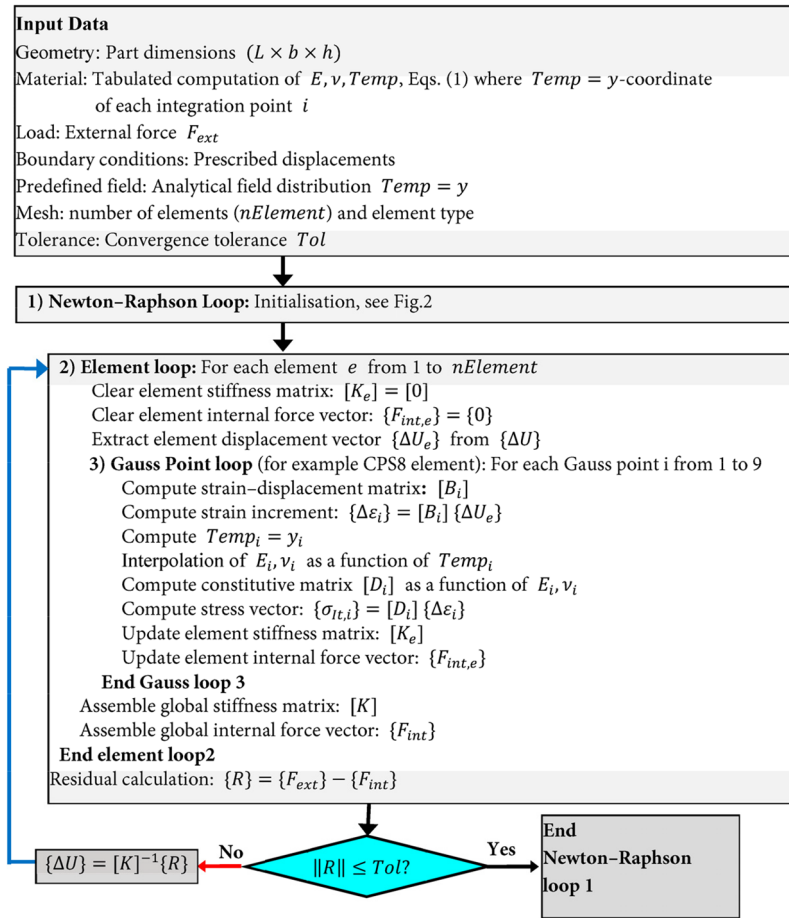


Figure 4: Flowchart of the equivalent thermal field method for linear elastic analysis.

4 Results and Validation

This section presents an application of the three methods introduced previously, followed by a comparative study to evaluate the performance, accuracy, and consistency of the proposed strategy.

The analyses are conducted using a linear elastic (*Static General*) approach in Abaqus [18]. Two simply supported structural elements subjected to transverse loads are considered: a 3D plate and a 2D beam. Here, the simply supported conditions equal to imposing the constraint $U_2 = 0$, where U_2 denotes the displacement component in the transverse direction (i.e., orthogonal to the support of both the plate and the beam). Thus, in the 3D plate $U_2 = 0$ is imposed along the four edges of the middle plane, whereas for the 2D beam $U_2 = 0$ is imposed at the two end nodes. These constraints are not sufficient to prevent the considered elements from rigid body motions; however, as it will be shown later, it was verified that, since the load acts only transversely and the elements are initially flat and operate in a linear environment, this constraint is enough to obtain meaningful results.

The configuration and loading of the 3D plate are presented in Fig. 5, while those of the 2D beam are shown in Fig. 6; the material and geometric properties of both structural elements are summarised in Table 1.

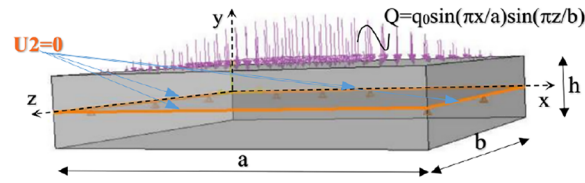


Figure 5: 3D plate ($a \times h \times b$) subjected to a sinusoidal load.

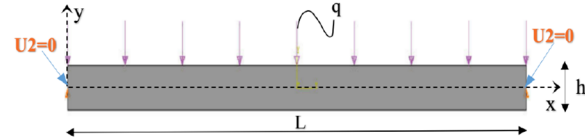


Figure 6: 2D beam ($L \times h$) subjected to a uniformly distributed load.

Table 1: Model geometry, materials, loads, and element types.

