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Effect of Euclidean and Geodesic Distance Models on Stochastic Buckling of Imperfect Cylindrical Shells

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ABSTRACT: Random geometric imperfections strongly influence the buckling resistance of thin cylindrical shells. In random field imperfection modeling, Euclidean and geodesic distances may generate different spatial correlation structures on closed cylindrical shells. This study examines whether such distance model differences are transmitted to nonlinear stochastic buckling response. A paired stochastic finite element framework is developed, in which Euclidean- and geodesic-distance imperfection fields are generated from common nodal white noise inputs and rescaled to the same prescribed root-mean-square (RMS) amplitude. Eight normalized correlation lengths are considered, with 200 paired samples for each case, and the corresponding critical buckling loads are evaluated. The results show that the field-level discrepancy increases with correlation length, but its transmission to buckling response is not proportional. For the present benchmark, the Euclidean distance effect is very small at short correlation lengths but becomes increasingly evident as the correlation length approaches the shell radius scale, indicating a correlation scale-dependent trend rather than a universal acceptability threshold.

KEYWORDS: Stochastic buckling of cylindrical shell; random geometric imperfection; Euclidean distance; geodesic distance; paired random field

1 Introduction

Thin-walled cylindrical shells are widely used in engineering structures because of their high load-carrying efficiency. However, their buckling resistance under axial compression is highly sensitive to initial geometric imperfections. In practice, such imperfections are unavoidable and may arise from manufacturing, assembly, transportation, or installation. As a result, experimentally observed buckling loads are often much lower than the ideal predictions for perfect shells [1–3]. Realistic imperfection modeling is therefore essential for numerical buckling assessment and knockdown-factor evaluation, and the influence of geometric imperfections on cylindrical shell buckling continues to receive considerable attention in recent numerical and probabilistic studies [4,5].

In numerical buckling analysis, geometric imperfections must first be represented before they can be imposed on a finite element shell model. For a manufactured shell, the imperfection pattern is a deterministic geometric feature; however, before measurement or across a population of nominally identical shells, its spatial distribution is uncertain and varies from specimen to specimen. Measured imperfection data provide

direct information on specific specimens [2,6–8], whereas deterministic patterns, such as critical mode imperfections or selected eigenmode combinations, prescribe only a limited set of assumed shapes [9,10]. To represent this spatial uncertainty, random field models have been widely used to describe geometric imperfections in stochastic shell buckling analyses [11–15]. Such random field based representations have continued to be adopted in more recent shell buckling and uncertainty studies [16–19]. A key component of these models is the spatial correlation structure, which characterizes the statistical dependence of the imperfection between different locations on the shell surface [20,21].

In distance-dependent random field models, the correlation between two points is determined by their separation distance and the prescribed correlation length. On a flat domain, this separation is naturally measured by the Euclidean distance. On a curved surface, however, different distance measures may be adopted. From the viewpoint of surface consistency, the geodesic distance is a natural choice because it measures separation along the shell surface. Accordingly, geodesic distance-based and manifold-based random field models have been developed for random fields on curved surfaces and shell structures [22–26].

For a closed cylindrical shell, distance-dependent random field modeling may use either the Euclidean distance in the embedding space or the geodesic distance along the cylindrical surface. The geodesic distance is surface consistent, whereas the Euclidean distance is simple to compute and naturally compatible with standard Euclidean radial kernels. Since the two distances assign different separations to the same pair of surface points, they may lead to different distance-dependent correlation structures and generated imperfection fields. However, it remains unclear how large this field-level difference is and whether it leads to meaningful changes in nonlinear buckling response.

To address the above issue, this study develops a paired stochastic finite element framework for cylindrical shells with root-mean-square (RMS)-controlled random radial imperfections. Euclidean- and geodesic-distance random fields are generated on the same shell mesh under common stochastic inputs and identical RMS-amplitude control. The paired imperfection fields are then imposed on nonlinear finite element models. Based on the paired results, the study quantifies both the imperfection field discrepancy and the critical load discrepancy, thereby assessing how distance model differences are transmitted from random field to nonlinear buckling response.

The main contributions of this study are summarized as follows:

- A paired stochastic finite element framework is developed to treat the distance model itself as a controlled variable in random imperfection-based shell buckling analysis.
- A field-to-response discrepancy assessment is introduced to quantify how Euclidean–geodesic differences in RMS-controlled imperfection fields are transmitted to nonlinear critical load differences.
- The response differences are examined across correlation scales to characterize the scale-dependent influence of the Euclidean-distance approximation.

The remainder of this paper is organized as follows. First, [Section 2](#) introduces the distance-dependent paired random imperfection model, and [Section 3](#) describes the corresponding stochastic finite element buckling model and analysis procedure. Based on this framework, [Section 4](#) examines the distance model effect at the imperfection field level, while [Section 5](#) further investigates how the field-level discrepancy is transmitted to the nonlinear buckling response. The response-oriented findings are then used in [Section 6](#) to discuss the implications of the Euclidean distance approximation. Finally, [Section 7](#) summarizes the main conclusions.

2 Distance Dependent Paired Random Field Generation

2.1 General Gaussian Random Field Generation

A Gaussian random field provides a probabilistic representation of spatially distributed uncertainty. After discretizing a geometric domain Ω into N points $\{\mathbf{p}_i\}_{i=1}^N$, a scalar random field can be represented by the random vector

$$\mathbf{Y} = [Y(\mathbf{p}_1), Y(\mathbf{p}_2), \dots, Y(\mathbf{p}_N)]^T. \quad (1)$$

For a Gaussian random field, this vector follows a multivariate Gaussian distribution:

$$\mathbf{Y} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{C}), \quad (2)$$

where $\boldsymbol{\mu}$ is the mean vector and $\mathbf{C}$ is the covariance matrix.

In distance-dependent random field modeling, the covariance between two discrete points is specified through a correlation function of their separation distance. For a stationary field with variance σ^2 , this covariance is written as

$$C_{ij} = \sigma^2 \rho(d_{ij}; c), \quad (3)$$

where d_{ij} is the adopted distance between $\mathbf{p}_i$ and $\mathbf{p}_j$, c is the correlation length, and $\rho(\cdot; c)$ is the prescribed correlation function. Thus, for fixed σ^2 , c , and $\rho(\cdot; c)$, the covariance matrix is determined by the selected distance matrix.

Given a covariance matrix, a Gaussian random field realization can be generated from a standard Gaussian vector $\boldsymbol{\xi} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$. Then a realization of the discretized field is obtained as

$$\mathbf{Y} = \boldsymbol{\mu} + \boldsymbol{\Phi} \boldsymbol{\Lambda}^{1/2} \boldsymbol{\Phi}^T \boldsymbol{\xi}, \quad (4)$$

where $\boldsymbol{\Lambda}$ is the diagonal eigenvalue matrix and $\boldsymbol{\Phi}$ is the corresponding orthonormal eigenvector matrix of the covariance matrix $\mathbf{C} = \boldsymbol{\Phi} \boldsymbol{\Lambda} \boldsymbol{\Phi}^T$. This formulation shows that the adopted distance model affects the generated random field through the covariance matrix.

2.2 Cylindrical Shell Random Field Model

The random geometric imperfection is defined on the nominal middle surface of a closed cylindrical shell with radius R and length L , as illustrated in Fig. 1. Using the surface coordinates (θ, z) , a point on the nominal middle surface is given by

$$\mathbf{x}_0(\theta, z) = (R \cos \theta, R \sin \theta, z)^T. \quad (5)$$

The imperfection is represented by a scalar radial amplitude field $w_0(\theta, z)$ defined on this nominal surface. Accordingly, the local radius of the imperfect shell is

$$R_w(\theta, z) = R + w_0(\theta, z), \quad (6)$$

and the corresponding point on the imperfect middle surface is

$$\mathbf{x}_w(\theta, z) = [(R + w_0(\theta, z)) \cos \theta, (R + w_0(\theta, z)) \sin \theta, z]^T. \quad (7)$$

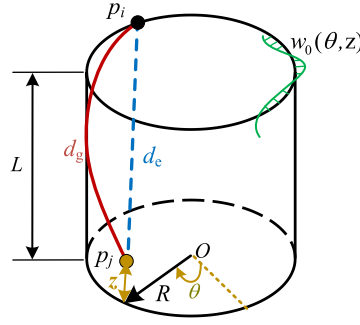


Figure 1: Schematic illustration of Euclidean chord distance and geodesic distance on a cylindrical surface.

