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ARTICLE



IECC-SAIN: Innovative ECC-Based Approach for Secure Authentication in IoT Networks

Younes Lahraoui¹, Jihane Jebrane², Youssef Amal¹, Saiida Lazaar¹ and Cheng-Chi Lee^{3,4,*}

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ABSTRACT: Due to their resource constraints, Internet of Things (IoT) devices require authentication mechanisms that are both secure and efficient. Elliptic curve cryptography (ECC) meets these needs by providing strong security with shorter key lengths, which significantly reduces the computational overhead required for authentication algorithms. This paper introduces a novel ECC-based IoT authentication system utilizing our previously proposed efficient mapping and reverse mapping operations on elliptic curves over prime fields. By reducing reliance on costly point multiplication, the proposed algorithm significantly improves execution time, storage requirements, and communication cost across varying security levels. The proposed authentication protocol demonstrates superior performance when benchmarked against relevant ECC-based schemes, achieving reductions of up to 35.83% in communication overhead, 62.51% in device-side storage consumption, and 71.96% in computational cost. The security robustness of the scheme is substantiated through formal analysis using the Automated Validation of Internet Security Protocols and Applications (AVISPA) tool and Burrows-Abadir-Needham (BAN) logic, complemented by a comprehensive informal analysis that confirms its resilience against various attack models, including impersonation, replay, and man-in-the-middle attacks. Empirical evaluation under simulated conditions demonstrates notable gains in efficiency and security. While these results indicate the protocol's strong potential for scalable IoT deployments, further validation on real-world embedded platforms is required to confirm its applicability and robustness at scale.

KEYWORDS: Industrial IoT; Elliptic Curve Cryptography (ECC); National Institute of Standards and Technology (NIST) curves; mapping; AVISPA; BAN logic; computational efficiency; security; scalable IoT deployments

1 Introduction

The exponential growth in Information and Communication Technology (ICT) has revolutionized the way we interact with and utilize data, facilitating seamless connectivity and empowering myriad applications across various sectors [1]. Central to this evolution is the Internet of Things (IoT), a paradigm that interconnects countless devices, enabling efficient automation, enhanced productivity, and enriched service delivery in both consumer and industrial contexts [2]. From smart homes that adjust environments based on occupant behavior to industrial sensors that optimize manufacturing processes in real-time, the IoT not only has streamlined operations but also has elevated standards of living and service provision [3].

However, this rapid digitization has been paralleled by a surge in cybersecurity threats, amplifying concerns about data integrity, privacy breaches, and system vulnerabilities. Cryptography emerges as a



¹Mathematics, Computer Science and Applications TEAM, Abdelmalek Essaâdi University, ENSA, Tangier, 90000, Morocco

²National School of Applied Sciences, Sultan Moulay Slimane University, Beni Mellal, 23000, Morocco

³Department of Library and Information Science, Fu Jen Catholic University, New Taipei City, 242062, Taiwan

⁴Department of Computer Science and Information Engineering, Asia University, Taichung City, 413, Taiwan

^{*}Corresponding Author: Cheng-Chi Lee. Email: cclee@mail.fju.edu.tw

cornerstone in mitigating these risks, offering robust mechanisms to secure sensitive information and communications. The IoT Threat Report 2023 by Securelist, backed by Kaspersky, underscores the importance of effective authentication mechanisms in the IoT space, revealing a 30% surge in attacks and over 1.5 billion incidents in the first half of 2023 [4]. This highlights the critical need for robust security measures to protect data exchanges across potentially insecure networks.

For instance, IoT authentication ensures that only authorized devices can communicate and access resources within a network. This authentication process hinges significantly on the efficiency and security of cryptographic protocols. Elliptic curve cryptography has emerged as a pivotal solution, offering superior performance advantages with shorter key lengths compared to traditional cryptographic methods. ECC's ability to provide equivalent security with smaller key sizes reduces computational overhead, making it particularly well-suited for resource-constrained IoT devices [5].

Against this backdrop, this paper presents a novel IoT authentication approach based on our previously proposed mapping and reverse operations [6] on elliptic curves over prime fields. We select NIST curves (192, 256, 384, 521) for their efficient arithmetic and low resource requirements [7]. It is worth noting that we also go beyond the 521-bit security level to demonstrate the scalability of our approach. Unlike existing benchmarks that rely heavily on point multiplication and suffer from significant performance degradation as security increases, our method maintains efficiency even at higher security levels. Our method focuses on optimizing authentication speed across varying key sizes, surpassing conventional point multiplication techniques that often bottleneck ECC-based systems [8]. Furthermore, we conduct a comprehensive security evaluation using AVISPA and BAN logic, supported by informal proofs against potential attacks. Through empirical assessments covering storage, communication, and computational costs, we demonstrate the efficiency and effectiveness of our proposed system in real-world scenarios.

2 Related Works

The Internet of Things (IoT) heavily depends on wireless networks for data collection by authorized users. Typically, communication happens between a central platform and various terminal nodes, where the platform sends commands to the nodes to gather and transmit data back. Mutual authentication between the platform and terminal nodes is crucial to ensure network security. Without this, unauthorized individuals could exploit the network for data theft or malicious activities. Moreover, terminal nodes need to authenticate themselves to other nodes to prevent unauthorized access, which could disrupt the network and deceive both the platform and legitimate nodes [9]. Therefore, mutual identity authentication is vital for maintaining the security of IoT systems. In scenarios where processing power and memory are constrained, ECC as a form of public key cryptography, is particularly suitable. ECC offers an efficient solution for secure communication in resource-limited environments, making it a viable option for enhancing IoT security [10].

The various ECC-based authentication schemes proposed in the literature utilize different methods, such as time-stamp techniques and certificate-based mutual authentication. However, certificate-based methods can be costly, as they require additional computational resources for servers and users to authenticate each other's identities. In contrast, the authors in [11] proposed an ECC-based authentication and key agreement scheme specifically for IoT environments. Their protocol facilitates secure communication between embedded devices and cloud servers by providing mutual authentication, allowing both the user and server to negotiate encryption keys collaboratively. They assert that their scheme meets all necessary security requirements and offers robust resistance to various well-known IoT attacks.

Despite the author's claim, studies in [12–15] identified significant security flaws and structural issues in their protocol. These issues include the failure of mutual authentication, ambiguity in session key establishment, vulnerability to offline password guessing, and susceptibility to insider and traceability attacks.

In response, each of these researchers proposed enhanced protocols, asserting that their improvements effectively address the identified vulnerabilities and fulfill the necessary security requirements. Further advancements include the work in [16], which proposed an anonymous authentication scheme that integrates a password validator to defend against known temporary information and DoS attacks. This scheme showcased ECC's potential in fortifying IoT devices against sophisticated attacks while maintaining a lightweight footprint. Another significant contribution is from [17] proposed a novel lightweight anonymous authentication protocol (LAAP) to meet security and efficiency requirements. This protocol uses ECC and dynamic pseudonyms to prevent traceable attacks caused by fixed identity identification and employs symmetric encryption to optimize the server's search for anonymous device information, reducing the time complexity from O(n) to O(1). Very recently, the authors in [18,19] proposed a mutual authentication scheme based on ECC and the U-Quark light hash function, which is known for its collision resistance. This approach not only retains the essential security features of ECC but also enhances performance, making it well-suited for the IoT environment. Despite continuous efforts, designing a resource-efficient and secure ECC-based authentication protocol for IoT edge devices remains a significant challenge. Similarly, in 2024, the authors in [6] proposed an innovative hash-based technique that embeds messages into an elliptic curve (EC) points before encryption, using a random parameter and a shared secret point from the generated through elliptic curve Diffie-Hellman protocol. The method's security was evaluated against various attack models, and its complexity and sensitivity were analyzed. A tag ensures message integrity, and the scheme meets criteria such as the strict avalanche criterion and linear complexity. Comprehensive analysis confirmed its effectiveness in maintaining data security and integrity. Another recent study, the authors in [20] proposed a two-factor authentication protocol for IoT-enabled Wireless Sensor Networks (WSNs), integrating ECC with a fuzzy verifier to enhance both security and usability in resource-constrained environments. Instead of relying on traditional deterministic password hashes, their scheme employs a fuzzy verifier approach, introducing randomness that enhances resistance to common attacks while preserving authentication reliability. Formal security validation using the Real-or-Random model and comparative analysis confirmed the scheme's efficiency, achieving a computation cost of 8.9569 ms, outperforming existing protocols in both performance and protection metrics. In a related contribution, the authors in [21] designed an authentication protocol optimized for military Internet of Drones (IoD) scenarios, addressing the need for secure, realtime communication under resource constraints. Their approach leverages Elliptic Curve Cryptography and independently managed session keys across various communication links to contain potential breaches and reduce computational load. To further strengthen security, the protocol integrates trust anchors, group key exchanges, and position verification techniques, ensuring resistance to attacks such as replay and denial-ofservice. In 2025, the authors in [22] introduced a lightweight authentication protocol specifically designed for the Internet of Medical Things (IoMT), addressing both security and user privacy in resource-constrained medical environments. Their approach ensures secure communication between body-connected devices while preserving patient confidentiality, and it is formally verified using the AVISPA tool. By leveraging smaller key sizes and efficient cryptographic techniques, the proposed protocol achieves 5 to 6 times greater computational efficiency compared to conventional methods such as ElGamal and Rivest-Shamir-Adleman (RSA), while resisting common threats found in existing schemes. Building on the work proposed in [6], we introduce a novel ECC-based mutual authentication protocol for IoT environments. Our protocol is designed to provide lightweight communication with reduced computational and storage costs while maintaining strong security properties. The security of our protocol has been formally verified using AVISPA and BAN logic, chosen based on a survey of 40 authentication protocols, which revealed that 25% use AVISPA and 36% use BAN logic [10]. Furthermore, our protocol has been tested informally against various attacks, including Distributed Denial of Service (DDoS), forward secrecy, impersonation replay attacks, and so on, demonstrating its robustness and reliability. Our proposed ECC-based authentication protocol addresses

