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Promoting Benchmarking and Best Practices for Municipal Solid Waste Management in Smart Cities using an Intuitionistic Fuzzy Evaluation Model Based on Relative Utility and Nonlinear Standardization

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ABSTRACT: Municipal solid waste management (MSWM) is a critical challenge in smart cities due to rapid urbanization, increasing waste volumes, and the complexity of waste streams. Smart cities strive to integrate advanced technologies, sustainability practices, and data-driven solutions into their MSWM strategies to ensure efficiency and environmental responsibility. However, decision-making in MSWM is often fraught with uncertainties arising from incomplete or imprecise information and subjective judgments. To address this, this study incorporates intuitionistic fuzzy sets (IFSSs) to handle uncertainties effectively, enabling a more robust and realistic evaluation framework. Subjective weights of criteria are calculated using the ranking comparison (RANCOM) method, capturing expert preferences while accounting for uncertainty in their judgments. Objective weights are derived through the indifference threshold-based attribute ratio analysis (ITARA), a data-driven approach that employs attribute thresholds to quantify significance. The evaluation based on relative utility and the nonlinear standardization (ERUNS) method integrates these weights to rank alternatives. By leveraging IFSSs, the proposed framework enhances the reliability of decision-making in uncertain environments, ensuring more accurate prioritization of MSWM strategies. This comprehensive approach identifies efficient, sustainable, and scalable waste management solutions, aligning with the goals of smart cities. It provides actionable insights for policymakers to implement optimized, resource-efficient, and environmentally friendly systems.

KEYWORDS: Municipal solid waste management; smart cities; sustainability; waste management strategies; intuitionistic fuzzy sets

1 Introduction

Rapid urbanization and population growth in recent decades have increased the problems of municipal solid waste management (MSWM). These are complex demand systems arising from smart cities, which integrate innovative technologies to improve everyday lives in urban environments. The lack of efficient MSWM requires proper management to mitigate the risks associated with cities and their inhabitants, as well as the negative impacts on the environment, economy, and human health. Large quantities of waste in modern cities are being generated due to soaring consumption levels, industrialization, and per capita lifestyles. This increasing rate of waste generation puts pressure on existing waste management structures, most of which are near or above their optimum working capacity. Thus, the problem is compounded by the fact that in modern urban centers space is very limited and traditional methods of disposal such as landfilling become uneconomical. Also, those practices emit greenhouse gases that foster climate change, thereby making it important to embrace sustainable practices.

Shahnazari et al. [1] examined how multi-criteria decision-making (MCDM) methods, which focus on the analytic hierarchy process (AHP) in particular, can assist the choice of energy recovery technologies. They also considered the environmental scorecard, which evaluates the environmental significance of impacts and operations when rating technologies for MSWM. This study has shown how MCDM can be used to make strategic decisions regarding energy recovery from municipal waste. Mujtaba et al. [2] employed three MCDM techniques to assess sustainable MSWM strategies in Lahore, Pakistan. They discussed their framework that combines environmental sustainability, social acceptance, and cost to identify efficient waste management decisions. The specific methods described in this research show how MCDM can solve local problems concerning municipal solid waste (MSW) in urban areas. Garcia-Garcia [3], as well as conducting a comprehensive qualitative review of the literature, identified and reviewed several MCDM models and techniques utilized in optimizing MSWM. These research findings also illustrated that methodologies such as TOPSIS, VIKOR, and AHP enable decision makers to make decisions with respect to multiple criteria. Recently, Rodrigues and Mondelli [4] applied multi-criteria analysis to evaluate integrated systems for MSWM in the city of Sao Paulo. The researchers demonstrated how MCDM may be used to determine alternative MSWM options relative to their environmental, technical, and economic attributes. Through two case studies, Yadav and Karmakar [5] demonstrated the utility of MCDM in achieving sustainable MSWM options. In the study, they illustrated how social environment and economic criteria can be incorporated into MCDM for directing waste management practices. Coban et al. [6] concluded a study on energy recovery for MSWM in cities. In this analysis, the authors assessed a range of energy recovery techniques by employing MCDM techniques while putting emphasis on environmental and economic viability. Biswas et al. discussed factors that influence circular economy effective under uncertainty [7]. Mir et al. [8] tried TOPSIS and VIKOR and utilized the refined models to propose a framework to augment the circular economy of MSWM. Their framework is focused on identifying solutions that incorporate concepts of reuse and recycling in waste management systems. Additionally, Dabić-Miletić and Raković applied the TOPSIS method to rank autonomous alternatives for intralogistics activities in sustainable warehouse systems development [9]. Finally, inefficient MSWM has potentially severe economic and social consequences, as well as environmental ones. Politically, economically, and socially, inefficient waste management disproportionately impacts the poor and reduces their quality of life by exposing them to health hazards. Smart cities are more of an opportunity that can help to revolutionize the collection and management of MSW through innovative use of technologies. The Internet of Things (IoT) allows real-time tracking of waste bins such that collection routes and, hence, operational costs are minimized. Artificial intelligence technology and machine learning can analyze waste patterns and forecast generation in order to identify areas where such waste can be recycled and used to secure more resources. The use of blockchain technology also holds promise in improving the

traceability of waste and its proper disposal. Current MSW systems, on the other hand, are not very flexible and innovative, which limits the efficient integration of such tools into the system. An approach to decision-making is important for closing the gap, as it offers frameworks by means of which waste management schemes can be compared and ranked. These approaches include technical, economic, environmental, and social acceptability criteria to guide stakeholders toward effective solutions.

The need for clean, healthy, and sustainable living means that smart MSWM requires new approaches that can incorporate innovation, information and communication technology, big data analysis, and smart technology. Consequently, decision-making regarding the most appropriate waste management strategy is challenging because there are often competing objectives, which include cost, efficiency, environmental impact, size, and social perceptions. Moreover, factors such as fluctuation in the professional judgement of experts, variance in the data used to support decisions, and changes in regulatory requirements are additional elements that increase complexity when making a decision. The application of decision-making models specific to smart city contexts, including those related to MSWM by municipal authorities, will enable them to deal with the multi-dimensional aspects of this challenge. In addition, these models can assist in improving the efficiency and circular economy characteristics not only of waste management systems, but also from an urban resilience and quality of life perspective. Therefore, this study is designed to examine a strong method of decision support to assess and rank probable MSWM solutions and provide useful suggestions for building better and more effective waste systems in urban environments.

In order to overcome these challenges, the present work introduces a holistic MCDM framework to assess the appropriateness of the MSWM strategies adapted to the context of smart cities. The ranking comparison (RANCOM) and indifference threshold-based attribute ratio analysis (ITARA) techniques are employed to combine the stakeholders' evaluations and statistical analysis. Uncertainties are addressed using intuitionistic fuzzy sets (IFSs), where imprecision and hesitation in the evaluations are reflected. Finally, the comparatively simple evaluation based on relative utility and nonlinear standardization (ERUNS) is used to make ranking decisions, making the assessment much fairer. A standard case has been developed based on six different MSWM strategies to examine the flexibility and feasibility of the proposed framework. This research aims to offer clear recommendations to policymakers as well as urban planners to work towards innovative and sustainable waste management solutions that are effective, cost-efficient, and scalable. Through the consideration of both the technological and social aspects of waste management, this contribution improves the implementation of smart cities and the further establishment of circular economy approaches.

1.1 Contributions

This study makes the following key contributions:

- The study proposes an MCDM framework to evaluate MSWM strategies, integrating subjective and objective weighting methods with advanced ranking techniques.
- Incorporates IFSs to address uncertainties in expert judgments and data variability, thereby enhancing the reliability and robustness of the evaluation process.
- Determines subjective weights using the RANCOM method, which captures expert preferences and their relative importance.
- Calculates through ITARA, a threshold-based technique that derives significance from data-driven insights.
- Tailors the evaluation framework to the context of smart cities, emphasizing sustainability, technological integration, and resource efficiency.
- Provides urban planners and policymakers with a decision-making tool to prioritize and implement waste management strategies aligned with smart city objectives.

- Supports change management toward more sustainable and circular waste management in cities, aiming to generate maximum environmental, economic, and social value.

1.2 Structure of the Paper

This paper is structured as follows to offer a structured analysis of MSWM strategies in smart cities through a holistic MCDM approach. [Section 2](#) discusses different MCDM approaches based on IFSs, and [Section 3](#) discusses key challenges and the need for solutions for MSWM. [Section 4](#) introduces the background of the proposed IFSs approach and the theoretical basis for handling imprecise information using membership, non-membership, and hesitation values. The essential elements of the proposed approach, such as subjective weighting by RANCOM, objective weighting by ITARA, and ranking by ERUNS, are discussed in [Section 5](#). [Section 5.3](#) offers an illustrative example of the proposed approach, which comprises identification of MSWM alternatives in smart cities, key criteria for evaluating MSWM, experimental results, sensitivity analysis, comparative analysis, limitations of the proposed model, and discussion about key results. Finally, [Section 6](#) concludes the study, summarizing its contributions and offering suggestions for the future to improve MSWM strategies in smart cities.

2 Literature Review

Zadeh introduced the concept of fuzzy sets [10], which was subsequently extended by Atanassov [11] through the development of intuitionistic fuzzy sets (IFSs). In contrast to conventional fuzzy sets, IFSs characterize each element by a membership degree μ and a non-membership degree ν , subject to the condition $0 \leq \mu + \nu \leq 1$. The remaining degree, $\pi = 1 - \mu - \nu$, represents the hesitation or indeterminacy degree and enables IFSs to explicitly capture uncertainty and incomplete information in the given data. Therefore, IFSs provide a more comprehensive representation of ambiguous information than conventional fuzzy sets. Owing to their ability to simultaneously model membership, non-membership, and hesitation information, IFSs have been widely applied in various fields, including pattern recognition, market trend analysis, and strategic planning [12]. A key methodological issue in IFS-based MCDM research is the aggregation and processing of intuitionistic fuzzy information, which is one of the major hurdles when carrying out a study to obtain accurate results. Researchers have discussed diverse ways of handling this problem. Xu [13] presented basic mathematical approaches for defining Intuitionistic Fuzzy Numbers (IFNs), and Xu and Yager [14] developed geometric approaches for MCDM using IFNs and extended their work to dynamic methods for analyzing multidimensional data originating from different places and times [15]. In addition, researchers are employing a variety of new approaches to modeling uncertainty (i.e., IF-methods such as those used by Xia and Xu [16], generalized IF-weighted methods such as those employed by Zhao et al. [17], and IF-Choquet integral methods as discussed by Tan and Chen [18]). Additionally, Alballa et al. [19] applied fuzzy parameterized possibility single-valued neutrosophic hypersoft expert systems for evaluating the management of solid waste through development of a multi-criteria decision making (MCDM) framework. Salamat et al. [20] offered a strong complex decision-making model that employed fairly operators and quaternion numbers within an intuitionistic fuzzy rough environment; this is intended to improve upon the existing aggregation of uncertain information. Thus, these models show the increasing demand for advanced techniques for modeling and aggregating uncertainties when addressing complex decision-making. As a result, the RANCOM method described in this manuscript represents a unique method for overcoming inaccurate assessments made by experts regarding alternatives in MCDM applications. This literature review aims to identify and discuss its development, applications, and comparative advantages based on recently published research. RANCOM was formulated by Więckowski et al. [21] for comparing criteria relevance

while considering experts' errors. The method uses inaccuracy modeling data to improve the decision-making procedure, which makes it different from the recognized technique, for example, AHP. Więckowski et al. [22] also demonstrated the use of RANCOM in housing location selection problems as a comparison with AHP. Their findings revealed that the way RANCOM addresses inaccuracies results in superior solutions in situations where professional opinions are ambiguous. To improve the model's flexibility, Więckowski et al. [23] proposed fuzzy RANCOM, which uses fuzzy logic to facilitate the control of uncertainties in the decision-making procedures. This adaptation provides flexibility and specificity in the conditions in Więckowski. The above developments have increased the capability of RANCOM in modeling and solving other multiple resource allocation decision-making problems. The RANCOM method has been widely implemented, primarily in the energy and sustainability industries. The comprehensive use of a procedure for selecting locations was made possible through the implementation of an interval-valued intuitionistic fuzzy distance measure by Rani et al. [24] in conjunction with RANCOM to utilize the potential of this methodology in order to address the complex issues associated with sustainable energy planning. In addition, Więckowski et al. [25], have combined RANCOM with ESP-SPOTIS as well as demonstrated the ability of the method to assist in developing environmentally friendly options for transportation; specifically, electric vehicles. However, Więckowski et al. [26] have compared RANCOM to several methods such as COPRAS, PROMETHEE and EDAS in terms of their application in sustainable energy development. As a result, after comparing RANCOM to the aforementioned MCDM methodologies, it has been shown that RANCOM provides many advantages. Shekhovtsov [27] assessed the efficacy of subjective weighting methods and proposed a new weight similarity coefficient, demonstrating that RANCOM was serving the purpose of obtaining better alignment with expert preferences than the other methods. ERUNS is a newly developed method, presented by Biswas et al. [28], to measure the performance of firms operating in the energy industry. The strength of this method is founded upon the nonlinear standardization process of the criteria, thus avoiding biases that may be produced by extreme values. The concept of relative advantage is also incorporated in the ERUNS framework, as it allows decision-makers to compare the level of performance between different alternatives systematically. The study also showed how the method can be used in decision-making situations in the energy sector where criteria interactions are nonlinear. Jiang et al. [29] extended the application area of ERUNS by proposing a new robust fuzzy decision support model for the performance assessment of the food supply chain. They showed that partners can rely on ERUNS to manage vagueness and uncertainty typical of supply chain data. With the support of ERUNS and the application of fuzzy logic, it was possible to increase the level of reliability of decisions made concerning efficiency and sustainability, which are often multi-parameter in nature. Olteanu et al. [30] used ERUNS together with a fuzzy-MEREC-AROMAN decision-making technique to determine priority investment sectors for Europe. The study (as shown in Table 1) has utilized ITARA as a starting point for extension and integration with various weighting and ranking methodologies in differing decision environments. While the research has provided important methodological advances, there are still many aspects that need to be treated simultaneously, including: uncertain expert assessment, subjective and objective weights of criteria, non-linear standardization, and evaluations based on relative utility. The developed framework integrates all of the previously mentioned areas into one decision-making process. Sensitivity analysis, comparative analysis, and 10,000 Monte Carlo simulations were performed to determine if the results were stable and robust.

Table 1: Comparative analysis of existing studies, identified limitations, and research gaps addressed by the proposed framework.