Two distance measures are considered for points on the cylindrical surface. For two nominal surface points $\mathbf{x}_{0,i}$ and $\mathbf{x}_{0,j}$, the Euclidean distance is the straight line separation in the embedding space, as indicated by the blue dashed line in Fig. 1:

$$d_{e,ij} = \|\mathbf{x}_{0,i} - \mathbf{x}_{0,j}\|_2. \quad (8)$$

In contrast, the geodesic distance measures the shortest separation along the cylindrical surface, as shown by the red solid line in Fig. 1:

$$d_{g,ij} = \sqrt{(R\Delta\theta_{ij})^2 + (z_i - z_j)^2}, \quad \Delta\theta_{ij} = \min(|\theta_i - \theta_j|, 2\pi - |\theta_i - \theta_j|). \quad (9)$$

Here, $\Delta\theta_{ij}$ is the shortest circumferential angle difference and therefore accounts for the closed circumferential direction. The two distances are nearly identical for nearby points but increasingly differ as the circumferential separation becomes larger.

For the two distance models, the covariance matrices are constructed from the same correlation function and correlation length, differing only in the adopted separation distance. For $s \in \{e, g\}$, the covariance matrix is defined by

$$\mathbf{C}_s(i, j) = \rho(d_{s,ij}; c). \quad (10)$$

Since $\rho(0; c) = 1$, both matrices have unit marginal variance, while their off-diagonal terms differ according to the Euclidean and geodesic distances assigned to each pair of surface points.

The resulting covariance matrices are checked for positive semidefiniteness before random field generation, because this property is not guaranteed when the squared-exponential kernel is constructed from geodesic distances on a curved or periodic domain [27,28]. Let the eigendecomposition of the symmetrized covariance matrix be

$$\mathbf{C}_s = \mathbf{V}_s \mathbf{\Lambda}_s \mathbf{V}_s^T, \quad \mathbf{\Lambda}_s = \text{diag}(\lambda_{s,1}, \lambda_{s,2}, \dots, \lambda_{s,N}). \quad (11)$$

When negative eigenvalues are present, they are truncated to zero to obtain a positive-semidefinite matrix:

$$\lambda_{s,k}^+ = \max(\lambda_{s,k}, 0), \quad \mathbf{C}_s^+ = \mathbf{V}_s \mathbf{\Lambda}_s^+ \mathbf{V}_s^T. \quad (12)$$

The relative magnitude of the removed negative spectrum is quantified by

$$r_{\text{neg}} = \frac{\sum_{\lambda_{s,k} < 0} |\lambda_{s,k}|}{\sum_{k=1}^N |\lambda_{s,k}|}. \quad (13)$$

The corresponding change in the covariance matrix is further measured by the relative Frobenius-norm perturbation

$$\eta_F = \frac{\|\mathbf{C}_s^+ - \mathbf{C}_s\|_F}{\|\mathbf{C}_s\|_F} = \left(\frac{\sum_{\lambda_{s,k} < 0} \lambda_{s,k}^2}{\sum_{k=1}^N \lambda_{s,k}^2} \right)^{1/2}. \quad (14)$$

Accordingly, the geodesic distance model used here refers to the PSD-projected discrete covariance matrix. Mathematically valid periodic or manifold-based alternatives exist [29], but they involve different covariance constructions; the present study therefore retains the same nominal squared-exponential form for both distance models and quantifies the projection-induced modification separately.

2.3 Paired RMS Controlled Field Generation

The Euclidean- and geodesic-distance fields are generated in pairs using a common stochastic input, so that their differences can be compared without the additional variability introduced by independent sampling. For the m -th paired sample, a common standard Gaussian vector is defined as

$$\boldsymbol{\xi}^{(m)} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}). \quad (15)$$

For $s \in \{\mathbf{e}, \mathbf{g}\}$, the corresponding raw Gaussian field is generated as

$$\mathbf{Y}_s^{(m)} = (\mathbf{C}_s^+)^{1/2} \boldsymbol{\xi}^{(m)} = \mathbf{V}_s (\boldsymbol{\Lambda}_s^+)^{1/2} \mathbf{V}_s^T \boldsymbol{\xi}^{(m)}. \quad (16)$$

Under this construction, identical covariance matrices would produce identical raw fields, so the paired difference reflects the effect of the two covariance structures under the same latent stochastic input.

The raw fields are subsequently centered and normalized to impose a common global RMS amplitude:

$$\widehat{\mathbf{Y}}_s^{(m)} = \frac{\mathbf{Y}_s^{(m)} - \overline{Y}_s^{(m)} \mathbf{1}}{\text{RMS}(\mathbf{Y}_s^{(m)} - \overline{Y}_s^{(m)} \mathbf{1})}, \quad s \in \{\mathbf{e}, \mathbf{g}\}, \quad (17)$$

where

$$\overline{Y}_s^{(m)} = \frac{1}{N} \mathbf{1}^T \mathbf{Y}_s^{(m)}, \quad \text{RMS}(\mathbf{v}) = \left(\frac{1}{N} \mathbf{v}^T \mathbf{v} \right)^{1/2}. \quad (18)$$

The final imperfection field is then given by

$$\mathbf{w}_s^{(m)} = a_w \widehat{\mathbf{Y}}_s^{(m)}, \quad s \in \{\mathbf{e}, \mathbf{g}\}, \quad (19)$$

where a_w is the prescribed RMS amplitude. Thus, $\mathbf{w}_e^{(m)}$ and $\mathbf{w}_g^{(m)}$ have zero mean and the same global RMS amplitude, allowing the paired comparison to focus on differences in spatial morphology rather than overall imperfection amplitude.

It is noted that the common random number construction determines the joint pairing of the two marginal random field models and is not unique. Consequently, sample-wise paired quantities are conditional on the adopted coupling, whereas the marginal statistics of each distance model are independent of the pairing choice. The adopted coupling therefore provides a controlled comparison under the same latent stochastic input.

3 Stochastic Finite Element Buckling Analysis

3.1 Benchmark Finite Element Model

A thin cylindrical shell subjected to axial compression is used as the benchmark structure [11], with its geometry and material properties summarized in Fig. 2. The nominal middle surface is discretized using Abaqus S4R four-node reduced-integration shell elements, with 5376 nodes and 5248 elements in the baseline mesh, corresponding to an approximate element size of 5 mm. The bottom edge is fully constrained, while the non-axial displacement components of the top edge are restrained and axial motion is permitted.

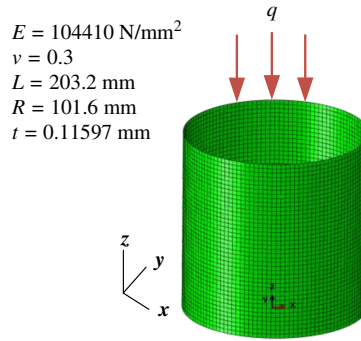


Figure 2: Benchmark cylindrical shell model, boundary conditions, and axial loading scheme.

The classical critical load of the corresponding perfect cylindrical shell is approximately $P_{th} = 5340$ N. A linear eigenvalue buckling analysis of the perfect-shell finite-element model gives $P_{FE} = 5346.43$ N, only 0.12% higher than the theoretical value, confirming the accuracy of the baseline finite-element representation.

For the subsequent nonlinear analyses of imperfect shells, the reference edge load is set to $q_{ref} = 8.365$ N/mm, corresponding to $P_{ref} = 2\pi R q_{ref} \approx 5340$ N = P_{th} . The load proportionality factor (LPF) therefore directly represents the applied load normalized by the classical perfect-shell critical load.

For the m -th paired sample generated from distance model $s \in \{e, g\}$, the RMS controlled random field $\mathbf{w}_s^{(m)}$ is imposed as a nodal radial imperfection. For a nominal node with cylindrical coordinates (R, θ_i, z_i) , the imperfect nodal coordinate is

$$\mathbf{x}_{s,i}^{(m)} = \left[\left(R + w_{s,i}^{(m)} \right) \cos \theta_i, \left(R + w_{s,i}^{(m)} \right) \sin \theta_i, z_i \right]^T. \quad (20)$$

The same finite element mesh, material properties, boundary conditions, and loading scheme are used for all imperfect shell realizations. Therefore, differences in the computed buckling responses can be attributed to the imposed imperfection fields rather than to changes in the finite element model or loading setup.

Because the shortest correlation length, $c = 5.08$ mm at $\alpha_c = 0.05$, is comparable to the baseline element size, mesh sensitivity of the nonlinear buckling response was assessed. For each of the eight correlation

lengths, 20 matched Euclidean–geodesic realizations were analyzed on both the baseline 128×41 and fine 256×82 meshes, with the latter approximately halving the element dimensions in both surface directions. As a supplementary check, six geodesic realizations were further analyzed on an extra-fine 512×164 mesh, including two each at $\alpha_c = 0.05$ and 0.10 , and one each at $\alpha_c = 0.30$ and 1.00 . The same imperfection realizations were transferred to the refined meshes by linear interpolation in the circumferential and axial directions.