existing security and efficiency gaps by enhancing cryptographic techniques and optimizing computational processes. This ensures robust mutual authentication with minimal resource consumption, making it well-suited for IoT applications. Overall, ECC-based authentication schemes effectively secure IoT devices by balancing strong security and computational efficiency. Our scheme builds on these principles, introducing improvements to meet the evolving security demands of IoT environments.

3 Elliptic Curves Cryptography and Secure Embedding Approach

This section provides essential background on ECC required to understand the proposed scheme. It covers key concepts such as point multiplication, solving modular quadratic equations, and data embedding into elliptic curve points.

3.1 Introduction to Elliptic Curve Cryptography over Prime Fields

Elliptic Curve Cryptography uses the algebraic structure of elliptic curves over finite fields for secure communication. An elliptic curve over a prime field \mathbb{F}_p (where p is a large prime) is defined by the equation:

$$y^2 = x^3 + ax + b \pmod{p}$$

where a and b are constants fulfills $-16(4a^3 + 27b^2) \neq 0$.

3.1.1 Points on Elliptic Curves

Points on the elliptic curve $E(\mathbb{F}_p)$ are pairs (x, y) that satisfy the curve equation, along with a point at infinity, O.

3.1.2 Key Operations

- 1. Point Addition: Given two points P and Q, their sum R = P + Q is another point on the curve. The point -P denotes the symmetric of P and verifies P + (-P) = O.
- 2. Point Doubling: Doubling a point P results in another point R = 2P.
- 3. Point Multiplication (PM): Multiplying a point P by an integer k (denoted kP) involves repeatedly adding P to itself. Efficient PM computation significantly impacts both performance and security. A comprehensive overview of PM algorithms optimized for resource-constrained devices is provided in [23].

3.1.3 Generator Point

A generator point *G* is a specific point on the elliptic curve used to generate all or almost all other points in the elliptic curve group through repeated addition. Table 1 describes various elliptic curve parameters used in alongside this paper.

Table 1: Domain Parameters of an Elliptic Curve over Prime Field \mathbb{F}_p

Parameter	Description
p	Prime integer defining the finite field \mathbb{F}_p .
a, b	Coefficients in the curve equation $y^2 = x^3 + ax + b$.
G	Generator point of the cyclic group $\langle G \rangle$.
n	Order of the generator point G (i.e., $nG = O$).

Comprehensive details on ECC can be found in [24].

3.2 Modular Quadratic Equation

The proposed mapping and reverse mapping operations require solving a modular quadratic equation of the form

$$x^2 + bx + c \equiv 0 \pmod{p},\tag{1}$$

with discriminant $\Delta = b^2 - 4c \pmod{p}$. A solution exists if and only if Δ is a quadratic residue modulo p. For NIST prime fields where $p \equiv 3 \pmod{4}$, this can be efficiently verified by evaluating the Legendre symbol [25]:

$$\left(\frac{\Delta}{p}\right) = \Delta^{\frac{p-1}{2}} \mod p. \tag{2}$$

If $\left(\frac{\Delta}{p}\right)$ = 1, then solutions exist and can be computed as $x = \frac{-b \pm r}{2} \mod p$, where $r = \Delta^{\frac{p+1}{4}} \mod p$. This procedure benefits from the arithmetic efficiency of NIST elliptic curve defined over prime fields, ensuring lightweight and fast computation within the proposed protocol.

3.3 Elliptic Curve Embedding Approach

In this section, we explore a secure method for embedding information into elliptic curves, originally proposed in [6], and it is applied in a novel way to support efficient and secure authentication. The approach focuses on integrating data within the curve's structure while ensuring robust security measures. Upon establishing an agreement between the server and the device on the elliptic curve domain parameters, as detailed in the forthcoming setup phase Section 4.1, we define

$$Mapp: \mathbb{F}_p \to E(\mathbb{F}_p),$$

which converts a given element m of the prime field \mathbb{F}_p into a point P_m on the elliptic curve $E(\mathbb{F}_p)$. This process involves using a secret point $P_s = (x_s, y_s)$ on the curve $E(\mathbb{F}_p)$. A line passing through P_s with slope $S = (y_s - m)x_s^{-1}$ is defined. One of the intersection points between this line and the elliptic curve is chosen as the mapping point P_m for m. If there are no intersection points other than P_s and $-P_s$, m is incremented by one, and the process is repeated until a point $P_m \in E(\mathbb{F}_p) \setminus \{\pm P_s\}$ is found. As highlighted in [6], the problem of finding the point P_m is equivalent to solving the following equation:

$$x^{2} + (x_{s} - S^{2})x + x_{s}^{-1}(-b + m^{2}) \equiv 0 \pmod{p}.$$
 (3)

Once a solution $x_m \neq x_s$ to Eq. (3) is found, the point P_m is defined as (x_m, y_m) , where the *y*-coordinate y_m is computed as $y_m \equiv Sx_m + m$. It is crucial to ensure that $x_m \neq x_s$ to guarantee that $P_m \neq \pm P_s$. Algorithm 1 provides comprehensive details on the embedding process.

Algorithm 1: Secure embedding algorithm

```
Input: Elliptic curve E(\mathbb{F}_p), secret point P_s = (x_s, y_s), m

Output: Mapping point P_m = (x_m, y_m) for integer m

Assert: BitLength(m) < BitLength(p) - 8;  // Bin(.): binary value

Update: m \leftarrow Integer(Bin(m) \parallel 00000000);  // Inetger(.): decimal value while no valid P_m is found do
```

Algorithm 1 (continued)

```
Calculate slope S = (y_s - m)x_s^{-1} \pmod{p}

Update then solve Eq. (3) for x

if Eq. (3) has solution x_m \neq x_s then

Compute y_m \equiv Sx_m + m \pmod{p}

return P_m = (x_m, y_m)

Increment m by 1
```

Now we explore how to reverse the mapping process. This method complements the secure embedding approach detailed previously, ensuring that the original integer $m \in \mathbb{F}_p$ can be accurately retrieved from a mapping point P_m elliptic curve $E(\mathbb{F}_p)$. We denote that process by:

```
Reverse: E(\mathbb{F}_p) \to \mathbb{F}_p
```

Given the shared elliptic curve $E(\mathbb{F}_p)$, a mapping point $P_m = (x_m, y_m)$, and the secret point $P_s = (x_s, y_s)$ used during P_m computation. To determine the original integer m, the reverse mapping process involves computing the slope of the line between the secret point and the mapping point, and then determining m from the computed slope and coordinates.

