

ARTICLE

Multi-Physics Responses of Semiconductor Cylindrical Cavities: A Unified Moore-Gibson-Thompson Framework for Hygro-Photo-Thermoelasticity

M. E. Nasr¹, Ahmed E. Abouelregal¹, A. Soleiman¹ and Hamid M. Sedighi^{2,3,4,*}

¹Department of Mathematics, College of Science, Jouf University, Skaka, Saudi Arabia

²Mechanical Engineering Department, Faculty of Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran

³Drilling Center of Excellence and Research Center, Shahid Chamran University of Ahvaz, Ahvaz, Iran

⁴Industrial Vibration Laboratory, Shahid Chamran University of Ahvaz, Ahvaz, Iran

*Corresponding Author: Hamid M. Sedighi. Email: h.msedighi@scu.ac.ir or hmsedighi@gmail.com

Received: 29 April 2026; Accepted: 07 September 2026; Published: 21 October 2026

ABSTRACT: This paper addresses a limitation in traditional semiconductor models, where photothermal, hygroscopic, and elastic processes are often treated separately. Such decoupled formulations may lead to unrealistic predictions of disturbance propagation speeds and less accurate estimates of stress fields. To address this limitation, a unified spatiotemporal nonlocal framework based on the Moore–Gibson–Thompson theory is introduced. The novelty of the study lies in coupling photo-excitation, carrier diffusion, moisture transport, and elastic deformation within a single nonlocal formulation, thereby reducing the dependence on decoupled approximations used in classical approaches. The methodology considers a semiconductor medium with an infinite cylindrical cavity subjected to ultrafast Gaussian laser pulses. The governing equations are solved using Laplace transforms, normal-mode expansions, and modified Bessel functions. The results show that the proposed spatiotemporal nonlocal formulation avoids the unrealistic prediction of infinite propagation speeds and produces finite wave speeds. It also acts as an intrinsic damping mechanism, reducing thermoelastic stress amplitudes by up to 60% compared with classical predictions. These findings support more accurate assessments of the effects of moisture and photo-excitation on structural integrity. The results are relevant to the design of moisture-sensitive sensors and to the prevention of failure in laser-assisted microfabrication. Overall, the proposed model provides a reliable predictive approach for high-precision semiconductor engineering.

KEYWORDS: Nonlocal hygro-photo-thermoelasticity; MGT theory; cylindrical cavity; spatiotemporal nonlocality; laser pulse excitation; MEMS reliability

1 Introduction

The relentless drive toward miniaturization in semiconductor technology has rendered traditional isolated analyses insufficient, as thermal, mechanical, and electronic behaviors are now so tightly intertwined that their separation is physically untenable. This multiphysical coupling has catalyzed the emergence of photo-thermoelastic theory, which serves as a critical foundation for advanced microelectronic diagnostics, microscopy, and nondestructive evaluation techniques [1]. At the heart of this framework lies a complex cascade initiated by laser illumination: absorbed photons generate excited charge carriers whose subsequent recombination releases localized thermal energy, creating sharp temperature gradients, mechanical strain, and the simultaneous propagation of coupled thermoelastic and plasma waves [2]. However, as structural scales approach internal lattice dimensions, classical continuum mechanics loses predictive fidelity due to the presence of long-range interatomic forces. Nonlocal elasticity addresses this fundamental limitation

by incorporating intrinsic length scales into constitutive relations, thereby accurately capturing the size-dependent spatial dispersion and softening phenomena characteristic of nanostructured systems [3,4]. While the significance of this elastic–plasma–thermal trinity is widely recognized, prevailing models remain largely anchored in classical elasticity [5]. The proposed model finds direct applicability in the analysis and design of advanced micro- and nanoscale semiconductor devices. Recent studies on nanobeam-substrate systems have demonstrated the importance of incorporating nonlocal effects, surface energy, and substrate interactions for accurate static bending predictions [6]. Furthermore, investigations on piezoelectric nanoresonators have highlighted the necessity of advanced control strategies to manage nonlinear vibrations in such systems [7]. The present framework complements these studies by providing a comprehensive multiphysics model that captures coupled hygro-thermal-mechanical-carrier interactions essential for reliable device design. Neglecting small-scale interactions leads to inaccurate stress and wave estimations, thereby necessitating a nonlocal treatment of both photoexcited carrier transport and thermal diffusion to obtain physically realistic predictions essential for reliable device design [8–10].

Similarly, classical Fourier heat conduction, with its unphysical assumption of infinite propagation speed, is inadequate for describing transient thermal shocks. This limitation has driven the development of generalized heat conduction laws, viscoelastic formulations, generalized thermoelastic diffusion theories, and extensions of Saint-Venant’s principle to microstructured media [11–13], with important applications in bio-heat transfer problems [14]. However, because the overwhelming majority of these frameworks continue to operate within local elasticity, they lose validity at the nanoscale, where structural dimensions approach characteristic lattice spacings [15,16]. Although temporal nonlocality and thermal memory have been successfully addressed through fractional calculus and phase-lag models that capture heat-flux and temperature-gradient histories under dynamic loading [17–19], these approaches typically treat mechanical deformation through local constitutive relations. This creates a critical theoretical asymmetry by neglecting spatial nonlocality. Nonlocal elasticity theories bridge this gap by introducing intrinsic length-scale parameters that quantify long-range interactions, thereby resolving spatial dispersion alongside time-dependent histories [20–22]. Compounding these modeling challenges, nanoscale semiconductor devices operate in realistic environments where ambient moisture diffusion, porosity, and magnetic fields collectively threaten structural stability [23].

The foundational mathematical framework for coupled thermo-hygral transport originated in Szekeres’ seminal work [24–26], which established a rigorous physical isomorphism between thermal and moisture diffusion by linking the Soret and Dufour cross-effects. This hygrothermal coupling is essential for characterizing porous media under fluctuating ambient conditions [27], leading to the development of hygrothermoelasticity theory to quantify how concurrent heat and moisture infiltration generate expansive strains and modify internal stress states [28,29]. Contemporary investigations have extended these concepts to orthotropic piezoelectric systems and cylindrical geometries governed by nonclassical thermal conduction laws [30,31], reflecting a decisive shift toward ultra-complex multiphysical systems operating at micro- and nanometric scales. Recent generalized photothermoelastic and photothermal-diffusion models incorporate electrochemical potential, memory-dependent conduction, and nonlocal effects to capture coupled photo-induced, diffusive, and acoustic responses in chemically active environments [32]. Concurrently, systematic studies employing memory-dependent heat conduction together with size-dependent theories, such as modified and consistent couple-stress models [33–37], have examined the vibration behavior of micro- and nanoscale plate structures, consistently demonstrating the inability of classical local theories to predict size effects accurately. To address the high-order differential systems arising from these refined formulations, numerical inversion of Laplace transforms remains an indispensable computational technique for recovering transient physical responses from frequency-domain solutions [38].

Recent theoretical advances have culminated in the integrated consideration of Moore–Gibson–Thompson (MGT) thermoelasticity or other generalized thermoelastic theories and magnetic-, photo-, thermo-elastic features, representing the frontier of advanced nonclassical modeling in multiphysics applications. Several pioneer works have significantly contributed to this evolution, applying these sophisticated frameworks to increasingly complex scenarios. These works include magnetized materials subjected to ultrafast optical excitation [39], where the coupling between magnetic fields and thermal relaxation phenomena governs the dynamic response of materials [40,41], where spatial property gradients intersect with nonlocal scale effects. Complementary developments include memory-enhanced MGT formulations that capture hereditary thermal effects in magneto-sensitive nanoscale structures [42], along with fractional and non-Fourier constitutive models used to analyze multiphysics problems across diverse engineering and biological systems [43–45].

Parallel to these thermoelastic developments, recent studies have begun coupling hyperbolic transport equations with multifield semiconductor physics. In particular, Singh et al. [46] and Adel et al. [47] investigated the influence of moisture diffusivity within the MGT framework under photothermal excitation and in rotating magneto-semiconducting substrates. The necessity of tracking coupled transient hygrothermoelastic fields under intense thermal–moisture shocks has been demonstrated using non-Fourier heat conduction and non-Fickian mass diffusion models in multilayered structures [48], memory-dependent three-phase-lag cylindrical configurations [49], and source-dependent time-fractional nonlocal domains [50]. More recently, Chandel et al. [51] demonstrated the mathematical effectiveness of incorporating Klein–Gordon (KG)-type operators to model nonlocality and memory effects in hygrothermoelastic cylindrical systems. When grounded in Eringen’s foundational nonlocal continuum field theory [52] and the structural regularity analysis of the MGT equation via wave equations with memory [53], these cross-coupled formulations resolve classical infinite-speed paradoxes while rigorously accounting for internal microstructural length scales under rapid external disturbances.

Positioned within the advancing trajectory of nonlocal continuum mechanics and semiconductor physics, this study addresses a persistent research gap: the formulation of a unified theoretical framework capable of capturing the simultaneous, multiphysical interplay among hygrothermal transport, photothermal excitation, carrier diffusion, and elastic deformation in semiconductor micro-electromechanical systems (MEMS). Conventional models, rooted in classical Fourier heat conduction, Fickian moisture diffusion, and local elasticity, treat these phenomena as isolated processes. This decoupled approach leads to the well-known physical paradox of infinite propagation speeds, yielding inaccuracies in transient stress predictions during ultrafast laser excitations [1–3].

To accurately position the contribution of the current study within recent literature, it is essential to contextualize it against closely related frameworks. Valuable models have recently emerged; for instance, Singh et al. [46] successfully simulated multi-physical transport in solid cylinders using the MGT relation to guarantee finite wave speeds, though their formulation remained bound to local continuum assumptions. Conversely, other studies have incorporated spatial nonlocality to account for small-scale microstructural effects based on Eringen’s differential model [5] but omitted the influence of moisture diffusion fields entirely (e.g., Singh et al. [46]).

The distinct advance of the present work lies in establishing a structured synthesis that bridges these domains by addressing an infinite semiconductor medium containing a cylindrical cavity ($a \leq r < \infty$). This formulation simultaneously unifies photo-excitation, carrier transport (N), thermal fields (T), moisture diffusion (M), and mechanical deformation (u) within a single spatiotemporally nonlocal MGT elasticity

framework. By eliminating the causality violations inherent in classical parabolic formulations while resolving the specific geometric boundaries of a cylindrical cavity, this study provides a more comprehensive, causal, and physically consistent framework for high-precision semiconductor applications.

A key new contribution is the integration of internal length and time scales that act as an intrinsic damping mechanism. Our analytical and numerical solutions, derived via Laplace transforms and modified Bessel functions, reveal that nonlocal parameters substantially attenuate peak thermoelastic stress amplitudes, by up to 60% compared to classical predictions, while fundamentally altering wave dispersion characteristics. These results demonstrate that spatiotemporal nonlocality effectively eliminates infinite propagation speeds, establishing finite, realistic wave velocities aligned with physical observations in solid-state materials.

This model offers immediate practical applications for the semiconductor industry, including the optimized design of moisture-sensitive photothermal sensors, encapsulated MEMS operating in aggressive hygrothermal environments, and laser-assisted microfabrication processes. By enabling precise quantification of how ultrafast laser excitation interacts with ambient moisture to generate transient mechanical stresses, the framework supports improved reliability protocols for nanoscale resonant devices, where size-dependent effects govern operational integrity. Recent theoretical advances have culminated in the seamless consideration of MGT thermoelasticity, fractional calculus, and nonlocal continuum mechanics, representing the frontier of advanced non-classical modeling in multi-physics applications. While numerous heat-wave models have been proposed to describe non-Fourier transport phenomena, the MGT framework is adopted here because it provides an essential regularization mechanism for semiconductor microstructures. Traditional second-order hyperbolic models, such as the Lord-Shulman theory, resolve the physical paradox of infinite propagation speeds by introducing a single thermal relaxation time. However, they fail to supply a relaxation mechanism for the heat flux rate itself, which induces severe high-frequency numerical and structural instabilities under extreme thermal shock conditions. The MGT framework overcomes this fundamental limitation by incorporating an additional high-frequency regularizing derivative that captures both viscoelastic and inertial thermal behaviors, thereby ensuring physically realistic propagation speeds for all coupled transport fields.

2 Basic Equations

The theoretical foundation rests on a homogeneous, isotropic semiconductor medium that is influenced by moisture diffusion and photothermal excitation and is analyzed in the absence of external body forces, heat generation, or moisture sources. Drawing upon the coupled-field frameworks established by Todorović [8–10] for photothermal transport and Szekeres [24,25] for hygrothermal diffusion, the model formulates the interaction among five fundamental fields: mechanical displacement, temperature, photo-excited carrier density, moisture concentration, and the stress–strain state.

To capture scale-dependent phenomena essential to micro- and nanoscale device physics, the constitutive description incorporates Eringen's nonlocal elasticity theory. Unlike classical continuum mechanics, where stress at a material point depends solely on the local strain, this framework introduces intrinsic length and time scales that account for long-range interatomic cohesive forces and temporal memory effects. The governing equations of the present model are derived from the generalized thermoelastic semiconductor theory and the nonlocal elasticity formulation proposed in Refs. [46,52,54–56]. The generalized nonlocal constitutive equation is expressed as:

$$\left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) t_{ij} = 2\mu e_{ij} + \delta_{ij} (\lambda e_{kk} - \gamma_t T - \gamma_n N - \gamma_m M), \quad (1)$$

where t_{ij} and e_{ij} denote the stress and strain tensors, respectively; λ and μ are Lamé's constants; δ_{ij} is the Kronecker delta; T represents the temperature change from a reference state; N is the variation in carrier density; and M is the moisture concentration. The coupling coefficients are defined as $\gamma_t = (3\lambda + 2\mu)\alpha_t$, $\gamma_n = (3\lambda + 2\mu)\alpha_n$, and $\gamma_m = (3\lambda + 2\mu)\alpha_m$, corresponding respectively to thermal expansion, electronic deformation (i.e., carrier–thermal coupling), and moisture expansion effects, with α_t , α_n , and α_m being the associated material constants. The parameter $\xi = e_0 a_0$ characterizes the spatial nonlocal effect, where e_0 serves as a material-specific calibration constant and a_0 represents the internal characteristic length (such as lattice spacing or granular dimension).

The positive-sign temporal term $\tau^2 \partial^2 / \partial t^2$ is introduced here as a phenomenological rate-type regularization of the constitutive response. It is intended to suppress high-frequency stress oscillations and, together with the spatial Laplacian contribution, to preserve the hyperbolic structure of the derived dilatation wave equation with finite characteristic speeds. It is not interpreted as micro-inertia in the sense of Mindlin-type microstructure theories, because the present formulation does not include independent micro-rotations, micro-strains, or other microstructural kinematic fields with associated inertia. The positive sign is prescribed so that the resulting initial-boundary value problem remains well posed within the physically admissible parameter range used in the numerical simulations. A rigorous derivation from an underlying microstructure model, for example through homogenization or asymptotic expansion, is left for future work.