3.2 Paired Random Imperfection Field Generation

The random field generation procedure described in [Section 2](#) is applied to the cylindrical shell model. In the present study, the squared-exponential correlation function is adopted:

$$\rho(d; c) = \exp \left[- \left(\frac{d}{c} \right)^2 \right], \quad (21)$$

where d denotes the adopted distance and c is the correlation length. The corresponding non-dimensional correlation length is defined as

$$\alpha_c = \frac{c}{R}. \quad (22)$$

Since the final imperfection amplitude is controlled by RMS normalization, the covariance matrices are constructed with unit marginal variance. The target RMS amplitude of the radial imperfection field is prescribed as $a_w/t = 0.5$. The same amplitude is used for all imperfection realizations generated from both distance models. Each field is shifted to zero mean and rescaled to this prescribed RMS amplitude before being imposed on the shell.

In the present study, eight non-dimensional correlation lengths are considered, i.e., $\alpha_c \in \{0.05, 0.10, 0.15, 0.20, 0.30, 0.50, 0.75, 1.00\}$. These values cover short-range, intermediate-range, and relatively long-range correlated imperfection patterns on the cylindrical surface. For each value of α_c , $N_s = 200$ paired samples are generated using the Euclidean- and geodesic-distance models. Since each paired sample produces two imperfect shell models, the total number of nonlinear finite element buckling analyses is 3200.

In the positive-semidefiniteness examination, the Euclidean distance covariance matrices remained positive semidefinite over the investigated range, whereas small negative eigenvalues occurred in the geodesic distance matrices at $\alpha_c = 0.75$ and 1.00 , requiring spectral projection. As summarized in [Table 1](#), the associated relative Frobenius-norm perturbations were very small, with a maximum value of 5.29×10^{-6} . Thus, the projection ensures a valid discrete covariance matrix while introducing only a negligible modification to the covariance structure used in the subsequent analysis.

Table 1: Spectral projection diagnostics for the geodesic distance covariance matrices.

α_c	n_{neg}	λ_{min}	r_{neg}	η_F
0.75	120	-8.25×10^{-7}	5.23×10^{-9}	2.24×10^{-9}
1.00	313	-3.52×10^{-3}	1.20×10^{-5}	5.29×10^{-6}

Note: n_{neg} denotes the number of negative eigenvalues before spectral projection, and λ_{min} is the minimum eigenvalue of the original covariance matrix.

3.3 Stochastic Nonlinear Buckling Analysis of Paired Samples

For each imperfect shell realization, a geometrically nonlinear buckling analysis is performed in Abaqus 2022 using the `Static, Riks` procedure with `NLGEOM=ON`. The initial, minimum, and maximum arc-length increments are 0.002, 1.0×10^{-6} , and 0.02, respectively, with a total arc-length scale factor of 1.0 and a maximum of 200 increments. The same analysis settings are used for all realizations generated from both distance models.

The critical load factor λ_{cr} was extracted automatically from the load proportionality factor (LPF) history. A frame i was identified as a candidate critical peak when $LPF_i \geq LPF_{i-1}$, $LPF_i > LPF_{i+1}$, $i \geq 3$, and $LPF_i \geq 0.05$, and the first frame satisfying these conditions was taken as λ_{cr} . This criterion excludes the initial loading stage and requires an explicit subsequent decrease in LPF. If no qualifying local maximum was detected, the global maximum of the available LPF history was specified as a fallback value. In the present database, all 3200 nonlinear analyses yielded a qualifying first local maximum, so no fallback or invalid cases occurred, and no cases required manual inspection. The robustness of the extraction rule was further examined by varying the LPF threshold to 0.02 and 0.10 and the minimum admissible frame to $i = 2$ and $i = 5$. None of these alternative settings changed the identified critical frame or the resulting λ_{cr} , indicating that the extracted critical load factors are insensitive to the tested peak-detection parameters.

The resulting critical load factors provide the basis for the paired response comparison. For the m -th stochastic input, the Euclidean- and geodesic-distance models yield a paired response $\lambda_{cr,e}^{(m)}$ and $\lambda_{cr,g}^{(m)}$, respectively. Their signed relative discrepancy is defined as

$$\Delta_{\lambda}^{(m)} = \frac{\lambda_{cr,e}^{(m)} - \lambda_{cr,g}^{(m)}}{\lambda_{cr,g}^{(m)}}. \quad (23)$$

A positive value of $\Delta_{\lambda}^{(m)}$ indicates a higher critical load factor predicted by the Euclidean distance model, whereas a negative value indicates a lower prediction.

The paired stochastic finite element procedure is summarized in Algorithm 1. For each correlation length and each Monte Carlo realization, the Euclidean- and geodesic-distance models are evaluated using the same white noise vector, the same RMS amplitude, and the same finite element settings.

Algorithm 1: Paired stochastic finite element buckling analysis

Require: Nominal FE model $\mathcal{M}_0$, distance matrices $\mathbf{D}_e$ and $\mathbf{D}_g$, normalized correlation lengths $\{\alpha_{c,k}\}_{k=1}^{N_c}$, target RMS amplitude a_w , number of paired samples N_s

Ensure: Paired critical load factors

- 1: **for** $k = 1, \dots, N_c$ **do**
 - 2: Set $c = \alpha_{c,k} R$.
 - 3: Construct $\mathbf{C}_e$ and $\mathbf{C}_g$.
 - 4: Apply spectral projection when required and compute the corresponding covariance square roots.
 - 5: **for** $m = 1, \dots, N_s$ **do**
 - 6: Generate a common white noise vector $\xi^{(m)}$.
 - 7: Generate paired RMS controlled fields $\mathbf{w}_e^{(m)}$ and $\mathbf{w}_g^{(m)}$.
 - 8: Impose the paired fields separately on the nominal shell model.
-

(Continued)

Algorithm 1 (continued)

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- 9: Perform Riks analyses and extract $\lambda_{\text{cr},e}^{(m)}$ and $\lambda_{\text{cr},g}^{(m)}$.
 - 10: **end for**
 - 11: Summarize the critical load statistics for $\alpha_{c,k}$.
 - 12: **end for**
-

4 Distance Model Effects at the Imperfection Field Level**4.1 Distance Induced Covariance Difference**

For two points with the same axial coordinate, the geodesic and Euclidean distances reduce to

$$\frac{d_g}{R} = \Delta\theta, \quad \frac{d_e}{R} = 2 \sin\left(\frac{\Delta\theta}{2}\right), \quad (24)$$

where $0 \leq \Delta\theta \leq \pi$. Thus, the two distances are asymptotically equivalent for small angular separations, but $d_e < d_g$ for finite circumferential separations. For the squared-exponential correlation function used in this study, this leads to $\rho(d_e; c) \geq \rho(d_g; c)$ for separated points.

Fig. 3 illustrates this distance-induced covariance difference. When the correlation length is short, both $\rho(d_e; c)$ and $\rho(d_g; c)$ decay rapidly, so the difference between the two distance measures $\Delta_\rho = \rho(d_e; c) - \rho(d_g; c)$ has little effect on the covariance matrix. As the correlation length increases, long-range circumferential correlations become more influential. In this regime, the Euclidean distance model assigns stronger correlations than the geodesic model to points that are far apart along the cylindrical surface. This covariance level difference provides the mechanism by which the distance model may alter the morphology of the generated random imperfection fields.

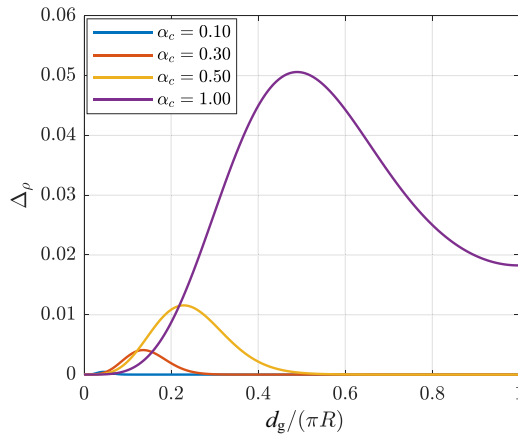


Figure 3: Distance-induced covariance difference for circumferentially separated points with the same axial coordinate.

4.2 Verification of Random-Field Representation

Because sample-wise centering and RMS normalization do not exactly preserve the nominal covariance structure of the underlying Gaussian fields, their effect on the spatial correlation was examined using 1000 realizations for each prescribed correlation length and distance model. The effective correlation length c_{eff} was obtained by least-squares fitting of the squared-exponential correlation function to the empirical correlation. For the raw Gaussian fields, c_{eff}/c remained approximately unity, whereas after centering and RMS normalization it decreased progressively at longer correlation lengths. For the geodesic model, c_{eff}/c

was 0.915, 0.836, and 0.768 at $\alpha_c = 0.50, 0.75$, and 1.00 , respectively, compared with 0.913, 0.831, and 0.753 for the Euclidean model. Thus, the normalization modifies the long-range correlation structure but affects the two distance models to a similar extent.