The reverse mapping process follows these steps:

1. **Slope Calculation:** The slope *S* of the line passing through the points $P_s = (x_s, y_s)$ and $P_m = (x_m, y_m)$ is computed as:

$$S \equiv (y_m - y_s)(x_m - x_s)^{-1} \pmod{p}$$

2. **Computing** m'**:** Use the slope S and the x-coordinate of the secret point P_s to compute the value m' as:

$$m' \equiv y_m - Sx_m \pmod{p}$$

3. **Removing the Last 8 Bits:** Convert m' to its binary form and remove the last 8 bits, which were appended during the secure embedding process to ensure a unique mapping. The truncated value represents the original integer m. Another way

```
m = \text{Integer}(Bin(m') \setminus \{\text{last 8 bits}\})
```

The algorithm for this reverse mapping approach is detailed in Algorithm 2.

Algorithm 2: Reverse mapping algorithm

```
Input: Elliptic curve E(\mathbb{F}_p), secret point P_s = (x_s, y_s), mapping point P_m = (x_m, y_m)

Output: Original integer m

Calculate slope S \equiv (y_m - y_s)(x_m - x_s)^{-1} \pmod{p}

Compute m' \equiv y_m - Sx_m \pmod{p}

m \leftarrow \text{Integer}(Bin(m') \setminus \{\text{last 8 bits}\})

return m
```

4 Our Protocol Explanation

Our protocol consists of four main phases: the setup phase, the registration phase, the login phase, and the authentication phase. Implemented using the Charm framework, it employs elliptic curve cryptography

for secure key exchange and authentication. Data transmission from the embedded devices *D* and the server is assumed to occur over a secure channel during the registration phase. In the following, we detail each phase and the corresponding steps involved in the protocol execution as depicted in Fig. 1. The parameters used in our protocol and their corresponding definitions are listed in Table 2.

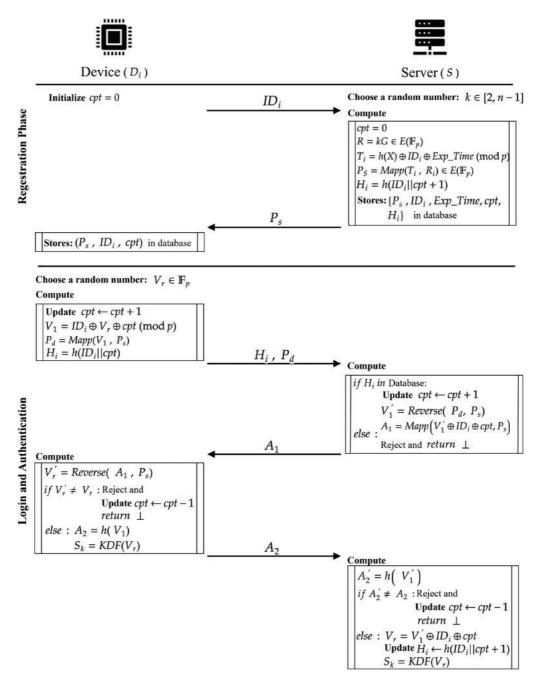


Figure 1: Proposed ECC-based authentication scheme for IoT.

Symbol	Description
D_i	Device i
S	Server
ID_i	Identifier of device <i>i</i>
X	The private key chosen by the server
k	Random number chosen by the server
EXP_Time	Expiration time for a specific device
V_r	Random number is chosen by the device
P_s	Computed value stored in server and device
\oplus	Exclusive OR (XOR) operation
	Concatenation
V_1	Computed by the device to establish identity proof
P_d	Computed by the device to ensure secure communication
A_1, A_2	Authentication values exchanged to verify identities
V_1'	Recomputed by the server to verify V_1
V'_r	Recomputed by the server to verify V_r
S_k	Session key

Table 2: Symbols and their descriptions used in the protocol.

4.1 Setup Phase

In the setup phase, both the server and the device agree on an elliptic curve to use, such as the standardized NIST curve prime192v1. In the proposed method, we specifically use elliptic curves defined over prime fields with characteristics greater than or equal to 5. The domain parameters for the chosen elliptic curve are detailed in Table 1.

4.2 Registration Phase

In the Registration Phase, the device D_i begins by sending its identifier ID_i to the server S. Both the device and the server create a counter (cpt) initialized to 0. Upon receiving ID_i , the server selects a random number k within the range [2, n-1], and their private key X. The server then performs a series of computations:

$$R = kG, (4)$$

$$T_i = h(X) \oplus ID_i \oplus Exp_Time \bmod p, \tag{5}$$

$$P_s = Mapp(T_i, R_i) \in E(\mathbb{F}_p), \tag{6}$$

$$H_i = h(ID_i||cpt+1). (7)$$

The server then stores the values $(P_s, ID_i, Exp_Time, cpt, H_i)$ in its database and sends P_s back to the device D_i . The device stores (P_s, ID_i, cpt) in its database for future use.

4.3 Login and Authentication Phase

The authentication phase involves mutual verification between the device and the server through a sequence of cryptographic operations and exchanges of various values.

During the Login and Authentication Phases, the device D_i initiates the process by updating its counter cpt and choosing a random number V_r from the finite field \mathbb{F}_p . It computes:

$$cpt \leftarrow cpt + 1,$$
 (8)

$$V_1 = ID_i \oplus V_r \oplus cpt \mod p, \tag{9}$$

$$P_d = Mapp(V_1, P_s) \in E(\mathbb{F}_p), \tag{10}$$

$$H_i = h(ID_i||cpt). (11)$$

The device then sends H_i and P_d to the server S. Upon receiving H_i and P_d , the server checks its database for a record of H_i . If it does not exist, the server rejects the request and returns a failure indicator (\bot). Otherwise, the server proceeds to compute:

$$V_1' = Reverse(P_d, P_s). \tag{12}$$

Additionally, the server updates its counter (cpt) to ensure synchronization with the device and then calculates:

$$cpt \leftarrow cpt + 1,$$
 (13)

$$A_1 = Mapp(V_1' \oplus ID_i \oplus cpt, P_s) \in E(\mathbb{F}_p). \tag{14}$$

The server then sends A_1 back to the device. Once the device receives A_1 it computes:

$$V_r' = Reverse(A_1, P_s). (15)$$

If V_r' does not match V_r , the device resets its counter to the previous state by setting $cpt \leftarrow cpt - 1$, rejects the authentication, and returns a failure indicator (\perp). If V_r' matches V_r , the device proceeds to compute:

$$A_2 = h(V_1), \tag{16}$$

and sends A_2 to the server. The device also derives the session key:

$$S_k = KDF(V_r),$$
 (17)

where KDF is a key derivation function. Upon receiving A_2 , the server computes:

$$A_2' = h(V_1'). (18)$$

If A'_2 does not match A_2 , the server resets its counter to the previous state by setting $cpt \leftarrow cpt - 1$, rejects the authentication, and returns a failure indicator (\bot). If A'_2 matches A_2 , the server updates H_i to $h(ID_i||cpt+1)$ and calculates:

$$V_r = V_1' \oplus ID_i \oplus cpt, \tag{19}$$

and derives the session key:

$$S_k = KDF(V_r). (20)$$

By adhering to these carefully structured steps, our ECC-based authentication protocol ensures robust mutual authentication between IoT devices and the central server, capitalizing on the computational efficiency and security strengths of elliptic curve cryptography.

5 Security Analysis and Performance Analysis of the Elliptic Curve Embedding Approach

In this section, we delve into the rigorous evaluation of the security and performance aspects of our proposed Elliptic Curve Embedding Approach (ECEA) for IoT authentication. Our approach builds upon the foundation laid by our previously proposed ElGamal ECC-based encryption [6], leveraging its inherent security properties and extending them to authenticate IoT devices securely.