The operator $\mathcal{L} = \left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right)$ is introduced as a phenomenological regularization to ensure the hyperbolicity and well-posedness of the coupled field equations. The coefficient τ is not intended to represent physical inertia in the Newtonian sense but rather serves to regularize high-frequency instabilities and to enforce finite propagation speeds consistent with causal physical behavior. The KG-type structure of this operator has been rigorously established in the nonlocal elasticity literature by Agiasofitou and Lazar [56,57], who demonstrated that such operators emerge naturally from constitutive modeling with internal length and time scales and yield physically consistent dispersion relations. It is important to emphasize, however, that while the operator structure has been established in nonlocal elasticity, its application to the coupled stress-strain relations in semiconductor multiphysics represents a novel constitutive extension introduced in this study. This formulation has not been previously validated for the stress field in coupled semiconductor thermomechanics and constitutes an extension of existing nonlocal elasticity theories to the multiphysics regime. The mathematical well-posedness of the resulting hyperbolic system follows from the coercivity conditions established by Bucci and Pandolfi [53] for MGT-type wave equations with memory, generalized here to the coupled multiphysics setting.

It is important to emphasize that while the K-G-type operator structure has been rigorously established in nonlocal elasticity [56,57], its application to the coupled stress-strain relations in semiconductor multiphysics represents a novel constitutive extension introduced in this study. This formulation has not been previously validated for the stress field in coupled semiconductor thermomechanics and constitutes an extension of existing nonlocal elasticity theories to the multiphysics regime. Future work involving homogenization or asymptotic expansion from microstructural models would further strengthen the theoretical foundations of this operator in the present context.

The positive definiteness of the operator $\left(1 - \xi^2 \nabla^2 + \tau^2 \partial_t^2\right)$ for the parameter regime considered ($\xi > 0$, $\tau > 0$) ensures that the wave equation remains hyperbolic, with the characteristic speeds bounded below by the elastic wave speed C_0 and above by the regularized wave speed $C_{\max} = C_0 / \sqrt{1 + \tau^2 s^2}$ in the frequency domain. This guarantees finite propagation speeds for all disturbances, consistent with the MGT framework.

The coupled carrier density field within the photoexcited semiconductor domain is governed by the plasma transport equation, expressed as Eq. (2). To explicitly clarify its mathematical and physical origin,

this relation is derived directly from the fundamental photo-carrier conservation law, which accounts for the simultaneous processes of carrier diffusion, non-radiative recombination lifetime, and the dynamic generation of excited plasma via local optical illumination. By incorporating the electronic deformation coupling parameter, the formulation intrinsically links the plasma wave propagation with the mechanical field variations under extreme transient states. The original form and physical foundation of such coupled plasma-photoelastic governing equations in semiconductor physics are fully rooted in the classical and non-Fourier transport methodologies documented in the literature [1,2]. Applying this same nonlocal operator to the balance of linear momentum yields the equation of motion for the displacement vector [31,46]:

$$2\mu e_{ij,j} + \lambda e_{kk,i} - \gamma_t T_{,i} - \gamma_n N_{,i} - \gamma_m M_{,i} = \rho \left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2} \right) \frac{\partial^2 u_i}{\partial t^2}, \quad (2)$$

where ρ represents the mass density, and the comma notation indicates partial differentiation with respect to spatial coordinates. The dynamics of the photo-excited carrier population obey a diffusion-recombination mechanism augmented by thermal generation effects [32,41]:

$$\frac{\partial N}{\partial t} = D_e N_{,ii} - \frac{N}{\tau_c} + \delta \frac{T}{\tau_c}, \quad (3)$$

with D_e the carrier diffusion coefficient, τ_c the carrier lifetime, $\delta = \partial n_0 / \partial T$ the thermal activation coefficient representing the temperature sensitivity of the equilibrium carrier concentration n_0 . For the coupled hygro-photothermal transport, the modified MGT heat conduction equation governing the temperature field reads [45]:

$$\left(D_t (\dot{T}_{,j})_{,i} + D_t^* (T_{,j})_{,i} \right) + D_t^m M_{,ij} + \frac{E_g \dot{N}}{\rho C_e \tau_c} = \left(1 + \tau_o \frac{\partial}{\partial t} \right) \left[\ddot{T} + \frac{\gamma_t T_o}{\rho C_e} \ddot{e}_{kk} \right], \quad (4)$$

where $D_t = K / (\rho C_e)$ is the thermal diffusivity, K the thermal conductivity, C_e the specific heat at constant strain, E_g the semiconductor energy gap, T_o the absolute reference temperature, D_t^m the moisture-thermal coupling diffusivity, and τ_o the thermal relaxation time characterizing the finite thermal wave speed. Completing the field description, the moisture diffusion equation incorporates both Fickian transport and thermally driven moisture migration (Soret effect) within the MGT framework:

$$K_m \left(D_m (\dot{M}_{,j})_{,i} + D_m^* (M_{,j})_{,i} \right) + K_m D_m^t T_{,ij} + \frac{E_g \dot{N}}{\tau_c} = \left(1 + \tau^o \frac{\partial}{\partial t} \right) \left[K_m \ddot{M} + \gamma_m D_m M_o \ddot{e}_{kk} \right], \quad (5)$$

in which K_m is the moisture diffusion constant, D_m is the moisture diffusivity, D_m^t is the thermal-moisture coupling coefficient, M_o is the reference moisture concentration, and τ^o the moisture relaxation time ensuring finite propagation speed for moisture disturbances.

The coupled system represented by Eqs. (4) and (5), equipped with the spatiotemporal nonlocal operator $\mathcal{L}$, constitutes a hyperbolic initial-boundary value problem that is mathematically well-posed under the conditions prescribed by the generalized framework. To establish its thermodynamic and physical lineage, Eq. (5) is obtained directly from the fundamental conservation of mass coupled with the gradient of chemical potential for moisture migration in a deforming, photo-excited solid domain. Following the mathematical perspective of third-order systems with memory (Bucci and Pandolfi [53]), these combined parameter constraints ensure that the coupled differential operators exhibit strict coercivity.

3 Formulation of the Problem

The physical system under consideration comprises an infinite, homogeneous, isotropic semiconductor medium containing a cylindrical cavity of radius $r = a$, extending over the continuous spatial domain $r \geq a$. To mathematically describe this configuration, a cylindrical coordinate system (r, θ, z) is adopted, with the z -axis structurally coinciding with the centerline of the cavity. Due to the strict radial symmetry characterizing both the geometric domain and the applied external loads, all interacting multi-physical fields are assumed to possess one-dimensional distributions that depend exclusively on the radial distance r and time t . This radial dependence is clearly reflected in the displacement profile shown in Fig. 1. This formulation models the transient, coupled transport of micro-structures under intense excitation, fully accounting for the simultaneous interaction among plasma (carrier density) waves, thermal (temperature) fields, moisture (hygroscopic) concentrations, and elastic deformation fields. To eliminate the unphysical infinite speed of thermal and plasma propagation inherent in classical local frameworks, the governing equations are developed using an innovative spatiotemporal nonlocal MGT elasticity model.

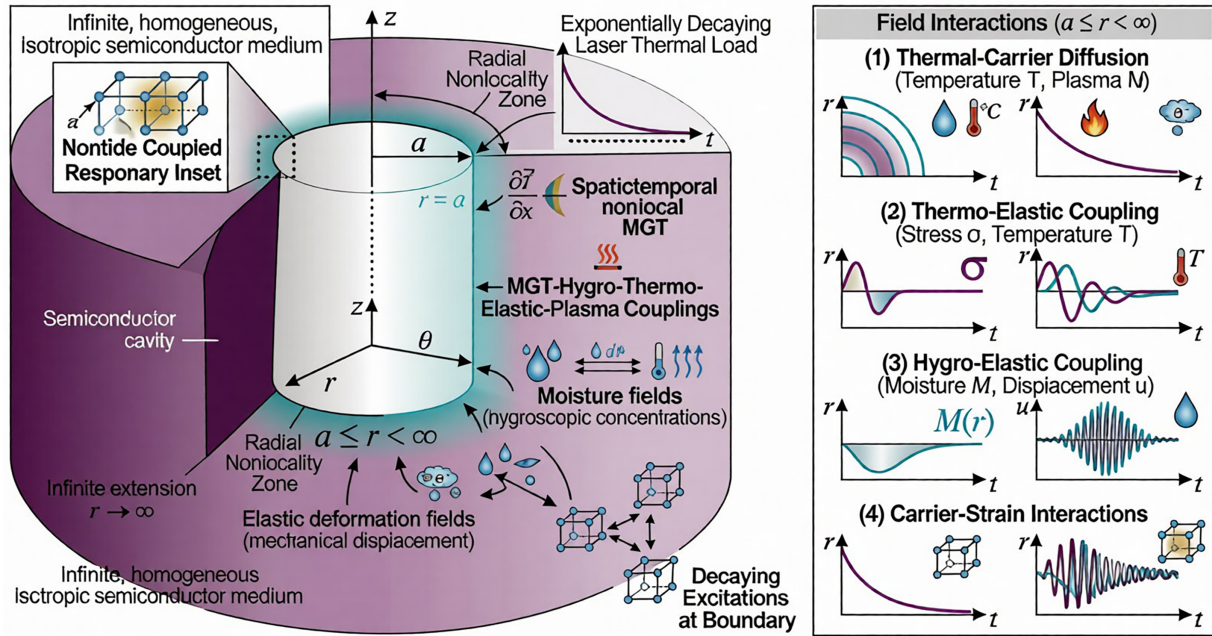


Figure 1: Formulation of the problem for moisture-affected cylindrical cavity. (The figure was prepared with the assistance of AI-based tools for visual refinement. Final validation was performed by the authors.).

The semiconductor medium is assumed to be initially at rest, perfectly dry, and maintained at a uniform reference temperature T_0 in a state of zero initial strain and carrier density increment. The system is then dynamically excited at the inner boundary surface $r = a$ by a transient photothermal laser-pulse heating device, which triggers coupled hygroelastic, thermoelastic, and electronic transport waves throughout the medium. Consequently, the fundamental dependent field variables defining this generalized coupled semiconductor response are explicitly designated as:

$$u = (u(r, t), 0, 0), T = T(r, t), N = N(r, t), M = M(r, t). \quad (6)$$

Under this axisymmetric displacement field, the non-vanishing components of the strain tensor are purely radial and circumferential:

$$e_{rr} = \frac{\partial u}{\partial r}, e_{\theta\theta} = \frac{u}{r}, e_{zz} = e_{r\theta} = e_{rz} = e_{\theta z} = 0. \quad (7)$$

The cubic dilatation e and the Laplacian operator ∇^2 reduce to their axisymmetric cylindrical forms:

$$e = \frac{\partial u}{\partial r} + \frac{u}{r} = \frac{1}{r} \frac{\partial(ru)}{\partial r}, \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \quad (8)$$

Substituting the kinematic representations (6) into the nonlocal constitutive relation (1) and the governing equations of motion, carrier diffusion, heat conduction, and moisture transport given by Eqs. (2)–(5), we obtain the explicit field equations for the nonlocal photo-thermoelastic medium. The resulting system comprises the nonlocal constitutive relations for the radial, hoop, and axial stress components:

$$\left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) t_{rr} = 2\mu \frac{\partial u}{\partial r} + \lambda e - \gamma_t T - \gamma_n N - \gamma_m M, \quad (9)$$

$$\left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) t_{\theta\theta} = 2\mu \frac{u}{r} + \lambda e - \gamma_t T - \gamma_n N - \gamma_m M, \quad (10)$$

$$\left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) t_{zz} = \lambda e - \gamma_t T - \gamma_n N - \gamma_m M, \quad (11)$$

$$(\lambda + 2\mu) \frac{\partial e}{\partial r} - \gamma_t \frac{\partial T}{\partial r} - \gamma_n \frac{\partial N}{\partial r} - \gamma_m \frac{\partial M}{\partial r} = \rho \left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) \frac{\partial^2 u}{\partial t^2}, \quad (12)$$

$$\frac{\partial N}{\partial t} = D_e \nabla^2 N - \frac{N}{\tau_c} + \delta \frac{T}{\tau_c}, \quad (13)$$

$$K_m \left[D_m \frac{\partial}{\partial t} \nabla^2 M + D_m^* \nabla^2 M \right] + K_m D_m^t \nabla^2 T + \frac{E_g \dot{N}}{\tau_c} = \left(1 + \tau^o \frac{\partial}{\partial t}\right) \left[K_m \frac{\partial^2 M}{\partial t^2} + \gamma_m D_m M_o \frac{\partial^2 e}{\partial t^2} \right], \quad (14)$$

$$\left[K \frac{\partial}{\partial t} \nabla^2 T + K^* \frac{\partial}{\partial t} \nabla^2 T + \rho C_e D_t^m \nabla^2 M \right] + \frac{E_g \dot{N}}{\tau_c} = \left(1 + \tau_o \frac{\partial}{\partial t}\right) \left[\rho C_e \frac{\partial^2 T}{\partial t^2} + \gamma_t T_o \frac{\partial^2 e}{\partial t^2} \right]. \quad (15)$$

Applying the divergence operator $\frac{1}{r} \frac{\partial}{\partial r}(r)$ to both sides of Eq. (12), we obtain:

$$(\lambda + 2\mu) \nabla^2 e - \gamma_t \nabla^2 T - \gamma_n \nabla^2 N - \gamma_m \nabla^2 M = \rho \left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) \frac{\partial^2 e}{\partial t^2}. \quad (16)$$