	2D Beam	3D Plate
Geometry	$L = 4 \text{ m}, h = 0.2 \text{ m}$	$a = 0.1 \text{ m}, h = 0.02 \text{ m}, b = 0.1 \text{ m}$
Materials	$E_c = 380 \text{ GPa}, E_m = 70 \text{ GPa},$ $\nu_c = \nu_m = 0.3$	$E_c = 427 \text{ GPa}, E_m = 70 \text{ GPa},$ $\nu_c = 0.17, \nu_m = 0.3$
Load	$q = 10 \text{ kN/m}$	$Q = 1000 \sin(\pi x/0.1) \sin(\pi z/0.1) \text{ kN/m}^2$
Boundary conditions	Simply supported ($U_2 = 0$) at both ends of the beam midline	Simply supported ($U_2 = 0$) at the four edges of the plate mid-surface
Element Types	CPS4 with 4 Gauss points and CPS8 with 9 Gauss points	C3D8 with 8 Gauss points and C3D20 with 27 Gauss points

The Fortran subroutine developed for the UMAT formulation and implemented for 3D simply supported plates is presented in [Appendix A.1](#). The constitutive relationship is formulated using a six-components (also dubbed as Voigt's) representation of stress, corresponding to a three-dimensional stress state, with the material stiffness expressed as a 6×6 constitutive matrix. For 2D simply supported beams, the formulation is simplified by reducing the order of these quantities to three components. However, the present *Equivalent thermal field method* was designed to improve the efficiency of material-property integration, and thus a Python script developed for this purpose, reported in [Appendix A.2](#), automatically generates a three-column text file containing the required material data. This file can then be directly imported into the "Properties" module using the "Read from File" command, enabling a faster and more reliable definition of material properties.

The results presented in [Tables 2–5](#) present the numerical values of the transverse deflection U_2 and of the normal stresses σ_{xx} as obtained for the three methods using both linear (C3D8 and CPS4 for the plate and the beam, respectively) and quadratic (C3D20 and CPS8 for the plate and the beam, respectively) elements. In detail, [Tables 2](#) and [3](#) present these results for the plate in a 3D environment, while [Tables 4](#) and [5](#) present the same results for the beam in a 2D ambient space.

Table 2: 3D plate deflection U_2 (10^{-6} m) at $(a/2, 0, b/2)$ as a function of power index p , the number of elements and the element type (quadratic: C3D20 and linear: C3D8).

Element Type	Number of Elements	Present			UMAT			Partition		
		$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$
C3D20	2	1.803	2.363	4.628	1.803	2.363	4.628	1.671	2.223	6.040
	4	1.856	2.424	4.771	1.856	2.424	4.771	1.796	2.380	5.288
	6	1.879	2.451	4.849	1.879	2.451	4.849	1.844	2.431	5.092
	8	1.894	2.470	4.900	1.894	2.470	4.900	1.844	2.458	5.039
	10	1.905	2.483	4.938	1.905	2.483	4.938	1.888	2.476	5.027
	12	1.914	2.494	4.969	1.914	2.494	4.969	1.901	2.489	5.031
	14			4.994			4.994			5.040
	16			5.016			5.016			5.051
C3D8	2	1.759	2.321	5.504	1.806	2.387	5.182	1.723	2.321	6.411
	4	1.810	2.380	5.034	1.829	2.402	4.883	1.789	2.380	5.296
	6	1.841	2.413	4.930	1.851	2.423	4.855	1.827	2.413	5.048
	8	1.861	2.435	4.918	1.867	2.441	4.873	1.850	2.435	4.985
	10	1.874	2.450	4.924	1.879	2.454	4.895	1.866	2.450	4.967
	12	1.885	2.462	4.939	1.888	2.465	4.918	1.878	2.462	4.968
	14	1.893		4.954	1.895	2.474	4.939	1.888		4.976

Note: Blank = Convergence of the present method reached.

Table 3: 3D plate stress σ_{xx} (MPa) at $(a/2, h/2, b/2)$ as a function of power index p , the number of elements and the element type (quadratic: C3D20 and linear: C3D8).

Element Type	Number of Elements	Present			UMAT			Partition		
		$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$
C3D20	2	6.772	7.999	10.470	6.772	7.999	10.470	5.985	6.690	6.152
	4	6.853	8.101	12.680	6.853	8.101	12.680	6.446	7.433	7.616
	6	6.880	8.133	13.220	6.880	8.133	13.220	6.616	7.689	8.761
	8	6.898	8.153	13.420	6.898	8.153	13.420	6.704	7.820	9.594
	10	6.910	8.167	13.520	6.910	8.167	13.520	6.759	7.902	10.210
	12		8.179	13.580		8.179	13.580		8.032	11.190
C3D8	2	3.444	3.580	3.645	3.736	4.081	5.018	3.431	3.580	3.560
	4	4.973	5.526	5.692	5.198	5.961	8.468	4.944	5.526	5.534
	6	5.580	6.332	7.134	5.743	6.661	10.020	5.555	6.332	7.017
	8	5.903	6.767	8.192	6.029	7.028	10.890	5.883	6.767	8.109
	10	6.104	7.039	8.978	6.206	7.255	11.440	6.087	7.039	8.917
	12	6.241	7.226	9.578	6.326	7.410	11.820	6.227	7.226	9.533
	14	6.341	7.363	10.050	6.414	7.522	12.090	6.329	7.363	10.010
	16	6.417	7.467	10.430	6.481	7.608	12.300	6.407	7.467	10.400
	18	6.477	7.550	10.740	6.534	7.676	12.460	6.468	7.550	10.720
	20	6.526	7.617	11.000	6.578	7.732	12.600	6.518	7.617	10.980

(Continued)

Table 3 (continued)

Element Type	Number of Elements	Present			UMAT			Partition		
		$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$
	22		7.672	11.220		7.777	12.710		7.672	11.200
	24			11.410			12.800			11.400
	26			11.570			12.880			11.560
	28			11.720			12.950			11.710
	30			11.850			13.010			11.840
	32			11.960			13.060			11.950
	34			12.060			13.110			12.050

Note: Blank = Convergence of the present method reached.

