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Heat Transfer Area Optimization for Heat Exchanger System

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ABSTRACT: This paper presents an allowable-tolerance-based group search optimization (AT-GSO), which combines the robust GSO (R-GSO) and the external quality design planning of the Taguchi method. AT-GSO algorithm is used to optimize the heat transfer area of the heat exchanger system. The R-GSO algorithm integrates the GSO algorithm with the Taguchi method, utilizing the Taguchi method to determine the optimal producer in each iteration of the GSO algorithm to strengthen the robustness of the search process and the ability to find the global optima. In conventional parameter design optimization, it is typically assumed that the designed parameters can be applied accurately and consistently throughout usage. However, for systems that are sensitive to changes in design parameters, even minor inaccuracies can substantially reduce overall system performance. Therefore, the permissible variations of the design parameters are considered in the tolerance-optimized design to ensure the robustness of the performance. The optimized design of the heat exchanger system assumes that the system's operating temperature parameters are specific. However, fixing the system operating temperature parameters at a constant value is difficult. This paper assumes that the system operating temperature parameters have an uncertainty error when optimizing the heat transfer area of the heat exchanger system. Experimental results show that the AT-GSO algorithm optimizes the heat exchanger system and finds the optimal operating temperature in the absence of tolerance and under three tolerance conditions.

KEYWORDS: Heat exchanger system; allowable tolerance-based group search optimization; Taguchi method; tolerance design

1 Introduction

Using tolerance optimization to design systems susceptible to parameter variations is very important in parameter design optimization. Traditional parameter design often assumes that parameters can be precisely realized and remain constant during operation. However, this ignores the fluctuations and uncertainties that may exist in real-world environments. In this case, even minor deviations can lead to significant degradation of system performance. Therefore, the purpose of tolerance-optimized design is to consider these variations and ensure the system remains robust within the permitted range.

Two types of tolerance design methods are commonly used to deal with changes in operating parameters and parameter specifications: robust parameter design and optimization design methods. Robust parameter design minimizes external factors' effects on product quality so that the product remains of high quality even under less-than-ideal environmental conditions [1]. The Taguchi method is a method to obtain robust parameters in heat exchanger systems [2-4]. However, this method only considers tolerance design. Although robust parameter values can be obtained, it may not be possible to ensure that the system's



overall performance is optimal. Optimization design methods use optimization algorithms to find the best parameter values to improve the system performance [5]. However, they usually ignore the robustness of these parameters to external variations or errors, resulting in a lack of robustness of the best parameters in a fluctuating environment.

A heat exchanger system is used to transfer heat between fluids of different temperatures. This system is commonly used in petroleum, chemical, metallurgy, power, light, food, and other industries [6]. The most common criteria for heat exchanger optimization are minimum initial cost, minimum operating cost, maximum efficiency, minimum pressure drop, minimum heat transfer area, and minimum weight or material [7]. Venkatesh et al. used a multi-objective genetic algorithm to solve the complex decision problem of cost, thermal resistance, and ultimate utility of heat exchanger systems [8]. Bianco et al. used a multi-objective genetic algorithm to minimize the heat transfer area of heat exchanger systems [9]. Yang et al. used an improved stochastic ranking evolutionary strategy algorithm to minimize the heat transfer area of heat exchanger systems [10]. Rani et al. used multi-objective optimization of opposition learning with beetle swarm algorithm to design a heat exchanger system to reduce five objective functions: total heat transfer rate, total weight, total mass flow rate, number of entropy generating units, and whole annual cost [11]. Gawai et al. used a modified, amended differential evolution algorithm to minimize the heat transfer area of the heat exchanger system [12]. Bakr et al. applied a genetic algorithm to optimize the design of a shell-and-tube heat exchanger to improve its thermal performance by maximizing heat transfer efficiency while minimizing pressure drop [13]. Zhang et al. enhanced the performance of a central heating system by optimizing the distribution of heat transfer area and the mass flow rates of working fluids, aiming to maximize the temperature of the cold fluid as the optimization objective [14]. Kharaji used constrained optimization algorithms to minimize heat transfer areas in shell-and-tube heat exchangers [7]. Shafiey Dehaj et al. applied a genetic algorithm to reduce heat transfer areas and improve the effectiveness of fin and tube heat exchangers [15]. Wu et al. used a genetic algorithm to design the spiral-wound heat exchanger and minimize its heat transfer areas [16]. From the survey of existing literature, many scholars have presented the heat exchanger system for minimizing heat transfer area. However, they assumed no uncertainty in the system's operating temperature parameters. Therefore, many scholars have assumed a constant value for the system's operating temperature during the optimization process. In industry, it is difficult to fix a system's operating temperature at a constant state, and the optimal parameters obtained by assuming a constant value for the system's operating temperature parameters are bound to have uncertain errors in practical applications [17]. Temperature cannot be accurately measured or controlled at every critical design point. However, this uncontrollable factor can be considered as a disturbance factor and integrated into the overall system design process. Although group search optimization (GSO) has demonstrated numerous successful applications, it suffers from poor search performance—its low accuracy and tendency to become trapped in local optima result in poor robustness. Therefore, Yang et al. used the experimental design approach to solve this problem and verified its performance [10]. However, it lacks tolerance capabilities. Thus, this paper focuses on the optimal design of the uncertainty system operating temperature parameters of the heat exchanger system through an allowable-tolerance-based group search optimization algorithm (AT-GSO) to achieve the minimum heat transfer area. The method proposed in this paper can consider the existence of uncertainty in the system operating temperature during the parameter optimization process, so that it can be applied in practice to reduce the impact caused by the error in the system operating temperature.