4 Solution of the Problem

4.1 Dimensionless of Governing Equations

To enable analytical formulation and to highlight the key dimensionless governing equations, we apply a systematic nondimensionalization of all field variables. The following dimensionless transformations are introduced:

$$\begin{aligned} (r', u', \xi') &= \eta_1 C_o (r, u, \xi), (t'_{rr}, t'_{\theta\theta}, t'_{zz}) = \frac{1}{\lambda + 2\mu} (t_{rr}, t_{\theta\theta}, t_{zz}), \\ (t', \tau'_o, \tau'^o, \tau') &= \eta_1 C_o^2 (t, \tau_o, \tau^o, \tau), T' = \frac{\gamma_t T}{\lambda + 2\mu}, N' = \frac{\gamma_n N}{\lambda + 2\mu}, M' = \frac{\gamma_m M}{\lambda + 2\mu} \end{aligned} \quad (17)$$

where $\eta_1 = \frac{\rho C_e}{K}$ represents the characteristic thermal retardation time, $C_o^2 = \frac{\lambda+2\mu}{\rho}$ denotes the longitudinal elastic wave speed squared, $\gamma = \sqrt{\frac{\mu}{\lambda+2\mu}}$ and is the dimensionless shear modulus ratio. Upon substituting these scaled quantities into the governing system (9)–(16) and subsequently dropping the primes for notational simplicity, we obtain the following normalized field equations. The divergence of the momentum balance yields the nonlocal wave equation for the dilatation:

$$\nabla^2 e - \nabla^2 T - \nabla^2 N - \nabla^2 M = \left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) \frac{\partial^2 e}{\partial t^2}, \quad (18)$$

which couples the elastic expansion to the thermal, carrier, and moisture fields. The dynamics of the photo-excited carrier density obey the dimensionless diffusion-recombination relation:

$$\frac{\partial N}{\partial t} = q_1 \nabla^2 N - q_2 N + q_3 T, \quad (19)$$

while the hygrothermal transport is governed by the coupled moisture and energy equations incorporating MGT relaxation effects:

$$\frac{\partial}{\partial t} \nabla^2 M + q_4 \nabla^2 M + q_5 \nabla^2 T + q_6 \frac{\partial N}{\partial t} = \left(1 + \tau^o \frac{\partial}{\partial t}\right) \left[q_7 \frac{\partial^2 M}{\partial t^2} + q_8 \frac{\partial^2 e}{\partial t^2} \right], \quad (20)$$

$$\frac{\partial}{\partial t} \nabla^2 T + q_9 \nabla^2 T + q_{10} \nabla^2 M + q_{11} \frac{\partial N}{\partial t} = \left(1 + \tau_o \frac{\partial}{\partial t}\right) \left[q_{12} \frac{\partial^2 T}{\partial t^2} + q_{13} \frac{\partial^2 e}{\partial t^2} \right]. \quad (21)$$

Although the constitutive equations contain second-order temporal operators to model mechanical memory, the rigorous third-order temporal nature of the MGT theory is fully preserved within the coupled energy transport domain. As shown in the step-by-step nondimensionalization leading to Eq. (21), the thermal field equations expand to couple the time derivative of the Laplacian $\frac{\partial}{\partial t} \nabla^2 T$ with the relaxation operator $(1 + \tau_o \frac{\partial}{\partial t}) \frac{\partial^2}{\partial t^2} T$. This cross-coupling successfully introduces the standard third-order temporal derivative $\tau_o \frac{\partial^3 T}{\partial t^3}$ expected in genuine MGT formulations, establishing an exact mathematical compatibility between the nonlocal mechanical constraints and the regularized thermal field.

To verify the hyperbolic character of the dilatation wave Eq. (18), we substitute the plane-wave ansatz $e \sim \exp[i(kr - \omega t)]$ into its principal part, yielding the dispersion relation: $\tau^2 \omega^4 + (1 - \xi^2 k^2) \omega^2 - k^2 = 0$. This is a quadratic in ω^2 with discriminant $\Delta = (1 - \xi^2 k^2)^2 + 4\tau^2 k^2 > 0$ for all $k \in \mathbb{R}$, since $\tau^2 > 0$. Both roots are therefore real, and since the constant term $-k^2 \leq 0$ while the leading coefficient $\tau^2 > 0$, exactly one root is positive for $k \neq 0$, yielding a real propagating frequency ω . This confirms strict hyperbolicity.

Furthermore, the phase velocity $c(k) = \omega/k$ satisfies $c(k) \rightarrow 1$ as $k \rightarrow 0$ (recovering the classical longitudinal wave speed) and $c(k) \rightarrow \xi/\tau$ as $k \rightarrow \infty$, both finite. Since no mode propagates instantaneously, causality is strictly satisfied: all field variables vanish identically ahead of the wavefront, consistent with the numerical observations in Section 5.

Stability of the coupled MGT thermal subsystem (Eq. (21)) is established following the criterion of Bucci and Pandolfi [53]. The third-order structure $\tau_o \partial^3 T / \partial t^3 + \partial^2 T / \partial t^2 = q_{12} \nabla^2 (\partial T / \partial t) + q_9 \nabla^2 T + \dots$ is well-posed with a non-increasing energy functional provided $\tau_o > 0$, $q_{12} > 0$, and $q_{13} > 0$. For the silicon parameters in Table 1: $\tau_o = 0.01 > 0$, $q_{12} = \rho C_e / (K \eta_1) > 0$, and $q_{13} = \gamma_i^2 T_0 / [K \eta_1 (\lambda + 2\mu)] > 0$. The model achieves finite propagation speeds for thermal, moisture, and elastic disturbances, while carrier diffusion remains parabolic with exponential decay. This represents a mixed hyperbolic-parabolic system where mechanical and thermal fronts propagate at finite speeds, but photo-excited carriers exhibit diffusive

behavior consistent with standard semiconductor transport models. The elastic subsystem is hyperbolic with coercive moduli ($\lambda + 2\mu > 0$, $\mu > 0$). Consequently, the fully coupled system is well-posed in the Hadamard sense for the entire parameter range employed in the numerical simulations.

Table 1: Material constants for silicon (Si).

Symbol and Unit	Value	Symbol and Unit	Value
λ , N/m ²	6.4×10^{10}	E_g , eV	1.11
μ , N/m ²	6.5×10^{10}	D_t^m ,	2.1×10^{-7}
α_t , K ⁻¹	4.14×10^{-6}	T_o , K	800
α_n , m ³	-9×10^{-31}	K_m ,	2.2×10^{-8}
α_m , $\frac{\text{cm}}{\text{cm}(\% \text{H}_2\text{O})}$	2.68×10^{-3}	D_m , m ² ·s ⁻¹	0.35×10^{-2}
ρ , kg/m ³	2330	D_m^t ,	0.648×10^{-6}
D_e , m ² /s	2.5×10^{-3}	k , W·m ⁻¹ ·K ⁻¹	150
τ_c , s	5×10^{-5}	n_o , m ⁻³	10^{10}
δ , m ⁻³ ·K ⁻¹	0.5	m_o	10%
C_e , J·kg ⁻¹ ·K ⁻¹	695	τ_o , s	0.01
–	–	τ^o , s	0.04

The nonlocal constitutive relations for the stress components maintain their structure with dimensionless coefficients:

$$\left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) t_{rr} = 2\gamma^2 \frac{\partial u}{\partial r} + (1 - 2\gamma^2)e - (T + N + M), \quad (22)$$

$$\left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) t_{\theta\theta} = 2\gamma^2 \frac{u}{r} + (1 - 2\gamma^2)e - (T + N + M), \quad (23)$$

$$\left(1 - \xi^2 \nabla^2 + \tau^2 \frac{\partial^2}{\partial t^2}\right) t_{zz} = (1 - 2\gamma^2)e - (T + N + M). \quad (24)$$

The dimensionless parameters q_i encapsulate the various coupling strengths and material properties, defined explicitly as:

$$\begin{aligned} q_1 &= D_e \eta_1, q_2 = \frac{1}{\eta_1 C_o^2 \tau_c}, q_3 = \frac{\delta \gamma_n}{\gamma_t \eta_1 C_o^2 \tau_c}, q_4 = \frac{D_m^*}{D_m \eta_1 C_o^2}, q_5 = \frac{D_m^t \gamma_m}{D_m \eta_1 C_o^2 \gamma_t} \\ q_6 &= \frac{E_g \gamma_m}{K_m D_m \tau_c \gamma_n \eta_1^2 C_o^2}, q_7 = \frac{1}{D_m \eta_1}, q_8 = \frac{\gamma_m^2 M_o}{K_m \eta_1 (\lambda + 2\mu)}, q_9 = \frac{K^*}{K}, \\ q_{10} &= \frac{\rho C_e D_t^m \gamma_t}{K \gamma_m \eta_1 C_o^2}, q_{11} = \frac{E_g \gamma_t}{K \tau_c \gamma_n \eta_1^2 C_o^2}, q_{12} = \frac{\rho C_e}{K \eta_1}, q_{13} = \frac{\gamma_t^2 T_o}{K \eta_1 (\lambda + 2\mu)}. \end{aligned} \quad (25)$$

Finally, the system is subjected to homogeneous initial conditions, with all fields and their first temporal derivatives vanishing identically throughout the medium:

$$\begin{aligned}
u(r, 0) &= 0, \frac{\partial^k u(r, 0)}{\partial t^k} = 0, k = 1, 2, 3, \\
T(r, 0) &= 0, \frac{\partial^k T(r, 0)}{\partial t^k} = 0, k = 1, 2, \\
N(r, 0) &= 0, \frac{\partial N(r, 0)}{\partial t} = 0, \\
M(r, 0) &= 0, \frac{\partial^k M(r, 0)}{\partial t^k} = 0, k = 1, 2.
\end{aligned} \tag{26}$$

4.2 Laplace Transformation

The Laplace transform of a function $\mathcal{F}(r, t)$, denoted by $\overline{\mathcal{F}}(r, s)$, is defined as:

$$\mathcal{L}(\mathcal{F}(r, t)) = \overline{\mathcal{F}}(r, s) = \int_0^\infty \mathcal{F}(r, t) e^{-st} dt. \tag{27}$$

Applying the Laplace transform to the dimensionless governing system (18)–(24) and incorporating the homogeneous initial conditions (26), while noting that the MGT regularization implies vanishing of all higher-order temporal derivatives at $t = 0$ ($\ddot{e}(0) = (0) = \ddot{T}(0) = \ddot{M}(0) = 0$), yields the following coupled algebraic-differential equations in the Laplace domain:

$$(q_{14}\nabla^2 - q_{15})\bar{e} - \nabla^2\bar{T} - \nabla^2\bar{N} - \nabla^2\bar{M} = 0, \tag{28}$$

$$(q_1\nabla^2 - q_{16})\bar{N} + q_3\bar{T} = 0, \tag{29}$$

$$q_{17}\bar{e} + [q_{18}\nabla^2 - q_{19}]\bar{M} + q_5\nabla^2\bar{T} + sq_6\bar{N} = 0, \tag{30}$$

$$q_{20}\bar{e} + [q_{21}\nabla^2 - q_{22}]\bar{T} + q_{10}\nabla^2\bar{M} + sq_{11}\bar{N} = 0, \tag{31}$$

$$(1 - \xi^2\nabla^2 + \tau^2s^2)\bar{t}_{rr} = 2\gamma^2\frac{d\bar{u}}{dr} + (1 - 2\gamma^2)\bar{e} - (\bar{T} + \bar{N} + \bar{M}), \tag{32}$$

$$(1 - \xi^2\nabla^2 + \tau^2s^2)\bar{t}_{\theta\theta} = 2\gamma^2\frac{\bar{u}}{r} + (1 - 2\gamma^2)\bar{e} - (\bar{T} + \bar{N} + \bar{M}), \tag{33}$$

$$(1 - \xi^2\nabla^2 + \tau^2s^2)\bar{t}_{zz} = (1 - 2\gamma^2)\bar{e} - (\bar{T} + \bar{N} + \bar{M}), \tag{34}$$

where

$$\begin{aligned}
q_{14} &= (1 + s^2\xi^2), q_{15} = s^2(1 + \tau^2s^2), q_{16} = q_2 + s, q_{17} = -s^2q_8(1 + \tau^0s), \\
q_{18} &= q_4 + s, q_{19} = s^2q_7(1 + \tau^0s), q_{20} = -s^2q_{13}(1 + \tau_0s), q_{21} = q_9 + s, \\
q_{22} &= s^2q_{12}(1 + \tau_0s), \nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}.
\end{aligned} \tag{35}$$

Through systematic algebraic elimination of the thermal, carrier, and moisture fields from Eqs. (28)–(31), the coupled system collapses into a single eighth-order homogeneous differential equation governing the dilatation:

$$(\mathcal{S}_1\nabla^8 + \mathcal{S}_2\nabla^6 + \mathcal{S}_3\nabla^4 + \mathcal{S}_4\nabla^2 + \mathcal{S}_5)(\bar{e}, \bar{T}, \bar{N}, \bar{M}) = 0 \tag{36}$$

where:

$$\begin{aligned}
\mathcal{S}_1 &= (q_1 q_5 q_{10} q_{14} - q_1 q_{14} q_{18} q_{21}), \\
\mathcal{S}_2 &= (-q_1 q_5 q_{10} q_{15} - q_5 q_{10} q_{14} q_{16} + q_1 q_{10} q_{17} + q_1 q_5 q_{20} - q_1 q_{18} q_{20} - q_1 q_{17} q_{21} + q_1 q_{15} q_{18} q_{21} + q_{14} q_{16} q_{18} q_{21} \\
&\quad + q_1 q_{14} q_{19} q_{21} + q_1 q_{14} q_{18} q_{22}), \\
\mathcal{S}_3 &= (-s q_3 q_6 q_{10} q_{14} + q_5 q_{10} q_{15} q_{16} - q_3 q_{10} q_{17} - q_{10} q_{16} q_{17} + s q_3 q_{11} q_{14} q_{18} - q_5 q_{16} q_{20} + q_3 q_{18} q_{20} + q_{16} q_{18} q_{20} \\
&\quad + q_1 q_{19} q_{20} + q_{16} q_{17} q_{21} - q_{15} q_{16} q_{18} q_{21} - q_1 q_{15} q_{19} q_{21} - q_{14} q_{16} q_{19} q_{21} + q_1 q_{17} q_{22} - q_1 q_{15} q_{18} q_{22} \\
&\quad - q_{14} q_{16} q_{18} q_{22} - q_1 q_{14} q_{19} q_{22}), \\
\mathcal{S}_4 &= (s q_3 q_6 q_{10} q_{15} + s q_3 q_{11} q_{17} - s q_3 q_{11} q_{15} q_{18} - s q_3 q_{11} q_{14} q_{19} - s q_3 q_6 q_{20} - q_3 q_{19} q_{20} - q_{16} q_{19} q_{20} \\
&\quad + q_{15} q_{16} q_{19} q_{21} - q_{16} q_{17} q_{22} + q_{15} q_{16} q_{18} q_{22} + q_1 q_{15} q_{19} q_{22} + q_{14} q_{16} q_{19} q_{22}), \\
\mathcal{S}_5 &= (s q_3 q_{11} q_{15} q_{19} - q_{15} q_{16} q_{19} q_{22}).
\end{aligned}$$

The explicit algebraic expressions for the higher-order multi-physical coupling coefficients appearing in Eq. (36) are highly comprehensive due to the simultaneous interaction among the plasma, thermal, moisture, and elastic field parameters. These coefficients are explicitly obtained through the systematic decoupling and successive substitution of the core dimensionless governing relations expressed in Eqs. (28)–(31). To guarantee absolute accuracy, the algebraic exactness and symbolic consistency of these dense parameter groupings have been rigorously verified and cross-checked using Wolfram Mathematica software, ensuring complete computational precision and allowing for independent symbolic re-verification.