A separate spatial resolution check was performed using the same baseline, fine, and extra-fine surface discretizations. For each prescribed correlation length and discretization, 1000 raw geodesic Gaussian field realizations were generated, and the empirical correlation was evaluated according to the cylindrical geodesic separation d_g . The corresponding c_{eff} was evaluated using the same fitting procedure.

Fig. 4a compares the empirical and prescribed correlation functions for the shortest correlation length, $\alpha_c = 0.05$. All three discretizations reproduce the prescribed decay closely, including the baseline grid for which the correlation length is comparable to the grid spacing. Over the complete correlation length range, c_{eff}/c remains close to unity for all three discretizations, as shown in Fig. 4b, with only small non-monotonic variations attributable to finite sample estimation. These results indicate that no systematic under-resolution of the prescribed raw-field correlation structure is observed on the baseline discretization.

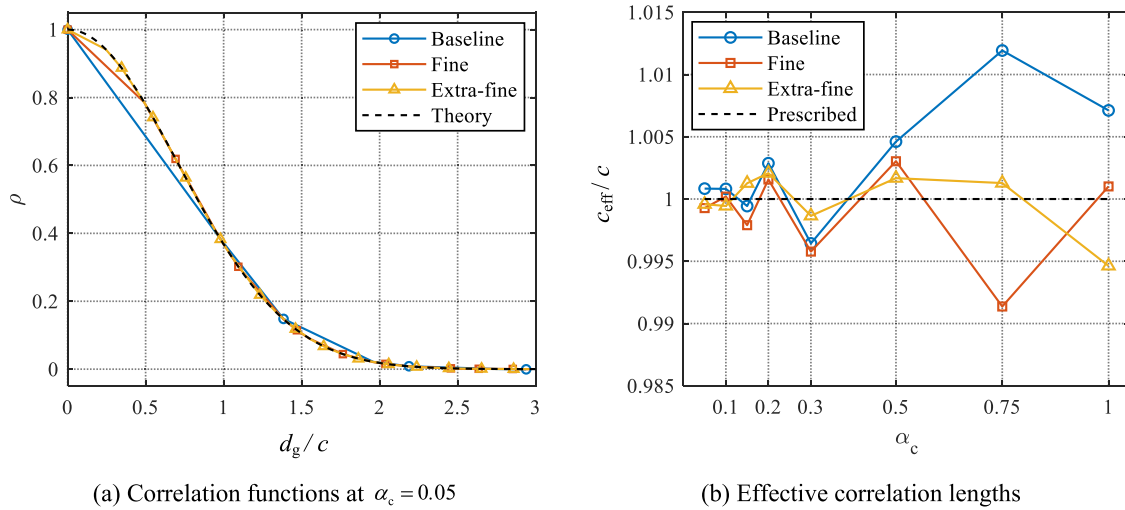


Figure 4: Spatial discretization assessment of the geodesic random field.

4.3 Representative Paired Imperfection Fields

Fig. 5 presents representative paired imperfection fields generated by the geodesic and Euclidean distance models using the same stochastic inputs and prescribed RMS amplitude. The comparison therefore highlights the morphology difference induced by the distance-dependent covariance models.

A direct visual comparison of the original imperfection fields shows that the two distance models produce very similar spatial patterns for all selected correlation lengths. For small α_c , the paired fields are almost indistinguishable, which is consistent with the local equivalence of the Euclidean and geodesic distances for nearby surface points. As α_c increases, the imperfection fields become smoother and more globally correlated, but the dominant spatial features remain nearly the same between the two distance models because the same white-noise vector is used in the paired generation.

The distance model effect is more clearly revealed by the difference field $(w_e - w_g)/t$. For $\alpha_c = 0.1$, the difference is extremely small and remains at the order of 10^{-4} . As the correlation length increases, the magnitude of the difference field increases, reaching the order of 10^{-2} for $\alpha_c = 0.5$ and becoming more visible at $\alpha_c = 1.0$, and the difference fields also show more coherent spatial patterns. This observation is consistent

with the covariance level difference discussed in [Section 4.1](#), where the two distance measures differ mainly in their treatment of long-range circumferential separation.

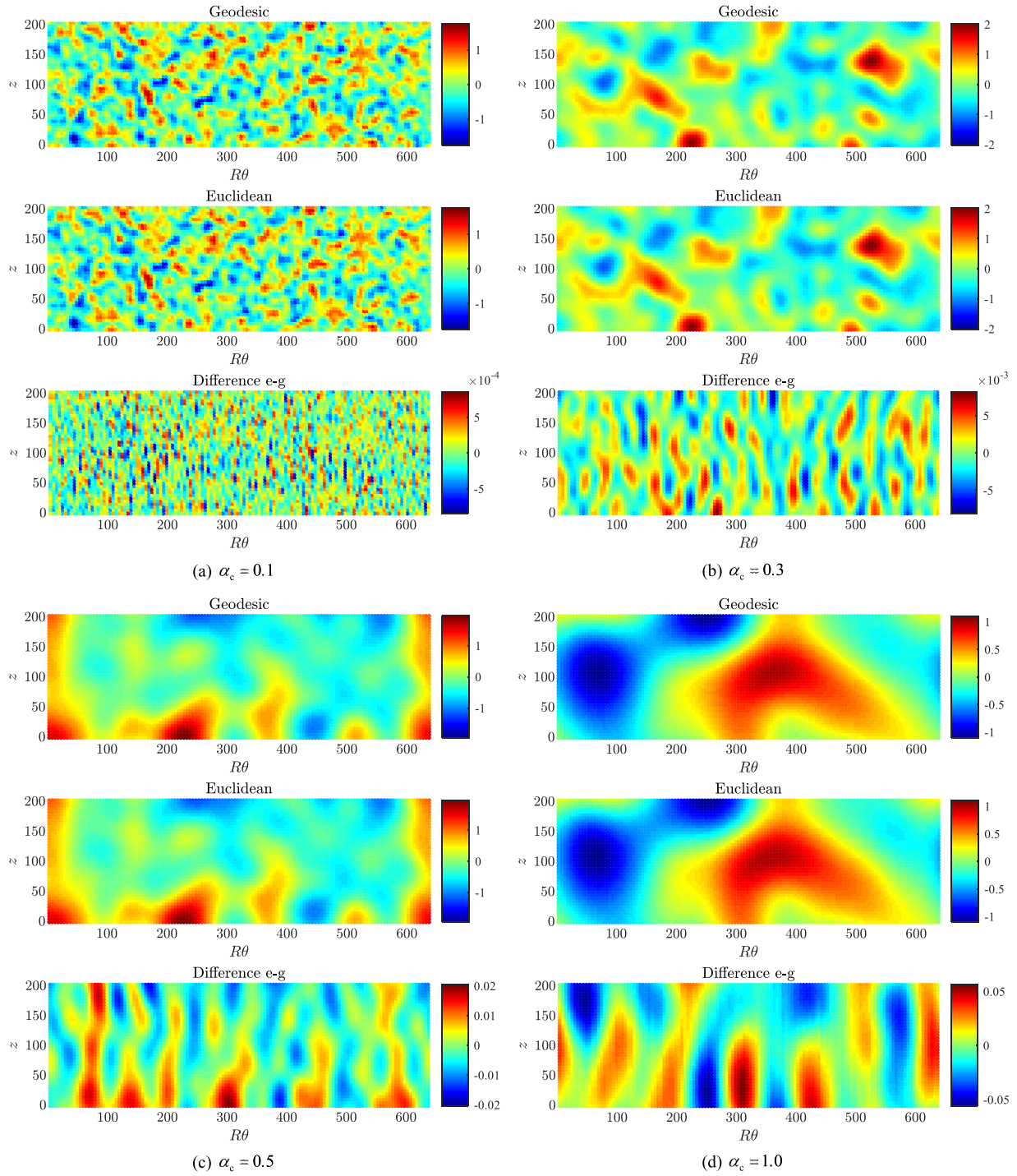


Figure 5: Representative paired imperfection fields generated by the geodesic and Euclidean distance models.