2 Problem Statement and Application Methods

2.1 Problem Statement

The heat exchanger is a kind of energy-saving equipment used to achieve heat transfer between materials, but also in the development and utilization of secondary energy, heat recovery, and energy saving of the main equipment. The chemical plant utilizes waste heat from the plant, which uses three heat exchangers to heat the material temperature from 100°C to 500°C. Conventional design methods focus on the optimization of individual heat exchangers, ignoring the interaction between heat exchangers in the heat exchanger network. The superstructure of the heat exchanger system is shown in Fig. 1. This paper investigates how to optimize the system operating temperatures T_1 and T_2 to minimize the total heat transfer area of the heat exchanger system, where there are uncertainties ΔT_1 and ΔT_2 in the system operating temperature parameters.

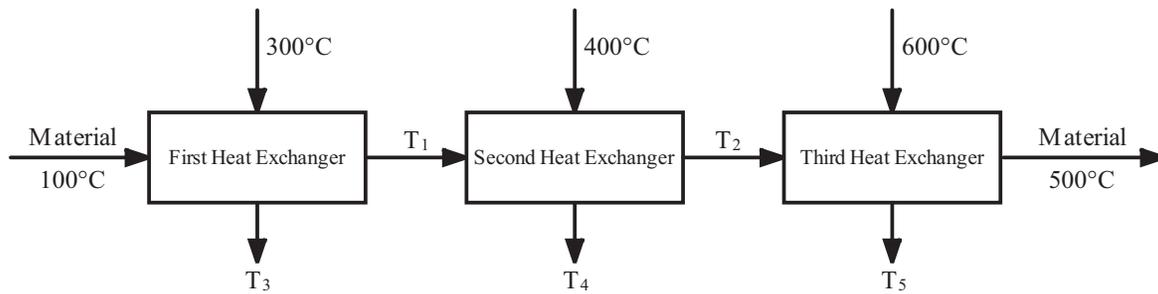


Figure 1: The heat exchanger system structure

Neglecting the change of specific heat of each material with temperature, according to the law of conservation of heat for the three heat exchangers, can be calculated as follows [18]:

$$mc_p (T_1 - 100) = mc_p (300 - T_3), \quad (1)$$

$$mc_p (T_2 - T_1) = mc_p (400 - T_4), \quad (2)$$

$$mc_p (500 - T_2) = mc_p (600 - T_5). \quad (3)$$

Assuming arithmetic mean temperature difference for each heat exchanger yields:

$$\Delta t_{m1} = \frac{(300 - T_1) + (T_3 - 100)}{2} = 300 - T_1, \quad (4)$$

$$\Delta t_{m2} = \frac{(400 - T_2) + (T_4 - T_1)}{2} = 400 - T_2, \quad (5)$$

$$\Delta t_{m3} = \frac{(600 - 500) + (T_5 - T_2)}{2} = 100. \quad (6)$$

The heat transfer area can be calculated according to the heat equation as follows:

$$A_i = \frac{Q_i}{K_i \Delta t_{mi}} = \frac{mc_p \Delta T_i}{K_i \Delta t_{mi}}, \quad (7)$$

where i is the number of the heat exchanger, A is the heat transfer area of the heat exchanger, Q is the amount of heat exchanged by the heat exchanger, K is the heat transfer coefficient of the heat exchanger, and Δt_m is the average value of temperature difference of the heat exchanger.

Thus, the single-objective optimization problem of the total heat transfer area of the heat exchanger system is described as follows [18]:

$$\text{Minimize } A = A_1 + A_2 + A_3 = \frac{100000(T_1 - 100)}{120(300 - T_1)} + \frac{100000(T_2 - T_1)}{80(400 - T_2)} + \frac{100000(500 - T_2)}{40 \times 100}, \quad (8)$$

where $100 \leq T_1 \leq 300$ and $T_1 \leq T_2 \leq 400$. The uncertainties ΔT_1 and ΔT_2 in the system operating temperature parameters is between 0% and 40%.

2.2 Allowable-Tolerance Based GSO Algorithm

The Group Search Optimization (GSO) algorithm relies on random searchers (Rangers) to conduct exploitation search operations. A single best-fit producer (Producer) is typically selected to perform exploration searches in a conventional GSO algorithm. Other group members are guided to the position with the highest fitness value through the producer-scrounger (PS) mechanism. However, selecting just one producer often causes the algorithm to get stuck in local optima, limiting its ability to explore more broadly. This paper utilizes the robust GSO (R-GSO) [10], which selects a fixed number of multiple producers combined with the Taguchi method to determine the optimal producer in each iteration to avoid trapping in local optima. The Taguchi method is a design of experiments approach rooted in statistical theory, which ensures robustness by organizing the minimal number of experiments across selected factors using an orthogonal array while accounting for uncontrollable factors in the system's environment that could affect quality characteristics [1]. The Taguchi method is used to enhance the global search capabilities of the GSO algorithm. This approach not only enhances the exploration search process but also significantly improves the robustness of solutions, expanding the search range and enhancing the algorithm's global optimization capacity. Additionally, for tolerance design optimization problems, this paper introduces the GSO algorithm with tolerance design capabilities, named AT-GSO, which merges the Taguchi method for external quality design planning with R-GSO to consider both robustness and optimal parameter design. Fig. 2 illustrates the flowchart of the AT-GSO algorithm.