Study	Main Idea	Methodological Framework	Application Area	Limitation	How the Proposed Work Addresses the Limitation
Hatefi [31]	Introduced the ITARA method to determine objective attribute weights by considering indifference thresholds and meaningful differences among attribute values.	ITARA	General multiple attribute decision-making	The method focuses mainly on objective weighting and does not incorporate uncertain expert judgments, subjective preferences, or a complete alternative-ranking mechanism.	The proposed framework extends the weighting process to an intuitionistic fuzzy environment, combines ITARA with RANCOM-based subjective weighting, and employs ERUNS for the final ranking of alternatives.
Lo et al. [32]	Developed an integrated assessment model for identifying potential product and system risks by combining ITARA with TOPSIS.	ITARA-TOPSIS	Product and system risk assessment	The approach does not explicitly model hesitation in expert assessments and relies on an objective weighting and distance-based ranking structure.	The proposed framework represents uncertainty through IFNs, integrates subjective and objective criterion weights, and applies nonlinear standardization and relative utility assessment through ERUNS.
Wu et al. [33]	Extended the ITARA-TOPSIS framework to group decision-making problems by incorporating R-number information.	R-number-based ITARA-TOPSIS	Multi-criteria group decision-making	Although uncertainty is considered, the framework does not use intuitionistic fuzzy information or integrate ranking-based subjective weights with nonlinear relative utility evaluation.	The proposed approach incorporates membership, non-membership, and hesitation information, combines RANCOM and ITARA weights, and uses ERUNS to obtain the final prioritization.
Simić et al. [34]	Proposed a type-2 neutrosophic ITARA-EDAS model for selecting sustainable petroleum transportation routes under uncertain information.	Type-2 neutrosophic ITARA-EDAS	Sustainable petroleum transportation	The framework addresses uncertainty but does not integrate a ranking-based subjective weighting mechanism with the objective ITARA weights or use nonlinear relative utility assessment.	The proposed framework combines RANCOM-based subjective preferences with ITARA-based objective information and employs ERUNS to capture nonlinear performance and relative utility.
Imran and Ullah [35]	Developed a circular intuitionistic fuzzy EDAS approach for decision-making in the automotive industry.	Circular intuitionistic fuzzy EDAS	Automotive industry	The study focuses primarily on alternative evaluation and does not integrate ITARA-based objective weighting, RANCOM-based subjective weighting, and ERUNS-based nonlinear utility evaluation.	The proposed framework establishes a complete decision process by combining intuitionistic fuzzy information, hybrid criterion weighting, nonlinear standardization, and relative utility-based ranking.
Ulutaş et al. [36]	Developed an integrated CCSD-ITARA-MARCOS approach for selecting suitable stackers in logistics systems.	CCSD-ITARA-MARCOS	Logistics equipment selection	The framework is mainly based on quantitative decision information and does not explicitly represent hesitation in expert assessments through intuitionistic fuzzy information.	The proposed approach uses IFNs to represent uncertain evaluations and combines subjective and objective criterion importance before applying a nonlinear ranking procedure.

(Continued)

Table 1 (continued)

Study	Main Idea	Methodological Framework	Application Area	Limitation	How the Proposed Work Addresses the Limitation
Wang et al. [37]	Combined dual probabilistic linguistic information with EDAS and ITARA for sustainable supplier selection.	Dual probabilistic linguistic ITARA-EDAS	Sustainable supplier selection	The approach handles linguistic uncertainty but does not integrate RANCOM-based subjective weighting or ERUNS-based nonlinear standardization and relative utility evaluation.	The proposed framework combines subjective ranking information with objective criterion variation and uses ERUNS to provide a nonlinear utility-based evaluation of the alternatives.
Liu et al. [38]	Integrated BWm and ITARA within a probabilistic hesitant fuzzy environment for multi-attribute decision-making.	BWm-ITARA and taxonomy method	General multi-attribute decision-making	Although subjective and objective weights are considered, the framework does not employ intuitionistic fuzzy representation or ERUNS-based nonlinear relative utility analysis.	The proposed framework uses IFNs to represent membership, non-membership, and hesitation, while ERUNS provides nonlinear standardization and relative utility-based ranking.
Khan et al. [39]	Developed an ITARA-VIKOR decision model based on 2-tuple linguistic q-rung picture fuzzy information.	2-tuple linguistic q-rung picture fuzzy ITARA-VIKOR	General decision analysis	The framework does not incorporate RANCOM-based subjective weighting or ERUNS-based nonlinear relative utility assessment.	The proposed framework integrates RANCOM and ITARA to balance subjective and objective information and applies ERUNS for nonlinear standardization and final prioritization.
Kazzni et al. [40]	Combined modified TOPSIS with ITARA for housing assistance and building assessment problems.	ITARA and modified TOPSIS	Housing assistance and building assessment	The framework does not explicitly represent expert hesitation and does not combine subjective and objective weights within an intuitionistic fuzzy decision environment.	The proposed approach incorporates IFNs for uncertain assessments and combines RANCOM and ITARA weights before ranking alternatives through ERUNS.
Wu et al. [41]	Developed an ITARA and ELECTRE III three-way decision model within a spherical fuzzy environment for customer selection.	Spherical fuzzy ITARA-ELECTRE III	Customer selection	The framework does not integrate RANCOM-based subjective weighting with ERUNS-based nonlinear standardization and relative utility evaluation.	The proposed framework combines subjective and objective weighting mechanisms with nonlinear standardization and relative utility assessment within a unified intuitionistic fuzzy decision process.
Proposed study	Develops an integrated decision-support framework for evaluating and prioritizing MSWM strategies under uncertain expert information.	IF-RANCOM-ITARA-ERUNS	MSWM in smart cities	Addresses the need for a unified framework that simultaneously considers uncertain expert assessments, subjective preferences, objective data variation, nonlinear standardization, relative utility, and decision robustness.	Integrates IFNs, RANCOM-based subjective weighting, ITARA-based objective weighting, and ERUNS-based ranking. The reliability of the results is further examined through sensitivity analysis, comparative analysis, and 10,000 Monte Carlo simulations.

3 Problem Statement

3.1 Challenges in MSWM

The disposal of MSW is now a major problem widely accepted throughout the world, especially in areas with increasing population density and expanded economic activity. This is highly crucial when smart cities seek to apply principles of sustainability, innovation, and technological solutions to address this problem. Since the field of waste management has expanded globally, modern cities are still encountering multifaceted problems; therefore, MSWM systems remain inefficient, environmentally harmful, and socially contentious. These challenges are due to past practices, inadequate infrastructure, and the absence of a harmonized system capable of assessing and determining feasible waste management options. Now, the key challenges in current MSWM are illustrated below.

- **Rising waste generation and complexity:** Rapid urbanization, population growth, and changing consumption patterns have increased MSW generation. MSW is a heterogeneous mixture of organic matter, paper, plastics, glass, metals, textiles, and other discarded materials. This diversity makes segregation, sorting, recovery, and recycling challenging, while inadequate management can lead to open dumping, uncontrolled disposal, and inappropriate burning, resulting in resource loss and adverse environmental and health impacts.
- **Environmental and health impacts:** Poor disposal of waste poses risks both to the environment and to health because waste has to go somewhere. Landfills release methane, which is a very strong greenhouse gas, and also leachate that contaminates soil and water. Incineration without using advanced emission controls lets pollutants go into the air, and this affects air quality and health as well.
- **Missed opportunities for resource recovery:** The inability to properly recycle and reuse plastic and metal items results in these materials ending up at dumpsites. These inefficiencies create an opportunity for wasted resources as well as limit our ability to calculate the true financial value of the waste we generate; thus hindering our efforts to develop and promote a circular economy.
- **Social resistance and acceptability issues:** The odor, pollution, and decreased property values resulting from the placement of large-scale dump sites and/or dump fires have limited public acceptance of such landfills; limiting the effectiveness of local government in managing MSW. The public's distrust of government and their disengagement with the issue have also contributed to the resistance.
- **Financial and technological barriers:** Advanced waste management technologies, including advanced technologies like waste-to-energy systems and IoT-based collection systems, have a lot of potential; however, they are expensive. In many cases, municipalities do not have sufficient funds to pay for the implementation of these new technologies, nor do they have the infrastructure necessary to support them.
- **Policy and regulatory challenges:** Conservation also requires a significant amount of money (financial investment) and most regional authorities can't afford to make this kind of commitment. Weak or ambiguous legislation combined with poor enforcement and poor regulations creates further complications.
- **Land scarcity in urban areas:** In many densely populated Asian cities, rapid urbanization and limited land availability pose significant challenges for conventional MSWM, particularly for land-intensive treatment and disposal facilities such as landfills and composting facilities. The high population density and competing demands for urban land can restrict the development and expansion of such infrastructure, increasing the need for waste management technologies that are compact, resource-efficient, and suitable for space-constrained urban environments.
- **Dynamic and evolving waste composition:** The composition of MSW is continuously changing because of variations in consumption patterns, technological development, urban lifestyles, and the

increasing use of electronic products, plastics, packaging materials, and other emerging waste streams. Such variations create uncertainty regarding waste quantities, material characteristics, recovery potential, and treatment requirements. Consequently, MSWM systems must possess sufficient flexibility and adaptability to accommodate changing waste characteristics and volumes. Conventional approaches with limited adaptability may therefore be inadequate for addressing the dynamic nature of modern MSW streams.

- **Lack of integrated decision-making frameworks:** MSWM includes waste segregation, collection, transportation, treatment, recycling, recovery, and final disposal. Effective MSWM requires these activities to be planned together while considering environmental impacts, economic costs, resource recovery, and public acceptance. However, many municipalities lack clear decision-making frameworks for evaluating and selecting appropriate waste management alternatives, which can lead to fragmented planning and inefficient waste management strategies.

3.2 Need for a Comprehensive Solution

Thus, it is necessary to propose a multifaceted evaluation model for waste management that should have an integrated approach for assessment results across multiple criteria. Such a framework should address the following questions:

- Which strategies offer the best balance of sustainability, cost-effectiveness, and scalability for smart cities?
- How can waste be transformed into a resource through effective recovery and recycling?
- What role can advanced technologies, such as IoT and waste-to-energy systems, play in enhancing the efficiency of waste management solutions?
- How can public engagement and policy alignment ensure successful implementation of sustainable strategies?

A review of the existing literature shows that MCDM methods have been increasingly applied to MSWM by considering diverse technical, economic, environmental, technological, and social criteria. However, differences remain in the way subjective expert preferences, objective data-driven information, and uncertainty are incorporated into the decision-making process. In particular, the imprecision and hesitation associated with expert judgments can affect the reliability of MSWM alternative evaluation. To address these issues, the present study develops an integrated intuitionistic fuzzy-based MCDM framework that explicitly captures uncertainty and hesitation in expert assessments while incorporating both subjective and objective perspectives. The RANCOM method is employed to determine subjective criterion weights based on expert preferences, whereas ITARA is used to derive objective criterion weights from the available information, and ERUNS is subsequently applied to rank the selected MSWM alternatives. The framework considers key factors such as economic cost, operational and technological performance, environmental impact, waste characteristics, resource recovery, scalability, and social acceptability. By integrating these dimensions, the proposed framework provides a systematic and comprehensive basis for evaluating competing MSWM strategies and offers policymakers and urban planners reliable insights for selecting efficient, sustainable, practical, and socially acceptable solutions that support the broader objectives of smart-city development.

4 Preliminaries

Definition 1 [10]: Let $\mathfrak{A}$ be the universal set. A fuzzy set $\mathfrak{J}$ in $\mathfrak{A}$ is defined as

$$\mathfrak{J} = \{(j, \omega_{\mathfrak{J}}(j)) : j \in \mathfrak{A}\},$$

where $\omega_z(j)$ denotes the membership degree of the element j in the universal set $\mathfrak{A}$.

Definition 2 [11]: An IFS in $\mathfrak{A}$ is defined as

$$\mathfrak{U}(c) = \{ \langle j, {}^P\Theta(j), {}^P\Lambda(j) \rangle | j \in \mathfrak{A} \} : \quad (1)$$

where ${}^P\Theta(j), {}^P\Lambda(j) \in [0, 1]$, such that $0 \leq {}^P\Theta(j) + {}^P\Lambda(j) \leq 1$ for all $j \in \mathfrak{A}$. ${}^P\Theta(j), {}^P\Lambda(j)$ represent membership degree and non-membership degree separately for some $j \in X$. We denote this pair as $\mathfrak{S} = ({}^P\Theta_{\mathfrak{S}}, {}^P\Lambda_{\mathfrak{S}})$, throughout this research and refer to it as an IFN, satisfying the conditions ${}^P\Theta_{\mathfrak{S}}, {}^P\Lambda_{\mathfrak{S}} \in [0, 1]$ and ${}^P\Theta_{\mathfrak{S}} + {}^P\Lambda_{\mathfrak{S}} \leq 1$.

Definition 3 [42]: Ranking IFNs is a crucial step in their application to practical decision-making problems. Let $\mathfrak{S} = ({}^P\Theta_{\mathfrak{S}}, {}^P\Lambda_{\mathfrak{S}})$ be an intuitionistic fuzzy number (IFN), where ${}^P\Theta_{\mathfrak{S}}$ and ${}^P\Lambda_{\mathfrak{S}}$ denote its membership and non-membership degrees, respectively. The score function of the IFN $\mathfrak{S}$ is defined as

$$S(\mathfrak{S}) = {}^P\Theta_{\mathfrak{S}} - {}^P\Lambda_{\mathfrak{S}}. \quad (2)$$

However, the score function alone may not be sufficient to distinguish or classify IFNs in some cases. Therefore, an accuracy function $H(\mathfrak{S})$ is introduced as a complementary measure to further distinguish IFNs with identical or closely related score values [43], which is defined as

$$H(\mathfrak{S}) = {}^P\Theta_{\mathfrak{S}} + {}^P\Lambda_{\mathfrak{S}}. \quad (3)$$

The next part of our work is dedicated to disclosing the basic principles of IFNs accumulation.

Definition 4 [14]: Let $\mathfrak{S}_1 = \langle {}^P\Theta_1, {}^P\Lambda_1 \rangle$ and $\mathfrak{S}_2 = \langle {}^P\Theta_2, {}^P\Lambda_2 \rangle$ be two IFNs, $\lambda > 0$ then

$$\mathfrak{S}_{1^c} = \left\langle {}^P\Lambda_1, {}^P\Theta_1 \right\rangle : \quad (4)$$

$$\mathfrak{S}_1 \vee \mathfrak{S}_2 = \left\langle \max\{{}^P\Theta_1, {}^P\Theta_2\}, \min\{{}^P\Lambda_1, {}^P\Lambda_2\} \right\rangle : \quad (5)$$

$$\mathfrak{S}_1 \wedge \mathfrak{S}_2 = \left\langle \min\{{}^P\Theta_1, {}^P\Theta_2\}, \max\{{}^P\Lambda_1, {}^P\Lambda_2\} \right\rangle : \quad (6)$$

$$\mathfrak{S}_1 \oplus \mathfrak{S}_2 = \left\langle {}^P\Theta_1 + {}^P\Theta_2 - {}^P\Theta_1 {}^P\Theta_2, {}^P\Lambda_1 {}^P\Lambda_2 \right\rangle : \quad (7)$$

$$\mathfrak{S}_1 \otimes \mathfrak{S}_2 = \left\langle {}^P\Theta_1 {}^P\Theta_2, {}^P\Lambda_1 + {}^P\Lambda_2 - {}^P\Lambda_1 {}^P\Lambda_2 \right\rangle : \quad (8)$$

$$\lambda \mathfrak{S}_1 = \left\langle 1 - (1 - {}^P\Theta_1)^\lambda, {}^P\Lambda_1^\lambda \right\rangle : \quad (9)$$

$$\phi_1^\lambda = \left\langle {}^P\Theta_1^\lambda, 1 - (1 - {}^P\Lambda_1)^\lambda \right\rangle : \quad (10)$$

Intuitionistic Fuzzy Dombi Operators

Theorem 1: [44] Let $\tilde{\alpha}_i = ({}^P\Theta_i, {}^P\Lambda_i)$, $i = 1, 2, \dots, n$, be a collection of intuitionistic fuzzy numbers (IFNs), where ${}^P\Theta_i$ and ${}^P\Lambda_i$ denote the membership and non-membership degrees, respectively. Let

$\mathbf{o}^v = (o_1^v, o_2^v, \dots, o_n^v)^T$ be the corresponding weight vector satisfying $o_i^v > 0$, $\sum_{i=1}^n o_i^v = 1$. Then, the intuitionistic fuzzy Dombi weighted average (IFDWA) operator is a mapping $\text{IFDWA}_{\mathbf{o}^v} : \mathcal{IFN}^n \rightarrow \mathcal{IFN}$, defined by

$$\text{IFDWA}_{\mathbf{o}^v}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \bigoplus_{i=1}^n o_i^v \tilde{\alpha}_i, \quad (11)$$

where $\oplus$ denotes the intuitionistic fuzzy Dombi sum operation.