Table 4: 2D beam deflection U_2 (10^{-4} m) at $(L/2, 0)$ as a function of power index p , the number of elements and the element type (quadratic: CPS8 and linear: CPS4).

Element Type	Number of Elements	Present			UMAT			Partition		
		$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$
CPS8	2	2.030	2.654	4.456	2.030	2.654	4.456	1.841	2.454	6.448
	4	2.038	2.654	4.433	2.038	2.654	4.433	1.956	2.596	5.144
	6	2.039	2.654	4.433	2.039	2.654	4.433	1.993	2.628	4.763
CPS4	2	1.668	2.164	4.915	1.768	2.321	4.332	1.636	2.164	5.742
	4	1.918	2.506	4.690	1.963	2.561	4.355	1.894	2.506	5.001
	6	1.981	2.586	4.562	2.005	2.612	4.389	1.964	2.586	4.705
	8	2.006	2.615	4.508	2.020	2.63	4.406	1.993	2.615	4.589
	10	2.018	2.629	4.482	2.027	2.639	4.415	2.008	2.629	4.534
	12	2.025	2.638	4.469	2.032	2.644	4.422	2.017	2.638	4.505
		(2.041)	(2.653)	(4.436)						

Note: Values in brackets denote the analytical deflections U_2 (10^{-4} m) at $(L/2, 0)$ provided by Li et al. [23].

Table 5: 2D beam stress σ_{xx} (MPa) at $(L/2, h/2)$ as a function of power index p , the number of elements and the element type (quadratic: CPS8 and linear: CPS4).

Element Type	Number of Elements	Present			UMAT			Partition		
		$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$
CPS8	2	3.934	4.643	5.756	3.934	4.643	5.756	3.348	3.671	3.181
	4	3.936	4.641	7.112	3.936	4.641	7.112	3.643	4.152	3.858
	6	3.937	4.641	7.425	3.937	4.641	7.425	3.749	4.317	4.500
	8			7.528			7.528			4.989
	10			7.571			7.571			5.356
	12			7.591			7.591			5.637

(Continued)

Table 5 (continued)

Element Type	Number of Elements	Present			UMAT			Partition		
		$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$	$p = 0.5$	$p = 1$	$p = 10$
CPS4	2	3.158	3.426	3.265	3.460	3.900	3.629	3.133	3.426	2.976
	4	3.652	4.135	4.046	3.863	4.508	5.557	3.626	4.135	3.862
	6	3.785	4.340	4.647	3.930	4.614	6.454	3.765	4.340	4.538
	8	3.839	4.429	5.105	3.947	4.644	6.891	3.823	4.429	5.034
	10	3.867	4.479	5.450	3.953	4.655	7.130	3.855	4.479	5.401
	12	3.885	4.512	5.718	3.956	4.660	7.274	3.875	4.512	5.682
	14		4.532	5.927		4.660	7.364		4.532	5.899
	16			6.096			7.426			6.074
	18			6.235			7.469			6.217
	20			6.351			7.501			6.337
	22			6.450			7.524			6.437
	24			6.534			7.542			6.524
	26			6.607			7.556			6.598
	28			6.671			7.567			6.663
	30			6.728			7.576			6.721
	32			6.778			7.584			6.772
	34			6.823			7.590			6.817
	36			6.863			7.595			6.858
	38			6.900			7.599			6.895
	40			6.933			7.602			6.929
		(3.940)	(4.641)	(7.627)						

Note: Blank = Convergence of the present method reached. Values in brackets denote analytical stresses σ_{xx} (MPa) at $(L/2, h/2)$ from Li et al. [23].

As it is expected from the physical and numerical points of view, we see that the deflections of both the plate and the beam (Tables 2 and 4) are represented by larger values as the number of elements grows, irrespective of the same elements being linear or quadratic. This is a well-known behaviour, since more elements imply a more refined description of the deformative process, thus a seeming ‘softer’ behaviour, which is only numerical and has no constitutive implication.

An analogous comment can be brought about the effect of using linear or quadratic elements, the latter better describing a deformation process and thus leading to a quicker convergence and to another seeming ‘softer’ behaviour, once again only numerical.

A strongly physical interpretation of the results comes from analysing the outputs for various exponents p of the power law distribution of functional grading reported in Eq. (1): indeed, the outputs indicate that the increase of the power index p from 0.5 to 10 implies that the metallic phase becomes predominant, leading to a reduction in the overall flexural stiffness. This is physically expected and corresponds to the fact that the ceramic phase is stiffer than the metallic one, thus in the parallel of the two rigidities the contribution of the stiffer one decreases in favour of the softer one, as it is well known.