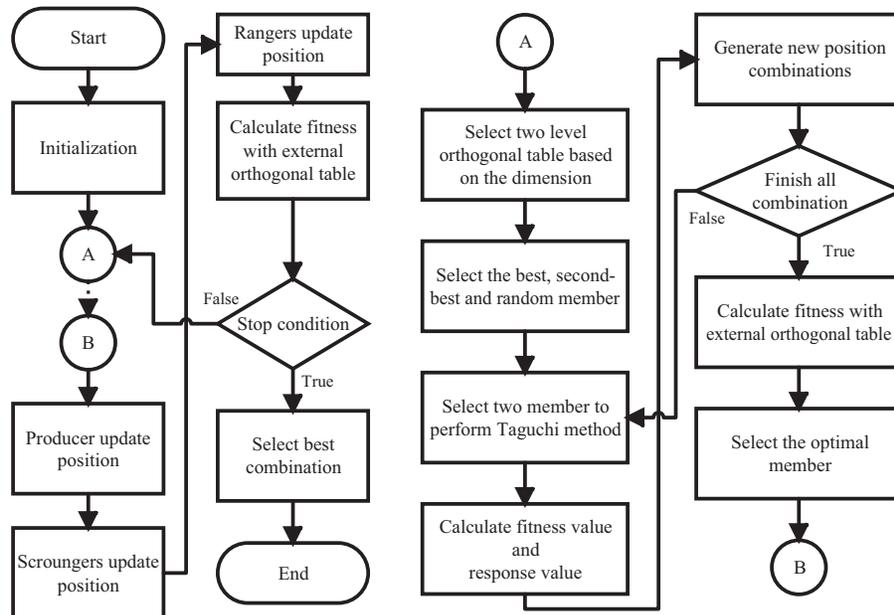


Figure 2: The flowchart of AT-GSO algorithm

Here is the process description of the AT-GSO algorithm. A parameter X_p is introduced to denote the number of producers, with a recommended range of $X_p \in [1, 10]$ in the initial setup of the AT-GSO algorithm. The AT-GSO algorithm selects the individuals with the highest two fitness scores as producers in each iteration, known as X_{p1} and X_{p2} , respectively. The remaining group members select a third producer, X_{p3} , to ensure diversity. The factor levels in the orthogonal experimental design are set by these three producers. As a result, three crossover types are chosen. X_{p1} and X_{p2} are assigned as the first and second levels in each iteration, along with combinations of X_{p1} , X_{p2} , and X_{p3} . An orthogonal table $L_m(2^{m-1})$ is selected, and three experiments are conducted for different level combinations of X_p . Where m denotes the number of rows of the orthogonal experiment; $m - 1$ denotes the number of columns of configurable decision parameters. Three new producers are generated after completing the Taguchi Method, X_{np1} , X_{np2} and X_{np3} , respectively. These new producers identify the optimal producer X_{op} for each iteration, enhancing both the robustness of the search process and the efficiency in locating global optima.

The tolerance design optimization problem is described as follows:

$$\text{Minimize } \{f(x) | x \in [l, u]\}, \text{ subject to the design constraints and the tolerance } \Delta x \text{ of } x, \tag{9}$$

where $f(x)$ is the object function, $x = (x_1, x_2, \dots, x_n) \in R^n$ is a set of vectors with n variables in the nominal value of the design, Δx is the tolerance of x , l , and u is the solution space of x .

The orthogonal table of three levels $L_m(3^{(m-1)/2})$ is used to build the external quality design planning to solve the tolerance design optimization problem, as shown in [Table 1](#).

Table 1: The external quality design planning for the tolerance design optimization problem

Level	Factor				
	x_1	x_2	x_3	...	x_n
1	$x_{1N} - \Delta x_1$	$x_{2N} - \Delta x_2$	$x_{3N} - \Delta x_3$...	$x_{nN} - \Delta x_n$
2	x_{1N}	x_{2N}	x_{3N}	...	x_{nN}
3	$x_{1N} + \Delta x_1$	$x_{2N} + \Delta x_2$	$x_{3N} + \Delta x_3$...	$x_{nN} + \Delta x_n$

The design nominal and tolerance values should be considered in practical design scenarios. This means that while achieving the optimal target value, the tolerances resulting from parameter variations can also be reduced. Thus, [Eq. \(9\)](#) is re-described as follows:

$$\text{Minimize } \{\mu(f(x \pm \Delta x)) \text{ and } \sigma(f(x \pm \Delta x))\}, \text{ subject to the design constraints} \tag{10}$$

where $\mu(f(x \pm \Delta x))$ is the average value of $f(x \pm \Delta x)$ in the tolerance experiment and $\sigma(f(x \pm \Delta x))$ is the standard deviation of $f(x \pm \Delta x)$ in the tolerance experiment.

The evolutionary algorithms assess and solve the problem based on fitness values rather than objective function values due to variations in component specifications in the optimization problems. The objective function method solely accounts for system performance $f(x)$ but neglects the effect of allowable error Δx of the design parameter x on system performance. However, the influence of the allowable error Δx in the design parameter x on system performance can be considered alongside the system's performance by applying the fitness function approach. Thus, choosing and designing system parameters using an adaptive function minimizes the sensitivity of the desired target value to changes in the specifications of specific

uncontrollable components. For the optimization problem where smaller values are preferred, Eq. (10) can be expressed using the fitness function as follows:

$$F = \mu (f (x \pm \Delta x)) + w\sigma (f (x \pm \Delta x)), \tag{11}$$

where w is the weight factor to balance between $\mu(f(x \pm \Delta x))$ and $\sigma(f(x \pm \Delta x))$.