Theorem 2: [44] Let $\tilde{\alpha}_i = ({}^P\Theta_i, {}^P\Lambda_i)$, $i = 1, 2, \dots, n$, be a collection of IFNs, and let $\mathbf{o}^v = (o_1^v, o_2^v, \dots, o_n^v)^T$ be a weight vector satisfying $o_i^v > 0$, $\sum_{i=1}^n o_i^v = 1$. For the Dombi parameter $m > 0$, the value obtained by applying the IFDWA operator is also an IFN and is given by

$$\begin{aligned} & \text{IFDWA}_{\mathbf{o}^v}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) \\ &= \left(1 - \frac{1}{1 + \left\{ \sum_{i=1}^n o_i^v \left(\frac{{}^P\Theta_i}{1 - {}^P\Theta_i} \right)^m \right\}^{1/m}}, \frac{1}{1 + \left\{ \sum_{i=1}^n o_i^v \left(\frac{1 - {}^P\Lambda_i}{{}^P\Lambda_i} \right)^m \right\}^{1/m}} \right). \end{aligned} \quad (12)$$

5 ITARA and RANCOM-ERUNS Algorithm Based on IF Environment

Consider an MCDM problem involving a alternatives, b evaluation criteria, and p decision-makers. Throughout this study, $i = 1, 2, \dots, a$, $j = 1, 2, \dots, b$, and $k = 1, 2, \dots, p$ denote the indices of the alternatives, evaluation criteria, and decision-makers, respectively. The alternatives are represented by ϑ_i^v , the evaluation criteria by $\mathcal{K}_j^\alpha$, and the decision-makers by $\mathcal{E}_k^S$. Each decision-maker $\mathcal{E}_k^S$ is assigned a positive importance weight ω_k , satisfying

$$\omega_k > 0, \quad \sum_{k=1}^p \omega_k = 1. \quad (13)$$

The alternatives ϑ_i^v ($i = 1, 2, \dots, a$) are evaluated with respect to the criteria $\mathcal{K}_j^\alpha$ ($j = 1, 2, \dots, b$) by the decision-makers $\mathcal{E}_k^S$ ($k = 1, 2, \dots, p$). The evaluation criteria are selected based on the requirements of the decision problem and consultation with the decision-makers. The evaluation provided by the k th decision-maker is expressed as an intuitionistic fuzzy number (IFN), denoted by

$$\mathfrak{G}_{ij}^{(k)} = \left\langle {}^P\Theta_{ij}^{(k)}, {}^P\Lambda_{ij}^{(k)} \right\rangle, \quad (14)$$

where ${}^P\Theta_{ij}^{(k)}$ and ${}^P\Lambda_{ij}^{(k)}$ denote the membership and non-membership degrees, respectively. These values satisfy

$$0 \leq {}^P\Theta_{ij}^{(k)} \leq 1, \quad 0 \leq {}^P\Lambda_{ij}^{(k)} \leq 1, \quad (15)$$

and

$$0 \leq {}^P\Theta_{ij}^{(k)} + {}^P\Lambda_{ij}^{(k)} \leq 1. \quad (16)$$

The hesitation degree associated with each assessment is defined as

$$\pi_{ij}^{(k)} = 1 - {}^P\Theta_{ij}^{(k)} - {}^P\Lambda_{ij}^{(k)}, \quad (17)$$

where $\pi_{ij}^{(k)}$ represents the degree of hesitation or indeterminacy associated with the judgment of decision-maker $\mathcal{E}_k^S$. Thus, IFNs enable decision-makers to simultaneously express membership, non-membership, and hesitation information, thereby providing a flexible representation of uncertainty and incomplete information during the evaluation process.

5.1 Algorithm

Step 1: Construct the intuitionistic fuzzy decision matrices. Each alternative $\mathcal{Y}_i^y$ ($i = 1, 2, \dots, a$) is evaluated with respect to each criterion $\mathcal{K}_j^\alpha$ ($j = 1, 2, \dots, b$) by each decision-maker $\mathcal{E}_k^S$ ($k = 1, 2, \dots, p$). The intuitionistic fuzzy decision matrix provided by the k th decision-maker is denoted by

$$\mathcal{D}^{(k)} = \left[\mathfrak{S}_{ij}^{(k)} \right]_{a \times b}, \quad k = 1, 2, \dots, p, \quad (18)$$

where $\mathfrak{S}_{ij}^{(k)}$ represents the intuitionistic fuzzy evaluation of alternative $\mathcal{Y}_i^y$ with respect to criterion $\mathcal{K}_j^\alpha$ provided by decision-maker $\mathcal{E}_k^S$. The structure of the intuitionistic fuzzy decision matrix for the k th decision-maker is represented as follows:

$$\mathcal{D}^{(k)} = \left[\begin{array}{c|cccc} & \mathcal{K}_1^\alpha & \mathcal{K}_2^\alpha & \dots & \mathcal{K}_b^\alpha \\ \hline \mathcal{Y}_1^y & \left\langle {}^P\Theta_{11}^{(k)}, {}^P\Lambda_{11}^{(k)} \right\rangle & \left\langle {}^P\Theta_{12}^{(k)}, {}^P\Lambda_{12}^{(k)} \right\rangle & \dots & \left\langle {}^P\Theta_{1b}^{(k)}, {}^P\Lambda_{1b}^{(k)} \right\rangle \\ \mathcal{Y}_2^y & \left\langle {}^P\Theta_{21}^{(k)}, {}^P\Lambda_{21}^{(k)} \right\rangle & \left\langle {}^P\Theta_{22}^{(k)}, {}^P\Lambda_{22}^{(k)} \right\rangle & \dots & \left\langle {}^P\Theta_{2b}^{(k)}, {}^P\Lambda_{2b}^{(k)} \right\rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathcal{Y}_a^y & \left\langle {}^P\Theta_{a1}^{(k)}, {}^P\Lambda_{a1}^{(k)} \right\rangle & \left\langle {}^P\Theta_{a2}^{(k)}, {}^P\Lambda_{a2}^{(k)} \right\rangle & \dots & \left\langle {}^P\Theta_{ab}^{(k)}, {}^P\Lambda_{ab}^{(k)} \right\rangle \end{array} \right]. \quad (19)$$

Each element of the intuitionistic fuzzy decision matrix is defined as

$$\mathfrak{S}_{ij}^{(k)} = \left\langle {}^P\Theta_{ij}^{(k)}, {}^P\Lambda_{ij}^{(k)} \right\rangle, \quad (20)$$

where ${}^P\Theta_{ij}^{(k)}$ and ${}^P\Lambda_{ij}^{(k)}$ denote the membership and non-membership degrees, respectively. The indices $i = 1, 2, \dots, a$, $j = 1, 2, \dots, b$, and $k = 1, 2, \dots, p$ refer to the alternatives, evaluation criteria, and decision-makers, respectively. Table 2 presents the linguistic terms used by the decision-makers to evaluate the MSWM alternatives, together with their corresponding intuitionistic fuzzy representations. Table 3 presents the professional backgrounds, roles, and responsibilities of the decision-makers involved in the evaluation process. The use of linguistic assessments enables decision-makers to express their judgments in an intuitive form while accommodating uncertainty and imprecise information.

Step 2: The relative importance of each decision-maker is determined using Eq. (21). For an IFN

$$\tilde{z}_{ij} = \left({}^P\Theta_{z_{ij}}, {}^P\Lambda_{z_{ij}} \right),$$

its score value is

$$S(\tilde{z}_{ij}) = {}^P\Theta_{z_{ij}} - {}^P\Lambda_{z_{ij}}.$$

Table 2: Linguistic scale and corresponding intuitionistic fuzzy numbers for evaluating MSWM alternatives.

Linguistic Terms	Abbreviations	IFNs
Exceptionally High Performance	$\mathcal{HEWC}$	$\langle 0.90, 0.05 \rangle$
Very High Performance	$\mathcal{MEWC}$	$\langle 0.85, 0.10 \rangle$
High Performance	$\mathcal{LEWC}$	$\langle 0.80, 0.15 \rangle$
Moderately High Performance	$\mathcal{HCF}$	$\langle 0.75, 0.20 \rangle$
Above-Average Performance	$\mathcal{MCF}$	$\langle 0.70, 0.25 \rangle$
Slightly Above-Average Performance	$\mathcal{LCF}$	$\langle 0.65, 0.30 \rangle$
Moderate Performance	$\mathcal{HEIR}$	$\langle 0.60, 0.35 \rangle$
Slightly Below-Average Performance	$\mathcal{MEIR}$	$\langle 0.55, 0.40 \rangle$
Below-Average Performance	$\mathcal{LEIR}$	$\langle 0.50, 0.45 \rangle$
Moderately Low Performance	$\mathcal{HSP}$	$\langle 0.45, 0.50 \rangle$
Low Performance	$\mathcal{MSP}$	$\langle 0.40, 0.55 \rangle$
Very Low Performance	$\mathcal{LSP}$	$\langle 0.35, 0.60 \rangle$

Table 3: Decision-makers involved in the evaluation of MSWM alternatives.

Profession	Role	Responsibilities
Waste Management Officer	Coordinator	Oversees the design and implementation of MSWM strategies. Ensures efficient waste collection, transportation, and disposal while considering sustainability requirements. Collaborates with relevant stakeholders to improve operational performance.
Urban Infrastructure Planner	Analyst	Evaluates the infrastructure available for MSWM. Assesses accessibility, scalability, and the integration of waste management facilities into the urban environment. Plans long-term improvements to address future requirements.
Environmental Specialist	Advisor	Evaluates the environmental effects of MSWM practices and monitors compliance with relevant environmental regulations. Recommends sustainable practices and technological improvements to reduce environmental impacts.

The obtained score values are then normalized to calculate the weight of each decision-maker as follows:

$$\varpi_j = \frac{\sum_{i=1}^k S(\tilde{z}_{ij})}{\sum_{\ell=1}^m \sum_{i=1}^k S(\tilde{z}_{i\ell})} = \frac{\sum_{i=1}^k ({}^P\Theta_{z_{ij}} - {}^P\Lambda_{z_{ij}})}{\sum_{\ell=1}^m \sum_{i=1}^k ({}^P\Theta_{z_{i\ell}} - {}^P\Lambda_{z_{i\ell}})}, \quad j = 1, 2, \dots, m. \quad (21)$$

In Eq. (21), $i = 1, 2, \dots, k$ denotes the index of the evaluation items, criteria, or assessments considered in determining the importance of each decision-maker, whereas $j = 1, 2, \dots, m$ denotes the index of the decision-makers. The auxiliary index $\ell = 1, 2, \dots, m$ is used in the denominator to sum the score values over all decision-makers and to avoid confusion with the fixed decision-maker index j . Here, k is the total number of evaluation items and m is the total number of decision-makers. Furthermore, $\tilde{z}_{ij}$ denotes the intuitionistic

fuzzy assessment associated with the i th evaluation item and provided by the j th decision-maker, while ω_j represents the normalized weight of the j th decision-maker.

Step 3: Construct the aggregated decision matrix $M = [M_{ij}]_{a \times b}$. The individual assessments provided by the decision-makers are combined using Eq. (22). The aggregation process incorporates the corresponding importance coefficients and produces a single collective intuitionistic fuzzy assessment for each alternative under each criterion.

$$= \left(1 - \frac{1}{1 + \left\{ \sum_{i=1}^n o^{\pi_i} \left(\frac{{}^P\Theta_i}{1 - {}^P\Theta_i} \right)^m \right\}^{\frac{1}{m}}}, \frac{1}{1 + \left\{ \sum_{i=1}^n o^{\pi_i} \left(\frac{1 - {}^P\mathcal{E}\mathcal{S}_i}{{}^P\mathcal{E}\mathcal{S}_i} \right)^m \right\}^{\frac{1}{m}}} \right) \quad i = 1, 2, \dots, a; j = 1, 2, \dots, b : \quad (22)$$

Here, o^{π_i} denotes the corresponding importance coefficient used in the aggregation process, while m is the parameter controlling the behavior of the aggregation operator. The first and second components of the resulting ordered pair represent the aggregated membership and non-membership information, respectively. The resulting matrix therefore summarizes the collective assessments of the decision-makers.

Step 4: Construct the aggregated score matrix. Each intuitionistic fuzzy element in the aggregated decision matrix is converted into a scalar score by applying Eq. (23).

$$\mathfrak{V}(\mathfrak{d}) = {}^P\Theta_{\mathfrak{S}} - {}^P\Lambda_{\mathfrak{S}} : \quad (23)$$

In Eq. (23), ${}^P\Theta_{\mathfrak{S}}$ and ${}^P\Lambda_{\mathfrak{S}}$ represent the aggregated membership and non-membership degrees, respectively. The score function measures the net degree of support associated with an intuitionistic fuzzy assessment. A larger value of $\mathfrak{V}(\mathfrak{d})$ indicates a more favorable evaluation.

Step 5: Determine the indifference threshold for each criterion. The indifference threshold represents the minimum difference in criterion scores that is considered meaningful when distinguishing among the alternatives. It is calculated using Eq. (24).

$$\mathcal{H}_j = \beta [\max_i (\mathfrak{V}(\mathfrak{d})_{ij}) - \min_i (\mathfrak{V}(\mathfrak{d})_{ij})] \quad j = 1, 2, \dots, b, \beta \in [0, 1] : \quad (24)$$

Here, $\mathcal{H}_j$ denotes the indifference threshold of criterion j , while β is a threshold parameter defined on the interval $[0, 1]$. The difference between the maximum and minimum score values reflects the total variation of the alternatives under the corresponding criterion.

Step 6: Calculate the normalized indifference threshold using Eq. (25).

$$\mathcal{Y}_j = \frac{\mathcal{H}_j}{\sum_{i=1}^r \mathfrak{V}(\mathfrak{d})_{ij}} \quad j = 1, 2, \dots, b \quad (25)$$

In Eq. (25), $\mathcal{Y}_j$ denotes the normalized indifference threshold associated with criterion j . This normalization expresses the threshold relative to the overall score information associated with the corresponding criterion and facilitates comparison across the criteria.

Step 7: Construct the ordered normalized score matrix. For each criterion, the elements of the normalized score matrix are arranged in ascending order such that $\tilde{\beta}_{ij} \leq \tilde{\beta}_{i+1,j}$. This ordering enables the successive differences between neighboring values to be determined systematically.

Step 8: Calculate the distance between consecutive elements of the ordered normalized score matrix using Eq. (26).

$$\mathfrak{X}_{ij} = \tilde{\beta}_{i+1,j} - \tilde{\beta}_{ij}, \quad i = 1, 2, \dots, a-1, \quad j = 1, 2, \dots, b. \quad (26)$$

Here, $\mathfrak{X}_{ij}$ represents the difference between two successive ordered normalized score values under criterion j . A larger value indicates greater separation between neighboring evaluations and, consequently, a stronger ability of the criterion to distinguish among the alternatives.

Step 9: Determine the significant differences by comparing $\mathfrak{X}_{ij}$ with the normalized indifference threshold $\mathcal{Y}_j$. The significant difference is calculated using Eq. (27).

$$\Omega_{ij} = \begin{cases} \mathfrak{X}_{ij} - \mathcal{Y}_j, & \text{for } \mathfrak{X}_{ij} > \mathcal{Y}_j \\ 0, & \text{for } \mathfrak{X}_{ij} \leq \mathcal{Y}_j \end{cases} \quad i = 1, 2, \dots, a; j = 1, 2, \dots, b : \quad (27)$$

The value of Ω_{ij} represents the portion of the successive difference that exceeds the corresponding indifference threshold. If $\mathfrak{X}_{ij} > \mathcal{Y}_j$, the difference contributes to the objective importance of criterion $\mathcal{K}^\alpha_j$. Otherwise, the difference is considered insignificant and is assigned a value of zero.

Step 10: Calculate the objective weights of the criteria. First, the significant differences associated with each criterion are combined through F_j . The normalized objective criterion weight is then obtained using Eq. (28).

$$\mathcal{N} = \frac{F_j}{\sum_{j=1}^t F_j} \quad j = 1, 2, \dots, b : \quad (28)$$

where

$$F_j = \left(\sum_{i=1}^{r-1} (\Omega_{ij})^\eta \right)^{\frac{1}{\eta}}.$$

Here, F_j represents the overall amount of significant variation associated with criterion j , while η is the parameter used to aggregate the significant differences. The value $\mathcal{N}_j$ represents the normalized objective weight obtained through the ITARA procedure. A criterion with greater significant variation receives a larger objective weight.