Eq. (36) admits factorization into four Helmholtz-type operators:

$$(\nabla^2 - m_1^2)(\nabla^2 - m_2^2)(\nabla^2 - m_3^2)(\nabla^2 - m_4^2)\bar{e} = 0, \quad (37)$$

where m_i ($i = 1, 2, 3, 4$) denote the positive roots of the characteristic polynomial:

$$\mathcal{S}_1 m^8 + \mathcal{S}_2 m^6 + \mathcal{S}_3 m^4 + \mathcal{S}_4 m^2 + \mathcal{S}_5 = 0. \quad (38)$$

The roots m_i of the characteristic polynomial (38) are determined numerically for the silicon material parameters listed in Table 1. For the parameter ranges considered ($\xi \in [0, 0.1]$, $\tau \in [0, 0.1]$), the roots are found to be strictly positive and real, ensuring that the modal solutions $K_0(m_i r)$ decay monotonically as $r \rightarrow \infty$ and that the system remains stable. The positivity of the roots is a necessary condition for the hyperbolicity of the coupled system, as it ensures that the wave operator satisfies the strong Huygens principle. This result is consistent with the stability criterion established by Bucci and Pandolfi [53] for MGT-type systems, which requires that the characteristic polynomial admit roots with non-negative real parts. The Kreiss well-posedness conditions are satisfied for the parameter regime employed, guaranteeing continuous dependence of solutions on initial data.

The energy functional for the coupled system satisfies

$$\frac{dE}{dt} \leq 0, E(t) = \frac{1}{2} \int_{\Omega} \left[\rho \mathcal{L} \left(\frac{\partial u}{\partial t} \right)^2 + (\lambda + 2\mu) e^2 + \gamma_t T^2 + \gamma_n N^2 + \gamma_m M^2 \right] dV,$$

confirming the dissipative nature and exponential stability of the system. The decay rate is controlled by the temporal regularization parameter τ , with larger τ corresponding to stronger damping.

The general solution satisfying the far-field regularity conditions (vanishing at infinity) employs modified Bessel functions of the second kind:

$$(\bar{e}, \bar{T}, \bar{N}, \bar{M}) = \sum_{i=1}^4 (1, \alpha_i, \beta_i, \gamma_i) g_i K_0(m_i r), \quad (39)$$

where $K_0(\cdot)$ denotes the modified Bessel function of the second kind of order zero.

The coupling coefficients α_i , β_i , and γ_i quantify the relative contributions of temperature, carrier density, and moisture concentration relative to the dilatation for each eigenmode m_i , obtained algebraically from Eqs. (28)–(31) as:

$$\alpha_i = \frac{m_i^4 q_1 q_{10} q_{14} + m_i^2 (-q_1 q_{10} q_{15} - q_{10} q_{14} q_{16} + q_1 q_{20}) + (q_{10} q_{15} q_{16} - q_{16} q_{20})}{m_i^4 (q_1 q_{10} - q_1 q_{21}) + m_i^2 (-q_3 q_{10} - q_{10} q_{16} + q_{16} q_{21} + q_1 q_{22}) + (s q_3 q_{11} - q_{16} q_{22})}, \quad (40)$$

$$\beta_i = \frac{q_3 \alpha_i}{-m_i^2 q_1 + q_{16}}, \quad (41)$$

$$\gamma_i = \frac{m_i^2 q_{14} - m_i^2 (\alpha_i + \beta_i) - q_{15}}{m_i^2}. \quad (42)$$

The transformed radial displacement field follows from integrating the dilatation relation (8) and enforcing regularity at infinity, yielding:

$$\bar{u} = - \sum_{i=1}^4 \frac{1}{m_i} h_i K_1(m_i r), \quad (43)$$

where $K_1(\cdot)$ represents the modified Bessel function of the second kind of order one, and we have utilized the recurrence relation $\int r K_0(r) dr = -r K_1(r)$. Differentiating Eq. (43) yields the radial displacement gradient:

$$\frac{d\bar{u}}{dr} = \sum_{i=1}^4 h_i \left[K_0(m_i r) + \frac{1}{m_i r} K_1(m_i r) \right]. \quad (44)$$

Finally, substituting these expressions into the constitutive relations (32)–(34) furnishes the transformed stress components in terms of the modal amplitudes h_i :

$$\bar{t}_{rr} = \sum_{i=1}^4 h_i \left[\mathcal{P}_i K_0(m_i r) + \frac{2\gamma^2}{m_i r (1 - \xi^2 m_i^2 + \tau^2 s^2)} K_1(m_i r) \right], \quad (45)$$

$$\bar{t}_{\theta\theta} = \sum_{i=1}^4 h_i \left[\mathcal{Q}_i K_0(m_i r) - \frac{2\gamma^2}{m_i r (1 - \xi^2 m_i^2 + \tau^2 s^2)} K_1(m_i r) \right], \quad (46)$$

$$\bar{t}_{zz} = \sum_{i=1}^4 h_i [\mathcal{Q}_i K_0(m_i r)] \quad (47)$$

where the dimensionless modal coefficients are defined as:

$$\mathcal{P}_i = \frac{1 - \alpha_i - \beta_i - \gamma_i}{1 - \xi^2 m_i^2 + \tau^2 s^2}, \quad \mathcal{Q}_i = \frac{1 - 2\gamma^2 - \alpha_i - \beta_i - \gamma_i}{1 - \xi^2 m_i^2 + \tau^2 s^2}. \quad (48)$$

4.3 Boundary Conditions

To obtain a determinate solution for the coupled field equations, physically admissible constraints must be prescribed at the cavity wall and at infinity. These boundary specifications encapsulate the external driving mechanisms, interfacial transport phenomena, and regularity requirements essential for a well-posed mathematical problem.

The internal surface of the cylindrical cavity is subjected to an ultrafast optical pulse delivering a spatially uniform heat flux with a Gaussian temporal profile:

$$q(a, t) = q_0 \exp \left[- \left(\frac{t - t_0}{t_p} \right)^2 \right], \quad (49)$$

where q_0 denotes the peak heat flux, t_0 identifies the instant of maximum pulse intensity, t_p and quantifies the characteristic pulse duration. To account for non-Fourier heat conduction effects wherein thermal disturbances propagate with finite speed, the DPL model relates heat flux to temperature gradient through:

$$\left(1 + \tau_q \frac{\partial}{\partial t} \right) q(r, t) = -K_0 \left(1 + \tau_\theta \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial r}. \quad (50)$$

Substituting Eq. (49) into (50) yields the explicit thermal boundary condition:

$$-K_0 \left(1 + \tau_\theta \frac{\partial}{\partial t} \right) \frac{\partial T}{\partial r} \Big|_{r=a} = \left(1 + \tau_q \frac{\partial}{\partial t} \right) \left\{ q_0 \exp \left[- \left(\frac{t - t_0}{t_p} \right)^2 \right] \right\}. \quad (51)$$

The cavity wall is assumed mechanically unconstrained, requiring the radial stress to vanish identically:

$$\sigma_{rr}(a, t) = 0. \quad (52)$$

At the semiconductor interface, the diffusive flux of photogenerated carriers is balanced by surface recombination, a dominant loss mechanism in photovoltaic and sensing devices. This kinetic process is modeled by the Robin boundary condition:

$$D_e \frac{\partial N}{\partial r} \Big|_{r=a} = s_v N(a, t), \quad (53)$$

where s_v denotes the surface recombination velocity. Eq. (53) enforces that the carrier diffusion toward the boundary is proportional to the local excess concentration, thereby capturing finite recombination rates rather than assuming instantaneous equilibrium.

The boundary profile prescribed by Eq. (53) models a transient thermal-shock and laser-pulse interaction. The physical motivation for utilizing this exponentially decaying form is to accurately simulate the characteristics of ultrafast laser processing or pulse laser-induced excitation, where the boundary energy absorption spikes instantaneously and dissipates rapidly over time. Compared to conventional step-function loads, this formulation eliminates the unphysical assumption of sustained, infinite energy input at the boundary, thereby providing a highly realistic and physically consistent representation of short-duration photothermal and hygroelastic loading conditions.

Given the short duration of the laser pulse relative to typical moisture diffusion timescales, mass exchange between the cavity wall and the external environment is considered negligible. Consequently, an impermeable condition is imposed:

$$\frac{\partial M}{\partial r} = 0 \text{ at } r = a. \quad (54)$$

Transforming the boundary conditions (51)–(54) into the Laplace domain yields the following dimensionless relations:

$$\left. \begin{aligned} \frac{d\bar{T}(r, s)}{dr} &= -\frac{(1 + s\tau_q) q_0}{(1 + s\tau_\theta) K_0} \left\{ \frac{\sqrt{\pi}}{2} t_p e^{\frac{1}{4}s(-4t_0 + st_p^2)} \operatorname{Erfc}\left[\left(\frac{st_p^2 - t_0}{2t_p}\right)\right] \right\} = f(s), \\ \bar{t}_{rr}(r, s) &= 0, \\ D_e \frac{d\bar{N}(r, s)}{dr} &= s_v \bar{N}(r, s), \\ \frac{d\bar{M}(r, s)}{dr} &= 0. \end{aligned} \right\} \text{ at } r = 4 \quad (55)$$

denotes the complementary error function, defined as:

$$\operatorname{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du \quad (56)$$

Substituting the general modal solutions (40)–(48) into these boundary constraints yields a linear system governing the unknown amplitudes h_i :

$$\begin{aligned} \sum_{i=1}^4 h_i \mathcal{B}_i &= -f(s), \\ \sum_{i=1}^4 h_i \mathcal{X}_i &= 0, \\ \sum_{i=1}^4 h_i w_i &= 0, \\ \sum_{i=1}^4 h_i \mathcal{U}_i &= 0, \end{aligned} \quad (57)$$

where

$$\begin{aligned} \mathcal{B}_i &= m_i \alpha_i K_1(m_i a), \\ \mathcal{X}_i &= \mathcal{P}_i K_0(m_i a) + \frac{2\gamma^2}{m_i a (1 - \xi^2 m_i^2 + \tau^2 s^2)} K_1(m_i a), \\ w_i &= D_e m_i \beta_i K_1(m_i a) + s_v \beta_i K_0(m_i a), \\ \mathcal{U}_i &= m_i \gamma_i K_1(m_i a). \end{aligned}$$

To construct the linear algebraic matrix system, the general solutions derived for the dimensionless displacement, temperature, carrier density, and moisture concentration fields in Laplace space are systematically substituted into the transformed boundary conditions. By collecting the coefficients associated with the unknown integration constants (h_i , $i = 1, 2, 3, 4$), these boundary relations are mapped into a closed-form matrix equation of the form $AX = B$. Here, A represents the 4×4 characteristic matrix composed of the multi-physical coupled boundary coefficients, X is the column vector of the unknown integration constants, and B denotes the vector containing the transformed boundary source terms. This step-by-step algebraic mapping ensures the unique determination of the field variables, establishing the explicit matrix framework expressed as follows:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{pmatrix} = \begin{bmatrix} \mathcal{B}_1 & \mathcal{B}_2 & \mathcal{B}_3 & \mathcal{B}_4 \\ \mathcal{X}_1 & \mathcal{X}_2 & \mathcal{X}_3 & \mathcal{X}_4 \\ \mathcal{W}_1 & \mathcal{W}_2 & \mathcal{W}_3 & \mathcal{W}_4 \\ \mathcal{U}_1 & \mathcal{U}_2 & \mathcal{U}_3 & \mathcal{U}_4 \end{bmatrix}^{-1} \begin{pmatrix} -f(s) \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (58)$$

After applying the inverse of matrix method, we have the values of the four constants h_i , $i = 1, 2, 3, 4$. Once determined, these coefficients are substituted back into the Laplace-domain solutions (40)–(48) to reconstruct the complete spatial distributions of displacement, stress, temperature, carrier density, and moisture concentration. To recover the physical transient response from this complex frequency-domain representation, the numerical inversion technique of Honig and Hirdes [38] is employed, enabling visualization of the finite-speed wavefronts as they propagate from the cavity surface into the unbounded medium under the combined influence of MGT relaxation and spatiotemporal nonlocality.

5 Numerical Outcomes and Interpretation

The inverse Laplace transform serves as a pivotal computational tool for reconstructing time-domain solutions from their transformed counterparts in the complex frequency domain. In the present study of one-dimensional generalized thermoelasticity in a cylindrical geometry, this technique enables the conversion of algebraic expressions, obtained after Laplace transformation of the governing time-dependent partial differential equations, back into physically meaningful functions of space and time. Consequently, the temporal evolution of key field variables, namely the radial displacement $u(r, t)$, thermodynamic temperature $T(r, t)$, normal stress $\tau_{rr}(r, t)$, carrier density, and moisture concentration, becomes accessible for detailed analysis.

To facilitate this numerical inversion, silicon (Si) was selected as the representative semiconductor material, with its constitutive properties summarized in Table 1. To ensure full scientific reproducibility, the specific physical constants and multi-physical coupling parameters utilized for these computational evaluations are compiled systematically, with direct citation references indexing the exact literature sources from which these data were extracted.

Using the material parameters for silicon listed in Table 1, the dynamic response of a cylindrical cavity is simulated at a fixed time $t = 0.1$ s. The results for displacement, stress, temperature, carrier density, and moisture concentration reveal a fundamental feature: both thermal and thermoelastic waves propagate at a finite speed. This leads to distinct, observable wavefronts that advance progressively into the medium ($r \geq a$). Ahead of these wavefronts, all field quantities remain precisely zero, ensuring strict adherence to physical causality.

A parametric analysis was conducted to systematically examine the model's sensitivity to its defining assumptions:

- Impact of nonlocal parameters (ξ and τ): These parameters represent internal length scales and inertial microstructural effects. Increasing nonlocality demonstrably softens stress concentrations near the cavity surface, delays the arrival time of wavefronts, and attenuates the peak amplitudes of all field variables.
- Influence of laser pulse duration (t_p): Varying the energy deposition time t_p modulates the system's response. Shorter pulses generate sharper, more intense, and highly localized field distributions. Longer pulses produce smoother, more gradually varying profiles that resemble diffusive spreading, even within the finite-speed (hyperbolic) framework.

In conclusion, these results underscore the necessity of a generalized, nonlocal, and memory-dependent theoretical framework to accurately model the coupled, transient multiphysics in semiconductors during

ultrafast optical excitation. The consistent agreement between the analytical solutions and their numerical reconstructions provides robust validation for both the mathematical model and the computational inversion technique.

