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Solid Waste Management: A MADM Approach Using Fuzzy Parameterized Possibility Single-Valued Neutrosophic Hypersoft Expert Settings

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ABSTRACT

The dramatic rise in the number of people living in cities has made many environmental and social problems worse. The search for a productive method for disposing of solid waste is the most notable of these problems. Many scholars have referred to it as a fuzzy multi-attribute or multi-criteria decision-making problem using various fuzzy set-like approaches because of the inclusion of criteria and anticipated ambiguity. The goal of the current study is to use an innovative methodology to address the expected uncertainties in the problem of solid waste site selection. The characteristics (or sub-attributes) that decision-makers select and the degree of approximation they accept for various options can both be indicators of these uncertainties. To tackle these problems, a novel mathematical structure known as the fuzzy parameterized possibility single valued neutrosophic hypersoft expert set ($\hat{\rho}$ -set), which is initially described, is integrated with a modified version of Sanchez's method. Following this, an intelligent algorithm is suggested. The steps of the suggested algorithm are explained with an example that explains itself. The compatibility of solid waste management sites and systems is discussed, and rankings are established along with detailed justifications for their viability. This study's strengths lie in its application of fuzzy parameterization and possibility grading to effectively handle the uncertainties embodied in the parameters' nature and alternative approximations, respectively. It uses specific mathematical formulations to compute the fuzzy parameterized degrees and possibility grades that are missing from the prior literature. It is simpler for the decision-makers to look at each option separately because the decision is uncertain. Comparing the computed results, it is discovered that they are consistent and dependable because of their preferred properties.

KEYWORDS

Hypersoft expert set; Sanchez's method; decision making; optimization; solid waste management; possibility grade; fuzzy parameterization

1 Introduction

Significant risks to human well-being are associated with the economy, society, and environment posed by the fast urban population growth. The buildup of material waste and the challenge of locating



appropriate sites for its management and disposal rank among the most urgent problems. Numerous issues, including a lack of space, poor infrastructure, and the environmental risks associated with inappropriate waste management, exacerbate this issue. To maintain sustainable urban development and the well-being of the populace and ecosystem, these issues require immediate attention [1,2]. The problem of solid waste management, which is commonly perceived as a multi-criteria decision-making (MCDM) problem [3,4], entails assessing multiple criteria to determine the best possible solutions. However, it is better to approach it as a multi-attribute decision-making (MADM) problem because of the complexity that multiple parameters and sub-parameters introduce. This enables a more methodical analysis that concentrates on particular characteristics that influence the results of decisions. Prioritizing features, balancing economic, environmental, and social factors, and incorporating stakeholder preferences are all important issues to take into consideration in this situation as they are essential to making wise decisions [5,6]. The main research challenges are (a) how to address expert concerns about approximating different options, (b) how to deal with the inherent ambiguity of attributes and sub-attributes, and (c) how to support experts in freely expressing their opinions, even when those opinions are somewhat vague. Fuzzy parameterization and possibility-graded settings are crucial tools for addressing these problems. These ideas offer a methodical approach to handling expert subjectivity and uncertainty in decision-making. The main objective of the study is to integrate fuzzy set-based numerical frameworks to address these research questions, providing a mathematical approach to expert judgment and model ambiguity. To give readers a better understanding of the suggested method, this section also provides an overview of the theoretical methodology and addresses the motivational factors and background of the study. For researchers, coping with informative ambiguity and uncertainty has been a critical issue. However, various structures, such as the single-valued neutrosophic set (SV-NS) suggested by Wang et al. [7], have been designed to handle this kind of imprecision and incompleteness. The SV-NS is a unique application of the neutrosophic set (NS) [8], allowing it to be used in a variety of academic disciplines. Although it requires more processing than established structures [9–11] its advantages, such as flexibility and dependability, set it apart. The three main parts true, indeterminate, and false belongings grades are operators that take the range $[0, 1]$ as a co-domain and the set of items under inspection as a domain. These parts are free as opposed to the aforementioned structures, and their sum must fall inside the range $[0, 3]$. There have been numerous studies on SV-NS reported up to this point. However, it is important to note the research done by the researchers [12–15], about the preparations of various measures and their use in different problems. Biswas et al. [16] have proposed a novel way for different techniques by using the various operations of SV-NS. Farid et al. [17,18] have covered usages of decision-related issues utilizing the aggregation operators of SV-NS. With time, it has been noticed that there are some real-world circumstances where specifications play a crucial role. As a result, the idea of soft sets (SS) [19] was developed to offer a parameterization tool for dealing with such circumstances. Then different structures have been developed by the combination of SS with neutrosophic set to form neutrosophic soft set (NSS) [20] and the single-valued neutrosophic soft set (SV-NSS) [21] are introduced since these two structures are unable to handle circumstances needing an approximate soft setting. By providing lower and upper approximations, Marei [22] applies the SV-NSS concept to rough sets to manage data roughness. Debnath [23] introduced a brand-new idea known as inverse SV-NSS to examine its use in decision-related issues by utilizing its operational advantages. When it came to handling situations when there were numerous expert opinions in a single soft set model, the soft set-like structures that had been developed for single expert opinions entirely broke down. This posed a significant obstacle when making decisions. The concept of a soft expert set (SeS) was provided by Alkhazaleh et al. [24] as a solution to the problem of managing different expert opinions in a model. In actuality, they included the viewpoints of several specialists in the domain set. The opinions of the experts have been

