



EDITORIAL

Introduction to the Special Issue on Analytical and Numerical Solution of the Fractional Differential Equation

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Fractional differential equations have garnered significant attention within the mathematical and physical sciences due to the diverse range of fractional operators available. Fractional calculus has demonstrated its utility across various disciplines, including biological modeling [1–5], applications in physics [6,7], most notably in the formulation of fractional diffusion equations, in robotics, and emerging areas such as intelligent artificial systems, among others. Numerous types of fractional operators exist, including those characterized by singular kernels, such as the Caputo and Riemann-Liouville derivatives [8,9]. It is important to highlight that the Riemann-Liouville derivative exhibits certain limitations; most notably, the derivative of a constant is not zero, which poses a significant inconvenience. To circumvent this issue, the Caputo derivative was introduced. Additionally, there are fractional derivatives with non-singular kernels, such as the Caputo-Fabrizio derivative [10] and the Atangana-Baleanu fractional derivative [11], each providing unique advantages for modeling purposes. Given the growing interest in utilizing fractional operators for various modeling scenarios, it is imperative to propose robust methodologies for obtaining both approximate and exact solutions. Consequently, this special issue emphasizes the exploration of diverse numerical schemes aimed at deriving approximate solutions for the models under consideration. Furthermore, analytical methods have also been discussed, providing additional avenues for obtaining exact solutions.

In this special issue, we received a total of 125 submissions, of which 113 were declined by the editorial office, and 12 successfully passed the editorial assessment. Subsequently, 12 papers underwent a rigorous review process, and after at least two rounds of evaluation by a minimum of three experts within the relevant domain, 10 papers were ultimately accepted for publication. This paper provides a comprehensive review of the accepted submissions.

In [1], Muhammad Farman et al. present a new fractional-order model and investigate the impact of smoking on the progression of asthma by using the Caputo operator to analyze different factors. To establish the existence and uniqueness of solutions, the authors use the Banach contraction principle; they further prove the positivity and boundedness of the model. Next, they present the equilibrium point of the fractional smoking model and establish its global stability using the Lyapunov function. In addition, the reproduction number is provided and its sensitivity assessed. Finally, a Newton-polynomial-based scheme is used to obtain numerical solutions and to illustrate the paper's main results with graphics.



In [12], Lee et al. proposed two hybrid numerical methods named the linear sine and cosine, known here as the L1-CAS method, and fast-CAS schemes for solving linear and nonlinear multi-term Caputo variable-order fractional partial differential equations. These methods combine CAS wavelet-based spatial discretization with L1 and fast algorithms in the time domain. A key feature of the approach is its ability to efficiently handle fully coupled space-time variable-order derivatives and nonlinearities through a second-order interpolation technique.

In [6], Kamran et al. developed the Laplace transform-based Chebyshev spectral collocation method to approximate the time fractional advection-diffusion equation, incorporating the Atangana-Baleanu Caputo derivative. The scheme developed in this paper can be considered as a numerical scheme for fractional differential equations, and can be used in numerous papers in the future, due to the stability and the convergence of the numerical scheme based on the findings of the paper.

In [2], Murad et al. proposed aims primarily to determine the prevalence of *Ascaris lumbricoides* infection among various risk factors, to assess blood parameters, levels of immunoglobulin E and interleukin-4, and to explore the relationship between ascariasis and asthma in affected individuals. Another objective is to examine a fractal-fractional mathematical model that describes the four stages of the life cycle of *Ascaris* infection, specifically within the framework of the Caputo-Fabrizio derivative, a derivative with a nonsingular derivative. The novelty is the utilisation of the fractional derivative, and it is noticed that this type of derivative can be used in modeling fractional models because it captures carefully the dynamic behaviors in real-world problems. An important point in this paper is the presentation of the Ulam-Hyers and Ulam-Hyers-Rassias Stability, which is a stability notion obtained using the existence and the uniqueness of the solution of the model under consideration.

In [13], Raza et al. have proposed an investigation related to a stochastic differential equation. As it is noticed in the literature, the stochastic differential equations are not introduced in fractional dynamics, the present investigation offers issue into this problem. The authors of this paper studied the dynamics of Leukemia. The population of cells has been divided into three subpopulations: susceptible cells, infected cells, and immune cells. To investigate the memory effects and uncertainty in disease progression, leukemia modeling is developed using stochastic fractional delay differential equations. The feasible properties of positivity, boundedness, and equilibria, i.e., Leukemia-free equilibrium and Leukemia present equilibrium of the model, were studied rigorously. The local and global stabilities and sensitivity of the parameters around the equilibria under the assumption of reproduction numbers were investigated. A numerical method for the stochastic fractional delayed model has been presented in this paper.