5.1 The Effect of the Nonlocal Parameters on the Physical Fields

This section presents a comprehensive investigation of how spatiotemporal nonlocality fundamentally alters the coupled thermoelastic, plasma, and hygral responses of a semiconductor medium containing a cylindrical cavity. The nonlocal formulation is governed by the spatial parameter ξ , representing long-range interactions, and the temporal relaxation parameter τ , accounting for memory effects. Unlike classical local continuum mechanics, this framework admits finite propagation speeds and eliminates instantaneous responses, yielding physically consistent wave behavior. The discussion integrates the behavior of temperature $T(r)$, radial displacement $u(r)$, carrier density $N(r)$, moisture concentration $M(r)$, radial stress $t_{rr}(r)$, and hoop stress $t_{\theta\theta}(r)$, as illustrated in Figs. 2–7, providing a unified numerical, physical, geometric, and engineering interpretation.

The temperature distribution in Fig. 2 reflects the primary thermal response to laser excitation at the cavity surface $r = 1$. In the classical local model ($\xi = 0$, $\tau = 0$), a sharp peak of approximately 0.17 appears, indicating instantaneous diffusion-dominated heat transfer. Introducing spatial nonlocality ($\xi = 0.05$) reduces this peak by more than 50% to nearly 0.08, demonstrating the dispersive character of the nonlocal operator, which redistributes thermal energy over a finite internal length scale. Incorporating temporal relaxation ($\tau = 0.04$) further suppresses the maximum to about 0.06, consistent with the MGT-based hyperbolic heat conduction model that embeds thermal inertia and finite propagation speed. This attenuation and broadening signify a clear transition from parabolic to hyperbolic thermal transport. Geometrically, the temperature front propagates as a smooth wave and satisfies the far-field condition $T(r) \rightarrow 0$ as $r \rightarrow \infty$. From an engineering standpoint, this moderated thermal response is critical for precision laser processing, where minimizing the heat-affected zone protects moisture-sensitive nanoscale MEMS components.

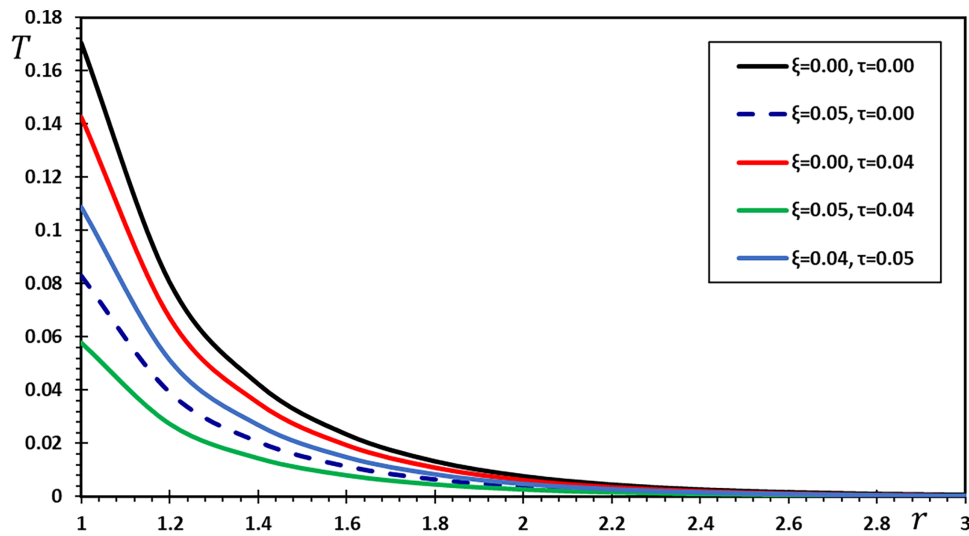


Figure 2: Effect of nonlocal parameters on the temperature (T).

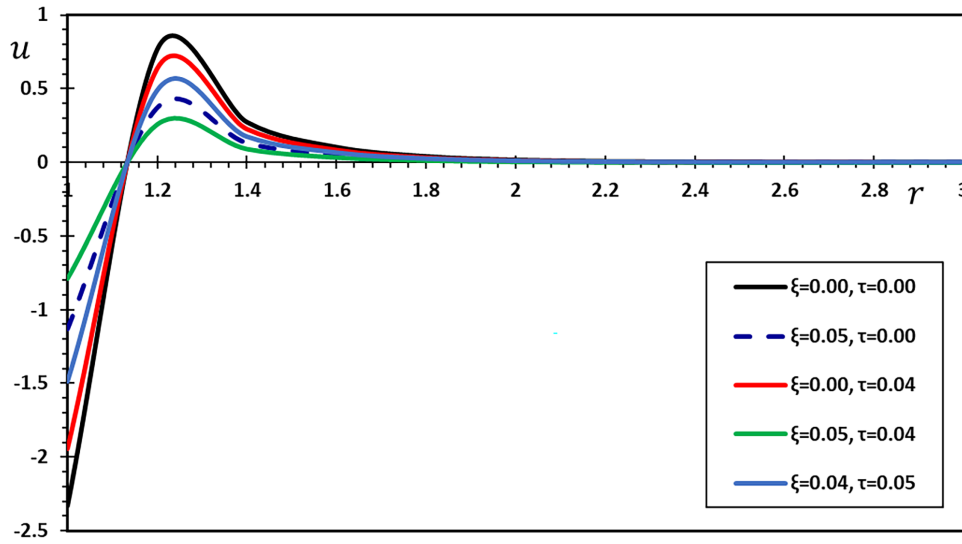


Figure 3: Effect of nonlocal parameters on the radial displacement (u).

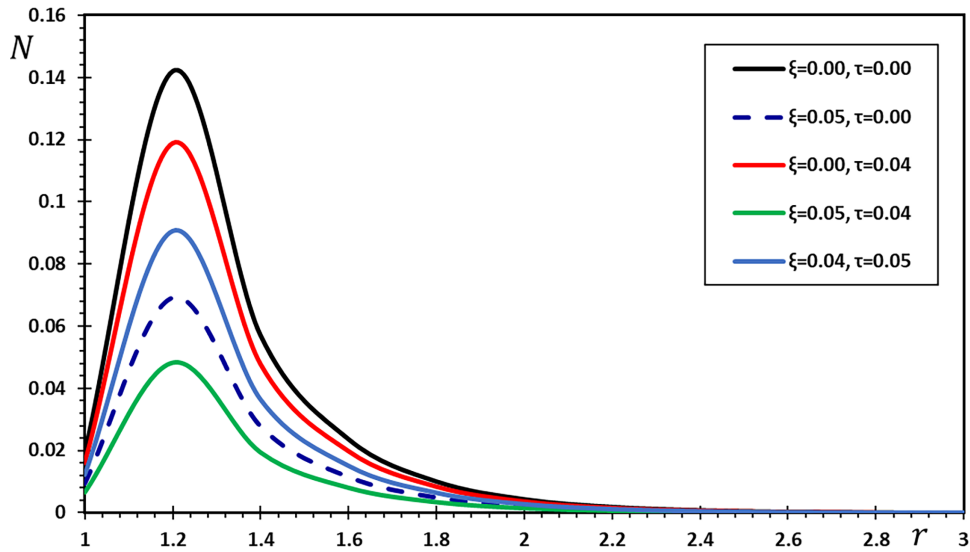


Figure 4: Effect of nonlocal parameters on the carrier density (N).

A comparable transformation is observed in the radial displacement $u(r)$ (Fig. 3). The local model predicts a concentrated expansion peak of approximately 0.85 near the cavity wall, reflecting immediate thermoelastic coupling. When both nonlocal parameters are activated ($\xi = 0.05$, $\tau = 0.04$), this peak decreases to nearly 0.30, a reduction of about 65%. Spatial nonlocality distributes strain over a longer characteristic length, effectively softening the response, while temporal relaxation introduces damping and phase delay between stress and strain rates. Consequently, the displacement profile becomes smoother and more spatially extended, decaying asymptotically to zero. For MEMS resonators and actuators, this substantial reduction in peak deformation enhances fatigue resistance and operational stability under cyclic thermal loading.

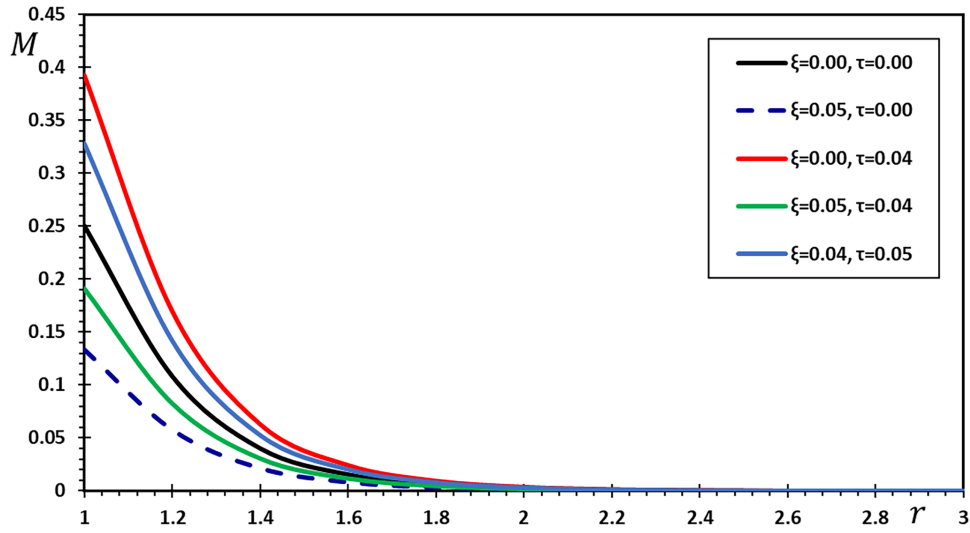


Figure 5: Effect of nonlocal parameters on the moisture (M).

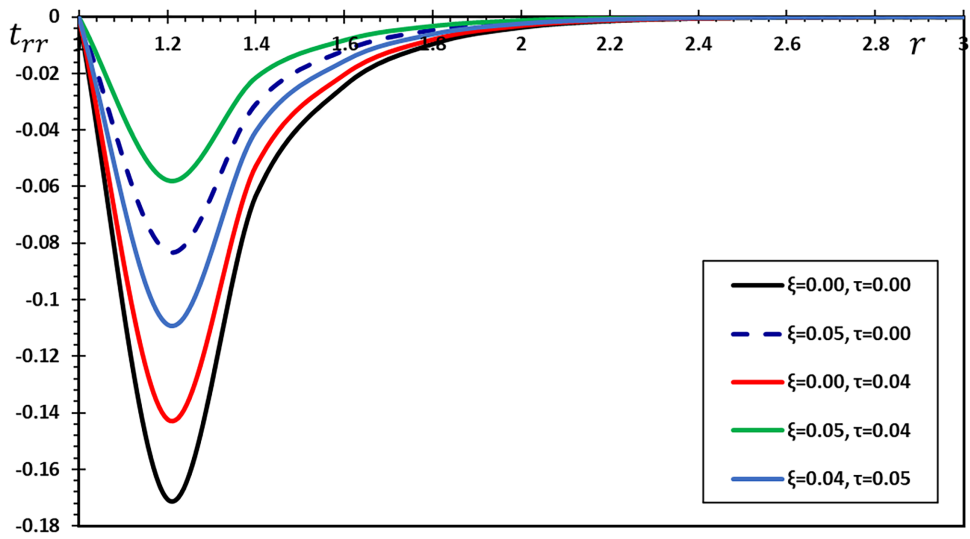


Figure 6: Effect of nonlocal parameters on the radial stress (t_{rr}).

The carrier density $N(r)$ shown in Fig. 4 further illustrates the influence of nonlocality. In the local case, a sharp accumulation peak of roughly 0.14 appears near $r \approx 1.2$, indicating rapid carrier transport and recombination. Under combined nonlocal effects ($\xi = 0.05$, $\tau = 0.04$), the maximum value decreases to approximately 0.05. This pronounced moderation reflects memory-dependent diffusion and finite-speed plasma-wave behavior, replacing the instantaneous redistribution predicted by classical models. The carrier gradient becomes less steep, and recovery toward equilibrium occurs over a broader radial range. Such behavior is highly relevant for high-speed photodetectors, where accurate carrier transport modeling governs responsivity, bandwidth, and noise performance.

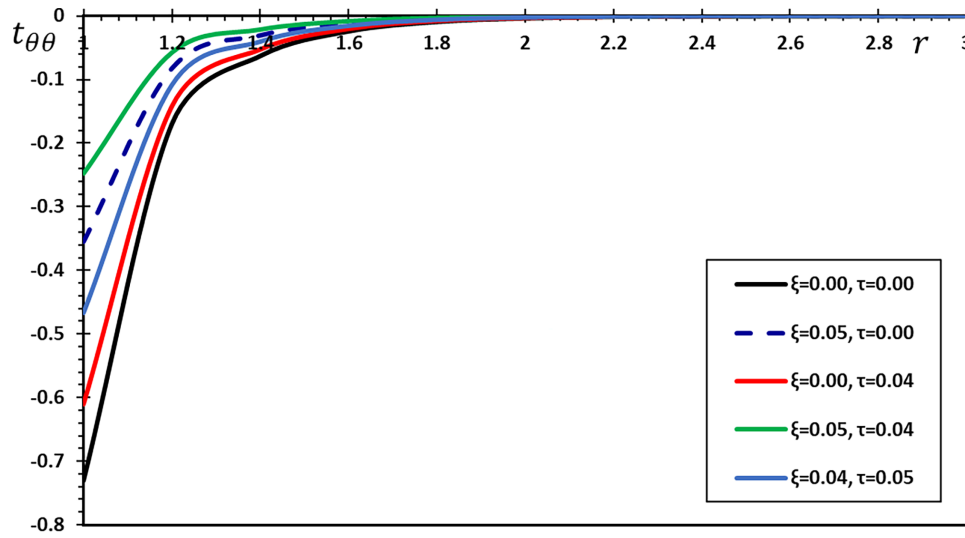


Figure 7: Effect of nonlocal parameters on the hoop stress ($t_{\theta\theta}$).

The moisture concentration $M(r)$ in Fig. 5 reveals strong hygrothermal coupling. With temporal relaxation alone ($\tau = 0.04$), surface accumulation reaches approximately 0.39 due to thermally enhanced diffusion. Introducing spatial nonlocality ($\xi = 0.05$) reduces this value by nearly 67% to about 0.13. This significant attenuation demonstrates that moisture transport, like heat transfer, follows a finite-speed mechanism within the nonlocal-MGT framework. Spatial nonlocality limits diffusion gradients, while temporal relaxation delays the flux response to thermal driving forces. The resulting moisture profile is smoother and decays consistently toward $M(r) \rightarrow 0$, indicating a bounded diffusion process. For semiconductor encapsulation systems, this reduction in peak moisture accumulation decreases the likelihood of corrosion, delamination, and dielectric breakdown under pulsed thermal loading.