Step 11: Construct the RANCOM matrix by comparing the relative positions of the criteria. The ranking information provided for the criteria is transformed into a pairwise comparison structure as follows.

$$= \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_n \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{matrix} & \begin{bmatrix} (\mathfrak{U}(\mathfrak{c})_{11}) & (\mathfrak{U}(\mathfrak{c})_{12}) & \dots & (\mathfrak{U}(\mathfrak{c})_{1n}) \\ (\mathfrak{U}(\mathfrak{c})_{21}) & (\mathfrak{U}(\mathfrak{c})_{22}) & \dots & (\mathfrak{U}(\mathfrak{c})_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (\mathfrak{U}(\mathfrak{c})_{n1}) & (\mathfrak{U}(\mathfrak{c})_{n2}) & \dots & (\mathfrak{U}(\mathfrak{c})_{nn}) \end{bmatrix} \end{matrix}$$

The elements of the RANCOM matrix are determined using the following significance function:

$$\mathfrak{U}(\mathfrak{c})_{ij} = \begin{cases} 1, & \Psi(C_i) < \Psi(C_j) \\ 0, & \Psi(C_i) > \Psi(C_j) \\ 0.5, & \Psi(C_i) = \Psi(C_j) \end{cases} \quad (29)$$

Here, $\Psi(C_j)$ denotes the significance or rank position of the j th criterion. A value of 1 indicates that criterion C_i is ranked ahead of criterion C_j , a value of 0 indicates the opposite relationship, and a value of 0.5 represents equal importance.

Step 12: Calculate the summation score $\mathfrak{Q}_j$ for each criterion using Eq. (30).

$$\mathfrak{Q}_j = \sum_{t=1}^b \mathfrak{U}_{jt}^{(c)}, \quad j = 1, 2, \dots, b. \quad (30)$$

In Eq. (30), $\mathfrak{Q}_j$ denotes the summation score of the j th criterion, obtained by summing its comparison values with respect to all b criteria. Here, $\mathfrak{U}_{jt}^{(c)}$ represents the comparison value between the j th and t th criteria.

Step 13: Determine the normalized subjective weight of each criterion using Eq. (31).

$$\mathfrak{Q}_j^s = \frac{\mathfrak{Q}_j}{\sum_{r=1}^b \mathfrak{Q}_r}, \quad j = 1, 2, \dots, b. \quad (31)$$

In Eq. (31), $\mathfrak{Q}_j^s$ denotes the normalized subjective weight of the j th criterion obtained using the RANCOM method. The auxiliary index $r = 1, 2, \dots, b$ is used in the denominator to sum the scores of all criteria. The normalization ensures that $\sum_{j=1}^b \mathfrak{Q}_j^s = 1$, so that the subjective criterion weights are expressed on a common scale and can subsequently be combined with the objective ITARA weights.

Step 14: Integrate the subjective and objective criterion weights. Eq. (32) combines the objective weights obtained from IF-ITARA and the subjective weights derived from IF-RANCOM. The final normalized criterion weights are subsequently calculated using Eq. (33).

$$\mathcal{M}_j = \frac{(\sigma)\mathcal{N} + (1 - \sigma)\mathfrak{Q}_{jj}^s}{2}, \quad \sigma \in [0, 1] j = 1, 2, \dots, b : \quad (32)$$

$$\mathcal{B}_j = \frac{\mathcal{M}_j}{\sum_{j=1}^t \mathcal{M}_j} j = 1, 2, \dots, b : \quad (33)$$

In Eq. (32), σ is the coefficient controlling the relative contribution of the objective and subjective weighting components, while $\mathcal{M}_j$ represents the integrated weight of criterion j . Eq. (33) normalizes these integrated values to obtain the final criterion weight $\mathcal{B}_j$. These final weights are subsequently used in the ERUNS ranking procedure.

5.2 ERUNS Method

The ERUNS method is employed to rank the alternatives after determining the final criterion weights. It uses nonlinear standardization and relative utility information to obtain the final appraisal score of each alternative.

Step 15: Normalize the score matrix $[\mathfrak{V}(\mathfrak{d})_{ij}]_{a \times b}$ to the interval $[v, R^\tau]$ using Eq. (34).

$$S_{ij}^{\mathfrak{x}} = \left(\frac{\mathfrak{V}(\mathfrak{d})_{ij}^{\min}}{\mathfrak{V}(\mathfrak{d})_{ij}} \right)^3 R^\tau + \frac{v}{\mathfrak{V}(\mathfrak{d})_{ij}^{\min}} \quad i = 1, 2, \dots, a; j = 1, 2, \dots, b : \quad (34)$$

Here,

$$\mathfrak{V}(\mathfrak{d})_{ij}^{\min} = \min_{1 \leq i \leq a} (\mathfrak{V}(\mathfrak{d})_{ij}).$$

The values of the interval $[v, R^\tau]$ are determined according to the requirements of the decision problem. The nonlinear transformation represented by $S_{ij}^{\mathfrak{x}}$ standardizes the original score information and prepares the decision matrix for the subsequent ERUNS calculations.

Step 16: Transform the standardized criterion values into a common minimization-oriented form according to the preference direction of each criterion. The transformed standardized decision matrix $\Phi = [\Phi_{ij}]_{m \times n}$ is obtained using Eq. (35).

$$\Phi_{ij} = \begin{cases} \max_{1 \leq r \leq m} (S_{rj}^{\mathfrak{x}}) + \min_{1 \leq r \leq m} (S_{rj}^{\mathfrak{x}}) - S_{ij}^{\mathfrak{x}}, & j \in \mathcal{K}_b^\alpha, \\ S_{ij}^{\mathfrak{x}}, & j \in \mathcal{K}_c^\alpha, \end{cases} \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n. \quad (35)$$

Here, $\mathcal{K}_b^\alpha$ denotes the set of benefit criteria, for which larger standardized values are preferred, whereas $\mathcal{K}_c^\alpha$ denotes the set of cost criteria, for which smaller standardized values are preferred. For a benefit criterion, the first branch of Eq. (35) reverses the preference direction while preserving the range of the standardized values. Cost criteria are retained without modification. Consequently, all transformed values Φ_{ij} follow a common minimization orientation, in which a smaller value represents a more preferable performance.

Step 17: Calculate the weighted standardized values Ξ_{bij} using Eq. (36).

$$\Xi_{bij} = \frac{\exp((\Phi_{ij})/\chi) \mathcal{B}_j}{\sum_{j=1}^n \exp((\Phi_{ij})/\chi) \mathcal{B}_j} \quad i = 1, 2, \dots, a; j = 1, 2, \dots, b : \quad (36)$$

where

$$(\Phi_{ij}) = \frac{\Phi_{ij}}{\sum_{j=1}^n \Phi_{ij}}, \quad j \in \{1, 2, 3, \dots, b\}.$$

In Eq. (36), $\mathcal{B}_j$ represents the final weight of criterion j , while χ is the parameter controlling the exponential transformation. The value Ξ_{bij} combines the standardized performance of alternative i with the relative importance of criterion j .

Step 18: Determine the utility degrees of the ideal and anti-ideal solutions. The utility degree of the i th alternative with respect to the ideal solution is calculated as

$$C_{\lambda i}^+ = \frac{\prod_{j=1}^b (\Phi_{ij})^{\Xi_{bj}^+}}{\sum_{j=1}^b \Xi_{bj}^+}, \quad i = 1, 2, \dots, a. \quad (37)$$

whereas the utility degree associated with the anti-ideal solution is calculated as

$$C_{\lambda i}^- = -\frac{\sum_{j=1}^b \Xi_{bj}^-}{\prod_{j=1}^b (\Phi_{ij})^{\Xi_{bj}^-}} - \min_{1 \leq i \leq a} \left(\frac{\sum_{j=1}^b \Xi_{bj}^-}{\prod_{j=1}^b (\Phi_{ij})^{\Xi_{bj}^-}} \right) + \max_{1 \leq i \leq a} \left(\frac{\sum_{j=1}^b \Xi_{bj}^-}{\prod_{j=1}^b (\Phi_{ij})^{\Xi_{bj}^-}} \right), \quad (38)$$

$$\Xi_{bj}^+ = \max_{1 \leq i \leq a} (\Phi_{ij} \cdot \mathcal{B}_j),$$

$$\Xi_{bj}^- = \min_{1 \leq i \leq a} (\Phi_{ij} \cdot \mathcal{B}_j), \quad j = 1, 2, \dots, b.$$

Here, Ξ_{bj}^+ and Ξ_{bj}^- represent the ideal and anti-ideal reference values associated with criterion j , respectively. The quantities $C_{\lambda i}^+$ and $C_{\lambda i}^-$ measure the relative utility of alternative i with respect to these two reference conditions.

Step 19: Normalize the utility degrees using Eq. (39).

$$(C_{\lambda i}^+) = \frac{C_{\lambda i}^+}{C_{\lambda i}^+ + C_{\lambda i}^-}$$

$$(C_{\lambda i}^-) = \frac{C_{\lambda i}^-}{C_{\lambda i}^+ + C_{\lambda i}^-} \quad i = 1, 2, \dots, a : \quad (39)$$

The normalized utility degrees express the relative contribution of the ideal and anti-ideal utility components on a common scale. These values are subsequently incorporated into the final appraisal function.

Step 20: Calculate the final appraisal score of each alternative using Eq. (40), where $R^r \in [0, 1]$ is the parameter controlling the relative contribution of the two normalized utility components.

$$AS_i = (C_{\lambda i}^+ + C_{\lambda i}^-) \frac{(1 + (C_{\lambda i}^+))^{R^r} (1 + (C_{\lambda i}^-))^{1-R^r} - (1 - (C_{\lambda i}^+))^{R^r} (1 - (C_{\lambda i}^-))^{1-R^r}}{(1 + (C_{\lambda i}^+))^{R^r} (1 + (C_{\lambda i}^-))^{1-R^r} + (1 - (C_{\lambda i}^+))^{R^r} (1 - (C_{\lambda i}^-))^{1-R^r}}, \quad i = 1, 2, \dots, a : \quad (40)$$

The value AS_i represents the final appraisal score of alternative $\mathcal{Y}_i$. The alternatives were ranked by their appraisals, and an option was selected for the best score and designated as the preferred option of MSWM. This methodology uses all the elements described above: uncertain expert opinions, objective data from the decision matrix, subjective criteria weights, non-linear standardization, and relative utility evaluations in one decision-making process. As illustrated in Fig. 1, there are four key phases that make up the methodology. These include intuitionistic fuzzy assessment, ITARA-based objective weighting of criteria, RANCOM-based integration of the criteria weights, and ERUNS-based ranking of the alternatives. Additional methods were also used to test the reliability of the results, including sensitivity testing, Monte Carlo Simulation, analysis of weight parameters, and comparison of results.

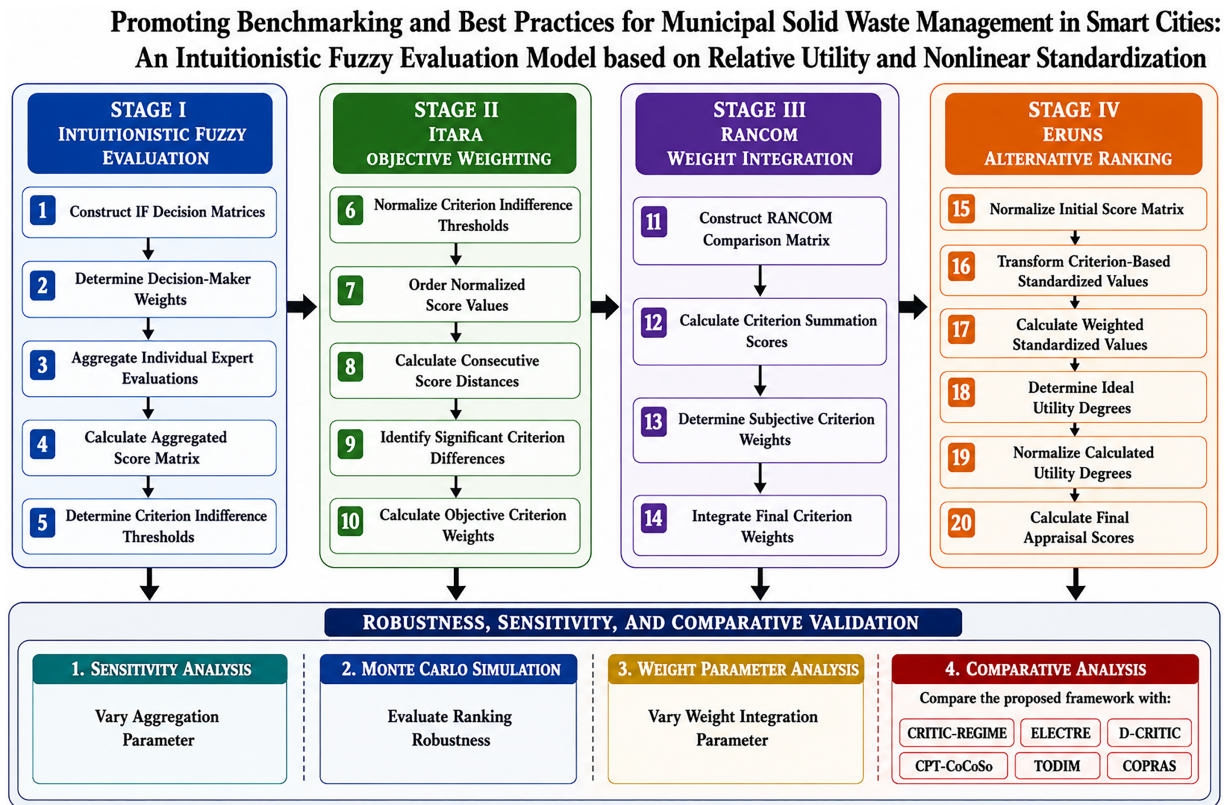


Figure 1: Overall methodological framework of the proposed model for evaluating MSWM strategies in smart cities.

5.3 MSWM Alternatives and Evaluation Criteria for Smart Cities

The determination of an optimal MSWM approach is contingent on defining alternative options and evaluating the criteria to be used in assessing them. This research identifies six possible MSWM strategies. Each of the six alternatives addresses waste treatment, resource recovery, recycling, disposal, or smart collection. A summary of each of the alternatives, including their key attributes, benefits and disadvantages, is presented in Table 4. Ten criteria will also be used to assess the six alternatives in terms of their relative merits. The ten criteria represent both practical and theoretical aspects of MSWM; namely economic, environmental, technical, operational, social, regulatory, geographical, and circular economy. Practical implications and expected outcomes of the use of each criterion are described in Table 5. Thus, the alternatives identified, along with the criteria developed, form the foundation upon which the evaluation and ranking of the alternatives will occur.

Table 4: MSWM alternatives considered for smart cities.

Alt.	MSWM Strategy	Description	Advantages	Disadvantages
θ_1	Waste-to-Energy (WTE) Plants	Converts non-recyclable waste into electricity or heat through thermal or biological processes.	Reduces landfill use; recovers useful energy; supports integrated waste management.	High investment and emission control costs; produces residual ash; requires a stable waste supply.