For example, suppose each producer has seven dimensions and two levels. The orthogonal table $L_8(2^7)$ is employed, as shown in Tables 2–4. The experiment is repeated three times for each row combination of the orthogonal experiment. After the orthogonal experiment is completed, three results are obtained for each row combination. Eq. (12) identifies which level had a more significant influence on each factor and ensures robustness in the search for the optimal value.

$$E_{fj} = -10 \cdot \log((J_{i(avg)}^2/J_{Max,avg}^2) + (J_{i(std)}^2/J_{Max,std}^2)), \quad j = 1, 2, \tag{12}$$

where $J_{i(avg)}$ is the average of the three results; $J_{i(std)}$ is the standard deviation of the three results; $J_{Max,avg}$ is the maximum average in the orthogonal experiment; $J_{Max,std}$ is the maximum standard deviation in the orthogonal experiment.

Table 2: Example of crossover performed using $L_8(2^7)$ orthogonal table for X_{p1} and X_{p2}

Exp.	Factor						
	A	B	C	D	E	F	G
1	X_{p1}						
2	X_{p1}	X_{p1}	X_{p1}	X_{p2}	X_{p2}	X_{p2}	X_{p2}
3	X_{p1}	X_{p2}	X_{p2}	X_{p1}	X_{p1}	X_{p2}	X_{p2}
4	X_{p1}	X_{p2}	X_{p2}	X_{p2}	X_{p2}	X_{p1}	X_{p1}
5	X_{p2}	X_{p1}	X_{p2}	X_{p1}	X_{p2}	X_{p1}	X_{p2}
6	X_{p2}	X_{p1}	X_{p2}	X_{p2}	X_{p1}	X_{p2}	X_{p1}
7	X_{p2}	X_{p2}	X_{p1}	X_{p1}	X_{p2}	X_{p2}	X_{p1}
8	X_{p2}	X_{p2}	X_{p1}	X_{p2}	X_{p1}	X_{p1}	X_{p2}

Table 3: Example of crossover performed using $L_8(2^7)$ orthogonal table for X_{p1} and X_{p3}

Exp.	Factor						
	A	B	C	D	E	F	G
1	X_{p1}						
2	X_{p1}	X_{p1}	X_{p1}	X_{p3}	X_{p3}	X_{p3}	X_{p3}
3	X_{p1}	X_{p3}	X_{p3}	X_{p1}	X_{p1}	X_{p3}	X_{p3}
4	X_{p1}	X_{p3}	X_{p3}	X_{p3}	X_{p3}	X_{p1}	X_{p1}
5	X_{p3}	X_{p1}	X_{p3}	X_{p1}	X_{p3}	X_{p1}	X_{p3}
6	X_{p3}	X_{p1}	X_{p3}	X_{p3}	X_{p1}	X_{p3}	X_{p1}
7	X_{p3}	X_{p3}	X_{p1}	X_{p1}	X_{p3}	X_{p3}	X_{p1}
8	X_{p3}	X_{p3}	X_{p1}	X_{p3}	X_{p1}	X_{p1}	X_{p3}

Table 4: Example of crossover performed using $L_8(2^7)$ orthogonal table for X_{p2} and X_{p3}

Exp.	Factor						
	A	B	C	D	E	F	G
1	X_{p2}						
2	X_{p2}	X_{p2}	X_{p2}	X_{p3}	X_{p3}	X_{p3}	X_{p3}
3	X_{p2}	X_{p3}	X_{p3}	X_{p2}	X_{p2}	X_{p3}	X_{p3}
4	X_{p2}	X_{p3}	X_{p3}	X_{p3}	X_{p3}	X_{p2}	X_{p2}
5	X_{p3}	X_{p2}	X_{p3}	X_{p2}	X_{p3}	X_{p2}	X_{p3}
6	X_{p3}	X_{p2}	X_{p3}	X_{p3}	X_{p2}	X_{p3}	X_{p2}
7	X_{p3}	X_{p3}	X_{p2}	X_{p2}	X_{p3}	X_{p3}	X_{p2}
8	X_{p3}	X_{p3}	X_{p2}	X_{p3}	X_{p2}	X_{p2}	X_{p3}

After selecting the optimal level for all factors, a new producer vector X_{npi} ($i = 1, 2, 3$) is generated. In the end, six producer vectors X_{p1} , X_{p2} , X_{p3} , X_{np1} , X_{np2} and X_{np3} are acquired, and their fitness values are calculated with external orthogonal table separately. The producer with the highest fitness is selected as the optimal producer X_{op} for the current iteration. Next, the original GSO algorithm update position process is executed.

2.3 Tolerance Design of Heat Exchanger System

Fixing a system's operating temperature at a constant state is difficult, and there is bound to be an uncertainty error in industry applications [17]. It is straightforward to find the optimal parameters without tolerance in a robust heat exchanger system, but considering tolerance tends to reduce the robustness of the system. This paper uses three tolerance values: $\pm 10\%$, $\pm 20\%$ and $\pm 40\%$. Since the heat exchanger system has two operator temperature parameters, the $L_9(3^2)$ external orthogonal table simulates the tolerance design problem. The two parameters of the heat exchanger system are defined as follows:

$$T = [T_1, T_2]. \tag{13}$$

The two parameters and tolerance levels of the heat exchanger system are shown in Table 5; the external orthogonal table is shown in Table 6. All two parameters must satisfy the following equation:

$$T = \{100 \leq T_1 \leq 300 \text{ and } T_1 \leq T_2 \leq 400\}. \tag{14}$$

Table 5: The control factors and levels of the heat exchanger system

Level	Factor	
	T_1	T_2
1	$T_{1N} - \Delta T_1$	$T_{2N} - \Delta T_2$
2	T_{1N}	T_{2N}
3	$T_{1N} + \Delta T_1$	$T_{2N} + \Delta T_2$