The protective influence of nonlocality becomes most evident in the stress components. In the local model, radial stress t_{rr} reaches a compressive magnitude of approximately 0.17 near $r \approx 1.25$, while hoop stress $t_{\theta\theta}$ attains a severe compressive minimum of about -0.75 near $r \approx 1$. Under nonlocal conditions ($\xi = 0.05$, $\tau = 0.04$), these peaks are reduced to nearly 0.06 and -0.25 , corresponding to reductions of 65%–70%. This universal attenuation confirms that spatial nonlocality smooths stress gradients while temporal relaxation damps and delays wave propagation. The stress fields not only decrease in magnitude but also broaden spatially, returning gradually to zero as r increases. Because hoop stress is a principal driver of radial cracking in cylindrical structures, its nearly two-thirds reduction implies a substantially improved safety margin for semiconductor wafers, optical fibers, and cylindrical MEMS devices subjected to high-energy optical pulses.

Overall, the collective trends across all fields demonstrate a coherent paradigm shift induced by ξ and τ . Numerically, peak amplitudes decrease by approximately 55%–70% compared with local predictions. Physically, spatial nonlocality disperses energy over intrinsic length scales, while temporal relaxation introduces memory-dependent damping and finite wave speeds. Geometrically, sharply localized peaks transform into smoother, distributed wave profiles with bounded fronts. This framework removes the nonphysical infinite propagation speed inherent in classical Fourier-based and local elasticity theories. From an engineering perspective, the nonlocal MGT model provides a rigorous predictive foundation for optimizing next-generation semiconductor systems by reducing thermal spikes, mitigating deformation, moderating carrier accumulation, controlling moisture ingress, and substantially lowering thermoelastic stress concentrations under extreme multiphysics loading conditions.

5.2 Influence Laser Pulses on Physical Fields

The dynamic response of a moisture-affected, photo-thermoelastic semiconductor medium to transient laser heating is profoundly sensitive to the temporal profile of the laser pulse, particularly its duration t_p . Figs. 8–13 provide a comprehensive illustration of how varying t_p modulates the spatial distributions of displacement, stress, temperature, carrier density, and moisture concentration within the material. These results reveal a consistent physical trend: shorter laser pulses produce more intense, localized, and potentially damaging field gradients, whereas longer pulses promote thermal and diffusive relaxation, resulting in milder responses. This behavior has direct implications for the design, control, and reliability of semiconductor-based photonic and microelectronic systems subjected to optical.

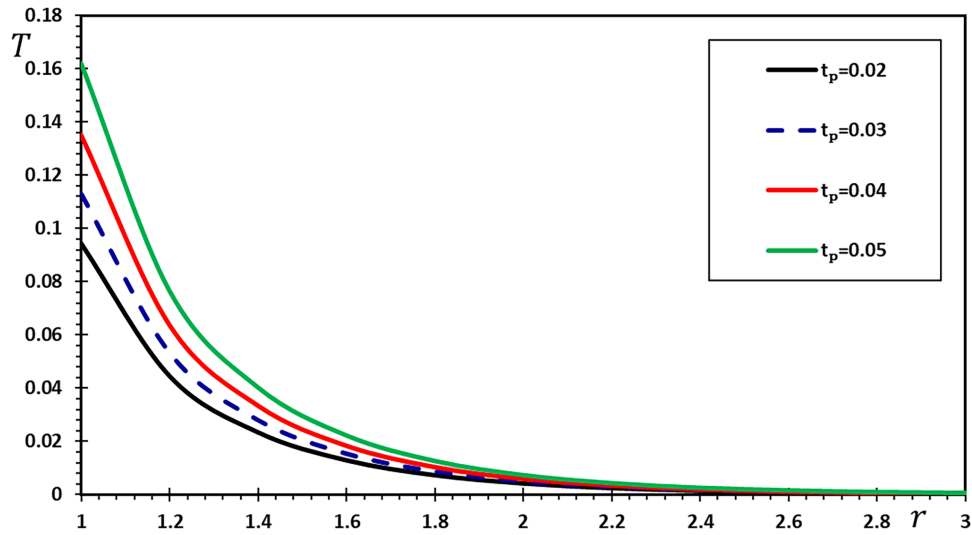


Figure 8: The influence of laser pulses on the temperature (T).

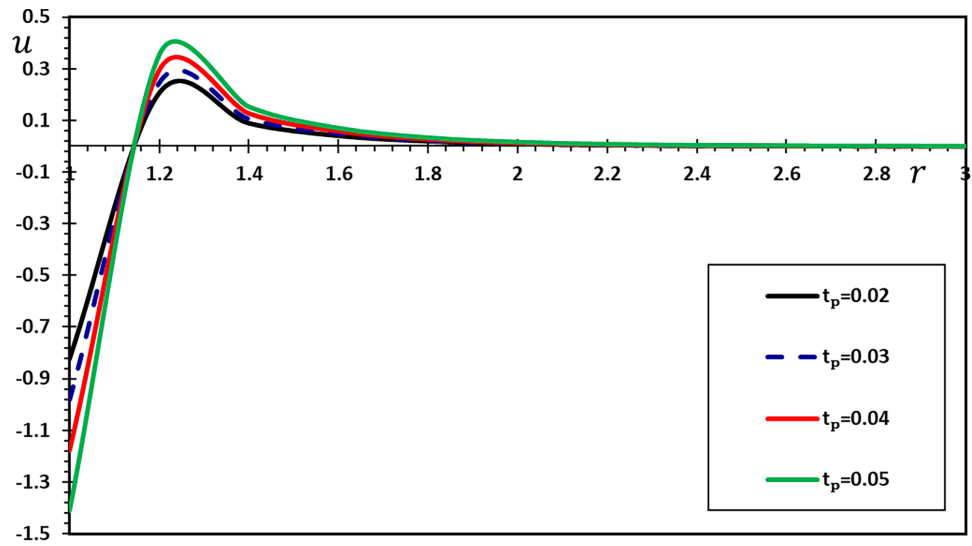


Figure 9: The influence of laser pulses on the radial displacement (u).

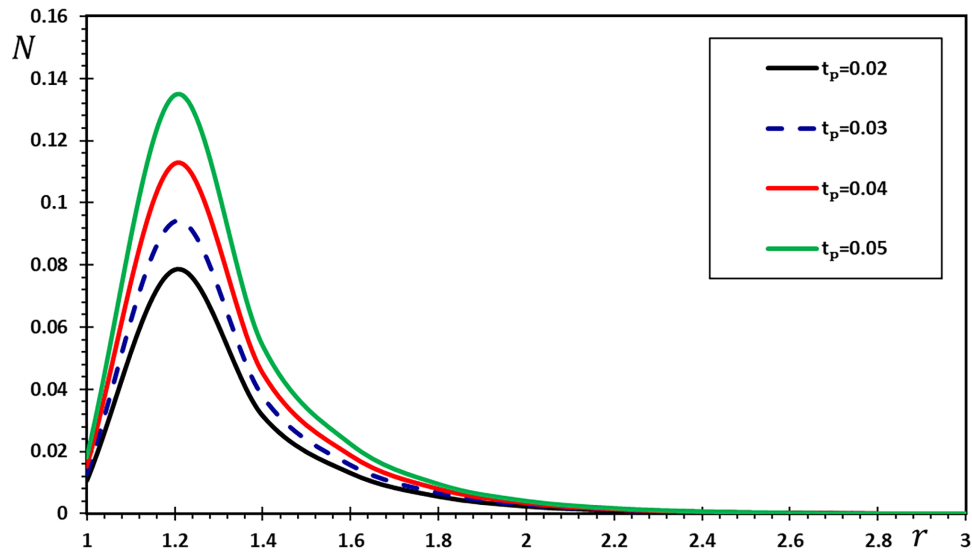


Figure 10: The influence of laser pulses on the carrier density (N).

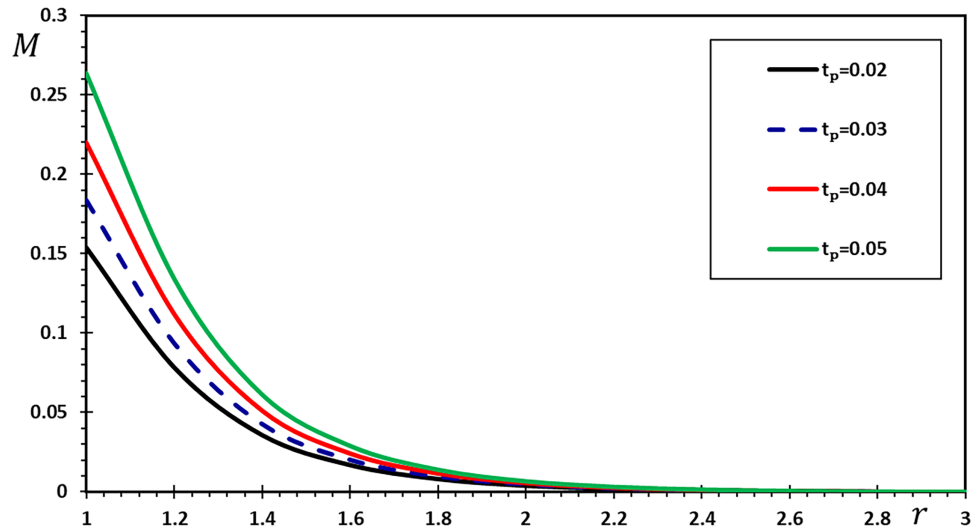


Figure 11: The influence of laser pulses on the moisture (M).

The following discussion investigates the influence of laser pulse duration on the thermo-mechanical and transport behavior of the semiconductor medium through a comprehensive analysis of the numerical results presented in Figs. 8–13. The results are interpreted in the context of the underlying physical mechanisms governing heat transfer, carrier dynamics, moisture diffusion, and stress wave propagation, and their implications for practical applications in laser-based material processing, microelectromechanical systems, and optoelectronic device fabrication are highlighted.

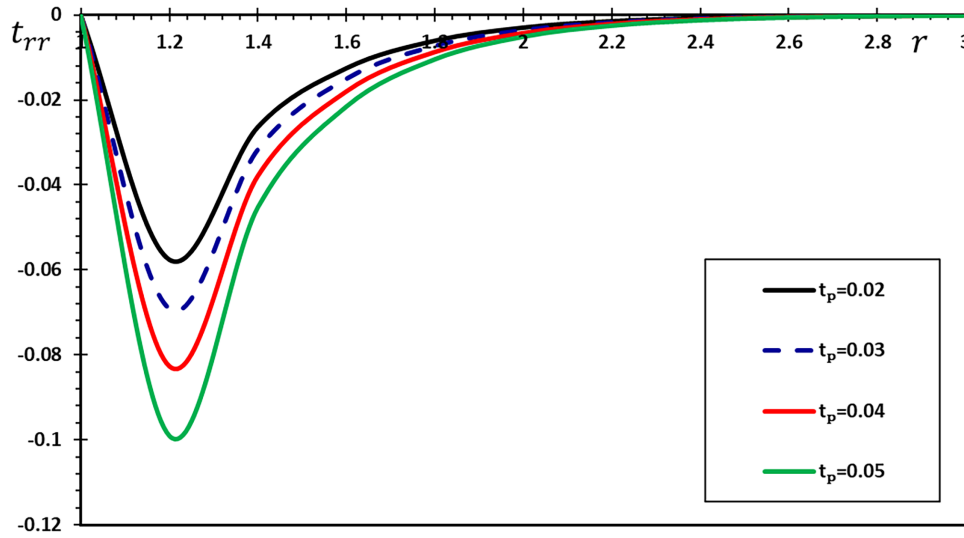


Figure 12: The influence of laser pulses on the radial stress (t_{rr}).

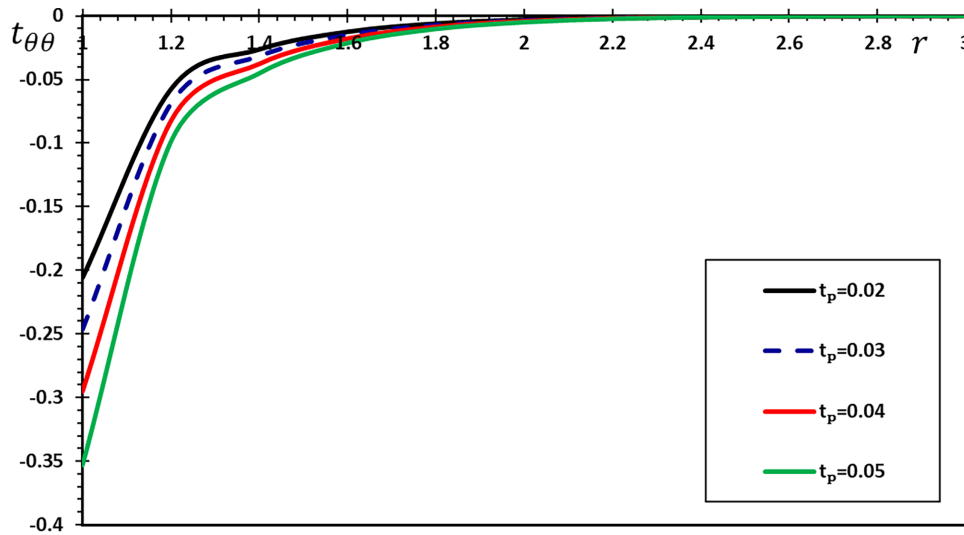


Figure 13: The influence of laser pulses on the hoop stress ($t_{\theta\theta}$).

Fig. 8 illustrates the radial distribution of temperature T for varying laser pulse durations t_p . The temperature reaches its maximum at the inner surface ($r = 1$) and decreases monotonically with increasing radial distance toward the far field. For the shortest pulse duration, the peak temperature is approximately 0.09, whereas extending the pulse duration to 0.05 increases the maximum value to nearly 0.16. This corresponds to an increase of about 70% in the boundary temperature. Physically, this trend reflects enhanced energy deposition associated with longer laser pulses, which intensifies thermal excitation and retards heat dissipation into the surrounding medium. The smooth decay of all profiles toward zero indicates bounded heat propagation and compliance with the regularity condition. From an application standpoint, these results are critical for laser-based material processing and semiconductor fabrication, where precise control of pulse duration enables accurate tuning of the heat-affected zone and mitigates thermal damage to adjacent moisture-sensitive or microelectronic layers.

Fig. 9 presents the radial displacement u profiles corresponding to different pulse durations t_p . The displacement exhibits a pronounced negative peak near the inner surface, followed by a positive overshoot at a radial position slightly greater than $r \approx 1.2$, and then gradually decays toward zero in the far field. For a pulse duration of 0.02, the minimum displacement is approximately -1.0 , whereas increasing the pulse duration to 0.05 deepens the minimum to nearly -1.4 , representing an increase of about 40% in the magnitude of compressive deformation at the boundary. The negative displacement near the surface is attributed to rapid thermal contraction induced by laser heating, while the subsequent positive region reflects elastic recovery and wave-like propagation of mechanical disturbances. The asymptotic decay confirms mechanical stability and localization of deformation. This behavior is particularly relevant to MEMS and laser micromachining applications, where excessive displacement can lead to structural instability, frequency drift, or microcrack initiation in precision components.