employed to manage the uncertainty in the parameters. These viewpoints have been interpreted with precision. Then Alkhazaleh et al. [25] also used this arrangement in a fuzzy environment by introducing a fuzzy soft expert set. This was the combination of a fuzzy and soft expert environment. In 2015, Broumi et al. [26] made use of the combination by developing the intuitionistic fuzzy soft expert set. After this Şahin et al. [27] developed the structure of the neutrosophic expert set. The soft set cannot deal with the domain having multiple arguments because it is only for dealing with a domain having a single argument. As a result, a new technique of function with the domain of multiple arguments, which is used to create the hypersoft set (HSS) [28] is suggested. These functions regard the order pairing of disjoint subclasses with attribute values as its domain and approximate alternatives. Kamaci [29] investigated the creation of a new combination of HSS and rough set associated procedures. The different researchers explored the different methods and aggregation operations in a neutrosophic HSS context [30,31]. Saqlain et al. [32] suggested some new structures by using the combination of SV-NS and HSS. Ihsan et al. [33,34] presented and applied concepts of neutrosophic soft sets with the combination of hypersoft expert sets (HSES) [35] in decision-related issues in the direction of authorizing the multiple crucial judgments of specialists. The contributions of researchers [36,37] concerning the use of aim-situation systems, combination methods of uncertain situations, and fuzzy forecasting methods are important and worthy of mention. To deal with the informational hesitant temperament, the idea of possibility degree [38] has been proposed as an alternative to the theory of probability. According to Zadeh [39], the fuzzy set (FS)'s various properties serve as the basis for the idea of possibility degree. Fedrizzi [40] managed numerous optimization modules in a mixed setting of FSs and the idea of possibility degree. The emphasis on the amplifying of distinct characteristics of these ideas was made by Dubois et al. [41]. Many academics are quite interested in the idea of possibility degree for measuring approximation-based uncertainty. To evaluate the study limitations and inevitable nature of the provided work, a few pertinent types of literature are evaluated in the following paragraphs. Fuzzy SS and the idea of possibility degree were merged by Alkhazaleh et al. [42] to define possibility fuzzy SS, where each approximation is given a specific possibility grade. The purpose of this grade is to evaluate how precise approximations are. They also talked about how to use it based on how similarity measures are created. The possibility intuitionistic fuzzy soft set was established by Bashir et al. [43] by expanding the idea of a possibility fuzzy soft set. By utilizing the suggested similarity measures between them, Karaaslan [44] contributed to the construction of the possible NSS and explained in different situations. In the literature, Zhao et al. [45] made a new combination of the idea of possibility degree with different structures of neutrosophic sets. They put up an algorithm by using the different operations of this new structure and used it for different purposes. Rahman et al. [46,47] developed some new structures of the idea of possibility degree. Their aggregation processes and similarity metrics were described. They also used the suggested ideas for decision-related issues like recognition and medical diagnosis. It has been noted that selectors may not always be certain of the attributes to select on a preference basis while making decisions. Put differently, they don't think the settings were chosen with certainty. Fuzzy parameterization, which is intended to evaluate the unsure attitude of decision-makers towards the choice of parameters, is used to manage this scenario. This evaluation is carried out by computing their approximations once a certain fuzzy membership grade has been attached to the parameters. Fuzzy parameterization was employed in a fuzzy soft set context by researchers [48–50] leading to the development of the fuzzy parameterized fuzzy soft set concepts. They talked about the applications for making decisions based on these put-up ideas. This concept was developed by Sulukan et al. [51] to form a new hybrid of fuzzy parameterization and the decision-related issues were covered based on performance values. In Deli's [52] discussion of several models employing these unsure attributes, he linked the concepts of parameterization and HSS. Qu et al. [53] addressed the resolution of several problems characterized by uncertainty, focusing on