(Continued)

Table 4 (continued)

Alt.	MSWM Strategy	Description	Advantages	Disadvantages
ϑ^{γ}_2	Smart Waste Collection Systems	Uses IoT-enabled bins and sensors to monitor fill levels and improve collection scheduling and route planning.	Reduces unnecessary trips; lowers fuel use and emissions; prevents bin overflow; supports data-based planning.	Requires digital infrastructure and maintenance; faces cybersecurity risks; limited mainly to waste collection.
ϑ^{γ}_3	Landfill with Methane Capture	Captures methane generated in engineered landfills for electricity generation or other energy uses.	Handles large waste volumes; recovers energy; offers a practical improvement over conventional landfilling.	Risk of leachate and methane leakage; requires large land areas and long-term monitoring.
ϑ^{γ}_4	Composting and Organic Waste Treatment	Biologically treats organic waste to produce compost and, in some systems, recover biogas.	Reduces organic waste disposal; improves soil quality; lowers landfill methane emissions.	Requires effective waste separation and process control; needs space; unsuitable for inorganic waste.
ϑ^{γ}_5	Recycling and Material Recovery Facilities	Separates and processes recoverable materials for reuse in manufacturing and other applications.	Conserves resources; reduces landfill disposal; supports material recovery and circular economy practices.	Depends on market conditions and waste separation; some materials are costly or difficult to recycle.
ϑ^{γ}_6	Incineration with Energy Recovery	Uses controlled high-temperature combustion to reduce waste volume and recover heat or electricity.	Greatly reduces waste volume; generates useful energy; destroys pathogens and harmful organic substances.	High capital and operating costs; requires strict emission control; may face public opposition.

Table 5: Key criteria and their types for evaluating MSWM strategies in smart cities.

Criterion	Criteria Name	Description	Why It Matters	Impact	Type
$\mathcal{K}^{\alpha}_1$	Cost-Effectiveness	Considers the investment, operating costs, and possible income associated with a waste management strategy.	Municipal budgets are often limited, so the selected option should remain financially practical over time.	Better cost performance allows funds to be used efficiently and supports wider implementation.	Cost
$\mathcal{K}^{\alpha}_2$	Environmental Impact	Examines emissions, air and water pollution, and possible effects on the surrounding environment.	Environmental performance is important for meeting sustainability targets and regulatory requirements.	A lower environmental burden helps protect public health and natural ecosystems.	Cost
$\mathcal{K}^{\alpha}_3$	Resource Recovery Efficiency	Looks at how effectively useful materials, compost, or energy can be recovered from the waste stream.	Good recovery performance reduces the need for virgin resources and makes better use of discarded materials.	Higher recovery can reduce disposal volumes while creating environmental and economic value.	Benefit

(Continued)

Table 5 (continued)

Criterion	Criteria Name	Description	Why It Matters	Impact	Type
$\mathcal{K}^{\alpha}_4$	Social Acceptability	Represents the level of public support for a strategy, particularly in relation to health, odor, appearance, and perceived risks.	Community support can make implementation easier, while strong opposition may delay a project.	Greater acceptance encourages public cooperation and reduces social and political barriers.	Benefit
$\mathcal{K}^{\alpha}_5$	Technological Feasibility	Considers whether the required technology is available, reliable, and suitable for existing urban infrastructure.	A practical and proven technology is generally easier to introduce and manage.	Higher feasibility can reduce technical failures, delays, and maintenance difficulties.	Benefit
$\mathcal{K}^{\alpha}_6$	Operational Complexity	Reflects the effort needed to operate and maintain the system, including staff, equipment, and technical skills.	Systems with simpler operating requirements are easier to manage, especially where technical expertise is limited.	Lower complexity can reduce operating problems, costs, and the likelihood of system failure.	Cost
$\mathcal{K}^{\alpha}_7$	Policy and Regulation Alignment	Considers how well a strategy complies with relevant waste, environmental, and emission regulations.	Regulatory compliance can support project approval and reduce legal difficulties.	Strong policy alignment improves long-term viability and lowers regulatory uncertainty.	Benefit
$\mathcal{K}^{\alpha}_8$	Scalability and Flexibility	Examines whether a system can respond to increasing waste volumes and changes in waste composition or urban conditions.	Cities continue to grow, so waste systems need to adjust to changing demands.	Greater adaptability supports long-term use and reduces the need for costly replacement or major modification.	Benefit
$\mathcal{K}^{\alpha}_9$	Land Use and Space Requirements	Considers the amount of physical space needed for facilities, processing activities, and related infrastructure.	Land availability is a major concern in densely populated urban areas.	Lower space requirements make a strategy easier to integrate into the existing urban environment.	Cost
$\mathcal{K}^{\alpha}_{10}$	Contribution to Circular Economy Goals	Examines how well a strategy supports waste reduction, reuse, recycling, and continued use of recovered resources.	Circular practices help conserve resources and reduce dependence on disposal-based waste management.	A stronger contribution supports resource efficiency and long-term urban sustainability.	Benefit

5.4 Experimental Results

Step 1: The results of the survey with the decision-makers are summarized in [Table 6](#). To express their opinions, decision-makers used the following linguistic expressions, which are shown in [Table 6](#).

Step 2: The weights of the decision-makers, denoting the expertise of each DM, have been incorporated in proportion to their set importance. This case involved three decision-makers. The IFS linguistic values of the different decision-makers are presented in [Table 7](#) and were established from [Eq. \(21\)](#).

Step 3: The specific IFNs are composed of systematically collected and categorized individual tables in relation to the viewpoints of the involved decision-makers. These were merged using the intuitionistic fuzzy weighted geometric (IFWG) operator in [Eq. \(22\)](#). The resulting aggregation is presented in [Table 8](#).

Table 6: Linguistic terms given by decision-makers.

Experts	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathcal{E}_1^S$	<i>HSP</i>	<i>LSP</i>	<i>MCF</i>	<i>HEWC</i>	<i>LEIR</i>	<i>MEIR</i>
	<i>LCF</i>	<i>MEWC</i>	<i>HEIR</i>	<i>LEWC</i>	<i>HCF</i>	<i>MSP</i>
	<i>MEIR</i>	<i>HSP</i>	<i>LSP</i>	<i>LCF</i>	<i>MEWC</i>	<i>HCF</i>
	<i>LEWC</i>	<i>MCF</i>	<i>HEIR</i>	<i>LSP</i>	<i>HSP</i>	<i>MEIR</i>
	<i>MEWC</i>	<i>HCF</i>	<i>LSP</i>	<i>LCF</i>	<i>HSP</i>	<i>LEIR</i>
	<i>MSP</i>	<i>HEWC</i>	<i>HCF</i>	<i>LEWC</i>	<i>LCF</i>	<i>MEWC</i>
	<i>LSP</i>	<i>HSP</i>	<i>MEIR</i>	<i>LCF</i>	<i>HEIR</i>	<i>LEWC</i>
	<i>MCF</i>	<i>MEWC</i>	<i>HSP</i>	<i>LEIR</i>	<i>MSP</i>	<i>HCF</i>
	<i>HCF</i>	<i>LEWC</i>	<i>LCF</i>	<i>MCF</i>	<i>LSP</i>	<i>MEIR</i>
	<i>LEIR</i>	<i>HCF</i>	<i>MCF</i>	<i>MEWC</i>	<i>HEWC</i>	<i>HSP</i>
$\mathcal{E}_2^S$	<i>MEWC</i>	<i>LSP</i>	<i>HSP</i>	<i>MCF</i>	<i>LEIR</i>	<i>LCF</i>
	<i>HSP</i>	<i>HEWC</i>	<i>LSP</i>	<i>MEIR</i>	<i>MSP</i>	<i>HCF</i>
	<i>LEWC</i>	<i>MEIR</i>	<i>MSP</i>	<i>HSP</i>	<i>LEIR</i>	<i>LCF</i>
	<i>HCF</i>	<i>MCF</i>	<i>LEIR</i>	<i>MEWC</i>	<i>HEWC</i>	<i>MSP</i>
	<i>MSP</i>	<i>HEWC</i>	<i>MEIR</i>	<i>LSP</i>	<i>LCF</i>	<i>HSP</i>
	<i>MEWC</i>	<i>HEIR</i>	<i>LEWC</i>	<i>HCF</i>	<i>LCF</i>	<i>MCF</i>
	<i>MCF</i>	<i>MEWC</i>	<i>HCF</i>	<i>HEIR</i>	<i>LSP</i>	<i>LEIR</i>
	<i>LEIR</i>	<i>HCF</i>	<i>HSP</i>	<i>MCF</i>	<i>LEWC</i>	<i>LSP</i>
	<i>HEWC</i>	<i>LEWC</i>	<i>MCF</i>	<i>LSP</i>	<i>MEIR</i>	<i>HCF</i>
	<i>LCF</i>	<i>MEIR</i>	<i>HCF</i>	<i>LEIR</i>	<i>MSP</i>	<i>HSP</i>
$\mathcal{E}_3^S$	<i>LSP</i>	<i>HSP</i>	<i>HEIR</i>	<i>MEWC</i>	<i>LEWC</i>	<i>MCF</i>
	<i>MCF</i>	<i>HCF</i>	<i>MEIR</i>	<i>LCF</i>	<i>HSP</i>	<i>LEIR</i>
	<i>MEWC</i>	<i>LCF</i>	<i>LEIR</i>	<i>HCF</i>	<i>MSP</i>	<i>LEWC</i>
	<i>LEIR</i>	<i>MSP</i>	<i>HSP</i>	<i>MCF</i>	<i>LSP</i>	<i>MEIR</i>
	<i>HCF</i>	<i>LEWC</i>	<i>HSP</i>	<i>MCF</i>	<i>LCF</i>	<i>LEIR</i>
	<i>HEWC</i>	<i>MEWC</i>	<i>LSP</i>	<i>HCF</i>	<i>MCF</i>	<i>LEWC</i>
	<i>MSP</i>	<i>HSP</i>	<i>MCF</i>	<i>LEIR</i>	<i>LCF</i>	<i>MEIR</i>
	<i>MEIR</i>	<i>HCF</i>	<i>LSP</i>	<i>HEWC</i>	<i>LEIR</i>	<i>LCF</i>
	<i>LEWC</i>	<i>HSP</i>	<i>LCF</i>	<i>MEIR</i>	<i>LSP</i>	<i>MCF</i>
	<i>HSP</i>	<i>MSP</i>	<i>LEWC</i>	<i>HEWC</i>	<i>MCF</i>	<i>HCF</i>

Table 7: Linguistic terms about decision-makers.

Decision-Makers	Qualification	Expertise	Aptitude	Experience	Weights
$\mathcal{E}_1^S$	<i>MSE</i>	<i>LSE</i>	<i>HTF</i>	<i>LCF</i>	0.3326
$\mathcal{E}_2^S$	<i>HCF</i>	<i>LTF</i>	<i>MCF</i>	<i>MES</i>	0.3706
$\mathcal{E}_3^S$	<i>LES</i>	<i>MTF</i>	<i>HES</i>	<i>LTF</i>	0.2968

Table 8: Aggregated decision matrix.

$\mathcal{H}^\alpha_i$	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathcal{H}^\alpha_1$	$\langle 0.7945, 0.1151 \rangle$	$\langle 0.4336, 0.5160 \rangle$	$\langle 0.6575, 0.2877 \rangle$	$\langle 0.8714, 0.0692 \rangle$	$\langle 0.7176, 0.2116 \rangle$	$\langle 0.6927, 0.2553 \rangle$
$\mathcal{H}^\alpha_2$	$\langle 0.6707, 0.2749 \rangle$	$\langle 0.8826, 0.0623 \rangle$	$\langle 0.5671, 0.3802 \rangle$	$\langle 0.7581, 0.1795 \rangle$	$\langle 0.6588, 0.2764 \rangle$	$\langle 0.6792, 0.2579 \rangle$
$\mathcal{H}^\alpha_3$	$\langle 0.8405, 0.0944 \rangle$	$\langle 0.6126, 0.3350 \rangle$	$\langle 0.4703, 0.4789 \rangle$	$\langle 0.7000, 0.2420 \rangle$	$\langle 0.7867, 0.1224 \rangle$	$\langle 0.7950, 0.1500 \rangle$
$\mathcal{H}^\alpha_4$	$\langle 0.7886, 0.1529 \rangle$	$\langle 0.7017, 0.2433 \rangle$	$\langle 0.5776, 0.3711 \rangle$	$\langle 0.8164, 0.1053 \rangle$	$\langle 0.7945, 0.1151 \rangle$	$\langle 0.5550, 0.3938 \rangle$
$\mathcal{H}^\alpha_5$	$\langle 0.8176, 0.1067 \rangle$	$\langle 0.8639, 0.0794 \rangle$	$\langle 0.5179, 0.4305 \rangle$	$\langle 0.6568, 0.2867 \rangle$	$\langle 0.6540, 0.2934 \rangle$	$\langle 0.5327, 0.4172 \rangle$
$\mathcal{H}^\alpha_6$	$\langle 0.8626, 0.0714 \rangle$	$\langle 0.8640, 0.0723 \rangle$	$\langle 0.7839, 0.1547 \rangle$	$\langle 0.8200, 0.1286 \rangle$	$\langle 0.7168, 0.2327 \rangle$	$\langle 0.8527, 0.0872 \rangle$
$\mathcal{H}^\alpha_7$	$\langle 0.6119, 0.3277 \rangle$	$\langle 0.7986, 0.1135 \rangle$	$\langle 0.7417, 0.2039 \rangle$	$\langle 0.6463, 0.3023 \rangle$	$\langle 0.6090, 0.3367 \rangle$	$\langle 0.7357, 0.1961 \rangle$
$\mathcal{H}^\alpha_8$	$\langle 0.6547, 0.2909 \rangle$	$\langle 0.8499, 0.0901 \rangle$	$\langle 0.4740, 0.4757 \rangle$	$\langle 0.8073, 0.1160 \rangle$	$\langle 0.7323, 0.1956 \rangle$	$\langle 0.6942, 0.2452 \rangle$
$\mathcal{H}^\alpha_9$	$\langle 0.8639, 0.0794 \rangle$	$\langle 0.8107, 0.1300 \rangle$	$\langle 0.7207, 0.2288 \rangle$	$\langle 0.6283, 0.3140 \rangle$	$\langle 0.4938, 0.4539 \rangle$	$\langle 0.7417, 0.2039 \rangle$
$\mathcal{H}^\alpha_{10}$	$\langle 0.6106, 0.3363 \rangle$	$\langle 0.6804, 0.2584 \rangle$	$\langle 0.8063, 0.1408 \rangle$	$\langle 0.8595, 0.0740 \rangle$	$\langle 0.8072, 0.1126 \rangle$	$\langle 0.6540, 0.2824 \rangle$

Step 4: The matrix scores displayed in Table 9 were calculated using Eq. (23).

Table 9: Decision matrix converting into score matrix.