More important, the results for the three methods are comparable, when not exactly coinciding, thus showing that the presented technique can easily replicate more worked approaches in a very simple way and with less computational effort and user's operation, as it will also be detailed later.

As per the stress component σ_{xx} (Tables 3 and 5), one can remark that the present technique provides the same values as the well-established UMAT, while it overestimates those given by the partition technique when using quadratic elements; the opposite happens when using linear elements. This is easily explained always by the fact that quadratic elements better and quicker describe the states of stress and deformation of the considered element, and, while UMAT is more refined right from the beginning and provides better approximations than the partition method (the reasons were described in Section 2), the opposite happens when considering linear elements.

The considerations about the number of elements are like those reported for the values of the transverse displacements, and will be considered in more detail in the next subsection; what is important to remark is that, apart from some numerical non-monotonicity that cancels out at convergence of the results, stress is larger for quadratic elements, which better describe a larger deformation, hence require a larger stress—and this is perfectly reasonable and corresponding to physical intuition and ordinary behaviour of linear elastic 'springs'.

On the other hand, as per the effect of the power law exponent p of the functional grading presented in Eq. (1), we may see that the value of the stress grows with p , and this is also physically expected and motivated: indeed, as already observed, the deformation grows, and, while on the one hand the global stiffness of the system is dominated by the 'softer' metallic phase, in order to maintain such a growing transverse deflection a much greater bending stress is needed.

It is remarkable that these considerations hold for both the plate and the beam elements investigated, which provides an indication of the robustness of the proposed technique for the static problems considered. As a further comparison with results in the literature, in Tables 4 and 5 the values from [23] are reported, to have a further insight on the validity of the proposed technique.

To verify that the unconstrained in-plane displacement components do not influence the numerical results, an additional calculation was performed for the beam using a conventional roller–pin configuration. Specifically, U_2 was constrained at the left end, while both U_1 and U_2 were constrained at the right end, thereby eliminating the possible rigid-body motions. As shown in Fig. 7, the resulting transverse displacement U_2 , including the mid-span deflection, is identical to that obtained using the original boundary conditions. The corresponding mid-span deflection is also the same as the value reported in Table 4. This verification confirms that the possible unconstrained in-plane rigid-body modes have no influence on the transverse response or on the numerical results reported in this study. Consequently, the original boundary conditions presented at the beginning of this section were retained.

4.1 Convergence Analysis of the Present Results

The convergence analysis of the present method is evaluated using the relative errors ΔU , $\Delta \sigma$:

$$\begin{aligned}\Delta \sigma &= \frac{\sigma_{xx}(\text{number of elements}) - \sigma_{xx}(\text{number of elements} - 2)}{\sigma_{xx}(\text{number of elements} - 2)} \\ \Delta U_2 &= \frac{U_2(\text{number of elements}) - U_2(\text{number of elements} - 2)}{U_2(\text{number of elements} - 2)}\end{aligned}\quad (3)$$

and highlights the efficiency of the quadratic elements C3D20, CPS8, which was somehow hinted on in the previous subsection when commenting the results presented in Tables 2–5.

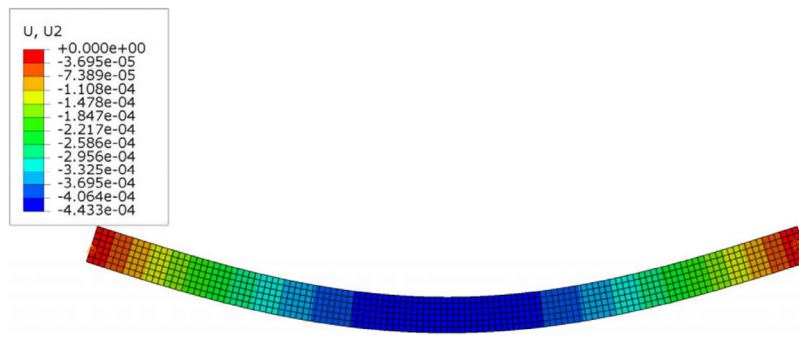


Figure 7: Beam for $p = 10$, U_2 constrained at the left end, both U_1 and U_2 constrained at the right end.

As shown in Figs. 8–11, a convergence criterion of 0.5% (used here as an illustrative value) is reached rapidly with these elements, particularly for the stresses; for instance, for $p = 10$, a maximum of only 10 C3D20 elements is required for the stresses and 14 for the deflections.