4.4 Global Paired Field Discrepancy

The representative fields in Fig. 5 provide a qualitative comparison of the two distance models. To quantify the field-level difference over all paired samples, the normalized paired field discrepancy is defined as

$$\varepsilon_w^{(m)} = \frac{\|\mathbf{w}_e^{(m)} - \mathbf{w}_g^{(m)}\|_2}{\|\mathbf{w}_g^{(m)}\|_2}. \quad (25)$$

Here, $\mathbf{w}_e^{(m)}$ and $\mathbf{w}_g^{(m)}$ are the Euclidean- and geodesic-distance induced imperfection fields generated from the same white noise vector. Since both fields are shifted to zero mean and rescaled to the same prescribed RMS amplitude, $\varepsilon_w^{(m)}$ measures the global difference between the paired spatial patterns rather than a difference in overall imperfection amplitude.

For each correlation length, the mean paired field discrepancy is computed as

$$\bar{\varepsilon}_w = \frac{1}{N_s} \sum_{m=1}^{N_s} \varepsilon_w^{(m)}, \quad (26)$$

where N_s is the number of paired samples. In addition to the mean value, the sample-wise distribution and the 5%–95% interval of ε_w are also examined.

Fig. 6 summarizes the distribution and statistical trend of ε_w , where the discrepancy is plotted in percentage form. For short correlation lengths, the paired field discrepancy is almost negligible. The boxplots are tightly concentrated near zero, and the mean value remains below 0.25% for $\alpha_c \leq 0.20$. This confirms that the Euclidean and geodesic covariance models generate nearly identical RMS-controlled imperfection fields when the correlation length is short.

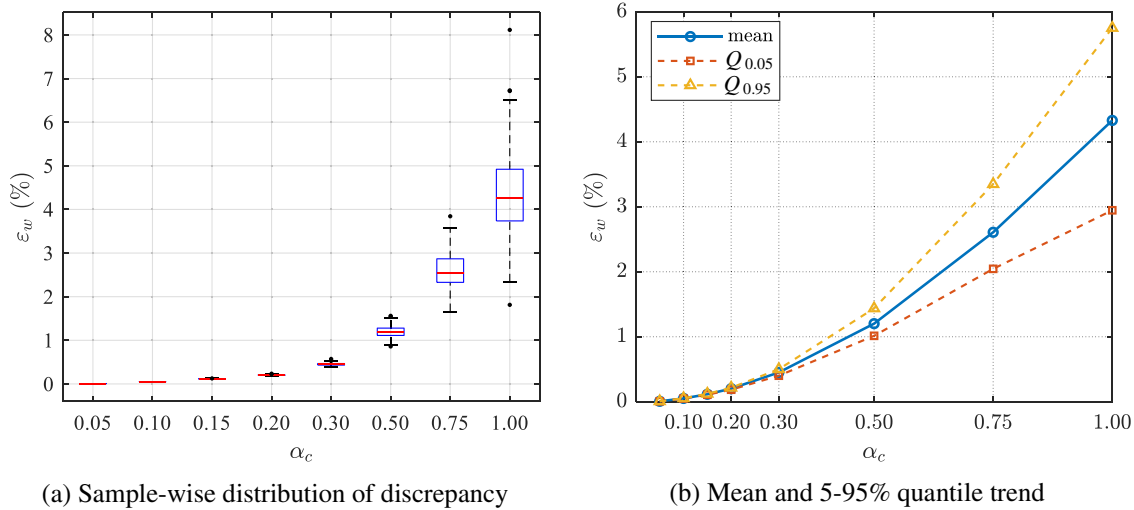


Figure 6: Paired imperfection field discrepancy between the Euclidean and geodesic models.

As α_c increases, the paired field discrepancy increases nonlinearly. The growth is still moderate at $\alpha_c = 0.30$, but becomes much more pronounced for $\alpha_c \geq 0.50$. The mean discrepancy reaches 1.201% at $\alpha_c = 0.50$, 2.607% at $\alpha_c = 0.75$, and 4.329% at $\alpha_c = 1.00$. Meanwhile, the 5%–95% interval also widens, indicating that the sample-to-sample variation of the distance model-induced field difference becomes larger

at longer correlation lengths. The boxplot at $\alpha_c = 1.00$ further shows that a few samples can reach field discrepancies above the main interquartile range.

This trend is consistent with the covariance level mechanism discussed in Section 4.1. At short correlation lengths, the rapid correlation decay limits the influence of the difference between chord and surface distances. As the correlation scale increases, long-range circumferential correlations become more influential and the Euclidean–geodesic field discrepancy becomes progressively more measurable. Thus, the field-level comparison establishes a clear correlation scale-dependent difference between the two distance models, which is examined at the buckling response level in the following section.

5 Transmission of Field-Level Discrepancy to Buckling Response

5.1 Critical Buckling Response and Statistical Characteristics

Fig. 7 shows representative LPF– U equilibrium paths for paired imperfect shells at $\alpha_c = 0.50$ and 1.00, where U denotes the axial displacement of the top edge. For each correlation length, one realization with median $|\Delta_\lambda|$ and one with the maximum $|\Delta_\lambda|$ are selected. The paired paths remain nearly coincident before buckling for the median cases, whereas a clearer separation of the first critical peaks appears in the maximum discrepancy cases, particularly at $\alpha_c = 1.00$. These representative results suggest that the distance model effect is reflected primarily in the critical peak, while the overall pre-buckling response remains largely unchanged.

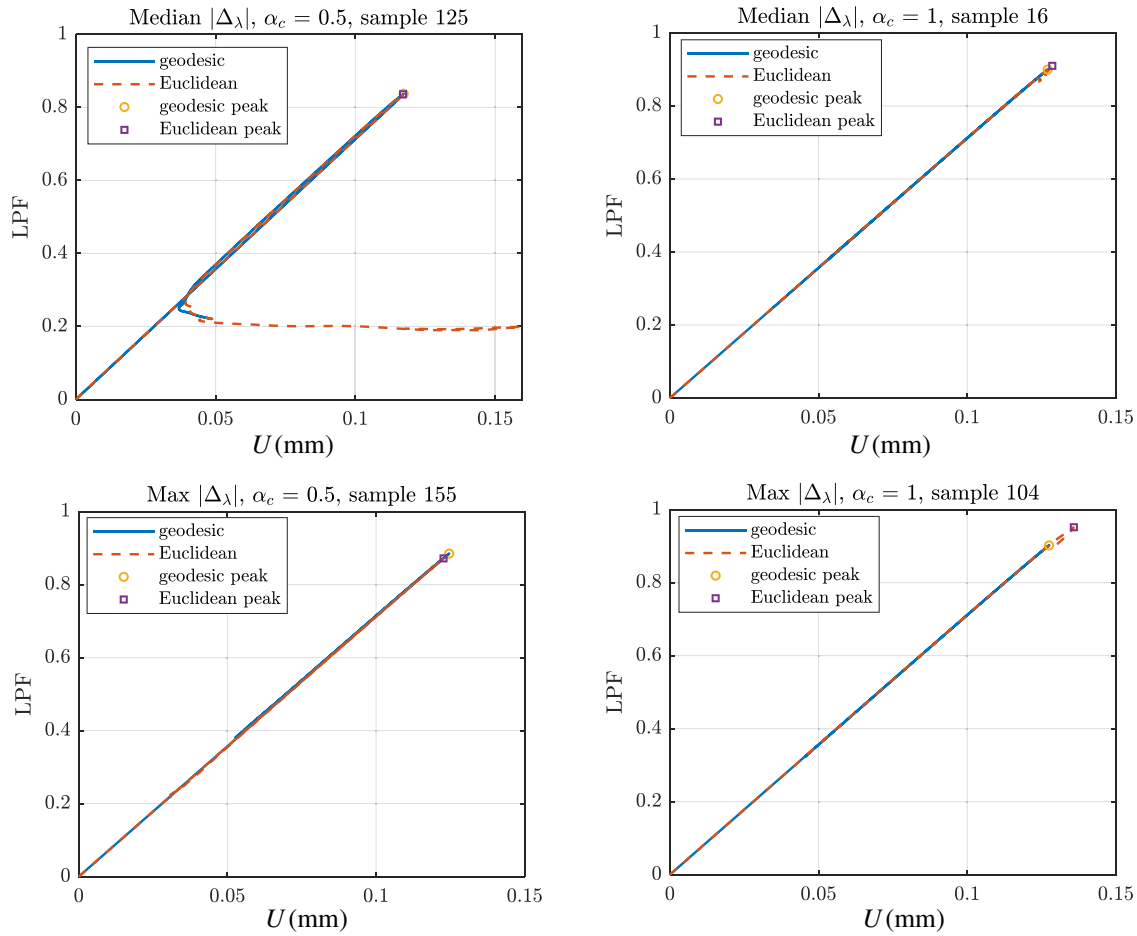


Figure 7: Representative LPF–displacement paths of paired imperfect cylindrical shells.