Lemma 1. Let $E(\mathbb{F}_p)$ an elliptic curve. For all elements m in \mathbb{F}_p and a point P_s in $E(\mathbb{F}_p)$. There exist an embedding point $P_m = Mapp(m, P_s) \in E(\mathbb{F}_p)$ if and only if there exist a third point P'_m in $E(\mathbb{F}_p)$ such that:

$$P_m = -(P_s + P_m'). \tag{21}$$

Proof: Let $P_m = Mapp(m, P_s)$ be the embedding point of $m \in \mathbb{F}_p$ based on a P_s point on the elliptic curve $E(\mathbb{F}_p)$. By construction, P_m is the intersection point of the line passing through P_s , where m is the y-intercept, such that $P_m \neq \pm P_s$. Given that the line already intersects the elliptic curve at P_s and P_m , P_m can be viewed as the sum of $-P_s$ and a point $-P'_m$, where P'_m is the third intersection point of the line with the curve. Hence,

$$P_m = -(P_s + P'_m).$$

On the other hand, suppose there exists a point P'_m that satisfies Eq. (21). Let $S = (y_s - y_m)(x_s - x_m)^{-1}$ mod p and $m = y_m - Sx_m \mod p$. Then, $P_m = Mapp(m, P_s)$ is the embedding point of m on the elliptic curve $E(\mathbb{F}_p)$ with respect to P_s . This is due to the fact that $-P_m = P_s + P'_m$. Hence, $-P_m$ is the symmetric point of P_m with respect to the x-axis. By the definition of point addition on an elliptic curve, $-P_m$ is the intersection point of the line passing through P_s and P'_m with the elliptic curve. This confirms that P_m is indeed the correct embedding point of m on the curve. \square

Theorem 1. Let $E(\mathbb{F}_p)$ be an elliptic curve, G the generator point of order n, and k a random number in [1, n-1]. The embedding approach acts as ElGamal ECC-based encryption with less computational overhead.

Proof: Consider G to be the generator point of $E(\mathbb{F}_p)$ with order n, and let k be a random number in [1, n-1]. Let $P_a = aG$ represent the public key of a receiver with a private key a. Suppose a message m < p is encoded as an integer and embedded into a point $P_m = Mapp(m, R)$, where $R = kP_a$. By Lemma 1, there exists a point P'_m such that:

$$P_m = -(P_m' + R) \tag{22}$$

$$= -(P_m' + kP_a) \tag{23}$$

$$=P_m^{\prime\prime}+k^{\prime}P_a, \tag{24}$$

where $k' \equiv -k \pmod n$ and $P''_m = -P'_m$ acts as another embedding point of the message m. The ElGamal ECC-based encryption of P''_m using the key k' calculated as $\operatorname{Enc}(m,k') = P''_m + k'P_a$. Thus, the process of embedding m into P_m mirrors the ElGamal encryption scheme, where the ciphertext comprises components dependent on the recipient's public key and a random key. This establishes the fundamental similarity between our proposed embedding approach and ElGamal ECC encryption. However, ElGamal ECC encryption requires additional operations after embedding the message in a point, specifically point multiplication and point addition. \square

As discussed in [6], the embedding approach successfully encodes a message (integer m < p) into a point on the curve with an average of two rounds. The algorithm's complexity has been demonstrated to be O(1) in terms of message size. Furthermore, sensitivity analysis reveals that small changes in m or the key (Point P_s) result in a 50% change rate, illustrating that the proposed scheme satisfies both the confusion property and the strict avalanche criterion [26,27]. Moreover, the proposed method successfully passes the NIST statistical

test suite, ensuring the randomness of the ciphertext from an adversary's perspective. By Theorem 1, we demonstrate how the embedding approach shares similarities with Elgamal ECC-based encryption. Thus, the embedding approach utilized in our proposed authentication method inherits the security properties of our previously proposed encryption scheme [6], particularly as elaborated in Section 5 for a comprehensive security analysis.

To demonstrate the authentication system's sensitivity to small changes in its inputs, ensuring security through significant variations in authentication parameters, we set an initial ID as a random number, then created 10 different ID_i values by changing one bit in the original ID, each linked to a device D_i . We kept all other parameters fixed $(k, X, cpt, Exp_time, EC$ -parameters, V_r). The resulting plot (see Fig. 2) illustrates distinct communication pathways for each device D_i based on normalized values of (x_s, x_d, x_{A_1}, A_2) , where x_s, x_d, x_{A_1} represent the x-coordinates of exchanged points P_s, P_d, A_1 . This highlights that multiple devices with almost identical IDs, but with one-bit difference, exhibit significant and unpredictable variations in the exchanged messages while interacting with a server having fixed parameters. This property holds for all inputs, thereby ensuring the integrity and robustness of the authentication protocol by preventing predictable outcomes from slight alterations. The empirical analysis of cryptographic operations using NIST curves demonstrates that our proposed mapping and reverse operations offer significantly higher efficiency compared to traditional point multiplication, which was implemented using the double-and-add algorithm exactly as in the tinyec library. Across all key sizes—192-bit, 256-bit, 384-bit, and 521-bit—mapping and reverse operations consistently achieve minimal execution times, as illustrated in Fig. 3. In contrast, point multiplication exhibits the highest execution times, especially noticeable even with smaller key sizes like 192-bit.

The computational overhead, assessed across the NIST elliptic curve with various key sizes compared to secp192r1 as illustrated in Table 3, which highlights how the execution time growth ratio for reverse and mapping operations remains manageable yet notably faster compared to the substantial growth observed with point multiplication. This disparity leads to slower authentication processes when relying on point multiplication. Conversely, our proposed mapping operations maintain a more linear growth pattern, ensuring efficiency even as key sizes increase.

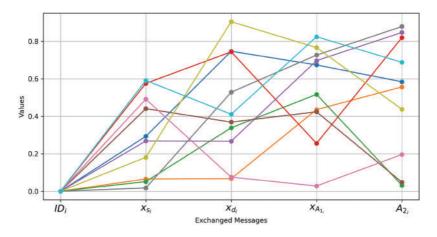


Figure 2: Normalized Data Pathways in Device-Server Communication during Authentication Protocol

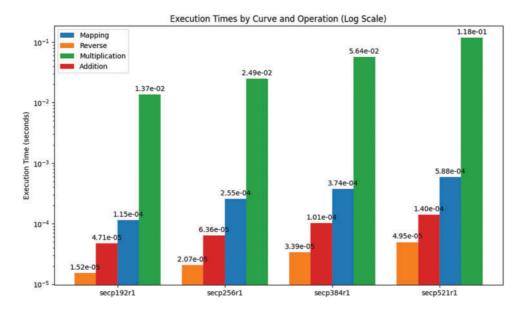


Figure 3: Comparative Analysis of Execution Times for Cryptographic Operations: Mapping, Reverse, Multiplication, Addition

Key Size (bits)	Reverse Ratio	Mapping Ratio	Addition Ratio	Multiplication Ratio
192	1.00	1.00	1.00	1.00
256	1.36	2.22	1.35	1.82
384	2.23	3.25	2.14	4.12
521	3.26	5.11	2.97	8.61

Table 3: Comparison of Execution Time Growth Ratios Across Key Sizes (relative to secp192r1)

Additionally, we compared the computational cost of point multiplication and our embedding algorithm on the NIST curve secp192r1. Point multiplication using the double-and-add algorithm requires up to 23,857 field operations with affine coordinates and 2500 with mixed coordinates (Jacobian and affine), reduced to 718 using the Fixed-Base Comb method, which trades off the storage of approximately 30 points [24]. In contrast, our embedding approach, which involves solving the quadratic Eq. (3) using the square root of the discriminant $\Delta^{\frac{p+1}{4}}$ [28], requires about 412 field operations on average without any additional storage requirements.

Note that the previously established comparison is motivated by the fact that all ECC-based authentication schemes rely on point multiplication as a core operation. While these schemes successfully enhance performance and security compared to those leveraging other cryptographic techniques like RSA, the point multiplication operation remains the most computationally intensive aspect. Our proposed embedding scheme is designed to offer improved performance while maintaining high security.

The findings show that our method is particularly suitable for reducing computational complexity in elliptic curve cryptographic applications, such as authentication, in resource-constrained IoT environments. By maintaining a delicate balance between security and performance while addressing the scalability challenges posed by larger key sizes, our method is well-suited for deployment in these environments.

6 Threat Model and Assumptions

This section outlines the security assumptions, attacker capabilities, and environmental constraints considered in the design of the proposed authentication protocol for IoT environments. We target typical IoT scenarios, where devices operate in open and potentially untrusted settings, often with limited computational, memory, and energy resources.

We assume that an adversary has full control over the communication channel between the device and the server. This includes the ability to intercept, eavesdrop, replay, inject, or alter transmitted messages. The adversary may act passively (observing protocol flows) or actively (modifying or forging messages). However, the adversary cannot break standard cryptographic assumptions, such as the collision resistance of SHA-256 or the intractability of the Elliptic Curve Discrete Logarithm Problem (ECDLP) over ECC-256. Moreover, it is assumed that the adversary cannot access the secure internal memory of honest IoT devices.

A stronger adversarial model is also considered, in which long-term secrets from the server side may become exposed. In such a setting, the focus shifts to assessing whether past session keys remain confidential. These assumptions are particularly relevant for environments that require session-level security guarantees even under partial compromise.