approaches that manage ambiguous and imprecise information. The contributions of scholars [54–56] are worth noting regarding the quantification of uncertainties, and indeterminacies.

The greatest environmental change caused by the massive increase in human population is urbanization, which has a detrimental effect on the environment's capacity to retain its natural stability. As per reports, a substantial quantity of solid waste, which is garbage collected from farms, businesses, municipalities, and other hazardous sources, is generated in every urban area due to various activities conducted by humans and other related species. For municipal administrations, figuring out how to handle and dispose of this much solid waste has proven to be a challenging task. The 3R (Reduce, Reuse, Recycle) method cannot, even when applied for this purpose, get rid of solid waste, including different kinds of materials that fall under specific categories. This specific kind of solid waste is typically handled by land-filling, incineration, and composting, among other well-known methods. But installing all these systems also means considering a lot of other factors, some of which might be social, political, economic, ecological, or environmental. Solid waste can contain a significant amount of organic compounds that can be used to create premium fertilizer for exceptional agricultural yields because it includes both industrial and agricultural waste. Similarly, setting up incinerators is a great way to generate energy, and you can dispose of the waste they generate by putting it in the appropriate type of landfill. Solid waste management and disposal have become recognized as intricate, multidisciplinary issues that need to be taken into account from social, technical, economic, and environmental perspectives.

Since several criteria and sub-criteria are involved, selecting suitable locations is also thought to be an essential step in putting any solid waste settlement techniques (SWST) into place (i.e., which are described in Fig. 1). Price, consistency, practicality, air problem mechanism, and its related factors were taken into consideration by Ekmekçioğlu et al. [57] and Yildirim et al. [58] while determining the best location for solid waste management.

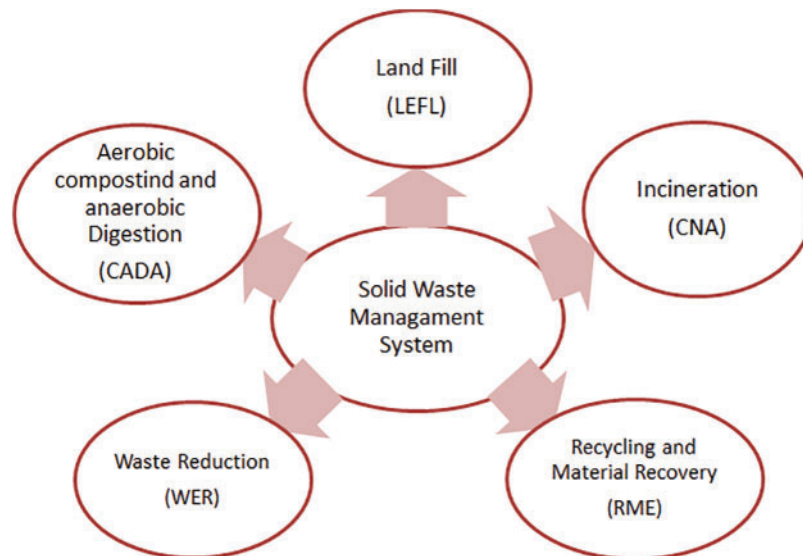


Figure 1: Solid waste management system

Since the participation of numerous attributes and sub-attributes, they have viewed the solid waste management challenge as an MCDM. Different techniques have been employed in many research studies to date to solve various environmental issues. In a Pythagorean hesitant fuzzy environment,