In [3], AL-Mekhlafi et al. proposed a cholera epidemic model described by the proportional fractional derivative. The authors proposed the equilibrium points and studied their stability. The existence of the solution has also been proposed in the present investigation. The authors used numerical schemes to approximate the solution of the model. The authors used the so-called Toufik-Atangana Method for the numerical scheme. Against the cholera, this paper proposes some interesting main findings. In other words, the authors presented a novel mathematical model to describe the progression of cholera by integrating fractional derivatives with both singular and non-singular kernels alongside stochastic differential equations over four distinct time intervals.

In [4], Ramaswamy et al. focused on a fractional model of Monkeypox virus disease described by the nonsingular fractional operator. The evaluation of this model determines the existence of two equilibrium states. These two stable points exist within the model and include a disease-free equilibrium and an endemic equilibrium. The disease-free equilibrium has undergone proof to demonstrate its stability properties. The authors found that the system remains stable locally and globally whenever the effective reproduction number remains below one. The effective reproduction number becoming greater than unity makes the

endemic equilibrium more stable both globally and locally than unity. In this paper, for the illustration of the main finding with graphics, the authors proposed a numerical scheme. The present paper is a good alternative for finding the approximate solutions for the fractional differential equations when the analytical solutions are not trivial.

In [14], Rayal et al. investigate the numerical performances of the nonlinear fractional Rössler attractor system under Caputo derivatives by designing the numerical framework based on Ultraspherical wavelets. The Caputo fractional Rössler attractor model is simulated into two categories, asymmetric and symmetric. The ultraspherical wavelets basis with suitable collocation grids is implemented for comprehensive error analysis in the solutions of the Caputo fractional Rössler attractor model, depicting each computation in graphs and tables to analyze how fractional order affects the model's dynamics. For the chaotic attractor and the novel numerical scheme, the present paper can be used for readers in the domain of chaotic behaviors.

In [15], Momani et al. described in their paper presented a model named the fractional Layla and Majnun model system. Note that a discrete system can enable the modeling and control of complicated processes more adaptively through the concept of versatility by providing system dynamics' descriptions with more degrees of freedom. And then, numerical approaches have become necessary and sufficient to be addressed and employed for benefiting from the adaptability of such systems for varied applications. The novelty is the use of the discrete fractional operator in the mathematical modeling, and proposing a numerical method to get the approximate solution.

In [7], Archana et al. proposed a paper aiming to find analytical solutions to the resulting one-dimensional differential equation of a cancer tumor model in the framework of time-fractional order with the Caputo-fractional operator employing a highly efficient methodology called the homotopy analysis transform method. Here, an analytical solution procedure has been proposed, named the homotopy analysis transform method. The method is convergent and stable and can be used for further investigation. A review of the analytical method has been proposed in the literature review of the present paper.

In [5], Khan et al. presented a mathematical model designed to investigate Tuberculosis disease. The fractional system's existence, uniqueness, and other relevant properties are shown. Then, we study the stability analysis of the equilibrium points of the epidemic model. A free equilibrium is locally asymptotically stable when the reproduction number is less than 1. Further, it is shown that the global asymptotic stability of the endemic equilibrium is obtained when the reproduction number is greater than 1. The existence of bifurcation analysis in the model is investigated, and it is shown that the system possesses the forward bifurcation phenomenon. Sensitivity analysis has been performed to determine the sensitive parameters that impact the reproduction number. In terms of modeling tuberculosis disease, this paper can be referenced.

In [16], Telksniene et al. propose a novel computational framework for the structural analysis of fractional differential equations involving iterated Caputo derivatives. The method proposed in the present investigation consists of transforming a primary fractional differential equation into a simple first-order differential equation well known in the literature. Note that the primary novelty of the proposed methodology lies in treating the structure of the integer-order component not as fixed, but as a parameterizable polynomial whose coefficients can be determined via global optimization. Furthermore, using particle swarm optimization, the framework identifies an optimal ordinary differential equation architecture by minimizing a dual objective that balances solution accuracy against a high-fidelity reference and the magnitude of the truncated residual series. For illustrations of the main findings of the paper, the authors considered the linear fractional differential and proposed its analytical solution, and tried to minimize RMSE in the optimization process. A second illustrative example is proposed in this last example, where the author proposed Root Mean Square Error, denoted by RMSE, in the optimization process for a nonlinear Riccati fractional differential equation, based on the numerical solution already proposed in the literature.

Conflicts of Interest: The authors declare no conflicts of interest to report regarding the present study.

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