The IoT devices are assumed to be constrained in terms of processing power and storage, and the initial registration phase is performed in a trusted and secure environment. Communications take place over insecure and asynchronous wireless networks. To maintain freshness and prevent replay without requiring strict time synchronization, we assume the use of a monotonically increasing counter (cpt) shared between the device and server. The server is trusted and possesses sufficient computational capacity to manage authentication requests from numerous devices simultaneously. This threat model establishes the foundation upon which the protocol's security properties—such as resistance to replay, impersonation, and man-in-the-middle attacks—will be rigorously evaluated in the subsequent formal and informal analysis sections.

7 Security Analysis of the Proposed Authentication

In this section, we aim to rigorously demonstrate the resilience of our proposed secure ECC-based authentication scheme against a range of prevalent attacks targeting the Internet of Things (IoT). To accomplish this, we will employ both informal and formal security analysis methodologies. Specifically, we will leverage the Automated Validation of Internet Security Protocols and Applications (AVISPA) tool and Burrows–Abadi–Needham (BAN) logic to thoroughly validate and verify our scheme's security properties. Through these methods, we will comprehensively assess the scheme's ability to withstand potential security threats.

7.1 Informal Security Analysis

This subsection provides an informal security analysis of our authentication scheme, demonstrating its resilience against several common threats: replay attacks, impersonation attacks, and man-in-the-middle attacks. Additionally, we discuss how the scheme ensures Perfect Forward Secrecy. We will show how our scheme effectively mitigates these threats, ensuring robust and secure authentication.

7.1.1 Device Anonymity

Threat: Tracking a specific IoT device over multiple sessions.

Mitigation: In the logging and authentication phases, it is essential for the server to identify which device is attempting to log in, enabling the authentication process based on the corresponding data of that identity. Some existing schemes [16] transmit the hash value of the device ID $(h(ID_i))$. While this

technique obscures the device's true identity, it still allows an adversary to track and gather information related to the same device whenever the same hashed ID is observed. This could enable the adversary to mount attacks, such as offline password guessing attacks, to deduce the original device ID or other sensitive information. In [15], the authors propose an approach to achieve anonymity by defining $PID_i = (X_i \oplus ID_i)$ during the registration phase. In the login and authentication phase, a random number N_1 is selected to compute $P_1 = N_1 \times G$, where G is the generator point of the curve. Subsequently, $PPID_i = PID_i \times P_1$ and P_1 are transmitted to the server. The server identifies the logging device by finding the corresponding record PID_i that relates to $PPID_i$ in its database. This is done by checking each existing PID_i record to verify if $PPID_i \stackrel{?}{=} PID_i \times P_1$. While this procedure effectively obscures the identity and achieves device anonymity, it incurs significant computational costs. In the proposed scheme, the device D_i sends $H_i = h(ID_i||cpt)$ to the server, which allows the server to retrieve the corresponding records from its database. Importantly, this value is never retransmitted by the device because the counter (cpt) is incremented with each successful authentication, leading to a new H_i value every time. It is worth noting that even small changes in the counter (cpt) result in a completely different H_i , a property ensured by the cryptographic hash function used. As a result, even if an adversary intercepts H_i , they cannot track the device across multiple sessions, thereby preserving the device's anonymity in our proposed scheme.

Effectiveness: Ensures unlinkability and anonymized authentication.

7.1.2 Perfect Forward Secrecy

Threat: Recovering past session keys after long-term key compromise.

Mitigation: The proposed authentication protocol ensures resilience against attacks targeting perfect forward secrecy (PFS) by protecting past session keys even if long-term keys are compromised. The protocol uses a unique random value V_r generated for each session, which is crucial for deriving session-specific values P_d , A_1 , and A_2 , all of which contribute to computing the session key. Since V_r is neither stored nor transmitted, an attacker cannot recover past V_r values or session keys, even if they possess the server's private key X and stored values P_s and ID_i . Additionally, new values P_d , A_1 , and A_2 cannot be used to derive or recompute older ones or V_r . Consequently, the proposed method ensures that each session key is independently derived and isolated, maintaining the security of past communications.

Effectiveness: Achieves strong session isolation and confidentiality over time.

7.1.3 Replay Attack

Threat: Reusing previously captured messages to gain unauthorized access.

Mitigation: Our authentication scheme is secured against replay attacks through the use of a counter (cpt) that is incremented with each authentication attempt. During the login phase, both the device and the server update their counters, which are then incorporated into the computation of the messages P_d and A_2 . This ensures that each session is uniquely identified by the current counter value. If an attacker intercepts and replays P_d and A_2 , the server's counter will not match the outdated counter value used in the replayed messages, resulting in a mismatch during the verification and subsequent rejection of the replayed attempt. Thus, the counter mechanism effectively invalidates replayed messages, providing robust security against replay attacks.

Effectiveness: Replay attempts are rejected due to counter mismatch.

7.1.4 Impersonation Attack

Threat: An attacker impersonates a legitimate device or server.

Mitigation: In our scheme, an attacker cannot impersonate either the device or the server due to the reliance on specific cryptographic values and session-specific information. To impersonate the device, the attacker would need the legitimate user's ID_i and the point P_s , a valid random number V_r , and the current counter value cpt, which are unknown to them. Consequently, the attacker cannot compute the correct P_d or generate a valid A_2 . To impersonate the server, the attacker would need to generate a valid A_1 , which requires knowledge of the server's secret values and the correct computation of P_d . Without these, the attacker cannot produce a valid response to the device's challenge. Thus, our scheme effectively prevents both device and server impersonation, ensuring secure authentication.

Effectiveness: Prevents both client-side and server-side impersonation through cryptographic binding of session-specific secrets.

7.1.5 Man-in-the-Middle Attack

Threat: Intercepting, modifying, or forging messages between device and server.

Mitigation: A Man-in-the-Middle (MitM) attack occurs when an attacker secretly intercepts and possibly alters the communication between two parties to gain unauthorized access or manipulate the data. For MitM attacks, the scheme's use of random values and counters ensures that each session is unique and secure. During the exchange, the counter cpt and random number V_r are securely embedded in an elliptic curve point P_d and incorporated in the hash value A_2 . Similarly, A_1 securely embeds V_r with the help of the shared secret point P_s . This ensures the integrity and confidentiality of the exchanged values. An attacker cannot alter or forge the messages without being detected, as they would need the same cryptographic keys and session-specific information. Thus, our scheme effectively prevents MitM attacks.

Effectiveness: Ensures message integrity and authenticity under active adversary conditions.

As shown in Table 4, the comparative analysis of the proposed scheme with other relevant schemes demonstrates comprehensive security coverage, outperforming other schemes in several critical areas. This comprehensive security performance underlines the effectiveness and robustness of the proposed scheme in various attack scenarios.

7.2 Formal Security Analysis

This subsection demonstrates the formal security analysis of the proposed methods utilizing the AVISPA tool and BAN Logic.

7.2.1 AVIPSA Tool

The Automated Validation of Internet Security Protocols and Applications tool is a comprehensive framework designed to rigorously assess network protocol security against well-known attack vectors. In our evaluation, we explicitly analyzed the resilience of the proposed protocol against Man-in-the-Middle (MITM) attacks, replay attacks, interception, and message modification attacks.

Utilizing the High-Level Protocol Specification Language (HLPSL), AVISPA allows for precise specification of protocol participants' actions and interactions. HLPSL, a role-based language, enables detailed specification of the behavior and roles of protocol entities. This high-level description is translated into an Intermediate Format (IF), which serves as the input for AVISPA's backend analyzers. This translation is essential for systematically analyzing the protocol's security properties.

	Chang et al. [12]	Wang et al. [13]	Rostampour et al. [15]	Panda and Chattopadhyay. [16]	Zhu et al. [17]	Proposed scheme
Impersonation attack	X	X	X	√	_	√
Replay attack	X	X	X	\checkmark	\checkmark	\checkmark
MITM attack	X	X	X	_	X	\checkmark
Forward secrecy	X	X	\checkmark	\checkmark	\checkmark	\checkmark
Mutual authentication	X	\checkmark	\checkmark	\checkmark	\checkmark	\checkmark
Dos attack	\checkmark	\checkmark	√	X	\checkmark	√
Anonymity	X	X	✓	X	✓	✓
Stolen verifier	X	X	✓	X	_	✓

Table 4: Security properties comparison of our proposed scheme with other relevant schemes (\checkmark : Secure, X: Insecure).