Fig. 8 summarizes the sample-wise distributions, mean values, coefficients of variation, and lower-tail critical load factors for the two distance models. The critical load statistics vary primarily with the normalized correlation length α_c . The mean critical load factor changes only slightly at short correlation lengths, increases rapidly from $\alpha_c = 0.15$ to 0.50, and then approaches a higher plateau. The lower-tail statistic $Q_{0.05}$ exhibits a similar trend, whereas the coefficient of variation (COV) is relatively large around $\alpha_c = 0.15$ –0.30 and decreases markedly as the correlation length increases. In comparison, the differences between the Euclidean and geodesic models are much smaller, with their distributions and statistical measures nearly overlapping over most of the investigated range. A slight separation becomes visible at $\alpha_c = 1.00$, where the Euclidean model gives marginally higher mean and lower-tail critical load factors.

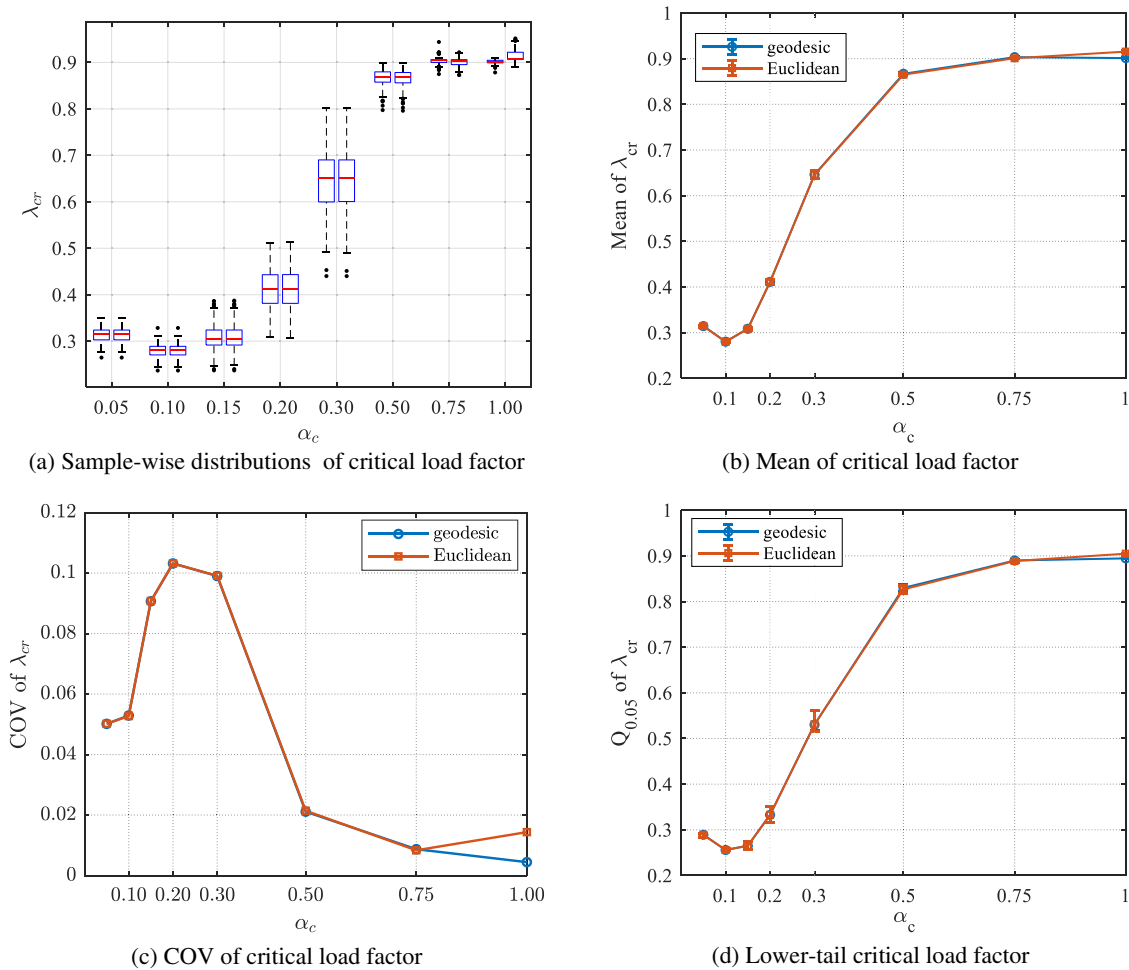


Figure 8: Critical buckling load statistics under the geodesic and Euclidean distance models.

To assess whether the response trends identified above are robust to mesh refinement, Fig. 9 compares the matched baseline–fine critical load factors. Mesh refinement reduces the critical load factors, with the largest reduction in the group mean occurring at the shortest correlation length. For the six geodesic realizations included in the supplementary extra-fine check, the changes from the fine to the extra-fine mesh remained below 4.5%, indicating reduced sensitivity under further refinement. Despite these changes in absolute values, the matched baseline–fine comparisons preserve the principal Euclidean–geodesic trends: the paired discrepancy remains nearly unchanged for $\alpha_c \leq 0.30$, while the tendency for the Euclidean model

to predict higher critical load factors at $\alpha_c = 1.00$ is retained, although the magnitude of the discrepancy remains sensitive to mesh refinement.

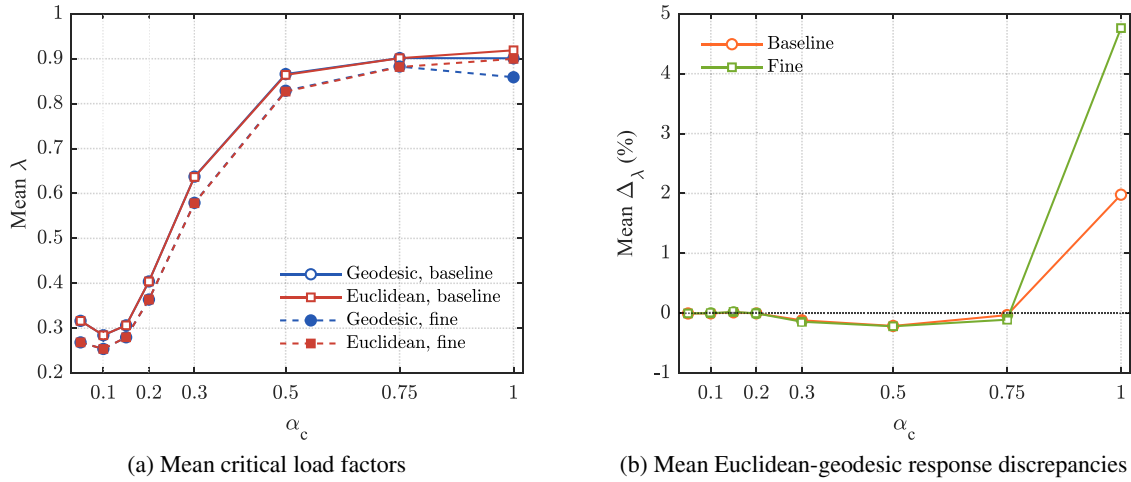


Figure 9: Mesh-refinement assessment based on matched imperfection realizations.

The Monte Carlo convergence results in Fig. 10 further show that the response statistics become increasingly stable as the ensemble size approaches the adopted $N_s = 200$. At $N_{\text{sub}} = 175$, the worst-case 95th-percentile relative error ε_{95} decreases to approximately 0.54% for the mean, and the corresponding errors in $Q_{0.05}$ are 1.69% and 1.31% for the geodesic and Euclidean models, respectively. Thus, the lower-tail statistic converges more slowly than the mean, while both distance models exhibit very similar convergence behavior.

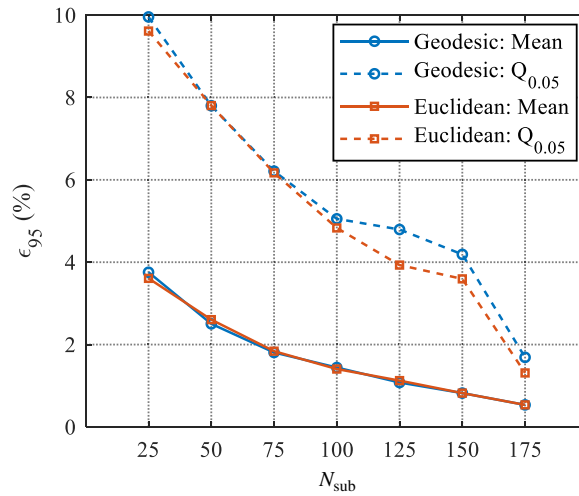


Figure 10: Monte Carlo convergence of the mean and lower-tail critical load factor statistics for the geodesic and Euclidean distance models. Here, ε_{95} denotes the worst-case 95th-percentile relative error among the eight prescribed correlation-length groups, with the complete 200-realization ensemble used as the reference.