researchers like using various techniques, Ghosh et al. [59] solved a multi-purpose material mobility issue for solid waste management. Luo et al. [60] used different techniques to evaluate the optimal selection of sites for the installation of an incineration plant. The choice of dump sites was covered by Liu et al. [61] utilizing an integrated strategy involving information skills. By using MADM, a contribution to recycling for environmentally friendly growth was made by Wang et al. [62]. It has been noted that the choice of solid waste site selection attributes and sub-attributes is illogical and vague. As a result, this issue has been covered extensively by authors who used decision-making techniques in contexts resembling fuzzy sets. But more notable and pertinent to this investigation are the studies of Rahimi et al. [63], Liu et al. [64], Mallick [65], Torabi-Kaveh et al. [66], Karasan et al. [67], Ali et al. [68], Hanine et al. [69,70], and Kahraman et al. [71]. To select a suitable solid waste site in uncertain environments, various techniques for decision-making have been applied in these structures. Max-min operators of neutrosophic hypersoft sets have recently been employed by Jafar et al. [72] to choose sites for SWST. A careful examination of the aforementioned research contributions reveals that the subsequent issues cannot be solved together by any analytical framework to solve the solid waste management issue:

(a). Since a typical theoretical approach offers an organized framework for the analysis of complex systems, it is necessary to address parameters and their sub-parametric values simultaneously. Using this method allows for the systematic consideration of interdependencies between parameters and their sub-levels, which facilitates more accurate problem modeling and assessment. This approach ensures a more comprehensive and effective decision-making process by allowing the identification of important factors, the measurement of their effects, and the creation of solutions that account for the intricate relationships between parameters.

(b). Since fuzzy parameterization enables the representation of imprecise information, it is essential for assessing the uncertain and ambiguous nature of multi-argument tuples. Each tuple's fuzzy membership grade allows for the quantification of the data's inherent uncertainty and vagueness. With values that are not strictly binary but rather fall within a range that reflects differing degrees of truth or relevance, this approach allows for a more flexible analysis. In situations of uncertainty, fuzzy parameterization enables more accurate modeling and decision-making by offering a nuanced understanding of complex systems.

(c). Since it measures how plausible or feasible these approximations are, the idea of possibility grades is crucial for assessing the degree to which decision-makers approximations are accepted. Rather than giving a definitive answer, possibility grades provide a means to handle uncertainty by assigning values that reflect how likely or possible a given approximation is. This enables more informed and adaptable decision-making in complex and uncertain environments by enabling decision-makers to evaluate and compare various options or solutions based on their potential effectiveness.

(d). A single-valued neutrosophic setting, or a three-dimensional membership function, allows decision-makers to independently express their opinions along various dimensions of analysis, capturing various aspects of the decision-making process. A composite "true grade" reflecting the decisions made by the decision-makers is produced by the function that results from the representation of individual criteria or attributes by each dimension. Because this method takes into account different viewpoints and levels of uncertainty, it can approximate the objects or scenarios being evaluated in a more nuanced manner.

The issue of choosing the solid waste site is a MADM problem, as was earlier mentioned. Thus, if the aforementioned crucial elements are taken into account as a whole, a reliable decision support

framework can be created, which ultimately results in the development of a new conceptual framework to satisfy this requirement. The purpose of this study is to present the concepts of $\hat{\rho}$ -set, which are fully able to address the previously described issues as one unit. These research's significant contributions include but are not limited to:

- (1). A unique mathematical structure, called $\hat{\rho}$ -set, is constructed and characterized to address the potential decision-making concerns related to the degree of possibility, employing a specific degree to parameters, and different settings like SV-NS and hypersoft approximate mapping. By addressing these challenges collectively, it is qualified to aid in the creation of a reliable selecting framework.
- (2). By calculating their pertinent fuzzy parameterized grades using the necessary criteria, the inspected attributes and sub-attributes uncertain behaviors are assessed.
- (3). A mathematical approach is used to establish the important possibility degrees of alternatives to check the calculation of substitutes related to attributes and sub-attributes depending upon the opinions offered by the experts.
- (4). Sanchez's method, a well-known strategy for making decisions, is altered in the $\hat{\rho}$ -set setting.
- (5). The $\hat{\rho}$ -set aggregates and the modified Sanchez's approach are combined to create a decision-assisted framework. A strong site selection algorithm is then proposed to control solid waste.