Table 6: The $L_9(3^2)$ external orthogonal table for tolerance design problem of the heat exchanger system

Exp.	Factor		
	T_1	T_2	A
1	$T_{1N} - \Delta T_1$	$T_{2N} - \Delta T_2$	A_1
2	$T_{1N} - \Delta T_1$	T_{2N}	A_2
3	$T_{1N} - \Delta T_1$	$T_{2N} + \Delta T_2$	A_3
4	T_{1N}	$T_{2N} - \Delta T_2$	A_4
5	T_{1N}	T_{2N}	A_5
6	T_{1N}	$T_{2N} + \Delta T_2$	A_6
7	$T_{1N} + \Delta T_1$	$T_{2N} - \Delta T_2$	A_7
8	$T_{1N} + \Delta T_1$	T_{2N}	A_8
9	$T_{1N} + \Delta T_1$	$T_{2N} + \Delta T_2$	A_9

3 Results and Discussions

This paper uses Eq. (2) to evaluate the performance of the heat exchanger system by the AT-GSO algorithm. Two types of experiments: heat exchanger system without tolerance and heat exchanger system with tolerance. Five independent experiments will be conducted on the hyperparameters of the AT-GSO algorithm. Since the heat exchanger system has two parameters, the dimension of the AT-GSO algorithm is 2, the search range of each dimension is [100, 400], the population size is 10, and the maximum iteration is 50. This paper evaluates the proposed algorithm's performance by comparing it with the fractional-order particle swarm optimization (FPSO) [19].

3.1 Experimental Results of AT-GSO for Designing of Heat Exchanger System

The AT-GSO hyperparameters are set as follows: the number of producers is 3, the percentage of rangers is 60%, the maximum search angle (θ_{max}) of producer is $\pi/\alpha^2 \times 1.8$, the maximum turning angle (α_{max}) of producer is $\theta_{max}/4$, the longest search distance of producer is 8, and the initial angle is $\pi/2$. Five independent experiments without tolerance will be conducted using this combination of AT-GSO hyperparameters and the results are shown in Table 7. The average A-value (the total heat transfer area) is 7049.2502; the standard deviation A-value is 1.71×10^{-3} . The results of the five FPSO independent experiments without tolerance are presented in Table 8. Fig. 3 shows the mean fitness value curves for the AT-GSO and FPSO algorithms, each averaged over five experiments. Both the AT-GSO and FPSO algorithms can find the optimal value without tolerance. However, as shown in Fig. 3, FPSO exhibits significant instability during the search process. Furthermore, Tables 7 and 8 demonstrate that AT-GSO outperforms FPSO and exhibits greater robustness.

Table 7: Five experimental results of heat exchanger system designed by AT-GSO

No.	Parameters of heat exchanger system		A	Average A	Standard deviation A
	T_1	T_2			
1	182.0114	295.5994	7049.2492	7049.2502	1.71×10^{-3}
2	181.864	295.4758	7049.2532		
3	182.0202	295.604	7049.2492		
4	182.019	295.6055	7049.2492		
5	181.9781	295.5414	7049.2500		

Table 8: Five experimental results of heat exchanger system designed by FPSO

No.	Parameters of heat exchanger system		A	Average A	Standard deviation A
	T_1	T_2			
1	181.7823	295.5998	7049.2548		
2	182.3430	295.8873	7049.2690		
3	182.5836	296.1037	7049.3099	7049.3113	1.03×10^{-1}
4	183.2305	295.1937	7049.4961		
5	182.0058	295.7262	7049.2532		