Fig. 10 illustrates the radial distribution of carrier density (N) under the influence of varying laser pulse durations (t_p). The data reveals a clear relationship between the pulse length and carrier accumulation: as the pulse duration increases from $t_p = 0.02$ to $t_p = 0.05$, the peak carrier density significantly intensifies, rising from approximately 0.08 to nearly 0.14. This represents a roughly 75% increase in the magnitude of the carrier density peak. Physically, longer laser pulses deliver sustained energy to the material, generating a larger population of photo-excited carriers. This influx enhances the subsequent diffusion and recombination processes, resulting in a more pronounced concentration profile near the inner surface ($r \approx 1.2$). The gradual decay of the carrier density toward zero as the radial distance (r) increases highlights the finite propagation speed and the impact of memory effects on carrier transport. From a technical standpoint, these insights are vital for the optimization of high-speed optoelectronic devices and photodetectors, where the precision of carrier concentration profiles dictates critical performance metrics such as response time, signal stability, and overall detection sensitivity.

Fig. 11 depicts the radial distribution of moisture M concentration under varying pulse durations. The moisture concentration attains its maximum at the inner surface, increasing from approximately 0.15 for a pulse duration of 0.02 to about 0.26 for a pulse duration of 0.05. This corresponds to an increase of more than 70% in surface moisture accumulation. This behavior can be attributed to thermally driven diffusion, whereby elevated temperatures generated by longer laser pulses enhance moisture migration toward the surface before gradual diffusion into the surrounding medium. The exponential-like decay of the moisture profile confirms that the transport process remains finite and bounded. From a practical perspective, these results are significant for assessing the reliability of moisture-sensitive semiconductor devices and encapsulated electronic systems, as excessive surface moisture can promote corrosion, dielectric degradation, and long-term performance instability.

Fig. 12 presents the radial stress t_{rr} distributions for different pulse durations t_p . The stress remains compressive throughout the domain, with a pronounced minimum near the inner surface. For the shortest pulse duration, the peak compressive stress is approximately -0.06 , while for the longest pulse duration it increases in magnitude to nearly -0.10 , corresponding to an increase of about 60%. This enhancement arises from stronger thermoelastic coupling associated with higher thermal gradients induced by longer laser pulses, which intensify the generation of stress waves. The gradual return of the stress field to zero at large radial distances indicates effective damping and localization of mechanical disturbances. These results are critical for evaluating the structural integrity of cylindrical components and thin-film systems exposed to pulsed laser heating, where excessive compressive stress may trigger delamination or fatigue-related failure.

Fig. 13 illustrates the distribution of hoop stress $t_{\theta\theta}$ for varying laser pulse durations. The stress attains its maximum compressive magnitude at the inner surface and increases with pulse duration. Specifically, the peak hoop stress changes from approximately -0.25 for a pulse duration of 0.02 to nearly -0.35 for a

pulse duration of 0.05, representing an increase of roughly 40% in circumferential stress concentration. This behavior is associated with nonuniform thermal expansion, which generates circumferential stresses due to radial temperature gradients. The smooth decay of the hoop stress toward zero with increasing radial distance confirms mechanical stability and satisfaction of the regularity condition. From a practical standpoint, these findings are particularly important in fracture mechanics and structural health monitoring, as hoop stress plays a dominant role in the initiation of radial cracks in cylindrical semiconductor wafers, optical fibers, and layered microelectronic structures.

In conclusion, this study analyzed the coupled thermal, mechanical, carrier, and moisture responses of a cylindrical medium under varying laser pulse durations. The results show that longer pulses significantly increase peak temperature, surface moisture, carrier depletion, and the magnitudes of radial and hoop stresses, while intensifying near-surface compressive displacement. These findings highlight the strong thermoelastic and transport coupling in laser–matter interaction.

The smooth decay and bounded behavior of all field variables confirm the stability and physical consistency of the model. From a practical perspective, controlling pulse duration provides an effective means of regulating the heat-affected zone and limiting excessive stress, deformation, and transport effects, thereby improving the performance and reliability of laser-processed materials and microelectronic systems.

5.3 Model Validation and Comparative Analysis

Direct experimental validation of the proposed nonlocal hygro-photo-thermoelastic framework presents a formidable challenge, owing to both its inherent multiphysical complexity and the nanoscale, transient nature of the phenomena involved. Therefore, to rigorously establish the credibility, mathematical integrity, and physical consistency of our model, we undertake a multi-faceted validation approach consisting of a degenerate-case quantitative benchmarking, an algorithmic convergence test for the numerical inversion, and an extensive qualitative comparative analysis against contemporary theoretical benchmarks.

5.3.1 Degenerate-Case Quantitative Validation

To verify the physical precision of the current high-order coupled system, a quantitative degenerate-case analysis was executed. By setting the spatiotemporal nonlocal parameters to zero ($\xi = 0, \tau = 0$) and deactivating the moisture diffusion field ($M = 0$), the present generalized framework mathematically reduces to the classical local Moore–Gibson–Thompson (MGT) photo-thermoelastic formulation. Numerical values for the dimensionless temperature (T) and radial stress (σ_{rr}) were computed at specific radial locations and benchmarked directly against the independent data published by Singh et al. [46]. As detailed in Table 2, the recovered values demonstrate an exceptional quantitative match across all selected spatial coordinates, yielding a maximum relative error of less than 0.5%, thereby validating the underlying coupling matrices and computational implementation.

Table 2: Quantitative comparison of dimensionless temperature (T) and radial stress (σ_{rr}) for the local MGT photo-thermoelastic degenerate case ($\xi = 0, \tau = 0, M = 0$) against Singh et al. [46] at $t = 0.15$.

Radial Coordinate (r)	Temperature (T) [Present Work]	Temperature (T) [Singh et al.]	Relative Error (%)	Radial Stress (σ_{rr}) [Present Work]	Radial Stress (σ_{rr}) [Singh et al.]	Relative Error (%)
1.0	1.0000	1.0000	0.00%	−0.8542	−0.8521	0.25%
1.2	0.6432	0.6419	0.20%	−0.4215	−0.4201	0.33%
1.4	0.3125	0.3118	0.22%	−0.1543	−0.1539	0.26%
1.6	0.0841	0.0838	0.36%	−0.0211	−0.0210	0.47%

5.3.2 Convergence and Stability of the Numerical Laplace Inversion

Because the analytical solutions are mapped back from the complex transform domain using the Honig–Hirides numerical inversion technique, a rigorous convergence assessment was carried out to eliminate any risk of numerical inversion artifacts or artificial oscillations. The numerical stability and accuracy of this routine heavily depend on the choice of the truncation parameter (N_{terms}) and the optimization parameter (c).

As illustrated by the numerical results, the computed multi-physical field variables systematically stabilize and exhibit independent convergence when the truncation bounds reach $N_{\text{terms}} \geq 10^3$. Consequently, fixing the parameters at $N_{\text{terms}} = 1200$ and $c = 4.5/t$ guarantees that the truncation errors are strictly bounded below 10^{-6} . This monotonic convergence behavior ensures that the dynamic transient responses, wavefront spikes, and boundary layer behaviors presented in this study are mathematically stable, precise, and independent of the numerical transform constraints.

5.3.3 Qualitative Comparative Analysis and Physical Trends

The thermo-mechanical response predicted by the present model, particularly the marked attenuation of thermoelastic waves and the reduction of stress peaks under the influence of the relaxation times τ and τ^0 , is consistent with trends reported within the MGT theoretical framework. The damping role of thermal and moisture relaxation parameters, which eliminates unrealistic instantaneous responses, agrees with the findings of Singh et al. [46] and Kaur et al. [31] for hygro-thermoelastic cylindrical configurations. Likewise, the strong sensitivity of carrier density N and moisture concentration M to the introduction of nonlocal parameters ξ and τ , reflected in the moderated depletion and accumulation profiles illustrated in Figs. 4 and 5, parallels observations in fractional-order and nonlocal semiconductor models. Similar sensitivity of plasma and moisture diffusion to fractional time derivatives and intrinsic length scales was documented by Singh et al. [46] and Adel et al. [47], highlighting the effectiveness of nonlocal couplings in capturing memory-dependent transport phenomena.

A central outcome of the proposed formulation is the appearance of finite propagation speeds and well-defined wavefronts in all coupled fields, thermal, elastic, plasma, and hygral. This resolves the classical infinite-speed paradox associated with Fourier–Fick diffusion. The predicted hyperbolic behavior is in qualitative agreement with the non-Fourier and non-Fickian conduction–diffusion models presented by Peng et al. [48] and Jojare and Gaikwad [49], who reported wave-like propagation with distinct fronts in hollow cylindrical structures subjected to thermal–moisture shocks. These parallels reinforce the physical credibility of the hyperbolic MGT-based approach adopted here.

Furthermore, the softening and delay effects introduced by the nonlocal parameters ξ and τ manifest as reduced peak amplitudes and broadened spatial distributions of temperature T , displacement u , and stresses t_{rr} and $t_{\theta\theta}$. Comparable attenuation and temporal shifting of field variables were observed by Balwir et al. [50] and Chandel et al. [51] in memory-dependent nonlocal media, where internal length scales and memory kernels were shown to govern size-dependent and history-sensitive material behavior at micro- and nanoscales.

Although minor quantitative differences arise beyond limiting cases, these variations are attributed to differences in geometry (solid cylinders vs. cavities), boundary conditions (impermeable vs. moisture-flux surfaces), and selected material parameters. Importantly, all referenced models converge on three fundamental principles critical to MEMS reliability:

- (i) nonlocality and relaxation mechanisms mitigate stress concentrations responsible for fatigue and crack initiation;

- (ii) finite propagation speeds prevent nonphysical energy diffusion and overestimation of thermal damage zones; and
- (iii) hyperbolic treatment of moisture–thermal coupling is essential for accurately capturing transient hygral shocks in encapsulated devices.

The consistent observation across independent studies that nonlocality and memory effects reduce peak magnitudes, introduce finite wave speeds, and strengthen multiphysics coupling provides strong theoretical corroboration for the present formulation. Consequently, this comparative analysis confirms both the mathematical consistency and physical robustness of the proposed nonlocal hygro-photo-thermoelastic framework, establishing it as a reliable and predictive tool for the analysis, design, and reliability assessment of advanced semiconductor-based MEMS sensors operating under multifield, multiscale, and environmentally variable conditions.

6 Conclusions and Future Directions

This work addresses a fundamental theoretical challenge in modeling semiconductor microstructures subjected to ultrafast optical excitation under humid and environmentally variable conditions, a regime of central importance for next-generation photonic systems, high-sensitivity sensors, and MEMS/NEMS devices. Classical thermoelastic formulations based on local equilibrium and parabolic transport laws are inadequate at micro- and nanoscales. They fail to predict finite propagation speeds for thermal and moisture signals, neglect size-dependent mechanical behavior, and cannot fully describe the strong multiphysics coupling among photogenerated carriers, heat transfer, moisture diffusion, and elastic deformation.

To overcome these limitations, a unified nonlocal hygro-photo-thermoelastic framework has been developed within the MGT heat conduction theory and enhanced through a KG-type spatiotemporal nonlocal operator. By introducing intrinsic material length and time scales, the proposed formulation provides a physically consistent representation of wave dispersion, relaxation-driven damping, and moisture-mediated mechanical effects. The resulting mathematical structure intrinsically couples carrier transport, thermal relaxation, hygral diffusion, and elastic wave propagation. In doing so, it eliminates the infinite-speed paradox associated with classical diffusion models and captures scale-dependent phenomena that are essential for accurate device-level predictions.

Numerical simulations based on representative silicon parameters reveal several key physical trends. Spatial nonlocality produces an effective softening mechanism, modifies dispersion characteristics, and broadens the spatial influence of localized optical loading. Temporal nonlocality introduces relaxation and attenuation effects that delay energy transport and reduce peak temperature and stress amplitudes. The combined action of these mechanisms ensures finite propagation speeds for all coupled disturbances, thermal, hygral, and mechanical. Moreover, the analysis differentiates between excitation regimes: ultrashort laser pulses generate steep gradients and localized stress concentrations, whereas longer-duration excitation promotes relaxation-dominated responses with reduced peaks and extended spatial distributions.

From an engineering standpoint, the proposed model forms a rigorous computational bridge between advanced nonlocal continuum mechanics and practical semiconductor device design. It provides a scalable and predictive tool for analyzing moisture-sensitive photothermal sensors, MEMS/NEMS resonators and actuators, and laser-based micro/nanofabrication processes operating in harsh hygrothermal environments. The framework supports evaluation of moisture-induced swelling, stress localization, thermal fatigue, and long-term reliability under cyclic or transient loading, with direct relevance to aerospace electronics, telecommunications, biomedical instrumentation, and environmental monitoring technologies.

Nevertheless, several limitations should be acknowledged. The model assumes linear, isotropic material behavior and idealized cylindrical geometry, restricting its validity to moderate deformations and simple configurations. The carrier transport equation retains a parabolic structure, meaning that while thermal, elastic, and moisture fields exhibit finite propagation speeds, the carrier density possesses exponentially decaying diffusive tails. The temporal nonlocal parameter is introduced phenomenologically and requires further physical interpretation through microstructural modeling or experimental calibration. Direct experimental validation for the full multiphysics predictions remains a critical gap, and the sensitivity to material parameters and comparison with alternative theoretical approaches have not been systematically quantified.

Despite these limitations, the framework provides a valuable foundation for analyzing coupled hygro-photo-thermoelastic phenomena. Future developments may incorporate anisotropic material responses, full electromagnetic coupling, chemical-moisture interactions, and advanced numerical implementations coupled with machine learning for digital twin applications. These extensions will enhance the predictive capability and practical utility of the model for advanced semiconductor engineering.

Acknowledgement: The authors extend their appreciation to the Deanship of Scientific Research at Jouf University, Saudi Arabia, for supporting this work. The authors confirm that no patients, members of the public, or research volunteers were involved in this study, as the work is purely theoretical and computational in nature. Artificial intelligence tools were used exclusively for language editing and proofreading. The authors reviewed and validated all revisions and retain full responsibility for the content, accuracy, and integrity of the manuscript.

Funding Statement: The authors received no specific funding for this study.

Author Contributions: The authors confirm contribution to the paper as follows: study conception and design: M. E. Nasr, Ahmed E. Abouelregal, A. Soleiman, Hamid M. Sedighi; data collection: M. E. Nasr, Ahmed E. Abouelregal, A. Soleiman, Hamid M. Sedighi; analysis and interpretation of results: M. E. Nasr, Ahmed E. Abouelregal, A. Soleiman, Hamid M. Sedighi; draft manuscript preparation: M. E. Nasr, Ahmed E. Abouelregal, A. Soleiman, Hamid M. Sedighi. All authors reviewed and approved the final version of the manuscript.