$\mathcal{H}^\alpha_i$	$\mathfrak{Y}_1$	$\mathfrak{Y}_2$	$\mathfrak{Y}_3$	$\mathfrak{Y}_4$	$\mathfrak{Y}_5$	$\mathfrak{Y}_6$
$\mathcal{H}^\alpha_1$	0.6794	-0.0823	0.3698	0.8022	0.5060	0.4374
$\mathcal{H}^\alpha_2$	0.3959	0.8202	0.1869	0.5786	0.3824	0.4213
$\mathcal{H}^\alpha_3$	0.7461	0.2776	-0.0087	0.4581	0.6644	0.6450
$\mathcal{H}^\alpha_4$	0.6356	0.4584	0.2065	0.7112	0.6794	0.1612
$\mathcal{H}^\alpha_5$	0.7109	0.7845	0.0875	0.3701	0.3606	0.1155
$\mathcal{H}^\alpha_6$	0.7913	0.7917	0.6292	0.6913	0.4841	0.7655
$\mathcal{H}^\alpha_7$	0.2841	0.6851	0.5378	0.3440	0.2722	0.5396
$\mathcal{H}^\alpha_8$	0.3638	0.7598	-0.0017	0.6913	0.5367	0.4490
$\mathcal{H}^\alpha_9$	0.7845	0.6807	0.4919	0.3143	0.0399	0.5378
$\mathcal{H}^\alpha_{10}$	0.2743	0.4220	0.6655	0.7855	0.6946	0.3716

Step 5: The indifference threshold values for the ten evaluation criteria were calculated using Eq. (24). A common threshold coefficient of $\beta = 0.2365$ was applied to all criteria. The resulting indifference-threshold values corresponding to the ten criteria are given by

$$\mathcal{H} = \begin{bmatrix} 0.2092 \\ 0.1498 \\ 0.1785 \\ 0.1301 \\ 0.1648 \\ 0.0727 \\ 0.0976 \\ 0.1801 \\ 0.1761 \\ 0.1209 \end{bmatrix} \quad (41)$$

Step 6: Once the $\mathcal{H}_j$ values are computed, the next step is to calculate the normalized indifference thresholds $\mathcal{Y}_j$ for the ten evaluation criteria using Eq. (25). The resulting criterion-wise normalized indifference thresholds, corresponding to criteria $j = 1, 2, \dots, 10$, are given by

$$\mathcal{Y} = \begin{bmatrix} 0.0771 \\ 0.0538 \\ 0.0642 \\ 0.0456 \\ 0.0679 \\ 0.0175 \\ 0.0367 \\ 0.0643 \\ 0.0618 \\ 0.0376 \end{bmatrix}. \quad (42)$$

Step 7: For each evaluation criterion, the normalized score values of the six alternatives are sorted in ascending order. Since the rows represent the evaluation criteria and the columns represent the alternatives, the sorting is performed row-wise. The resulting ordered normalized matrix is denoted by $[\tilde{\beta}_{ji}]$, where $j = 1, 2, \dots, 10$ denotes the criteria and $i = 1, 2, \dots, 6$ denotes the alternatives.

$$[\tilde{\beta}_{ji}] = \begin{bmatrix} -0.0823 & 0.3698 & 0.4374 & 0.5060 & 0.6794 & 0.8022 \\ 0.1869 & 0.3824 & 0.3959 & 0.4213 & 0.5786 & 0.8202 \\ -0.0087 & 0.2776 & 0.4581 & 0.6450 & 0.6644 & 0.7461 \\ 0.1612 & 0.2065 & 0.4584 & 0.6356 & 0.6794 & 0.7112 \\ 0.0875 & 0.1155 & 0.3606 & 0.3701 & 0.7109 & 0.7845 \\ 0.4841 & 0.6292 & 0.6913 & 0.7655 & 0.7913 & 0.7917 \\ 0.2722 & 0.2841 & 0.3440 & 0.5378 & 0.5396 & 0.6851 \\ -0.0017 & 0.3638 & 0.4490 & 0.5367 & 0.6913 & 0.7598 \\ 0.0399 & 0.3143 & 0.4919 & 0.5378 & 0.6807 & 0.7845 \\ 0.2743 & 0.3716 & 0.4220 & 0.6655 & 0.6946 & 0.7855 \end{bmatrix}. \quad (43)$$

Step 8: The difference between the entries of the ordered normalized matrix is then obtained using Eq. (26).

$$\mathfrak{X}_{ij} = \begin{bmatrix} 0.4521 & 0.1954 & 0.2863 & 0.0453 & 0.0280 & 0.1451 & 0.0119 & 0.3655 & 0.2744 & 0.0974 \\ 0.0676 & 0.0135 & 0.1805 & 0.2519 & 0.2451 & 0.0621 & 0.0598 & 0.0852 & 0.1776 & 0.0504 \\ 0.0687 & 0.0255 & 0.1869 & 0.1773 & 0.0095 & 0.0742 & 0.1939 & 0.0877 & 0.0459 & 0.2435 \\ 0.1734 & 0.1573 & 0.0194 & 0.0438 & 0.3408 & 0.0258 & 0.0018 & 0.1546 & 0.1428 & 0.0291 \\ 0.1227 & 0.2417 & 0.0817 & 0.0318 & 0.0736 & 0.0004 & 0.1455 & 0.0685 & 0.1039 & 0.0909 \end{bmatrix} \quad (44)$$

Step 9: Eq. (27) is applied to compute the difference between the entries of the matrix $\mathfrak{X}_{ij}$ and $\mathcal{Y}_j$, which are represented in the matrix Ω_{ij} .

$$\Omega_{ij} = \begin{bmatrix} 0.3750 & 0.1417 & 0.2221 & 0 & 0 & 0.1276 & 0 & 0.3011 & 0.2125 & 0.0597 \\ 0 & 0 & 0.1163 & 0.2063 & 0.1772 & 0.0446 & 0.0232 & 0.0209 & 0.1158 & 0.0127 \\ 0 & 0 & 0.1228 & 0.1317 & 0 & 0.0566 & 0.1572 & 0.0233 & 0 & 0.2058 \\ 0.0963 & 0.1035 & 0 & 0 & 0.2730 & 0.0082 & 0 & 0.0903 & 0.0810 & 0 \\ 0.0456 & 0.1879 & 0.0176 & 0 & 0.0057 & 0 & 0.1088 & 0.0042 & 0.0421 & 0.0533 \end{bmatrix} \quad (45)$$

Step 10: The final weights of the ITARA method are derived using the values of Ω_{ij} , as given in Eq. (28).

$$\mathcal{N} = \begin{bmatrix} 0.1563 & 0.0918 & 0.1015 & 0.0927 & 0.1232 & 0.0553 & 0.0721 & 0.1259 & 0.0948 & 0.0865 \end{bmatrix} \quad (46)$$

Step 11: The RANCOM matrix is constructed by conducting pairwise comparisons of the criteria positions, as shown below.

$$= \begin{bmatrix} 0.5000 & 1.0000 & 0 & 0 & 1.0000 & 1.0000 & 0 & 1.0000 & 1.0000 & 0 \\ 0 & 0.5000 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 1.0000 & 1.0000 & 0.5000 & 0 & 1.0000 & 1.0000 & 0 & 1.0000 & 1.0000 & 0 \\ 1.0000 & 1.0000 & 1.0000 & 0.5000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0 \\ 0 & 1.0000 & 0 & 0 & 0.5000 & 1.0000 & 0 & 1.0000 & 1.0000 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 0.5000 & 0 & 0 & 1.0000 & 0 \\ 1.0000 & 1.0000 & 1.0000 & 0 & 1.0000 & 1.0000 & 0.5000 & 1.0000 & 1.0000 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 & 1.0000 & 0 & 0.5000 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5000 & 0 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.5000 \end{bmatrix}. \quad (47)$$

Step 12: The summed criteria weights (Ω) are computed using the formula provided in Eq. (30).

$$\begin{bmatrix} 5.5000 & 1.5000 & 6.5000 & 8.5000 & 4.5000 & 2.5000 & 7.5000 & 3.5000 & 0.5000 & 9.5000 \end{bmatrix} \quad (48)$$

Step 13: The subjective weight of the criteria is determined using the formula in Eq. (31).

$$\begin{bmatrix} 0.1100 & 0.0300 & 0.1300 & 0.1700 & 0.0900 & 0.0500 & 0.1500 & 0.0700 & 0.0100 & 0.1900 \end{bmatrix} \quad (49)$$

Step 14: The combined weights, obtained by integrating the objective weights from ITARA and the subjective weights from RANCOM, are computed using Eq. (32), with $\sigma = 0.5$.

$$\mathcal{M}_j = \begin{bmatrix} 0.0666 & 0.0305 & 0.0579 & 0.0657 & 0.0533 & 0.0263 & 0.0555 & 0.0490 & 0.0262 & 0.0691 \end{bmatrix}, \quad (50)$$

The normalized combined criterion weights satisfy $\sum_{j=1}^{10} \mathcal{M}_j = 1$. The resulting normalized weights are given by

$$\mathcal{M} = \begin{bmatrix} 0.1332 & 0.0610 & 0.1158 & 0.1314 & 0.1066 & 0.0526 & 0.1110 & 0.0980 & 0.0524 & 0.1382 \end{bmatrix}. \quad (51)$$

Fig. 2 provides a visual representation of all types of weight comparisons.

5.5 ERUNS Method

Step 15: Eq. (34) is used to normalize the matrix $\lambda = [\mathfrak{V}(\mathfrak{d})_{ij}]_{r \times o}$ to the interval $[v, R^r]$.

$$\begin{bmatrix} 0.1929 & 0.5760 & 0.1951 & 0.3220 & 0.2437 & 0.4577 & 0.9751 & 0.4028 & 0.1983 & 1.000 \\ 1.000 & 0.2395 & 0.4799 & 0.4682 & 0.2146 & 0.4573 & 0.3646 & 0.1929 & 0.2364 & 0.6714 \\ 0.3366 & 1.000 & 1.000 & 0.8856 & 1.000 & 0.6759 & 0.5033 & 1.000 & 0.3367 & 0.3814 \\ 0.1594 & 0.3810 & 0.3269 & 0.2786 & 0.4839 & 0.5783 & 0.8223 & 0.2161 & 0.4932 & 0.2997 \\ 0.2595 & 0.5954 & 0.2235 & 0.2958 & 0.4946 & 1.000 & 1.000 & 0.2847 & 1.000 & 0.3590 \\ 0.2949 & 0.5417 & 0.2311 & 1.000 & 0.9305 & 0.4853 & 0.5012 & 0.3376 & 0.3075 & 0.7668 \end{bmatrix}$$

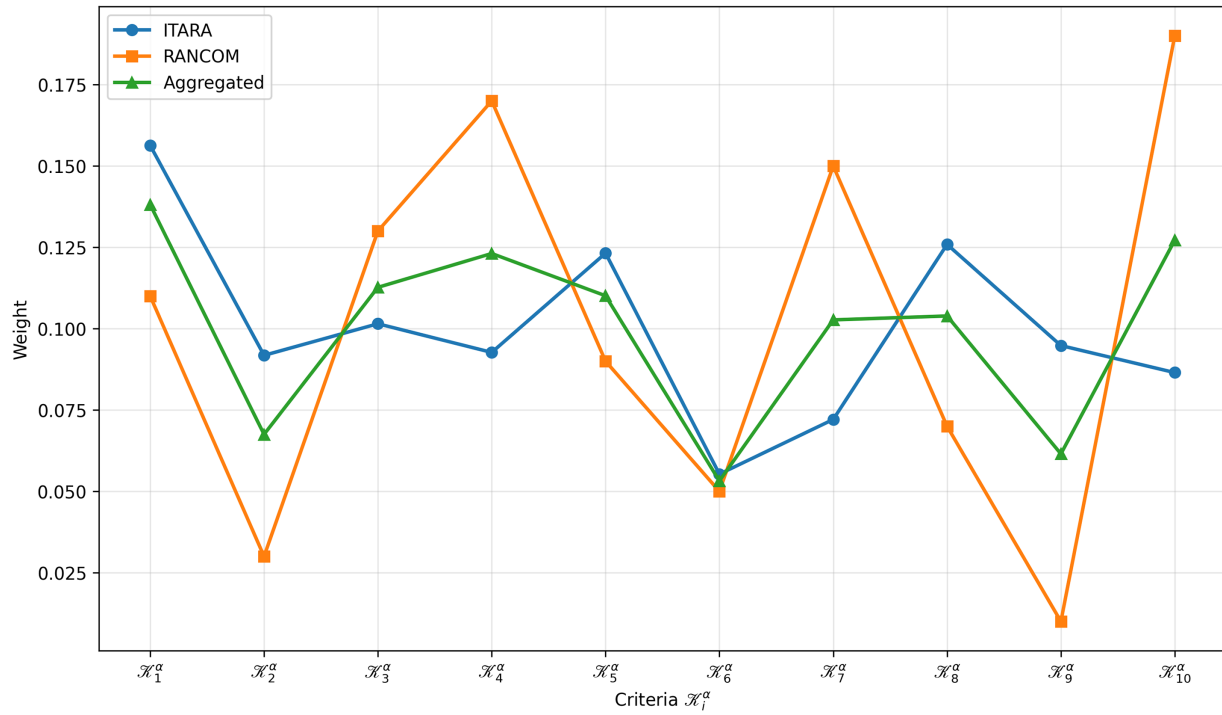


Figure 2: Comparison of weights.

Step 16: The standardized criterion values obtained in Step 15 were transformed according to their preference directions using Eq. (35). Based on the criterion classification used in this study, the cost-type criteria are $\mathcal{K}_1^\alpha$, $\mathcal{K}_2^\alpha$, $\mathcal{K}_6^\alpha$, and $\mathcal{K}_9^\alpha$, whereas the benefit-type criteria are $\mathcal{K}_3^\alpha$, $\mathcal{K}_4^\alpha$, $\mathcal{K}_5^\alpha$, $\mathcal{K}_7^\alpha$, $\mathcal{K}_8^\alpha$, and $\mathcal{K}_{10}^\alpha$. Accordingly, the standardized values associated with the benefit-type criteria were transformed, whereas those associated with the cost-type criteria were retained unchanged. The resulting transformed standardized decision matrix is presented in Table 10.

Table 10: Transformed standardized decision matrix.

$\mathcal{K}_1^\alpha$	$\mathcal{K}_2^\alpha$	$\mathcal{K}_3^\alpha$	$\mathcal{K}_4^\alpha$	$\mathcal{K}_5^\alpha$	$\mathcal{K}_6^\alpha$	$\mathcal{K}_7^\alpha$	$\mathcal{K}_8^\alpha$	$\mathcal{K}_9^\alpha$	$\mathcal{K}_{10}^\alpha$
0.1929	0.5760	1.0000	0.9666	0.9809	0.4577	0.3995	0.8002	0.1983	0.2997
1.0000	0.2395	0.7252	0.8204	1.0000	0.4573	1.0000	1.0000	0.2364	0.6383
0.3366	1.0000	0.1951	0.4029	0.2146	0.6759	0.8713	0.1929	0.3367	0.9283
0.1594	0.3810	0.8782	1.0000	0.7407	0.5783	0.5522	0.9869	0.4932	1.0000
0.2595	0.5954	0.9815	0.9928	0.7300	1.0000	0.3646	0.9183	1.0000	0.9507
0.2949	0.5417	0.9739	0.2786	0.2942	0.4853	0.8733	0.8654	0.3075	0.5430

Step 17: The transformed standardized decision matrix was normalized row-wise to ensure that the criterion values for each alternative are expressed on a common scale. The normalized value corresponding to the i th alternative and j th criterion was calculated using

$$\bar{\delta}_{ij} = \frac{\delta_{ij}}{\sum_{j=1}^{10} \delta_{ij}}, \quad i = 1, 2, \dots, 6, \quad (52)$$

where $\tilde{z}_{ij}$ denotes the transformed standardized value of alternative i under criterion j , and $\tilde{\bar{z}}_{ij}$ denotes the corresponding normalized value. Since the six rows represent the six alternatives and the ten columns represent the ten evaluation criteria, the normalization is performed across the ten criteria within each row. The resulting normalized transformed decision matrix is presented in Table 11.

Table 11: Normalized transformed decision matrix.

$\mathcal{K}_1^\alpha$	$\mathcal{K}_2^\alpha$	$\mathcal{K}_3^\alpha$	$\mathcal{K}_4^\alpha$	$\mathcal{K}_5^\alpha$	$\mathcal{K}_6^\alpha$	$\mathcal{K}_7^\alpha$	$\mathcal{K}_8^\alpha$	$\mathcal{K}_9^\alpha$	$\mathcal{K}_{10}^\alpha$
0.0329	0.0981	0.1703	0.1646	0.1671	0.0779	0.0680	0.1363	0.0338	0.0510
0.1405	0.0337	0.1019	0.1153	0.1405	0.0643	0.1405	0.1405	0.0332	0.0897
0.0653	0.1940	0.0379	0.0782	0.0416	0.1311	0.1690	0.0374	0.0653	0.1801
0.0235	0.0563	0.1297	0.1477	0.1094	0.0854	0.0816	0.1458	0.0729	0.1477
0.0333	0.0764	0.1259	0.1274	0.0937	0.1283	0.0468	0.1178	0.1283	0.1220
0.0540	0.0993	0.1784	0.0510	0.0539	0.0889	0.1600	0.1586	0.0563	0.0995

The values in each row of the normalized transformed decision matrix sum to one, subject to rounding to four decimal places. The resulting matrix is subsequently used in the remaining ERUNS calculations.