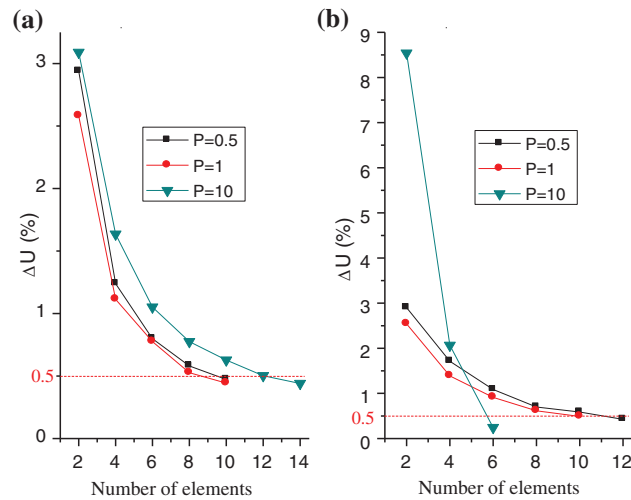


Figure 8: Convergence of deflections with respect to the number of elements: (a) C3D20, (b) C3D8.

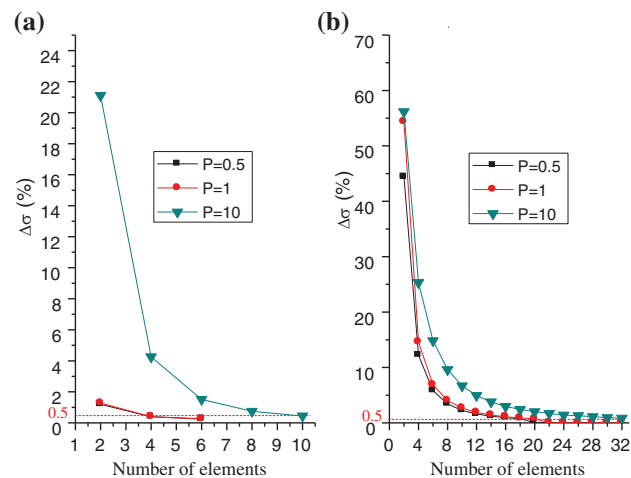


Figure 9: Convergence of stresses with respect to the number of elements: (a) C3D20, (b) C3D8.

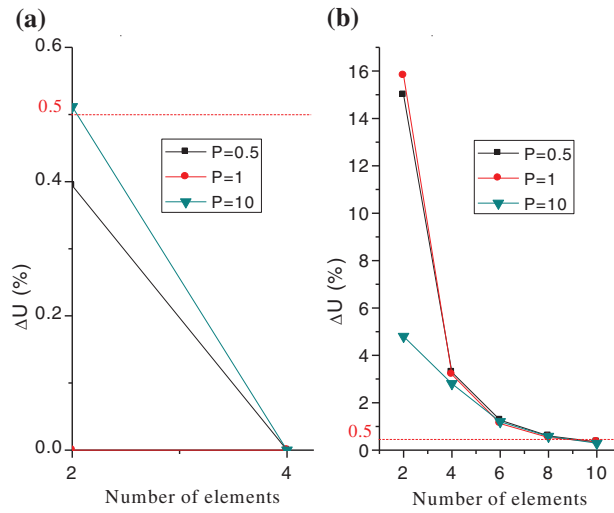


Figure 10: Convergence of deflections with respect to the number of elements: (a) CPS8, (b) CPS4.

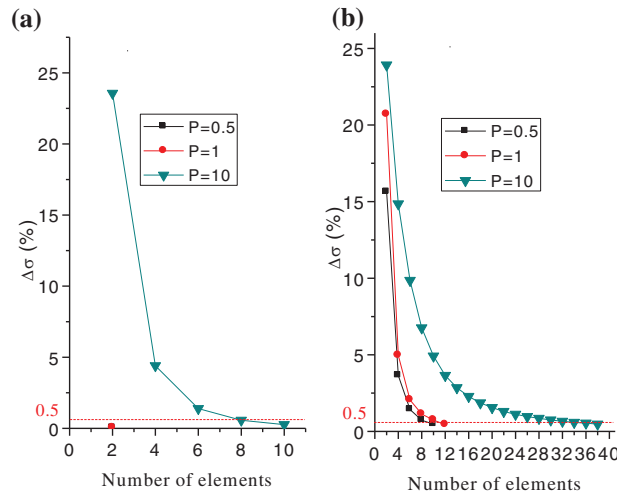


Figure 11: Convergence of stresses with respect to the number of elements: (a) CPS8, (b) CPS4.

In contrast, linear elements (C3D8, CPS4) exhibit significantly slower convergence, especially for the stress. Figs. 8–11 also indicate that the required mesh density for these elements is strongly dependent on the power index p . For CPS4 elements, the number of elements required increases substantially, from 12 to 38 as p varies from 0.5 to 10. Moreover, the lowest convergence value achieved with C3D8 for $p = 10$ remains slightly above the criterion (i.e., $0.83\% > 0.5\%$). Therefore, the use of linear elements constitutes a limitation of the present method.