The backend of the AVISPA tool comprises four main components, each using different methodologies to analyze the protocol. These are the SAT-based Model-Checker (SATMC), which uses satisfiability-solving techniques; the Tree-Automata-based Protocol Analyzer (TA4SP), which employs tree automata; the Onthe-Fly Model-Checker (OFMC), which performs dynamic analysis; and the Constraint Logic-based Attack Searcher (CL-AtSe), which uses constraint-solving techniques to identify potential attacks. For protocols involving XOR operations, such as ours, OFMC and CL-AtSe are particularly suitable, as SATMC and TA4SP do not support XOR operations.

The AVISPA tool produces an Output Format (OF) that provides a detailed security analysis of the protocol. A result indicating that the protocol is SAFE confirms its robustness against specified attacks, such as MITM and replay attacks, thereby validating the protocol's security properties.

In our proposed protocol, we utilized HLPSL to define the protocol's roles, environment, and security objectives as depicted in Fig. 4. This includes specifying the actions and interactions of each participant, setting up instances of each role, constructing the entire protocol session, and establishing goals such as confidentiality to ensure sensitive information remains protected, and authentication to verify the legitimacy of the data exchanged between entities.

The security goals are integral to the protocol's design and are articulated using HLPSL constructs such as **secrecy** and **authentication**. These constructs ensure that secret values remain undisclosed and unauthorized access to sensitive information is prevented, while also validating the legitimacy of the entities involved.

As illustrated in Fig. 5, the OFMC and CL-AtSe backends verified that the protocol is **SAFE** under the specified analysis conditions. The OFMC backend's statistics demonstrate its efficiency by detailing the parse time, search time, visited nodes, and search depth. Meanwhile, the CL-AtSe backend provides metrics on the number of analyzed and reachable states. These results indicate that our protocol has undergone a comprehensive analysis and has been deemed secure against the potential threats evaluated by the AVISPA tool.

```
role Device_A(A:agent,B:agent,G:text,Vr:text,H:hash_func,SND,RCV:channel(dy))
played by A
def=
              local
                           State:nat,cpt:nat,Kb:public_key,K:text,Exp:text,X:text,Secret:text
              init
                          State := 0
              transition
                           1. State=0 /\ RCV(start) =|> State':=1 /\ SND(A)
                          2. State=1 /\ RCV({H(X'.Exp'.A).exp(K',G)}_inv(Kb')) =|> State':=2 /\
                                     SND\left(H\left(A^{cpt}\right),\left\{xor\left(xor\left(A,Vr^{\prime}\right).cpt\right).H\left(X^{\prime}.Exp^{\prime}.A\right).exp\left(K^{\prime},G\right)\right\}_{inv}\left(Kb^{\prime}\right)\right\}
                           4. State=2 /\ RCV({xor({{xor(A,Vr).{H(X.Exp.A).exp(K,G)}_inv(Kb)}_inv(Kb).{H(X.Exp.A)
                                      .exp(K,G)]_inv(Kb), inv(Kb), xor(A.cpt)).{H(X.Exp.A).exp(K,G)}_inv(Kb)}_inv(Kb))=>
              State':=3 /\ Secret':=new() /\ secret(Secret',sec_2,{A,B}) /\
               SND\left(\left\{Secret'\right\}_H\left(\left\{xor\left(\left\{xor\left(A,Vr\right).\left\{H\left(X.Exp.A\right).exp\left(K,G\right)\right\}_inv\left(Kb\right)\right\}_inv\left(Kb\right).\left\{H\left(X.Exp.A\right)\right\}_inv\left(Kb\right)\right\}_inv\left(Kb\right)\right\}_inv\left(Kb\right)\right\}_inv\left(Kb\right)\right\}_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb\right)_inv\left(Kb
              . \exp(K,G) \}_{inv}(Kb) \}_{inv}(Kb) , A) . \{H(X.Exp.A) . \exp(K,G) \}_{inv}(Kb) \}_{inv}(Kb) . \{H(X.Exp.A) . \exp(K,G) \}_{inv}(Kb) \}_{inv}(Kb) . A) . A)
              .exp(K,G)}_inv(Kb)}_inv(Kb))) /\ request(B,A,Kb,Secret)
end role
role Device B(A:agent,B:agent,G:text,R:text,K:text,H:hash func,X:text,Exp:text,SND,RCV:channel(dy))
played by B
def=
             local
                          State:nat.cpt:nat.Kb:public key.Vr:text.Secret:text
                           State := 0
              transition
                          1. State=0 /\ RCV(A) =|> State':=1 /\ Kb':=new() /\ SND({H(X.Exp.A).exp(K,G)} inv(Kb'))
                          3. State=1 /\ RCV(H(A ^ cpt), {xor(xor(A,Vr').cpt).{H(X.Exp.A).exp(K,G)}_inv(Kb)}_inv(Kb))
                                      /\ SND({xor({{xor(xor(A,Vr').cpt).{H(X.Exp.A).exp(K,G)}_inv(Kb).}_inv(Kb).{H(X.Exp.A)
               . \exp(K,G) \}_{inv}(Kb) \}_{inv}(Kb) , xor(A.opt)) . \{H(X.Exp.A) . \exp(K,G) \}_{inv}(Kb) \}_{inv}(Kb))
                            5.  State=2 \ / \ RCV(\{Secret'\}_H(\{\{xor(A,Vr).\{H(X.Exp.A).exp(K,G)\}_inv(Kb)\}_inv(Kb)\}_inv(Kb)) \} 
                                        . \{H(X.Exp.A).exp(K,G)\}_inv(Kb)\}_inv(Kb), A). \{H(X.Exp.A).exp(K,G)\}_inv(Kb)\}_inv(Kb)
                                        . \{ \texttt{H(X. Exp.A).exp(K,G)} \} \underline{inv(Kb)} \underline{inv(Kb)}) ) > \texttt{State'=3 /} \texttt{ secret(Secret', sec_2, \{A,B\}) /} 
                                      witness (B, A, Kb, Secret)
end role
```

Figure 4: HLPSL Code Specification for our proposed authentication protocol analyzed by the AVISPA tool.

```
% ATSE
% OFMC
                                             % Version of 2006/02/13
% Version of 2006/02/13
                                            SUMMARY
SUMMARY
                                             SAFE
SAFE
                                            DETAILS
DETAILS
                                             BOUNDED_NUMBER_OF_SESSIONS
BOUNDED_NUMBER_OF_SESSIONS
                                             TYPED_MODEL
/home/span/span/testsuite/results/avispa-test-YJ.if
                                            PROTOCOL
GOAL
                                             /home/span/span/testsuite/results/avispa-test-YJ.if
as_specified
                                            GOAL
BACKEND
                                             As Specified
OFMC
COMMENTS
                                            BACKEND
STATISTICS
                                             CL-AtSe
parseTime: 0.00s
searchTime: 0.01s
                                            STATISTICS
                                             Analysed: 10 states
visitedNodes: 16 nodes
                                             Reachable: 6 states
depth: 7 plies
```

Figure 5: Analysis result of OFMC and CL AtSc security checkers on our protocol.

7.2.2 BAN Logic

SK

To complement the automated validation, we employed BAN logic to formally reason about the mutual authentication, session key establishment, and message integrity properties of our protocol.

We present the constructs and primary postulates of BAN logic in Tables 5 and 6, respectively.

	-
Construct	Description
$P \equiv X$	<i>P</i> believes that <i>X</i> is true
$P \vartriangleleft X$	P sees X and is capable of reading and repeating X after decryption
$P \sim X$	P once said X
$P \Rightarrow X$	P has jurisdiction over X
$\sharp(X)$	X is fresh
(X,Y)	<i>X</i> and <i>Y</i> are part of a single statement
$\langle X \rangle_Y$	X is combined with Y
$\{X\}_K$	X is encrypted under the key K
$(X)_K$	X is hashed using the key K
$P \stackrel{K}{\longleftrightarrow} Q$	P and Q share the key K
$P \stackrel{X}{\rightleftharpoons} Q$	X is a secret shared between P and Q
$K^+ \mapsto P$	P has a public key K^+

Table 5: Constructs used in BAN logic.

Table 6: Primary postulates of BAN logic.