Steps 18, 19 and 20: Table 12 represents the usefulness values for the ideal function, the anti-ideal function, and the utility function. The appraisal scores of the alternatives were derived with $R^\tau = 0.5$.

Table 12: Final evaluation matrix.

Alternative	C_3^-	C_3^+	$\Theta(C_3^+)$	$\Theta(C_3^-)$	AS	Ranking
ϑ^{γ_1}	0.530738	0.149321	0.780430	0.219570	0.663751	2
ϑ^{γ_2}	0.596219	0.132922	0.817701	0.182299	0.696359	1
ϑ^{γ_3}	0.450595	0.175879	0.719256	0.280744	0.617416	4
ϑ^{γ_4}	0.449700	0.176229	0.718452	0.281548	0.616857	5
ϑ^{γ_5}	0.487768	0.162475	0.750132	0.249868	0.639815	3
ϑ^{γ_6}	0.390062	0.203174	0.657516	0.342484	0.577677	6

5.6 Sensitivity Analysis

The sensitivity analysis plays a crucial role in decisions that contain aspects of uncertainty or decisions that deal with systems having high levels of complexity and require sensitivity analysis, as it determines the impact that changes in input parameters will have on overall results. This analysis contributes to the verification of the results and identifies significant components, which increases the measures of model reliability, assists in managing risk, and improves decision-making by increasing operational dependability. The general ranking of the alternatives is preserved when varying the model parameters (R^τ) and χ that affect the ranking, as demonstrated in Table 13. Table 14 provides the frequency of the six alternatives' dominance by means of a Monte Carlo Simulation for ten thousand iterations under various scenarios. According to these results, the proposed methodology demonstrates a high level of ranking stability. Thus, there was an optimal solution provided via the methodology; ϑ^{γ_2} occurred in 9884 (out of ten thousand) iterations or approximately 98.84%. This result is consistent with the deterministic ranking $\vartheta^{\gamma_2} > \vartheta^{\gamma_1} > \vartheta^{\gamma_5} > \vartheta^{\gamma_3} > \vartheta^{\gamma_4} > \vartheta^{\gamma_6}$, confirming that the identification of the best alternative remains highly stable under perturbations in the evaluation information. To further examine the robustness of the ITARA weighting procedure, a sensitivity analysis was conducted by varying the indifference-threshold parameter β from

0.1000 to 1.0 by fixing other parameters as follows: $\eta = 2.8927$, $\sigma = 0.6071$, $\chi = 2.4853$, and $R^\tau = 0.7$. The ITARA criterion was then recalculated for each scenario, and the ranking of each criterion and alternative was assessed based on these new weights. The results can be seen in Table 15, with the analysis demonstrating small to moderate changes in the weights when varying beta, resulting in changes in the positioning of closely ranked criteria/alternatives; however, the overall decision structure remained consistent and $\mathcal{V}_2$ retained the first place in all examined scenarios. This suggests that the weights developed using the ITARA criterion responded to the variation of the indifference threshold; however, as the principal decision outcome remained unchanged, this demonstrates an additional measure of the proposed framework's robustness and reliability across different parameters.

Table 13: The effect of parameters R^τ and χ on the outcome of the alternatives.

χ	k	$(\mathcal{V}_1)$	$(\mathcal{V}_2)$	$(\mathcal{V}_3)$	$(\mathcal{V}_4)$	$(\mathcal{V}_5)$	$(\mathcal{V}_6)$	Ranking
$\chi = 10$	$R^\tau = 0.2$	0.3264	0.6433	0.2491	0.1210	0.2583	0.1209	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.4$	0.3297	0.6354	0.2553	0.1237	0.2632	0.1182	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.6$	0.3211	0.6481	0.2438	0.1192	0.2597	0.1225	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.8$	0.3308	0.6397	0.2515	0.1206	0.2614	0.1198	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 1$	0.3249	0.6445	0.2487	0.1221	0.2578	0.1213	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
$\chi = 100$	$R^\tau = 0.2$	0.3283	0.6412	0.2529	0.1203	0.2601	0.1189	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.4$	0.3226	0.6379	0.2461	0.1232	0.2645	0.1205	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.6$	0.3317	0.6458	0.2502	0.1215	0.2593	0.1197	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.8$	0.3274	0.6386	0.2479	0.1209	0.2628	0.1218	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 1$	0.3252	0.6407	0.2494	0.1231	0.2587	0.1184	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
$\chi = 500$	$R^\tau = 0.2$	0.3306	0.6436	0.2475	0.1208	0.2612	0.1199	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.4$	0.3258	0.6362	0.2512	0.1220	0.2624	0.1206	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.6$	0.3289	0.6404	0.2483	0.1216	0.2607	0.1188	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.8$	0.3228	0.6375	0.2506	0.1205	0.2591	0.1222	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 1$	0.3298	0.6467	0.2471	0.1224	0.2634	0.1210	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
$\chi = 1000$	$R^\tau = 0.2$	0.3235	0.6416	0.2517	0.1226	0.2616	0.1215	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.4$	0.3312	0.6384	0.2477	0.1218	0.2604	0.1192	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.6$	0.3262	0.6369	0.2500	0.1201	0.2592	0.1208	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.8$	0.3291	0.6441	0.2465	0.1230	0.2621	0.1185	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 1$	0.3247	0.6422	0.2495	0.1213	0.2581	0.1194	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
$\chi = 2000$	$R^\tau = 0.2$	0.3304	0.6400	0.2504	0.1209	0.2618	0.1220	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.4$	0.3272	0.6367	0.2473	0.1234	0.2599	0.1183	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.6$	0.3255	0.6451	0.2509	0.1222	0.2630	0.1217	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.8$	0.3285	0.6373	0.2482	0.1217	0.2600	0.1200	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 1$	0.3232	0.6440	0.2506	0.1223	0.2573	0.1198	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
$\chi = 5000$	$R^\tau = 0.2$	0.3321	0.6422	0.2521	0.1211	0.2620	0.1207	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.4$	0.3284	0.6395	0.2487	0.1236	0.2608	0.1195	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.6$	0.3267	0.6473	0.2515	0.1223	0.2619	0.1218	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 0.8$	0.3293	0.6387	0.2494	0.1208	0.2602	0.1203	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$
	$R^\tau = 1$	0.3243	0.6452	0.2510	0.1230	0.2598	0.1209	$\mathcal{V}_2 > \mathcal{V}_1 > \mathcal{V}_5 > \mathcal{V}_3 > \mathcal{V}_4 > \mathcal{V}_6$

Table 14: Dominance frequencies of the optimal alternative over 10,000 Monte Carlo simulations.

Weighting Method	Ranking Method	ϑ_1	ϑ_2	ϑ_3	ϑ_4	ϑ_5	ϑ_6
Entropy	MAUT	1785	6087	367	556	799	406
	TOPSIS	1850	6243	327	482	751	347
	VIKOR	969	1662	4769	691	1145	764
	EDAS	1829	6122	382	530	762	375
	MARCOS	1896	6168	348	473	739	376
	CoCoSo	1859	6119	371	498	769	384
	WASPAS	1903	6187	339	478	730	363
	RANCOM-ERUNS	1918	6602	174	403	594	309
CRITIC	MAUT	1919	6263	314	446	699	359
	TOPSIS	1973	6360	290	400	642	335
	VIKOR	1042	1819	4508	657	1177	797
	EDAS	1952	6310	304	421	664	349
	MARCOS	1989	6334	293	405	642	337
	CoCoSo	1961	6290	301	429	673	346
	WASPAS	2011	6356	282	397	632	322
	RANCOM-ERUNS	2009	6808	143	314	461	265
ITARA	MAUT	1302	7781	117	223	372	205
	TOPSIS	1233	7931	108	206	331	191
	VIKOR	1048	4360	2087	719	987	799
	EDAS	1266	7860	113	217	348	196
	MARCOS	1211	7969	100	203	338	179
	CoCoSo	1244	7879	106	221	356	194
	WASPAS	1187	8021	98	193	328	173
	RANCOM-ERUNS (Proposed)	28	9884	6	18	34	30

Table 15: Criteria-weight and alternative-ranking sensitivity for different values of β .

Scenario	β	$\mathcal{N}_1$	$\mathcal{N}_2$	$\mathcal{N}_3$	$\mathcal{N}_4$	$\mathcal{N}_5$	$\mathcal{N}_6$	$\mathcal{N}_7$	$\mathcal{N}_8$	$\mathcal{N}_9$	$\mathcal{N}_{10}$	Criteria Ranking	Alternative Ranking
S ₁	0.10	0.1359	0.0694	0.1154	0.1225	0.1104	0.0514	0.1026	0.1027	0.0642	0.1256	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_3 >$ $\mathcal{N}_5 > \mathcal{N}_8 > \mathcal{N}_7 > \mathcal{N}_2 >$ $\mathcal{N}_9 > \mathcal{N}_6$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_4 >$ $\vartheta''_3 > \vartheta''_6$
S ₂	0.20	0.1374	0.0681	0.1135	0.1229	0.1103	0.0527	0.1027	0.1035	0.0622	0.1267	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_3 >$ $\mathcal{N}_5 > \mathcal{N}_8 > \mathcal{N}_7 > \mathcal{N}_2 >$ $\mathcal{N}_9 > \mathcal{N}_6$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_3 >$ $\vartheta''_4 > \vartheta''_6$
S ₃	0.30	0.1394	0.0664	0.1112	0.1234	0.1099	0.0542	0.1027	0.1047	0.0600	0.1281	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_3 >$ $\mathcal{N}_5 > \mathcal{N}_8 > \mathcal{N}_7 > \mathcal{N}_2 >$ $\mathcal{N}_9 > \mathcal{N}_6$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_3 >$ $\vartheta''_4 > \vartheta''_6$
S ₄	0.40	0.1418	0.0641	0.1085	0.1238	0.1091	0.0561	0.1027	0.1063	0.0577	0.1299	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_5 >$ $\mathcal{N}_3 > \mathcal{N}_8 > \mathcal{N}_7 > \mathcal{N}_2 >$ $\mathcal{N}_9 > \mathcal{N}_6$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_3 >$ $\vartheta''_4 > \vartheta''_6$
S ₅	0.50	0.1448	0.0612	0.1053	0.1241	0.1081	0.0584	0.1026	0.1081	0.0551	0.1323	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_8 >$ $\mathcal{N}_5 > \mathcal{N}_3 > \mathcal{N}_7 > \mathcal{N}_2 >$ $\mathcal{N}_6 > \mathcal{N}_9$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_3 >$ $\vartheta''_4 > \vartheta''_6$
S ₆	0.60	0.1482	0.0576	0.1019	0.1244	0.1065	0.0614	0.1023	0.1103	0.0522	0.1352	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_8 >$ $\mathcal{N}_5 > \mathcal{N}_7 > \mathcal{N}_3 > \mathcal{N}_6 >$ $\mathcal{N}_2 > \mathcal{N}_9$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_3 >$ $\vartheta''_4 > \vartheta''_6$
S ₇	0.70	0.1523	0.0532	0.0980	0.1248	0.1044	0.0654	0.1018	0.1128	0.0485	0.1390	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_8 >$ $\mathcal{N}_5 > \mathcal{N}_7 > \mathcal{N}_3 > \mathcal{N}_6 >$ $\mathcal{N}_2 > \mathcal{N}_9$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_3 >$ $\vartheta''_4 > \vartheta''_6$
S ₈	0.80	0.1575	0.0477	0.0924	0.1254	0.1019	0.0711	0.1011	0.1158	0.0429	0.1441	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_8 >$ $\mathcal{N}_5 > \mathcal{N}_7 > \mathcal{N}_3 > \mathcal{N}_6 >$ $\mathcal{N}_2 > \mathcal{N}_9$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_3 >$ $\vartheta''_4 > \vartheta''_6$
S ₉	0.90	0.1649	0.0402	0.0834	0.1269	0.0987	0.0799	0.1006	0.1200	0.0339	0.1516	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_8 >$ $\mathcal{N}_7 > \mathcal{N}_5 > \mathcal{N}_3 > \mathcal{N}_6 >$ $\mathcal{N}_2 > \mathcal{N}_9$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_5 > \vartheta''_3 >$ $\vartheta''_4 > \vartheta''_6$
S ₁₀	1.00	0.1777	0.0270	0.0671	0.1298	0.0929	0.0955	0.1004	0.1272	0.0178	0.1647	$\mathcal{N}_1 > \mathcal{N}_{10} > \mathcal{N}_4 > \mathcal{N}_8 >$ $\mathcal{N}_7 > \mathcal{N}_6 > \mathcal{N}_5 > \mathcal{N}_3 >$ $\mathcal{N}_2 > \mathcal{N}_9$	$\vartheta''_2 > \vartheta''_1 > \vartheta''_3 > \vartheta''_5 >$ $\vartheta''_4 > \vartheta''_6$

The visual depiction of the results is presented using multiple graphs, as seen in Figs. 3–5.

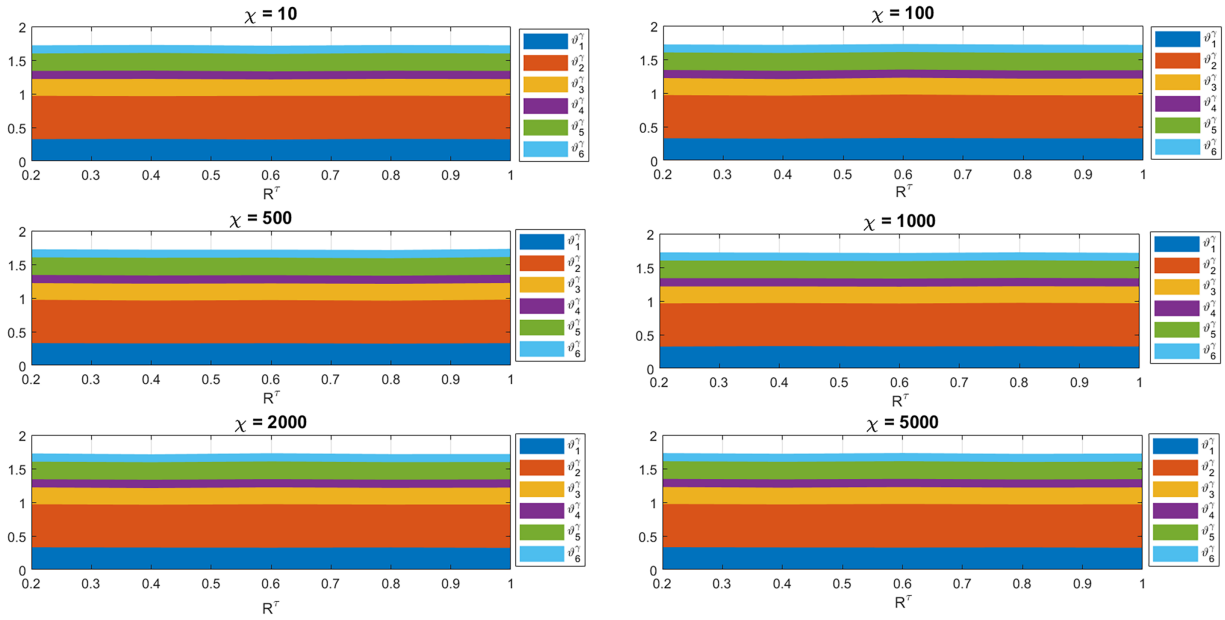


Figure 3: Area graph representation of alternative rankings across R^τ for each χ .