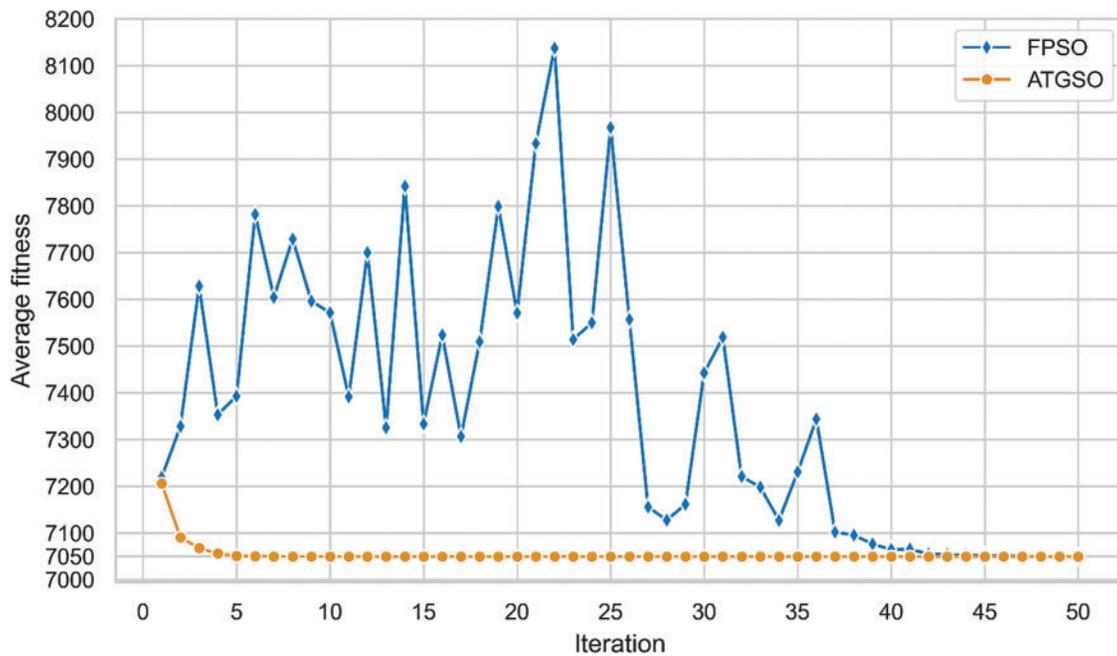


Figure 3: The fitness curve of five experiments of heat exchanger system without tolerance designed by AT-GSO

3.2 Experimental Results of AT-GSO for Tolerance Design of Heat Exchanger System

This paper uses three tolerance values: $\pm 10\%$, $\pm 20\%$ and $\pm 40\%$. The AT-GSO hyperparameters with $\pm 10\%$ tolerance are set as follows: the number of producers is 3, the percentage of rangers is 60%, the maximum search angle (θ_{max}) of producer is $\pi/\alpha^2 \times 1.8$, the maximum turning angle (α_{max}) of producer is $\theta_{max}/4$, the longest search distance of producer is 8, and the initial angle is $\pi/2$. Five independent experiments with $\pm 10\%$ tolerance will be conducted using this combination of AT-GSO hyperparameters and the results are shown in Table 9. The average A-value (the total heat transfer area) is 7049.2502; the standard deviation A-value is 1.71×10^{-3} . The results of the five FPSO independent experiments are presented in Table 10. Fig. 4 shows the mean fitness value curves for the AT-GSO and FPSO algorithms, each averaged over five experiments. Both the AT-GSO and FPSO algorithms can find the optimal value with $\pm 10\%$ tolerance. However, as shown in Fig. 4, FPSO exhibits instability during the search process. Furthermore, Tables 9 and 10 demonstrate that AT-GSO outperforms FPSO and exhibits greater robustness.

Table 9: Five experimental results of heat exchanger system with $\pm 10\%$ tolerance designed by AT-GSO

No.	Parameters of heat exchanger system		A	Average A	Standard deviation A
	T_1	T_2			
1	175.2682	285.1128	7195.1064	7195.1085	2.54×10^{-3}
2	175.0003	285.0609	7195.1121		
3	175.2554	285.113	7195.1064		
4	175.1989	284.9998	7195.1094		
5	175.2429	285.1157	7195.1064		

Table 10: Five experimental results of heat exchanger system with $\pm 10\%$ tolerance designed by FPSO

No.	Parameters of heat exchanger system		A	Average A	Standard deviation A
	T_1	T_2			
1	179.4212	286.3664	7195.1508	7195.2222	1.13×10^{-1}
2	174.7302	284.9428	7195.1312		
3	174.7168	285.5181	7195.2027		
4	175.1358	284.6135	7195.1666		
5	177.0154	286.0285	7195.4592		

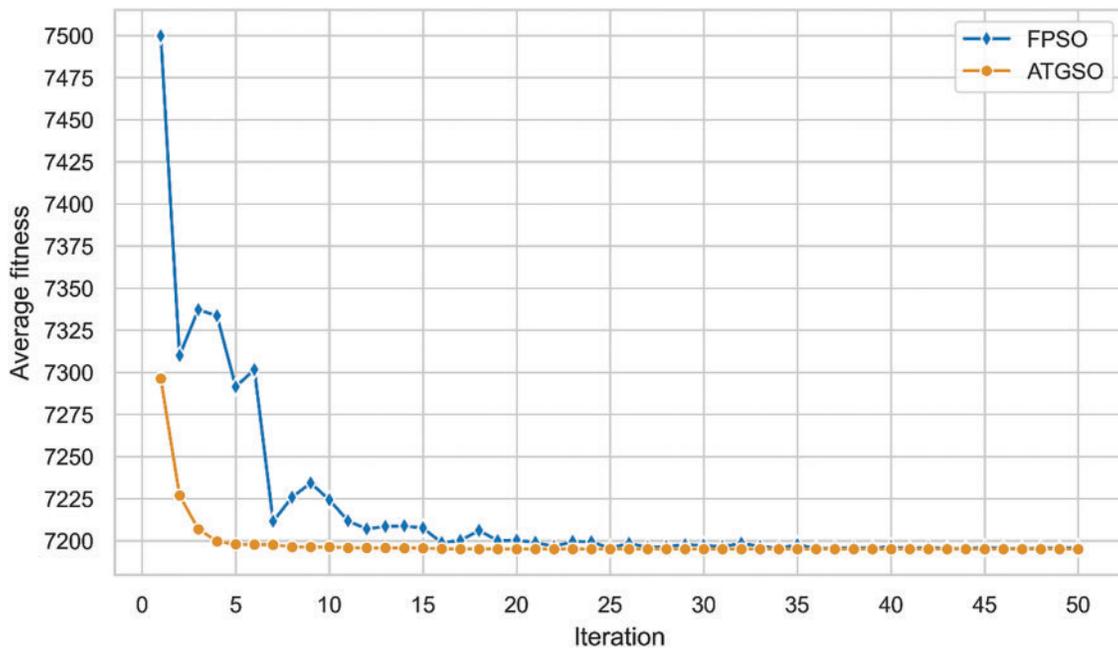


Figure 4: The fitness curve of five experiments of heat exchanger system with $\pm 10\%$ tolerance designed by AT-GSO