4.2 Comparative Study

The horizontal analysis of the results shown in Tables 2–5 reveals that for the quadratic elements C3D20 and CPS8 the deflections and stresses obtained by the proposed method are identical to those from the UMAT approach, regardless of mesh density. This confirms the effectiveness of the equivalent thermal method, in which temperatures are evaluated at all Gauss points. However, the relative error compared to the partition method is substantial, reaching 84% for the 2D beam with $p = 10$ and 4 CPS8 elements, and remains significant (35%) even after numerical convergence. Moreover, the analytical deflections U_2 and

stresses σ_{xx} reported by Li et al. [23] in Tables 4 and 5 show very good agreement with the results obtained using the proposed equivalent thermal-field method with quadratic CPS8 elements, providing additional validation of its accuracy for both deflection and stress predictions. As an additional independent validation, a 3D plate case with $p = 2$, which is not included in Tables 2 and 3, was compared with the analytical solution of Vel and Batra [24]. For $p = 2$, C3D20 elements through the thickness, the proposed method gives $\sigma_{xx}(a/2, h/2, b/2) = 9.943$ MPa and $U_2(a/2, 0, b/2) = 3.339 \times 10^{-6}$ m, compared with 10.261 MPa and 3.351×10^{-6} m, respectively, from Vel and Batra [24]. The corresponding relative differences are approximately 3.2% for the stress and 0.36% for the deflection, further supporting the accuracy of the proposed method.

For the linear elements C3D8 and CPS4, the results obtained by the proposed method diverge from the UMAT approach, with a stress discrepancy reaching 39% for 6 CPS4 elements. Although mesh refinement (up to 38 elements) reduces this gap, the error remains significant at approximately 10%. In contrast, the results of the present method show strong agreement with the partition method. This consistency arises because, within the proposed approach and using linear elements, the temperature is calculated at the element centre and then assigned to all Gauss points. Consequently, the temperature-dependent mechanical properties E , ν are linearly interpolated at the Gauss points, whereas their actual evolution follows a non-linear power law (except for $p = 1$). It should be noted that the partition method relies on a similar principle, where properties are pre-calculated at the mid-surface of each layer and uniformly assigned to the entire horizontal elements. The slight discrepancy observed between the equivalent thermal and partition methods for these elements is explained by the incompatibility between the non-linear material properties and the linear interpolation employed. This hypothesis is corroborated by the specific case of a linear power law (with $p = 1$), for which the results become identical. Consequently, the use of linear elements represents an inherent limitation of the proposed method.

A quantitative comparison of the three approaches was performed for the 3D plate using identical geometry, mesh, material properties, loading, boundary conditions, and analysis settings. As reported in Table 6, all three models use 43,200 C3D20 elements and converge in one load increment with one equilibrium iteration, corresponding to a linear static analysis. The proposed method requires 412.90 s of CPU time, compared with 492.70 s for the UMAT approach, corresponding to a reduction of approximately 19.3%. More importantly, the model preparation time is substantially reduced, from approximately 26 min for the partition method and 35 min for the UMAT approach to approximately 9 min for the proposed method. In addition, unlike the UMAT approach, the proposed method does not require the installation and configuration of an external Fortran compilation environment, such as Intel Fortran and Visual Studio. These results demonstrate that the main advantage of the proposed method lies in reducing model preparation and implementation effort while maintaining comparable computational requirements.

Table 6: Quantitative comparison of the proposed, UMAT, and partition methods for the 3D plate.

Quantity	Present	UMAT	Partition
Elements	43,200	43,200	43,200
Element type	C3D20	C3D20	C3D20
.cae size (MB)	5.77	5.77	5.80
.inp size (MB)	17.24	17.21	17.32
Total CPU time (s)	412.90	492.70	434
Number of increments	1	1	1
Equilibrium iterations	1	1	1
Model preparation time (min)	~9	~35	~26

5 Conclusions

In this work, special attention has been devoted to the numerical modelling of FGM structures using Abaqus. The evaluation of existing approaches, notably the layer-wise partition and UMAT subroutines, highlights significant limitations in terms of implementation complexity, model preparation effort, and computational time. To address these challenges, an alternative strategy based on an equivalent thermal field has been proposed. This approach provides a more streamlined, time-efficient, and accessible framework for FGM modelling.

The comparative study shows that the results that we obtained by using quadratic elements are virtually identical to those that we obtained by subroutine-based methods. These findings confirm the robust capability of the proposed approach to accurately capture the mechanical behaviour of FGM structures, in terms of both deflections and stresses. Furthermore, the convergence analysis underscores the superior performance of quadratic elements, which achieve high accuracy with a significantly reduced mesh density.

However, the use of linear elements (C3D8, CPS4) represents a notable limitation of the proposed method. These elements exhibit slower convergence, particularly for stress evaluations, and show a persistent discrepancy compared to the UMAT reference, even after extensive mesh refinement.

Although the equivalent thermal-field concept can, in principle, be extended to FGM structures exhibiting geometrically linear or nonlinear elastic behaviour, provided that the positions of the integration points along the material-gradient direction are known, the scope of the present study is limited to linear elastic static analyses of beams and plates with a one-dimensional power-law variation of material properties along the thickness direction and without simultaneous physical thermal loading. The applicability of the method to multidirectional FGMs, arbitrary three-dimensional material distributions, material nonlinearities, and dynamic or transient loading remains to be assessed. Future work will therefore focus on investigating such extensions.