Overall, the marginal response statistics indicate that variation with the prescribed correlation length is substantially greater than the differences between the two distance models.

5.2 Paired Response Discrepancy between Distance Models

Although the marginal response statistics characterize the overall differences between the two distance models, they do not reveal the sample-wise response changes enabled by the paired construction. These changes are examined directly using the signed relative response discrepancy $\Delta_\lambda^{(m)}$ defined in Eq. (23).

Fig. 11a summarizes the distribution of Δ_λ over the investigated correlation lengths. At short correlation lengths, the distributions are tightly concentrated around zero, indicating small paired response differences for most realizations. Although isolated larger discrepancies occur, the central distributions remain very narrow for $\alpha_c \leq 0.20$. As the correlation length increases, the paired response discrepancy becomes more scattered. For $0.30 \leq \alpha_c \leq 0.75$, the median remains close to zero or slightly negative, while both positive and negative discrepancies are observed. The negative tail becomes more pronounced at $\alpha_c = 0.75$, where several realizations yield noticeably lower critical load factors under the Euclidean distance model. Hence, no consistent directional bias is observed over this range, and the response difference remains dependent on the particular imperfection realization.

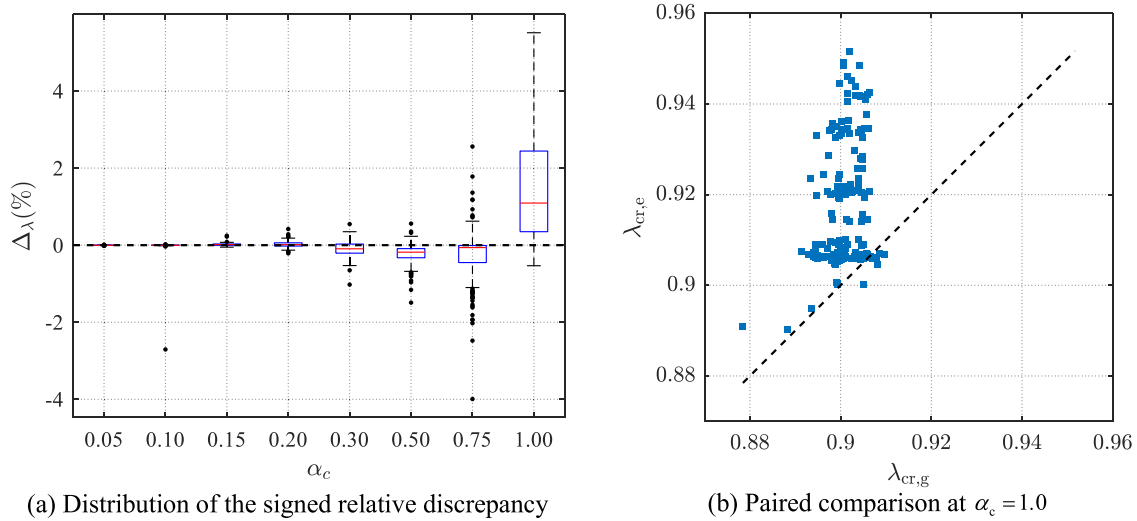


Figure 11: Paired response discrepancy between the Euclidean and geodesic distance models.

The clearest directional tendency occurs at $\alpha_c = 1.00$. The distribution of Δ_λ shifts predominantly to the positive side, and the paired scatter plot in Fig. 11b shows that most realizations lie above the equality line $\lambda_{cr,e} = \lambda_{cr,g}$. Thus, under the adopted pairing, the Euclidean distance model tends to predict a higher critical load factor than the geodesic model at the largest correlation length considered. The scatter plot further shows that this tendency is present across individual paired realizations rather than being limited to a difference between the marginal group statistics.

Overall, the paired analysis reveals a clear change in response behavior with correlation length: the discrepancies are small and concentrated near zero at short correlation lengths, mixed in sign at intermediate correlation lengths, and predominantly positive at $\alpha_c = 1.00$.

5.3 Relation between Field-Level and Response-Level Discrepancies

The preceding results show that the paired field discrepancy ε_w increases with the normalized correlation length, whereas the response-level discrepancy does not increase proportionally. Fig. 12 therefore compares ε_w with the absolute and signed response discrepancies for all paired samples.

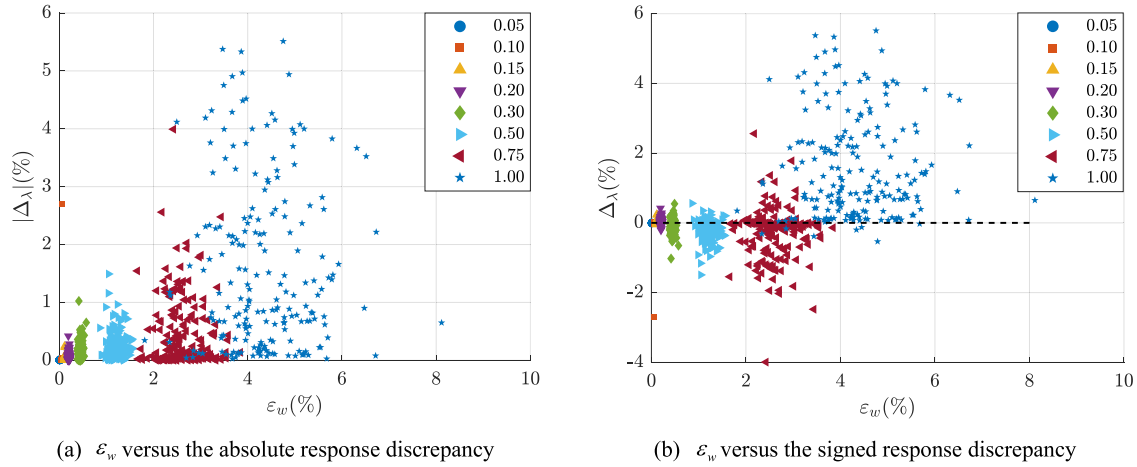


Figure 12: Relation between the paired imperfection field discrepancy and response-level discrepancy.

For short correlation lengths, the samples cluster near the origin, indicating negligible differences at both the field and response levels. As α_c increases, both ε_w and the possible range of $|\Delta_\lambda|$ increase. However, the relation remains highly scattered: realizations with similar ε_w can produce substantially different $|\Delta_\lambda|$, including cases where a field discrepancy of several percent results in only a very small critical load factor difference. Thus, the global field discrepancy alone is insufficient to predict the response discrepancy of an individual realization.

The signed comparison in Fig. 12b further shows that the response difference is realization dependent. Both positive and negative values occur over the intermediate correlation length range, whereas the distribution shifts predominantly to the positive side at $\alpha_c = 1.00$. Hence, neither the magnitude nor the sign of Δ_λ can be inferred directly from ε_w .

The confidence intervals in Table 2 quantify the finite-sample uncertainty of the paired response measures. At $\alpha_c = 1.00$, $\bar{\Delta}_\lambda = 1.576\%$ with a 95% bootstrap confidence interval of 1.377%–1.780%, while $Q_{0.95}(|\Delta_\lambda|) = 4.302\%$ has a corresponding interval of 4.028%–4.906%. In addition, $P(\Delta_\lambda > 0) = 96.5\%$ with a 95% Wilson interval of 92.95%–98.29%. These intervals show that the predominantly Euclidean higher tendency at $\alpha_c = 1.00$ is well resolved with respect to finite sample uncertainty, although the directional probability remains conditional on the adopted coupling. At shorter correlation lengths, the practical interpretation is governed primarily by the small magnitude of the response discrepancy rather than statistical significance alone.