The AT-GSO hyperparameters with $\pm 20\%$ tolerance are set as follows: the number of producers is 10, the percentage of rangers is 90%, the maximum search angle (θ_{max}) of producer is $\pi/\alpha^2 \times 1.6$, the maximum turning angle (α_{max}) of producer is $\theta_{max}/6$, the longest search distance of producer is 2, and the initial

angle is $\pi/3$. Five independent experiments with $\pm 20\%$ tolerance will be conducted using this combination of AT-GSO hyperparameters and the results are shown in Table 11. From Table 11, it is learnt that AT-GSO did not search for the best parameter in one out of five experiments. The average A-value (the total heat transfer area) is 7380.1461; the standard deviation A-value is 7.69×10^{-1} . The results of the five FPSO independent experiments are presented in Table 12. Fig. 5 shows the mean fitness value curves for the AT-GSO and FPSO algorithms, each averaged over five experiments. Table 12 shows that the FPSO algorithms fail to find the optimal value with $\pm 20\%$ tolerance, falling into a local optimum solution. As shown in Fig. 4, FPSO exhibits instability during the search process. Furthermore, Tables 11 and 12 demonstrate that AT-GSO outperforms FPSO.

Table 11: Five experimental results of heat exchanger system with $\pm 20\%$ tolerance designed by AT-GSO

No.	Parameters of heat exchanger system		A	Average A	Standard deviation A
	T_1	T_2			
1	161.3939	264.3195	7517.614		
2	221.4265	364.7216	7347.1441		
3	221.7663	365.1105	7345.2903	7380.1461	7.69×10^{-1}
4	222.0226	364.6672	7345.3919		
5	221.7659	365.1276	7345.2897		

Table 12: Five experimental results of heat exchanger system with $\pm 20\%$ tolerance designed by FPSO

No.	Parameters of heat exchanger system		A	Average A	Standard deviation A
	T_1	T_2			
1	161.7287	264.7395	7517.6765		
2	157.5516	263.7430	7518.8142		
3	160.4889	263.6374	7517.6896	7517.9037	5.1×10^{-1}
4	160.3982	263.7787	7517.7220		
5	161.2059	264.2711	7517.6158		

The AT-GSO hyperparameters with $\pm 40\%$ tolerance are set as follows: the number of producers is 3, the percentage of rangers is 60%, the maximum search angle (θ_{max}) of producer is $\pi/\alpha^2 \times 1.8$, the maximum turning angle (α_{max}) of producer is $\theta_{max}/4$, the longest search distance of producer is 8, and the initial angle is $\pi/2$. Five independent experiments with $\pm 40\%$ tolerance will be conducted using this combination of AT-GSO hyperparameters and the results are shown in Table 13. The average A-value (the total heat transfer area) is 7049.2502; the standard deviation A-value is 1.71×10^{-3} . The results of the five FPSO independent experiments with $\pm 40\%$ tolerance are presented in Table 14. Fig. 3 shows the mean fitness value curves for the AT-GSO and FPSO algorithms, each averaged over five experiments. Both the AT-GSO and FPSO algorithms can find the optimal value without tolerance. However, as shown in Fig. 6, FPSO exhibits significant instability during the search process. Furthermore, Tables 13 and 14 demonstrate that AT-GSO outperforms FPSO and exhibits greater robustness.

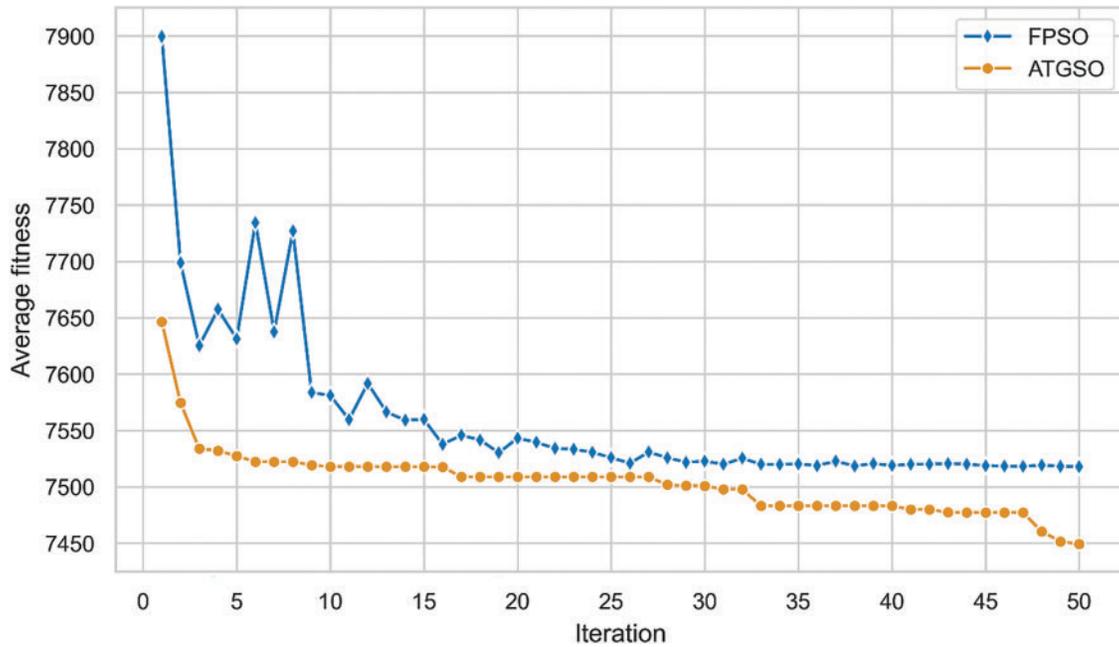


Figure 5: The fitness curve of five experiments of heat exchanger system with ±20% tolerance designed by AT-GSO