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Availability of Data and Materials: The data that were generated are on the one hand replicable by using the numerical subroutines presented in the Appendix and, on the other hand, will be made available by the authors upon request.

Ethics Approval: Not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

A.1 UMAT Fortran Subroutine for Three-Dimensional Plate Models

SUBROUTINE UMAT(STRESS, STATEV, DDSDD, SSE, SPD, SCD, RPL,

```

1 DDSDDT, DRPLDE, DRPLDT, STRAN, DSTRAN, TIME, DTIME,
2 TEMP, DTEMP, PREDEF, DPRED, CMNAME, NDI, NSHR, NTENS,
3 NSTATV, PROPS, NPROPS, COORDS, DROT, PNEWDT, CELENT,
4 DFGRD0, DFGRD1, NOEL, NPT, LAYER, KSPT, KSTEP, KINC)
C
  INCLUDE 'ABA_PARAM.INC'
C
  CHARACTER*80 CMNAME
  INTEGER NDI, NSHR, NPROPS
  INTEGER NOEL, NPT, LAYER, KSPT, KSTEP, KINC
  INTEGER NTENS, NSTATV
C ----Declaration-----
  DOUBLE PRECISION STRESS(NTENS)
  DOUBLE PRECISION STATEV(NSTATV)
  DOUBLE PRECISION DDSDDDE(NTENS, NTENS)
  DOUBLE PRECISION STRAN(NTENS)
  DOUBLE PRECISION DSTRAN(NTENS)
  DOUBLE PRECISION E, NU, MU, LAMBDA
  DOUBLE PRECISION Em, Ec, num, nuc, Y, H, ETA, P
C
  Em = 7.0D7
  Ec = 4.27D8
  num = 0.30D0
  nuc = 0.17D0
  H = 0.02D0
  P = 1.0D0
C
  Y = COORDS(2)
  ETA = (Y/H) + 0.5D0
  E = Em + (Ec-Em) *ETA**P
  NU = num + (nuc-num)*ETA**P
C
  MU = E/(2.D0*(1.D0 + NU))
  LAMBDA = E*NU/((1.D0 + NU)*(1.D0-2.D0*NU))
C
  DDSDDDE = 0.D0
  DDSDDDE(1, 1) = LAMBDA + 2.D0*MU
  DDSDDDE(2, 2) = DDSDDDE(1,1)

```

```

        DDSDE(3, 3) = DDSDE(1,1)
        DDSDE(1, 2) = LAMBDA
        DDSDE(1, 3) = LAMBDA
        DDSDE(2, 1) = LAMBDA
        DDSDE(2, 3) = LAMBDA
        DDSDE(3, 1) = LAMBDA
        DDSDE(3, 2) = LAMBDA
        DDSDE(4, 4) = MU
        DDSDE(5, 5) = MU
        DDSDE(6, 6) = MU
C
    STRESS = MATMUL(DDSDE, DSTRAN)
C
    RETURN
END

```

A.2 Python Script Used to Generate Properties Text File

```

import math
# - - - PARAMETERS - - -
nElement = 12
p = 0.5
Em, Ec = 7e+7, 380e+6
NUm, NUc = 0.3, 0.3
H = 0.2
eta = [-math.sqrt(0.6), 0, math.sqrt(0.6)]
Hc = H/nElement
# - - - WRITE MATERIAL PROPERTIES TO A HEADERLESS 3-COLUMN TEXT FILE - -
with open("Properties_E,Nu,Temp_Ziane.txt", "w") as f:
    for e in range(1, nElement + 1):
        for i in range(len(eta)):
            y = -H/2 + (e-1) * Hc + (Hc/2) * (1 + eta[i])
            ratio = (y/H + 0.5)**p
            Ei = Em + (Ec - Em) * ratio
            NUi = NUm + (NUc - NUm) * ratio
            f.write( f"{Ei:.15e} {NUi:.15e} {y:.15e}\n")