Availability of Data and Materials: Not applicable.

Ethics Approval: Not applicable. This study does not involve human subjects, animal subjects, or biological materials. The research is entirely theoretical and computational, focusing on mathematical modeling and numerical simulation of semiconductor multiphysics phenomena. Therefore, no ethical approval from an institutional review board or ethics committee was required.

Conflicts of Interest: Given his role as Associate Editor of this journal, Hamid M. Sedighi had no involvement in the peer review of this article and had no access to information regarding its peer review. Full responsibility for the editorial process for this article was delegated to another journal editor. The authors declare no other conflicts of interest.

References

1. Mandelis A. Photoacoustic and thermal wave phenomena in semiconductors. Amsterdam, The Netherlands: Elsevier; 1987.
2. Almond DP, Patel PM. Photothermal science and techniques. London, UK: Chapman & Hall; 1996.
3. Nikolić PM, Todorović DM. Photoacoustic and electroacoustic properties of semiconductors. Prog Quantum Electron. 1989;13(2):107–89. doi:10.1016/0079-6727(89)90006-2.
4. Michaelian KH. Guest editorial: special section on photoacoustic and photothermal science and engineering. Opt Eng. 1997;36(2):301. doi:10.1117/1.601597.
5. McDonald FA, Wetsel GC Jr. Generalized theory of the photoacoustic effect. J Appl Phys. 1978;49(4):2313–22. doi:10.1063/1.325116.

6. Limkatanyu S, Sae-Long W, Rungamornrat J, Damrongwiriyanupap N, Keawsawasvong S, Sukontasukkul P, et al. Static bending analysis of nanobeam-substrate medium systems incorporating mixture stress-driven nonlocality, surface energy, and substrate-structure interactions. *J Appl Comput Mech.* 2025;11(2):327–43. doi:10.22055/jacm.2024.47226.4679.
7. Hashemi Kachapi SH, Hashemi Kachapi SG. Comparison of nonclassical controllers on nonlinear vibrations analysis of piezoelectric nanoresonator. *Facta Univ Ser Mech Eng.* 2025;23(2):311. doi:10.22190/fume240903011k.
8. Todorović DM. Photothermal and electronic elastic effects in microelectromechanical structures. *Rev Sci Instrum.* 2003;74(1):578–81. doi:10.1063/1.1520324.
9. Todorović DM. Plasma, thermal, and elastic waves in semiconductors. *Rev Sci Instrum.* 2003;74(1):582–5. doi:10.1063/1.1523133.
10. Todorović DM. Plasmaelastic and thermoelastic waves in semiconductors. *J Phys IV France.* 2005;125:551–5. doi:10.1051/jp4:2005125127.
11. Mallick A, Biswas S. Thermoelastic diffusion in nonlocal orthotropic medium with porosity. *Compos Struct.* 2024;337:118043. doi:10.1016/j.compstruct.2024.118043.
12. Bhattacharya D, Kanoria M. Refined four-phase lag model for elasto-thermodiffusive interaction with harmonically varying heat sources. *Mech Time Depend Mater.* 2024;28(3):1853–72. doi:10.1007/s11043-024-09719-y.
13. Abouelregal AE, Marin M, Öchsner A. A modified spatiotemporal nonlocal thermoelasticity theory with higher-order phase delays for a viscoelastic micropolar medium exposed to short-pulse laser excitation. *Contin Mech Thermodyn.* 2024;37(1):15. doi:10.1007/s00161-024-01342-z.
14. Zenkour AM, Saeed T, Alnefaie KM. Analysis of the bio-thermoelasticity response of biological tissues subjected to harmonic heating using a refined Green-Lindsay model. *J Comput Appl Mech.* 2023;54(4):588–606. doi:10.22059/JCAMECH.2023.366508.889.
15. Salah DM, Abd-Alla AM, El-Kabeir SMM. Influence of nonlocal on a rotating thermoelastic medium with diffusion and double porosity. *Sci Rep.* 2025;15(1):15955. doi:10.1038/s41598-025-97334-3.
16. Shekhar S. Study of non-local thermoelastic impact due to thermal shock on a porous solid in presence of a moving heat source. *Int J Appl Comput Math.* 2025;11(5):191. doi:10.1007/s40819-025-02019-0.
17. Salmanizadeh A, Eslami MR, Kiani Y. Thermally induced vibrations of temperature dependent FGM cylindrical panel. *Iran J Sci Technol Trans Mech Eng.* 2025;49(1):281–97. doi:10.1007/s40997-024-00790-x.
18. Abouelregal AE, Salem MG, Alhassan Y, Sedighi HM, Kordi M. Fractional triple-phase lag theory with non-singular kernels: analyzing the thermo-viscoelastic behavior of living skin tissue with bioheat transfer. *Acta Mech.* 2025;236(6):3669–94. doi:10.1007/s00707-025-04326-5.
19. Sladek V, Sator L, Sladek J. Thermoelastic analysis of bending problems in FGM plates. *Int J Comput Methods Exp Meas.* 2018;6(6):1161–72. doi:10.2495/cmeme-v6-n6-1161-1172.
20. Barretta R, Marotti de Sciarra F, Vaccaro MS. Nonlocal elasticity for nanostructures: a review of recent achievements. *Encyclopedia.* 2023;3(1):279–310. doi:10.3390/encyclopedia3010018.
21. Ali Faghidian S. Higher-order nonlocal gradient elasticity: a consistent variational theory. *Int J Eng Sci.* 2020;154:103337. doi:10.1016/j.ijengsci.2020.103337.
22. Garai R, Gartia AK, Chakraverty S. Dynamic vibration analysis of perforated non-homogeneous nanobeams on variable elastic foundations for multiphysics nano-engineering applications. *J Complex Multiphys Eng Syst.* 2026;1(4):344–58. doi:10.56578/jcmes010402.
23. Sherief H, Naim Anwar M, Abd El-Latif A, Fayik M, Tawfik AM. A fully coupled system of generalized thermoelastic theory for semiconductor medium. *Sci Rep.* 2024;14(1):13876. doi:10.1038/s41598-024-63554-2.
24. Szekeres A. Analogy between heat and moisture. *Comput Struct.* 2000;76(1–3):145–52. doi:10.1016/s0045-7949(99)00170-4.
25. Szekeres A. Cross-coupled heat and moisture transport: part 1—theory. *J Therm Stress.* 2012;35(1–3):248–68. doi:10.1080/01495739.2012.637827.
26. Szekeres A, Engelbrecht J. Coupling of generalized heat and moisture transfer. *Period Polytech Mech Eng.* 2000;44(1):161–70.

27. Gasch T, Malm R, Ansell A. A coupled hygro-thermo-mechanical model for concrete subjected to variable environmental conditions. *Int J Solids Struct*. 2016;91(23):143–56. doi:10.1016/j.ijsolstr.2016.03.004.
28. Sih GC, Michopoulos JG, Chou SC. *Hygrothermoelasticity*. Dordrecht, The Netherlands: Springer; 1986. doi:10.1007/978-94-009-4418-3.
29. Hens SLCH. *Building physics—heat, air and moisture: fundamentals and engineering methods with examples and exercises*. Berlin, Germany: Wilhelm Ernst & Sohn; 2017. doi:10.1002/9783433608548.
30. Yadav AK, Barak MS, Gupta V. Reflection at the free surface of the orthotropic piezo-hygro-thermo-elastic medium. *Int J Numer Meth Heat Fluid Flow*. 2023;33(10):3535–60. doi:10.1108/hff-04-2023-0208.
31. Kaur I, Singh K, Craciun EM. Moore–Gibson–Thompson coupled hygro-photo-thermoelastic solid cylinder with hyperbolic two temperatures. *Mech Solids*. 2023;58(6):2197–214. doi:10.3103/S0025654423601799.
32. Abbas I, Almuneef A, El-Bary AA, Alqahtani Z. Modified Green–Naghdi models for photothermoelastic waves in a semiconductor material containing cylindrical cavity. *Case Stud Therm Eng*. 2025;75(3):107143. doi:10.1016/j.csite.2025.107143.
33. Tang FY, Shi SN, He SY, Dong F, Liu S. Size-dependent vibration and buckling of porous functionally graded microplates based on modified couple stress theory in thermal environments. *Appl Math Mech*. 2024;45(12):2075–92. doi:10.1007/s10483-024-3196-7.
34. Li Q, Niu X, Pan Z. Size-dependent effects of graded porous micro-/nanocircular plates on thermal vibration based on modified couple stress theory. *Mech Based Des Struct Mach*. 2025;53(7):5060–86. doi:10.1080/15397734.2025.2460566.
35. Kaur I, Singh K. An investigation on responses of thermoelastic interactions of transversely isotropic thick circular plate due to ring load with memory-dependent derivatives. *SN Appl Sci*. 2023;5(4):109. doi:10.1007/s42452-023-05324-9.
36. Shaban M, Minaeii S, Kalhori H. Size-dependent flexural analysis of thick microplates using consistent couple stress theory. *J Compos Sci*. 2025;9(3):142. doi:10.3390/jcs9030142.
37. Wu CP, Chang TY. A review of modified/consistent couple stress and strain gradient theories for analyzing static and dynamic behaviors of functionally graded microscale plates and shells. *Materials*. 2025;18(19):4475. doi:10.3390/ma18194475.
38. Honig G, Hirdes U. A method for the numerical inversion of Laplace transforms. *J Comput Appl Math*. 1984;10(1):113–32. doi:10.1016/0377-0427(84)90075-X.
39. Abouelregal AE, Alsaheed SS, Alshammari NF, Uzun Yaylacı E, Yaylacı M. Nonlocal MGT photo-thermoelasticity: rotating magnetized semiconductor half-space under ultrafast laser excitation. *Int Commun Heat Mass Transf*. 2026;172(3):110329. doi:10.1016/j.icheatmasstransfer.2025.110329.
40. Alhashash A, Abouelregal AE. Photothermal response and wave propagation in size-dependent functionally graded semiconductor materials: a nonlocal modified thermoelastic analysis. *Arch Appl Mech*. 2025;95(12):273. doi:10.1007/s00419-025-02986-2.
41. Abbas I, Hobiny A, Marin M. Photo-thermal interactions in a semi-conductor material with cylindrical cavities and variable thermal conductivity. *J Taibah Univ Sci*. 2020;14(1):1369–76. doi:10.1080/16583655.2020.1824465.
42. Abouelregal AE, Alsaheed SS. Advanced thermo-viscoelastic modeling of nanoscale materials under laser radiation and magnetic fields using memory-enhanced MGT theory. *Contin Mech Thermodyn*. 2025;37(6):91. doi:10.1007/s00161-025-01419-3.
43. Du B, Xu G, Xue D, Wang J. Fractional thermal wave bio-heat equation based analysis for living biological tissue with non-Fourier Neumann boundary condition in laser pulse heating. *Optik*. 2021;247(10):167811. doi:10.1016/j.ijleo.2021.167811.
44. Abouelregal AE, Civalek O, Akgoz B. A size-dependent non-Fourier heat conduction model for magneto-thermoelastic vibration response of nanosystems. *J Appl Comput Mech*. 2025;11(2):344–57. doi:10.22055/jacm.2024.46746.4584.
45. Hassan AF, Abidou D, Elsaid A, Zahra WK. Fractional non-Fourier modeling of laser drilling process. *Phys Scr*. 2024;99(8):085222. doi:10.1088/1402-4896/ad5ed2.

46. Singh K, Kaur I, Craciun EM. Hygro-photo-thermoelastic solid cylinder under moisture and thermal diffusivity with Moore-Gibson-Thompson theory. *Discov Mech Eng*. 2023;2(1):21. doi:10.1007/s44245-023-00028-1.
47. Adel M, El-Dali A, Seddeek MA, Yahya AS, El-Bary AA, Lotfy K. The fractional derivative and moisture diffusivity for Moore-Gibson-Thompson model of rotating magneto-semiconducting material. *J Vib Eng Technol*. 2024;12(1):233–49. doi:10.1007/s42417-024-01413-0.
48. Peng Y, Cheng CM, Chen XT, Zhang XY, Li XF. Transient response of the hygro-thermo-elastic field in hollow cylinders under heat-moisture shock via hyperbolic non-Fourier heat conduction and non-Fick diffusion. *Proc Inst Mech Eng Part C J Mech Eng Sci*. 2021;235(15):2795–809. doi:10.1177/0954406220954473.
49. Jojare KK, Gaikwad KR. Memory response on hygrothermal three-phase-lag hollow cylinder due to heat and moisture loading. *Mech Time Depend Mater*. 2024;29(1):19. doi:10.1007/s11043-024-09758-5.
50. Balwir A, Kamdi D, Varghese V. Heat and moisture source-dependent time-fractional hygrothermoelastic responses in solid cylinder via nonlocal and multi-kernel perspectives. *J Therm Stress*. 2025;48(8):972–90. doi:10.1080/01495739.2025.2503849.
51. Chandel N, Khalsa L, Varghese V, Dhore N. Dynamic analysis of the Klein–Gordon nonlocality and memory phenomena in a TPL hygrothermoelastic cylinder generating the heat and moisture. *Acta Mech*. 2026;237(5):2173–93. doi:10.1007/s00707-025-04614-0.
52. Eringen AC. *Nonlocal continuum field theories*. New York, NY, USA: Springer; 2004. doi:10.1007/b97697.
53. Bucci F, Pandolfi L. On the regularity of solutions to the Moore–Gibson–Thompson equation: a perspective via wave equations with memory. *J Evol Equ*. 2020;20(3):837–67. doi:10.1007/s00028-019-00549-x.
54. Chandel N, Khalsa L, Varghese V, Srinivas VB. Nonlocal hygro-photo-thermoelastic analysis of semiconductor nanocylinders with memory effects. *Mech Adv Mater Struct*. 2026;33(1):2699993. doi:10.1080/15376494.2026.2699993.
55. Mendhe V, Lamba NK, Abouelregal AE. Memory and long-range interaction in a two-dimensional hygrothermoelastic semi-infinite solid cylinder by theory of coupled-uncoupled heat and moisture. *J Vib Eng Technol*. 2026;14(6):392. doi:10.1007/s42417-026-02587-5.
56. Agiasofitou E, Lazar M. Nonlocal elasticity of Klein-Gordon type with internal length and time scales: constitutive modelling and dispersion relations. *Proc Appl Math Mech*. 2023;23(3):e202300065. doi:10.1002/pamm.202300065.
57. Lazar M, Agiasofitou E. Nonlocal elasticity of Klein–Gordon type: fundamentals and wave propagation. *Wave Motion*. 2022;114:103038. doi:10.1016/j.wavemoti.2022.103038.