Table 13: Five experimental results of heat exchanger system with ±40% tolerance designed by AT-GSO

No.	Parameters of heat exchanger system		A	Average A	Standard deviation A
	T_1	T_2			
1	183.4763	337.4131	6627.3791	6627.3971	4.77×10^{-2}
2	183.468	337.4117	6627.3942		
3	183.4252	337.3241	6627.4797		
4	183.484	337.3731	6627.3663		
5	183.4948	337.2556	6627.3659		

Table 14: Five experimental results of heat exchanger system with ±40% tolerance designed by FPSO

No.	Parameters of heat exchanger system		A	Average A	Standard deviation A
	T_1	T_2			
1	183.1962	336.5194	6628.0124	6627.7023	2.01×10^{-1}
2	183.5304	336.7452	6627.6544		
3	183.2722	337.3388	6627.7744		
4	183.6906	337.4513	6627.4987		
5	183.3865	337.5855	6627.5716		

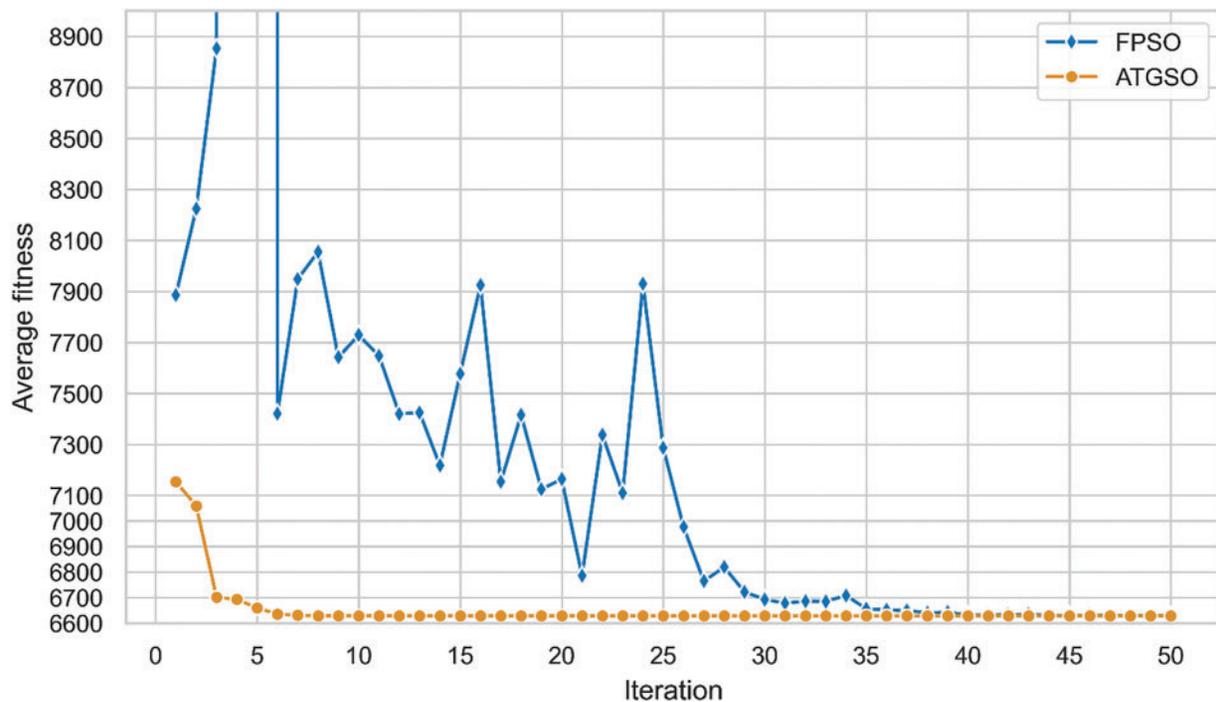


Figure 6: The fitness curve of five experiments of heat exchanger system with $\pm 40\%$ tolerance designed by AT-GSO

4 Conclusions

This paper proposed the AT-GSO algorithm, which combines the R-GSO algorithm and external quality design planning of the Taguchi method to optimize the heat exchanger system to minimize the total heat transfer area. The AT-GSO algorithm has a tolerance design capability. In the industry, keeping the operating temperature of a heat exchanger system in a constant state is not easy. This paper allows three tolerance values for operating temperature: $\pm 10\%$, $\pm 20\%$ and $\pm 40\%$. Experimental outcomes indicate that the AT-GSO algorithm proficiently optimizes the heat exchanger system and finds the optimum operating temperature without and with all three tolerances. The method proposed in this paper can take into account the existence of uncertainties in the parameters during the parameter optimization process so that it can reduce the impact caused by the errors in the parameters in practical applications. In future work, the method proposed in this paper can be applied to different algorithms and different applications. While tolerances were introduced for operating temperatures, other potential uncertainties (e.g., flow rate variations, fluid properties, or fouling factors) were not considered. In addition, this paper currently minimizes only the total heat transfer area, while other objectives are not included in the optimization process.

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