```

References

1. Malkiya Rasalin Prince R, Meher RS, Arulkirubakaran D, Debnath T, Immanuel ID, Jayaprakash P, et al. Comprehensive review on lanthanum based thermal barrier coating materials for high temperature applications. *Mater Today Proc.* 2022;68(10):1769–74. doi:10.1016/j.matpr.2022.10.054.
2. Han Z, Yin Y, Bai Y, Dong H, Yang T, Ma W. Optimized ZrO₂-based thermal barrier coatings via solution precursor plasma spraying with enhanced phase stability. *Surf Coat Technol.* 2025;513:132455. doi:10.1016/j.surfcoat.2025.132455.
3. Iqbal A, Moskal G. Recent development in advance ceramic materials and understanding the mechanisms of thermal barrier coatings degradation. *Arch Comput Meth Eng.* 2023;30(8):4855–96. doi:10.1007/s11831-023-09960-7.
4. Ren S, Sun Y, Yan H, Zhang X, Bao Y, Lv K. A review on the evolution of thermal and environmental barrier coating systems and their high-temperature degradation mechanisms in advanced aero-engines. *Materials.* 2026;19(11):2413. doi:10.3390/ma19112413.
5. Zhang Y, Cheng Y, Yan P, Feng G, Zhang P. Thermal and environmental barrier coatings for next-generation aerospace applications: a comprehensive state-of-the-art review. *Open Ceram.* 2026;27(19):101015. doi:10.1016/j.oceram.2026.101015.
6. Chen BY, Gao PH, Zhao DM, Li Q, Naumov A, Aleksander T, et al. Research progress in bond coats for thermal barrier coatings: from material design, preparation processes to post-treatment. *Rev Mater Res.* 2026;2(7):100250. doi:10.1016/j.revmat.2026.100250.
7. Sobhy M, Zenkour AM. Porosity and inhomogeneity effects on the buckling and vibration of double-FGM nanoplates via a quasi-3D refined theory. *Compos Struct.* 2019;220:289–303. doi:10.1016/j.compstruct.2019.03.096.
8. Al-Furjan MSH, Habibi M, Ghabussi A, Safarpour H, Safarpour M, Tounsi A. Non-polynomial framework for stress and strain response of the FG-GPLRC disk using three-dimensional refined higher-order theory. *Eng Struct.* 2021;228:111496. doi:10.1016/j.engstruct.2020.111496.
9. Du C, Zheng Y, Zhou X, Zhang D. Rigid-flexible coupling dynamic modeling of a rotating functionally graded material rectangular Mindlin plate. *Mech Eng.* 2021;43(6):849–60.
10. Du C, Gao X, Zhang D, Zhou X. Dynamic analysis of a rotating FGM beam with the point interpolation method. *Int J Comput Methods.* 2022;19(7):2141013. doi:10.1142/s0219876221410139.
11. Du C, Xu N, Li L, Yu C, Zhang D. Dynamic analysis of a hub-FGM micro-beam based on meshless method in thermal environment. *Eur J Mech A.* 2026;117(5):105971. doi:10.1016/j.euromechsol.2025.105971.
12. Alnujaie A, Sayyad AS, Hadji L, Tounsi A. Buckling and free vibration analysis of multi-directional functionally graded sandwich plates. *Struct Eng Mech.* 2022;84(6):813–22. doi:10.12989/sem.2022.84.6.813.
13. Alkunte S, Fidan I, Naikwadi V, Gudavasov S, Ali MA, Mahmudov M, et al. Advancements and challenges in additively manufactured functionally graded materials: a comprehensive review. *J Manuf Mater Process.* 2024;8(1):23. doi:10.3390/jmmp8010023.
14. Patel M. A comprehensive review of functionally graded materials and their ballistic impact performance: current status and future challenges. *Next Mater.* 2025;8:100704. doi:10.1016/j.nxmater.2025.100704.
15. Ziane N, Meftah SA, Belhadj HA, Tounsi A, Adda Bedia EA. Free vibration analysis of thin and thick-walled FGM box beams. *Int J Mech Sci.* 2013;66:273–82. doi:10.1016/j.ijmecsci.2012.12.001.
16. Sadowski T, Birsan M, Pietras D. Multilayered and FGM structural elements under mechanical and thermal loads. Part I: comparison of finite elements and analytical models. *Arch Civ Mech Eng.* 2015;15(4):1180–92. doi:10.1016/j.acme.2014.09.004.
17. Ziane N, Meftah SA, Ruta G, Tounsi A. Thermal effects on the instabilities of porous FGM box beams. *Eng Struct.* 2017;134(1):150–8. doi:10.1016/j.engstruct.2016.12.039.
18. Dassault Systemes Simulia Corp. ABAQUS user's manual, ver. 2017. Providence, RI, USA: Dassault Systèmes Simulia Corp.; 2017.
19. Jrad H, Mars J, Wali M, Dammak F. An extended finite element method for modeling elastoplastic FGM plate-shell type structures. *Struct Eng Mech.* 2018;68(3):299–312.

20. Jrad H, Mars J, Wali M, Dammak F. Geometrically nonlinear analysis of elastoplastic behavior of functionally graded shells. *Eng Comput.* 2019;35(3):833–47. doi:10.1007/s00366-018-0633-3.
21. Burlayenko VN, Sadowski T. Free vibrations and static analysis of functionally graded sandwich plates with three-dimensional finite elements. *Meccanica.* 2020;55(4):815–32. doi:10.1007/s11012-019-01001-7.
22. Kessentini A, Allouch M, Jrad H, Mars J, Ben Said L, Alhadri M, et al. Mathematical formulation and numerical simulation of the mechanical behavior of ceramic/metal (TiB/Ti) FG sheets subjected to spherical indenter. *Mathematics.* 2024;12(2):209. doi:10.3390/math12020209.
23. Li XF, Wang BL, Han JC. A higher-order theory for static and dynamic analyses of functionally graded beams. *Arch Appl Mech.* 2010;80(10):1197–212. doi:10.1007/s00419-010-0435-6.
24. Vel SS, Batra RC. Exact solution for thermoelastic deformations of functionally graded thick rectangular plates. *AIAA J.* 2002;40(7):1421–33. doi:10.2514/2.1805.