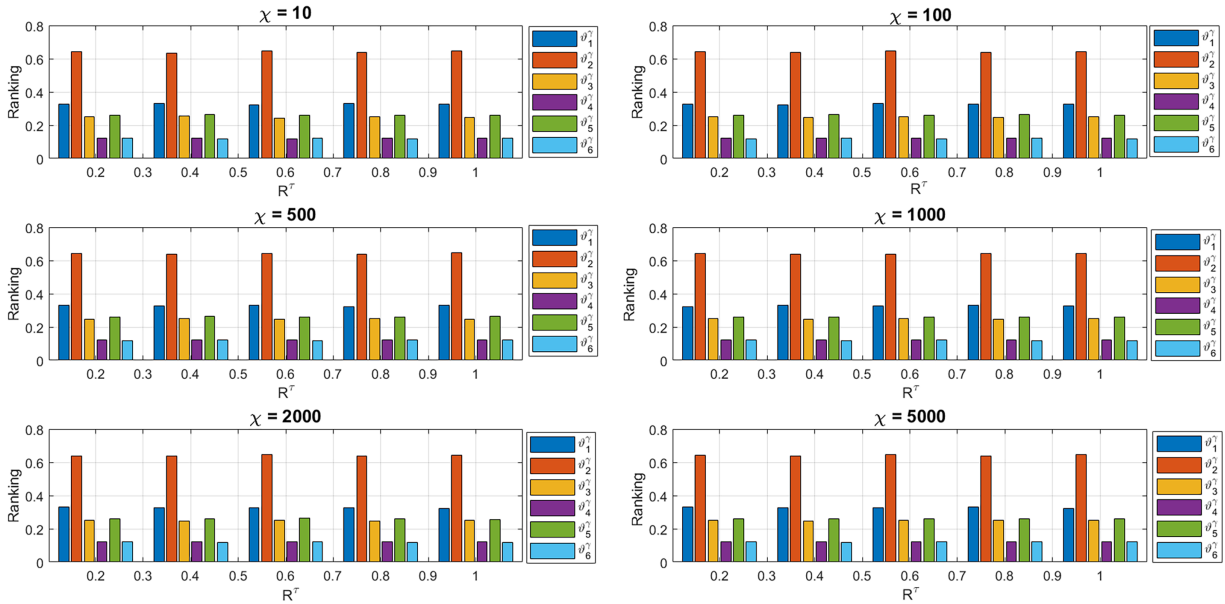


Figure 4: Bar graph representation of alternative rankings across R^τ for each χ .

5.7 Comparative Analysis

A comparative analysis of the proposed methodology against existing methods is performed to demonstrate its capabilities. As exhibited in Table 16, it is clear that all methods give close rankings to the alternatives, with ϑ_2^τ ranking as the most preferred alternative across all the methodologies. The findings indicate that the alternatives are ranked as $\vartheta_2^\tau > \vartheta_1^\tau > \vartheta_5^\tau > \vartheta_3^\tau > \vartheta_4^\tau > \vartheta_6^\tau$, with ϑ_2^τ identified as the

optimal alternative. In particular, the proposed method, which combines ITARA and RANCOM-ERUNS, is also identified ϑ^{γ}_2 as the most preferred, similar to other methods including CRITIC-MARCOS, ELECTRE, D-CRITIC, CPT-CoCoSo, WASPAS and COPRAS. These outcomes show that the ranking of the alternatives under the proposed framework is consistent with the results generated from different established methods. Furthermore, ϑ^{γ}_2 remains the best alternative in all analyses, further validating the effectiveness of the decision-making model.

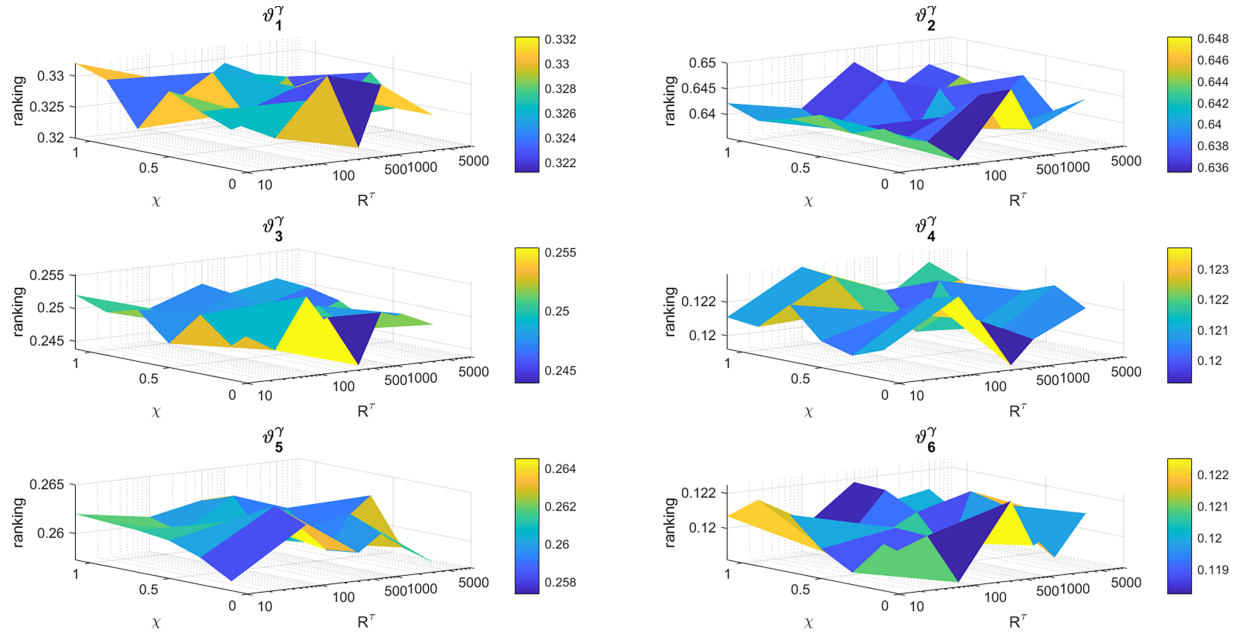


Figure 5: Behavior of each alternative across R^{τ} and χ .

Table 16: Comparison of the newly proposed with already existing.

Authors	Methodology	Ranking of Alternatives	Optimal Alternative
Akram et al. [45]	VIKOR method	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_4 > \vartheta^{\gamma}_6 > \vartheta^{\gamma}_3$	ϑ^{γ}_2
Rouyendegh [46]	ELECTRE model	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_4 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_3 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_6$	ϑ^{γ}_2
Senapati and Yager [47]	WPM	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_4 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_3 > \vartheta^{\gamma}_6$	ϑ^{γ}_2
Zhang and Wei [48]	D-CRITIC and CPT-CoCoSo	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_3 > \vartheta^{\gamma}_4 > \vartheta^{\gamma}_6$	ϑ^{γ}_2
Ali [49]	CRITIC-MARCOS	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_3 > \vartheta^{\gamma}_6 > \vartheta^{\gamma}_4$	ϑ^{γ}_2
Keshavarz-Ghorabae et al. [50]	WASPAS	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_3 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_4 > \vartheta^{\gamma}_6$	ϑ^{γ}_2
Stanujkić and Karabašević [51]	WASPAS method	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_3 > \vartheta^{\gamma}_4 > \vartheta^{\gamma}_6$	ϑ^{γ}_2
Kumari and Mishra [52]	COPRAS method	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_6 > \vartheta^{\gamma}_4 > \vartheta^{\gamma}_3$	ϑ^{γ}_2
Proposed	ITARA and RANCOM-ERUNS	$\vartheta^{\gamma}_2 > \vartheta^{\gamma}_1 > \vartheta^{\gamma}_5 > \vartheta^{\gamma}_3 > \vartheta^{\gamma}_4 > \vartheta^{\gamma}_6$	ϑ^{γ}_2

5.8 Marginal Limitations of the Model

While the proposed MCDM framework demonstrates significant potential in evaluating MSWM strategies for smart cities, it is not without certain limitations. The key limitations are outlined below:

- While relying on IFSs to combat uncertainty is a fairly robust solution, it may introduce subjectivity during the determination of membership, non-membership, and hesitation degrees. This procedure

heavily relies on the direction of specialists; therefore, it is not always free from biasing factors within the study.

- The three methods—RANCOM, ITARA and ERUNS—entail extensive computational approaches and procedures. This computational complexity is likely to present some scalability problems, especially in cases where the proposed approach is applied on larger problems which have an expanded parameter space or more extensive systems with many criteria and alternatives.
- Sensitivity of the model depends on the data and its quality that is used in the study. It is especially acutely sensitive to missing or inaccurate data, which raises the importance of data quality and data accuracy.
- The feasibility of the proposed framework is a crucial consideration; however, it does not incorporate dynamic changes in waste generation, population growth, or regulatory changes. This restriction compromises its flexibility over time to incorporate such evolving factors.

Nevertheless, these limitations do not diminish the advantages of the proposed framework, which can serve as a systematically integrated decision-making approach in the field of MSWM for smart cities.

5.9 Discussion

This paper addresses the identified research gaps by highlighting the scarcity of practical approaches to choosing the most suitable MSWM strategies for smart cities. It is argued that the proposed MCDM framework supported by IFS provides a practical and systematic methodology for evaluating waste management options. The framework integrates the approaches RANCOM, ITARA, and ERUNS. While these approaches rely on expert knowledge for decision-making, they also confirm that the assessment is performed from an analytical view. The illustrative example demonstrated the usability of the proposed framework by comparing six waste management alternatives based on several evaluation criteria, including costs, environmental consequences, implementation and technical feasibility. In the RANCOM method, decision criteria were weighted subjectively according to expert judgement, so domain priorities were also included. Similarly, the ITARA method provided the quantification of objective weights for the analyzed criteria by assessing attribute thresholds. Lastly, the ERUNS method integrated the developed weights and applied nonlinear standardization and relative utility measures that enabled the ranking of the available alternatives.

The analysis concluded that smart waste collection (with IoT) was the most effective alternative. It achieved this success because of the possibility of optimizing waste collection operations through the use of real-time data; minimizing the amount of fuel consumed by vehicles and reducing operational costs; improving the cleanliness of urban areas; the scalability of smart waste collection to meet urban needs; and alignment with technological aspects of Smart Cities. Further, IoT will allow for adaptation to new technologies and innovations, providing a “forward-thinking” approach to solving problems. While other alternatives (such as waste-to-energy and recycling) were successful in recovering value from waste streams through resource recovery, they had relatively high operating complexity and initial investments associated with implementation. Therefore, while these alternatives are beneficial for sustainability and contribute to a circular economy, they were considered less viable than smart waste collection technology in the context of a smart city. In contrast, traditional strategies such as landfills with methane capture have environmental implications and take up significant amounts of land space, although they are cost-efficient. The discussion highlights the advantages of the framework in addressing uncertainty. The inclusion of IFS enables the representation of hesitation and imprecision in expert judgment; important characteristics when considering subjective evaluation criteria in real-world applications. However, there are limitations to the model, including increased computational complexity and an increased reliance upon quality data. Therefore, when applying this model to larger data sets or dynamic urban environments, it will likely require

additional computational resources and/or some methodological refinement. Overall, the developed framework provides a robust and flexible tool for the evaluation of MSWM strategies. The results demonstrate how this type of decision support system can provide guidance to policymakers and urban planners to make informed and sustainable decisions regarding MSWM. Future research should include: developing methods to integrate dynamic real-time data into the models; increasing the number of stakeholders involved in the evaluation process; and addressing current computational limitations so that the models can be more scalable in a variety of urban settings.

5.10 Practical Implications for Scientific and Technical Professionals

Practical applications of this framework involve many mathematical steps; however, end-users do not have to know all of these details. End-users who work in MSWM will primarily need to determine the appropriate waste management options (alternatives), develop the criteria for evaluating those options, and enter their assessment using the defined linguistic terms. All remaining calculation steps (including converting the entered assessments into intuitionistic fuzzy numbers, determining subjective and objective weightings of each criterion, non-linear normalization of each criterion, and producing an overall ranking of the assessed alternatives) can be done in a variety of environments such as a spreadsheet, a programming environment, or in a computer-based decision support system. For example, from a practical viewpoint, natural scientist users may utilize the resultant weights and ranks of the alternatives to evaluate differences in how environmentally sustainable MSWM alternative(s) perform. Computer science students/graduates can also develop the procedural steps as a decision support tool that they can integrate into a municipality's database(s) or other city-wide information systems. Additionally, operations research students/graduates may utilize the same structural process for solving other multiple criteria type problems but would only need to replace the alternatives, criteria, and evaluation data with new data without modifying the underlying mathematics used. Therefore, the proposed methodology can be utilized as a decision support tool where the user is responsible for defining the problem, entering data, and interpreting results while the procedural calculations occur automatically within the structure.

6 Conclusion

This study presents a comprehensive framework for evaluating MSWM strategies in the context of smart cities, addressing the complex challenges of waste generation, environmental sustainability, and technological integration. By combining subjective and objective decision-making tools, the framework provides a balanced and reliable method for prioritizing waste management alternatives. The use of IFSs effectively handles uncertainties in expert judgements and data variability, enhancing the robustness of the evaluation process. The application of the proposed framework in the illustrative example demonstrated its practicality and adaptability. Through the integration of RANCOM, ITARA, and ERUNS methods, the framework captured both expert knowledge and data-driven insights, enabling a holistic assessment of six MSWM strategies. The results identified smart waste collection systems (IoT-based solutions) as the most suitable alternative for smart cities, excelling in scalability, efficiency, and alignment with technological and sustainability goals. Other strategies, such as waste-to-energy plants and recycling systems, showed strong performance but were less optimal due to higher costs and complexity. While the framework successfully addresses many challenges in MSWM, limitations such as dependency on data quality and computational complexity highlight areas for improvement. Future research should explore dynamic real-time data integration, stakeholder involvement, and expanded applicability to diverse urban contexts. This study contributes to the development of sustainable and efficient waste management solutions, providing actionable insights for policymakers and urban planners to optimize MSWM in smart cities.

6.1 Future Directions and Outlook

Future studies will be able to build upon the proposed theoretical model for more dynamic, data-driven, and computationally sophisticated decision-making environments. There is potential for the integration of multiple types of mathematical/operations research models to include stability analysis, stochastic programming, robust and multi-criteria optimization, optimal control theory, and game-theory models that could account for changing conditions and for resources that have multiple constraints. For example, regime-switching stochastic control with memory has the capability to make decisions based on historical data as well as on how the system state has changed over time. Machine learning and statistical approaches hold much promise as well. Optimization-based feature selection and sparse non-linear modeling could assist in identifying key decision variables (criteria), removing redundant variables, and reducing the computational complexity of decision-making. Predictive modeling would allow for this framework to project future waste generation, operational expenses, treatment demand, and environmental impact; similar to current trends in using data-driven approaches in assessing sustainable practices. In addition, future studies could expand the use of the present intuitionistic fuzzy model through even more flexible uncertainty representations and/or more data-driven approaches to decision making. The combination of interval-valued q-rung orthopair fuzzy soft sets with Aczel-Alsina operators provides additional degrees of freedom in both the representation and aggregation of very complex uncertain evaluations. Furthermore, the combination of q-rung orthopair fuzzy rough numbers with data-driven approaches could provide additional degrees of freedom when combining expert evaluations with the patterns found within large datasets. This extension allows the application of the present framework to solve complex urban issues associated with smart cities that contain heterogeneous, incomplete, or extensive amounts of data. The present Monte Carlo study using 10,000 iterations could also be supplemented with bootstrap resampling, Bayesian inference, and extended validation studies outside the confines of other cities/datasets. The proposed framework can be implemented as an automated decision-support tool linked to smart city databases, IoT sensors, GIS systems, and digital twins. With these developments, practitioners would be able to focus their efforts on defining the problem and interpreting results while computational procedures are carried out automatically. Ultimately, the integration of mathematical modeling with machine learning/statistics/optimization and/or real-time data will potentially extend the scope of this framework into fields such as renewable energy, healthcare, transportation/logistics/water management/finance/reliability assessment. As such, there is an opportunity to illustrate the practical applications of mathematics/natural sciences in addressing complex environmental/societal issues.

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