5.4 Circumferential Spectral Interpretation

To further characterize the differences in imperfection morphology, a circumferential Fourier decomposition was performed at each axial location and accumulated over the shell height. Let $\mathcal{W}_{s,n}$ denote the modal energy of the imperfection field generated by distance model $s \in \{e, g\}$ at circumferential wave number n , and let $\mathcal{W}_{\Delta,n}$ denote the corresponding modal energy of the paired difference field $\mathbf{w}_e - \mathbf{w}_g$. The normalized modal energy fraction and modal discrepancy contribution are defined as

$$\widehat{\mathcal{W}}_{s,n} = \frac{\mathcal{W}_{s,n}}{\sum_n \mathcal{W}_{s,n}}, \quad f_n = \frac{\mathcal{W}_{\Delta,n}}{\sum_n \mathcal{W}_{g,n}}. \quad (27)$$

With the same Fourier normalization used for all fields, Parseval's relation gives $\sum_n f_n = \varepsilon_w^2$, so f_n represents the contribution of circumferential wave number n to the global paired field discrepancy. The

difference in normalized modal energy fractions between the two distance models is further defined as

$$\Delta \widehat{\mathcal{W}}_n = \widehat{\mathcal{W}}_{\epsilon,n} - \widehat{\mathcal{W}}_{g,n}. \quad (28)$$

Fig. 13 presents the sample mean values of f_n and $\Delta \widehat{\mathcal{W}}_n$, expressed as percentages, over the paired realizations at selected correlation lengths. Fig. 13a shows that the Euclidean–geodesic field discrepancy becomes increasingly dominated by low circumferential wave numbers as the prescribed correlation length increases. At $\alpha_c = 0.30$, the modal discrepancy remains small, whereas progressively larger contributions from several low-order circumferential components appear at longer correlation lengths and become most pronounced at $\alpha_c = 1.00$. This behavior is consistent with the increasing difference between the Euclidean chord distance and the surface geodesic distance at large circumferential separations, which alters the long-range covariance structure and is reflected in the large-scale spatial organization of the imperfection fields.

Table 2: Summary of paired field-level and response-level discrepancies.

α_c	$\bar{\epsilon}_w$ (%)	$Q_{0.95}(\epsilon_w)$ (%)	$\bar{\Delta}_\lambda$ (%)	Median $ \Delta_\lambda $ (%)	$Q_{0.95}(\Delta_\lambda)$ (%)	$P(\Delta_\lambda > 0)$ (%)
0.05	0.007	0.007	−0.004 [−0.0039, −0.0034]	0.003	0.007 [0.0056, 0.0082]	0.5 [0.09, 2.78]
0.10	0.051	0.053	−0.017 [−0.0442, −0.0026]	0.004	0.014 [0.0125, 0.0158]	30.0 [24.07, 36.68]
0.15	0.115	0.122	0.017 [0.0124, 0.0212]	0.019	0.059 [0.0482, 0.0662]	72.0 [65.41, 77.76]
0.20	0.201	0.215	0.019 [0.0090, 0.0291]	0.040	0.160 [0.1094, 0.1955]	60.0 [53.08, 66.54]
0.30	0.451	0.500	−0.092 [−0.1182, −0.0667]	0.133	0.432 [0.3550, 0.5042]	30.0 [24.07, 36.68]
0.50	1.201	1.436	−0.218 [−0.2531, −0.1838]	0.194	0.712 [0.5533, 0.8041]	12.5 [8.61, 17.80]
0.75	2.607	3.348	−0.259 [−0.3546, −0.1644]	0.135	1.564 [1.3128, 1.9328]	19.5 [14.61, 25.54]
1.00	4.329	5.751	1.576 [1.3767, 1.7798]	1.093	4.302 [4.0283, 4.9059]	96.5 [92.95, 98.29]

Note: Values in brackets denote 95% confidence intervals. Percentile-bootstrap intervals based on 10,000 resamples are reported for $\bar{\Delta}_\lambda$ and $Q_{0.95}(|\Delta_\lambda|)$, while Wilson intervals are reported for $P(\Delta_\lambda > 0)$.

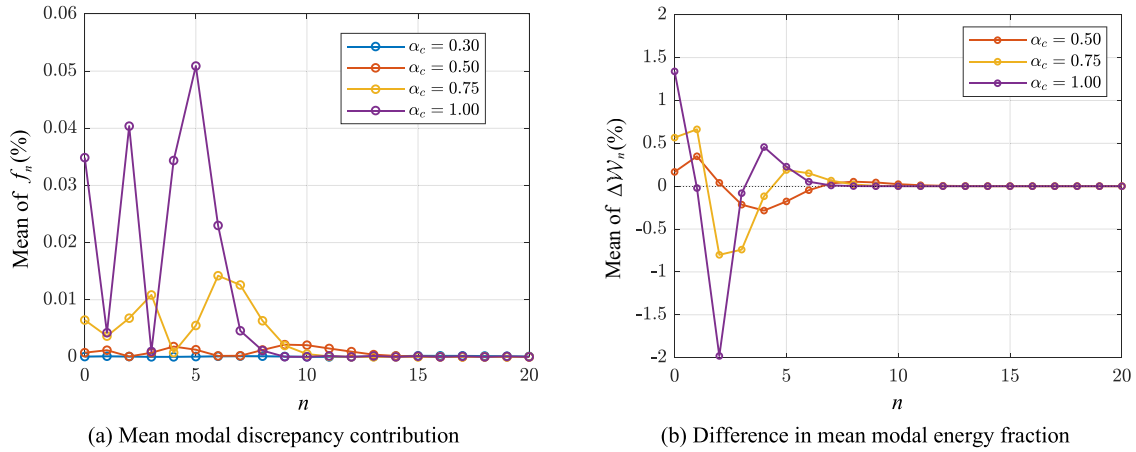


Figure 13: Circumferential spectral characteristics of the Euclidean–geodesic imperfection difference.

Fig. 13b further shows that the long correlation difference involves a redistribution of modal energy among several low-order circumferential components rather than a change in a single dominant wave number. The positive and negative changes are largest at $\alpha_c = 1.00$ and decay rapidly toward zero at higher wave numbers. This redistribution provides a plausible morphology-level explanation for the stronger distance model effect observed at the largest prescribed correlation length. However, the present spectral analysis does not identify a unique circumferential mode associated with either the sign or magnitude of Δ_λ . Because

the circumferential spectrum retains modal energy but not circumferential phase or the axial location of individual components, these spatial characteristics may also contribute to the realization-dependent buckling response.

The spectral results also help explain why the global discrepancy measure ε_w alone does not fully characterize the response difference. Although ε_w quantifies the overall magnitude of the Euclidean-geodesic field difference, it does not distinguish how that difference is distributed among spatial scales. The transmission of the field discrepancy to the nonlinear buckling response can therefore vary among realizations with similar values of ε_w .

6 Implications of the Euclidean-Distance Approximation

For the present benchmark, the two distance models give very similar buckling responses at short prescribed correlation scales. For $\alpha_c \leq 0.20$, $Q_{0.95}(|\Delta_\lambda|)$ remains below approximately 0.16%, and the marginal critical load factor distributions are nearly coincident. The discrepancy increases at longer correlation scales, with $Q_{0.95}(|\Delta_\lambda|)$ reaching approximately 1.56% at $\alpha_c = 0.75$ and 4.30% at $\alpha_c = 1.00$. The signed difference remains realization-dependent over most of the investigated range, while at $\alpha_c = 1.00$ the Euclidean model gives the higher critical load factor in 96.5% of the paired realizations. This tendency is consistent with the increasingly pronounced long-range covariance and low-order spectral differences between the two distance models.

These results indicate a clear dependence of the Euclidean-distance effect on the prescribed correlation scale. When the correlation scale is short relative to the shell radius, the Euclidean and geodesic distance models produce very similar buckling responses. As the correlation scale increases and long-range circumferential correlation becomes more important, the influence of the distance definition becomes progressively more evident. The Euclidean-distance approximation is therefore most consequential for imperfection fields with large spatial correlation scales, whereas its effect remains limited for the short-correlation cases considered here.

7 Conclusions

This study examined the effect of distance model selection on stochastic buckling of cylindrical shells with RMS-controlled geometric imperfections using paired Euclidean- and geodesic-distance random fields. The main conclusions are as follows:

- The Euclidean-geodesic imperfection field difference increases with correlation length. At longer correlation scales, the field discrepancy is dominated by low-order circumferential components, reflecting increasing differences in large-scale circumferential morphology.
- The critical load factor statistics vary strongly with correlation length, whereas the Euclidean and geodesic models produce similar marginal response distributions over most of the investigated range.
- The paired response difference is negligible at short correlation scales and mixed in sign at intermediate scales, while a clear Euclidean-higher tendency emerges at the largest correlation scale, where the discrepancy magnitude retains some mesh sensitivity.
- The field-level discrepancy is not transmitted proportionally to the nonlinear buckling response, indicating that buckling sensitivity is influenced by the spatial organization of the imperfection difference, not solely by its global magnitude.

The quantitative findings are specific to the present cylindrical shell, RMS-normalized radial imperfections, squared-exponential covariance model, axial compression loading, and investigated range $\alpha_c \leq 1.00$. The geodesic covariance model is used in its PSD-projected discrete form, and the paired response measures

depend on the adopted common random number coupling. In addition, the normalized fields do not exactly retain the nominal covariance at large α_c , and the response discrepancy at the largest correlation scale retains some mesh sensitivity. Further studies involving other covariance models, imperfection amplitudes, shell geometries, boundary conditions, loading cases, and longer correlation lengths would be needed to establish broader engineering criteria.

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