Session key

Postulate	Description
Message meaning rule	$\frac{P \equiv P\stackrel{K}{\rightleftharpoons}Q, P\vartriangleleft\langle X\rangle_{K}}{P \equiv Q \sim X}$
Freshness concatenation rule	$\frac{P \equiv \sharp(X)}{P \equiv \sharp(X,Y)}$
Belief rule	$\frac{P \equiv(X), P \equiv(Y)}{P \equiv(X, Y)}$
Nonce verification rule	$\frac{P \equiv\sharp(X), P \equiv Q \sim X}{P \equiv Q \equiv X}$
Jurisdiction rule	$\frac{P \equiv Q \Rightarrow X, P \equiv Q \equiv X}{P \equiv X}$
Session key rule	$\frac{P \equiv \sharp(X), P \equiv Q \equiv X}{P \equiv P\overset{K}{\longleftrightarrow} Q}$

The process to prove the authentication protocol using BAN logic is as follows:

- 1. Define the goals, which need to be achieved by the authentication system.
- 2. The proposed authentication protocol should be idealized in the formal language of BAN logic.
- 3. Set the initial state of the protocol by mentioning the various assumptions.

4. Apply BAN logic constructs and postulates to prove or achieve the goals set in the first step. The proof of the proposed authentication protocol using BAN logic is described as follows: **Goals:**

Goal 1:
$$S \equiv S \stackrel{SK}{\longleftrightarrow} D_i$$

Goal 2:
$$S| \equiv D_i | \equiv S \stackrel{\mathcal{SK}}{\longleftrightarrow} D_i$$

Goal 3:
$$D_i | \equiv S \stackrel{\mathcal{SK}}{\longleftrightarrow} D_i$$

Goal 4:
$$D_i | \equiv S | \equiv S \stackrel{SK}{\longleftrightarrow} D_i$$

—The idealized form of the authentication phase of the proposed protocol is given by:

M1:
$$D_i \rightarrow S : \{ID_i, P_d, A_2\}$$

M2:
$$S \to D_i : \{P_s, A_1\}$$

—The following are the assumptions made to achieve the defined goals:

A1:
$$S \equiv \sharp(V_r)$$

A2:
$$D_i = \sharp(h(V_1))$$

A3:
$$S| \equiv S \stackrel{P_s}{\longleftrightarrow} D_i$$

A4:
$$D_i | \equiv S \stackrel{P_d}{\longleftrightarrow} D_i$$

A5:
$$S| \equiv D_i \Rightarrow V_r$$

A6:
$$S| \equiv D_i \Rightarrow V_1$$

A7:
$$S| \equiv S \stackrel{\mathcal{SK}}{\longleftrightarrow} D_i$$

A8:
$$D_i | \equiv S \stackrel{\mathcal{SK}}{\longleftrightarrow} D_i$$

A9:
$$S| \equiv D_i | \equiv S \stackrel{SK}{\longleftrightarrow} D_i$$

—The above-mentioned postulates and assumptions are applied to the idealized form: M1 and M2 to achieve the defined goals as follows.

M1:
$$D_i \rightarrow S : \{ID_i, P_d, A_2\} : \{ID_i, V_1, A_2\}$$

According to the assumption A1 and the freshness concatenation rule:

$$S| \equiv \sharp (\{ID_i, V_1, A_2\})$$

Using the assumptions A4, A6, and the message meaning rule:

$$|S| \equiv D_i | \sim (\{ID_i, V_1, A_2\})$$

Using the assumptions A5 and A9 and the jurisdiction rule:

$$|S| \equiv h(V_1')$$

M2:
$$S \to D_i : \{P_s, A_1\}$$

Using the assumptions A2, A3, and the nonce verification rule:

$$D_i|\equiv S|\equiv h(V_1')$$

According to the session key rule:

$$D_i | \equiv D_i \stackrel{\mathcal{SK}}{\longleftrightarrow} S$$

From the goal definition:

$$D_i | \equiv S \stackrel{\mathcal{SK}}{\longleftrightarrow} D_i$$

This completes the proof. The BAN logic analysis confirmed that both communication parties can trust the established session key and are mutually authenticated, reinforcing the results obtained through AVISPA. This dual-layer verification provides strong assurance of the protocol's ability to withstand critical security threats in IoT environments.

8 Performance Analysis

The proposed ECC-based authentication protocol was rigorously evaluated to assess its computational efficiency, communication overhead, storage requirements, and security robustness. To ensure consistency and rigor in the evaluation, the proposed protocol employs a 256-bit elliptic curve (ECC-256) alongside SHA-256 for cryptographic operations. The selection of SHA-256 is based on recommendations from NIST, which highlight its suitability for IoT environments due to its optimal balance between computational efficiency and cryptographic strength, particularly in resource-constrained devices. Although NIST has not deprecated SHA-256 in favor of SHA-3—both being considered secure—SHA-256 remains the most practical and efficient option for the majority of current IoT deployments [29]. Similarly, all relevant recent protocols included in the comparative analysis were standardized to use ECC-256 and SHA-256. When evaluating computational complexity, lightweight operations such as exclusive OR (XOR) and string concatenation were excluded.

8.1 Experimental Setup

The results presented here were obtained following the implementation of the protocols on a system with the following specifications as shown in Table 7: Ubuntu 16 and Python 2.6 in a virtual machine (VM) adapted to the Charm framework. Web sockets were employed for communication between the embedded device *D* and the server *S*. All experiments were conducted on a MacBook Air with a 1.6 GHz Intel i5 processor, 8 GB of RAM, running macOS 12.2. Notably, all experiments were performed on a single core of the processor.

Table 7: Experimental setup.

Configuration	Type
Computing Environment	Desktop, Server
Server Hardware	1.6 GHz Intel Core i5 dual-core
Server Primary Memory	8 GB 1600 MHz DDR3
Server Operating System	Ubuntu 16.04, 64 bits
Web Socket	V10.3

(Continued)

Configuration	Туре
Framework	CHARM-CRYPTO v0.40

8.2 Computational Cost

In our analysis of the computation cost, we define the execution times of various cryptographic operations as follows:

- T_p : Time required for a point multiplication operation.
- T_{Mapp} : Execution time for mapping and processes using ECC.
- T_{Rev} : Execution time for reverse processes using ECC.
- T_{\oplus} : Duration needed to perform an XOR operation.
- T_{\parallel} : Time taken for data concatenation.
- T_h : Execution time for performing a hash operation.

Our experimental findings indicate the following execution times for T_p , T_{Mapp} , T_{Rev} , and T_h : 0.0326, 0.0001347, 0.00000905, and 0.0074 s, respectively. During the execution of the protocol, the registration phase incurs a computation cost of $2T_h + T_p + T_{Mapp}$. In the login and authentication phase, the session key $S_k = KDF(V_r)$ is treated as a hash function due to its fixed output length and deterministic nature. Consequently, the computation cost for this phase is $6T_h + 2T_{Mapp} + 2T_{Rev}$. Therefore, the overall computational cost of the proposed protocol is $8T_h + T_p + 3T_{Mapp} + 2T_{Rev}$, which corresponds to approximately 92.22 ms. A detailed breakdown of the computational costs is provided in Table 8.

Scheme Login and authentication phase Registration phase **Total** Chang et al. [12] 53.4174 153.0011 206.4185 Wang et al. [13] 54.8872 128.214 183.1098 Rostampour et al. [15] 318.3577 62.2872 256.0705 Panda and Chattopadhyay [16] 12.5998 316.3012 328.901 Zhu et al. [17] 58.1685 91.1478 149.3163 Ours 47.53 44.68 92.22

Table 8: The comparison of computation overhead (ms).

Based on the comparison Table 8, our proposed protocol demonstrates a significant improvement in computational efficiency relative to other recent protocols. With a total computation overhead of 92.22 ms, our protocol is markedly more efficient than those listed in the comparison. Specifically, it achieves a reduction of approximately 61.1% compared to the average total computation cost of the other protocols.

The registration phase of our protocol incurs a cost of 47.53 ms, which is more efficient compared to some other protocols. Notably, the efficiency gains are particularly evident in the login and authentication phase, where our protocol's computation overhead is only 44.68 ms, significantly lower than that of other protocols. For example, the protocol by [16] has a computation cost of 316.3012 ms, and Ref. [15] shows 256.0705 ms for the same phase.

This reduction is primarily attributable to our protocol's design, which incorporates only a single point multiplication T_p in the registration phase, unlike other protocols that may involve more complex operations.

This efficiency is further illustrated in Fig. 6, which shows that our protocol achieves the lowest computation overhead values in both the registration and authentication phases. This reduced overhead is crucial for time-sensitive applications, as it enables faster and more responsive authentication processes. In comparison, other schemes exhibit higher computation overheads, which can result in delays or increased resource consumption. Thus, the streamlined design of our protocol not only improves computational efficiency but also makes it a more optimal choice compared to existing solutions.

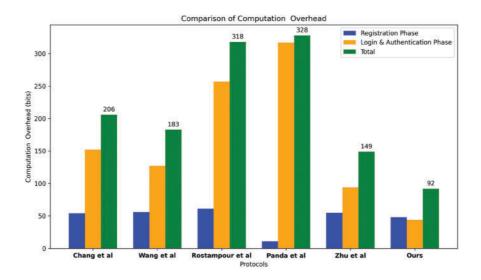


Figure 6: Performance comparison of different schemes in terms of computational overheads: Chang et al. [12], Wang et al. [13], Rostampour et al. [15], Panda and Chattopadhyay [16], and Zhu et al. [17].

8.3 Communication Overhead

The communication overhead was assessed by measuring the size of the authentication messages exchanged between IoT devices and the server during the registration, login, and authentication phases. In our proposed authentication protocol, the data transmitted between the device and the server consists of three types of messages:

- 1. **Registration Phase:** The message msg1 includes ID and P_s . The size of ID used in our protocol is 160 bits. The size of P_s , representing the elliptic curve point (x_s, y_s) , is 512 bits (256 bits for each coordinate). Therefore, the total communication cost for this phase is:
- msg1 = 160 bits + 512 bits = 672 bits.
- 2. **Login and Authentication Phase:** Two messages are transmitted:
- $-msg2 = h(ID||cpt) + P_d$, where h(ID||cpt) is a hash function output of 256 bits (using SHA-256) and P_d is an elliptic curve point of 512 bits. Thus, the total size of msg2 is 256 bits + 512 bits = 768 bits.
- $-msg3 = A_1 + A_2$, where A_1 and A_2 are both elliptic curve points of 512 bits each. Therefore, the total size of msg3 is 512 bits + 512 bits = 1024 bits.

The total communication overhead for both phases is the sum of the sizes of all transmitted messages: msg1 + msg2 + msg3 = 672 bits + 768 bits + 1024 bits = 2464 bits.

Based on the comparison Table 9, our proposed protocol exhibits superior performance in terms of communication overhead compared to other recent protocols. Specifically, our protocol achieves the lowest registration phase communication overhead, totaling 672 bits. This is notably more efficient than the protocol by [13], which has a registration phase overhead of 768 bits.

Scheme	Registration phase	Login and authentication phase	Total
Chang et al. [12]	1024	2560	3584
Wang et al. [13]	768	2304	3072
Rostampour et al. [15]	3072	768	3840
Panda and Chattopadhyay [16]	1280	2304	3584
Zhu et al. [17]	1280	1792	3072
Ours	672	1792	2464

Table 9: The comparison of communication overhead (bits).

In the login and authentication phase, our protocol's communication overhead is 1792 bits, which aligns with the protocol size presented by [17]. However, our protocol achieves a lower total communication overhead of 2464 bits, compared to 3072 bits in theirs. This represents a reduction of approximately 19.8% in total communication cost when compared to the protocol by [17].

Overall, the reduced registration phase overhead and competitive login and authentication phase size contribute to the lower total communication cost of our protocol. This reduction in message size is critical for IoT environments, where bandwidth and energy efficiency are of utmost importance as illustrated in Fig.7, making our protocol a more efficient choice relative to existing solutions.

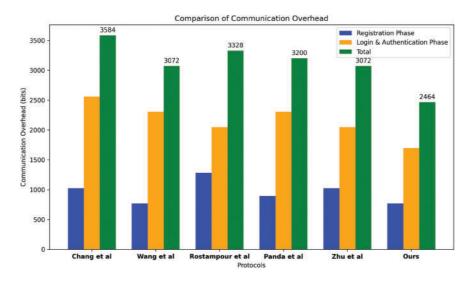


Figure 7: performance of different schemes in terms of communication overheads: Chang et al. [12], Wang et al. [13], Rostampour et al. [15], Panda and Chattopadhyay [16], and Zhu et al. [17].

8.4 Storage Cost

For our proposed protocol, the storage cost is analyzed in two phases: the registration phase and the login and authentication phase.

1. Registration Phase:

- *Device*: During the registration phase, the device stores the identity ID (160 bits) and the value of P_s (512 bits) received from the server, resulting in a total storage cost of **672** bits.
- *Server*: Concurrently, the server stores the values of P_s (512 bits), ID (160 bits), and EXP_time (160 bits) in its database, totaling **832** bits.

2. Login and Authentication Phase:

- Device: In this phase, the device temporarily stores the values of V_r (160 bits), V_1 (160 bits), and S_k (256 bits), leading to a total temporary storage cost of **576**bits.
- Server: The server temporarily stores the values P_s (512 bits), P_d (512 bits), V_r' (512 bits), ID (160 bits), h(ID||cpt) (256 bits), A_2 (256 bits), V_r (512 bits), and S_k (256 bits), leading to a total storage cost of **2976** bits.

The overall total storage cost for both the device and the server across all phases is 5056 bits.

However, in the relevant literature [13,16,17], the focus is typically on the storage cost of the device rather than the server, due to the limited memory capacity of IoT devices compared to the vast storage resources of cloud servers. Therefore, to ensure a fair and accurate comparison with other protocols, we consider only the device's storage cost in our analysis. The detailed results, highlighting the device's storage requirements, are presented in Table 10.

Scheme	Registration phase	Login and authentication phase	Total
Chang et al. [12]	1024	1280	2304
Wang et al. [13]	768	1280	2048
Rostampour et al. [15]	768	768	1536
Panda and Chattopadhyay [16]	1536	1793	3329
Zhu et al. [17]	768	1280	2048
Ours	672	576	1248

Table 10: Comparison of device storage overhead (bits)

As shown in Fig. 8, our proposed protocol offers notable improvements in device storage overhead compared to other recent protocols, both during the registration and login/authentication phases. The reduction in storage requirements is critical for IoT devices, where memory capacity is typically limited. This efficiency allows our protocol to effectively minimize storage usage while maintaining robust security and functionality

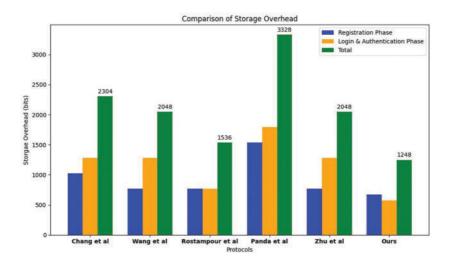


Figure 8: performance of different schemes in terms of device storage overheads: Chang et al. [12], Wang et al. [13], Rostampour et al. [15], Panda and Chattopadhyay [16], and Zhu et al. [17].

The evaluation of our proposed authentication protocol highlights its significant advantages in both security and lightweight performance compared to existing protocols. By effectively reducing communication, computation, and storage overheads, our scheme not only ensures robust protection against a variety of attacks but also maintains user anonymity and service availability. These attributes make it particularly well-suited for secure authentication in practical deployment scenarios.

Furthermore, the protocol's efficiency in resource-constrained environments, achieved through substantial reductions in overheads, underscores its practicality. This balance of security and performance positions our protocol as an optimal solution for applications where both resource efficiency and strong security measures are critical, making it a superior choice for modern authentication needs.

9 Conclusion and Perspectives

This article introduces a novel ECC-based authentication protocol for IoT designed to enhance security while optimizing performance. Through the application of ECC, our protocol achieves superior security with reduced computational, communication, and storage overheads. A thorough security analysis, conducted both informally and formally using AVISPA and BAN logic, confirms the protocol's resilience against various attacks.

Our performance evaluation underscores the protocol's efficiency, demonstrating significantly lower computational, communication, and storage costs compared to existing relevant authentication protocols. These findings highlight that our protocol is not only secure but also highly practical for real-world applications, particularly in IoT environments where resource constraints are a significant concern.

Future work will focus on applying our authentication protocol in real-world scenarios to validate its practical effectiveness and reliability. Additionally, we aim to extend our protocol for integration with blockchain technology, leveraging its decentralized nature to further enhance security and trust in various applications. By continually refining and adapting our approach, we aim to advance secure and efficient authentication mechanisms in an increasingly interconnected digital landscape.

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