Three main sections comprise the remainder of the paper. To make sure readers are familiar with the fundamental terms used throughout the study, [Section 2](#) offers a review of key terminology and concepts from the pertinent literature. In [Section 3](#), the methodology is presented in detail, including the steps and real-world application of the suggested technique, making it easy to understand how the approach functions. [Section 4](#) presents a discussion of the obtained results, comparison, and sensitivity analysis. Lastly, [Section 5](#) highlights the study's contributions and possible ramifications while providing a succinct synopsis of the research findings and key conclusions.

2 Fundamental Knowledge

This part attempts to go over a few keywords for a correct comprehension of the study that is being presented. The subsequent parts of the paper represent Θ , $\hat{\omega}$, Υ , and Ξ as the initial universe of objects, collection of parameters, collection of experts and their opinions, respectively.

Definition 2.1. [7] Consider three different functions β_s , β_n , and β_i defined from Θ to unit closed interval. These are the representation for membership, non-membership and indeterminate functions respectively. If the condition $0 \leq \beta_s(\varpi) + \beta_i(\varpi) + \beta_n(\varpi) \leq 3$ is satisfied for $\varpi \in \Theta$. Then a set $\mathbb{N}_{SV} = \{\varpi, < \beta_s(\varpi), \beta_i(\varpi), \beta_n(\varpi) > \mid \varpi \in \Theta\}$ is named as a SV-NS set.

Definition 2.2. [35] If $\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3, \dots, \hat{\omega}_n$, are sub-attribute-valued non-overlapping collections corresponding n different attributes and let Ω be the cartesian product of $\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3, \dots, \hat{\omega}_n$, and $\Pi = \Omega \times \Upsilon \times \Xi$, then HSES represented by Γ can be defined as $\Gamma = \{(\delta, \pi_\tau) \mid \delta \in \Pi\}$ where $\pi_\tau : \Pi \rightarrow \Theta$.

3 Materials and Methods

This section provides a quick description of the stages included in the approved methodology.

3.1 Notions of Proposed Theoretical Model, i.e., $\hat{\rho}$ -set

This research area aims to characterize and elucidate the basic ideas behind the Λ -set. To help with understanding, illustrative examples will be provided in addition to these concepts.

Definition 3.1. If $\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3, \dots, \hat{\omega}_n$, are sub-attribute-valued non-overlapping collections corresponding n different attributes and $\Omega = \hat{\omega}_1 \times \hat{\omega}_2 \times \hat{\omega}_3 \times \dots \times \hat{\omega}_n$, and $\Pi = \Omega_F \times \Upsilon \times \Xi = \{\pi_1, \pi_2, \pi_3, \dots, \pi_n\}$, and $\Omega_F = \{t_1/\xi(t_1), t_2/\xi(t_2), t_3/\xi(t_3), \dots, t_n/\xi(t_n)\}$ is a fuzzy set over Ω such that $\xi : \Omega \rightarrow [0, 1]$, then a $\hat{\rho}$ -set Λ can be stated as: $\Lambda = \{\pi, \Psi(\pi)\}$ with $\Psi : \Pi \rightarrow (PSVN)_\Theta$ defined by

$\Psi(\pi) = \{(\alpha/\eta(\alpha), \vartheta(\pi)(\alpha)) : \alpha \in \Theta; \pi \in \Pi\}$. Thus a $\hat{\rho}$ -set Λ can be characterized as

$\Lambda(\pi) = \{(\pi, \{(\alpha/\eta(\pi)(\alpha), \vartheta(\pi)(\alpha)) : \alpha \in \Theta\}) : \pi \in \Pi\}$. Where $\eta : \Pi \rightarrow (PSVN)_\Theta$ and $\vartheta : \Pi \rightarrow F_\Theta$ and the symbols $\eta(\pi)(\alpha), \vartheta(\pi)(\alpha)$ represent for the neutrosophic and possibility values of α , respectively.

Example 3.1. Consider the collection of computers as a set of universe $\Theta = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$. A commercial bank wants to buy a computer for its use in the office. Bank manager calls an expert committee $\Upsilon = \{\dot{e}_1, \dot{e}_2\}$ having expertise in computer selection. These experts decide some parameters $\hat{\omega} = \{p_1 = RAM, p_2 = ROM\}$ to buy this thing. After the division of the parameters into sub-parameters with different classes such as Ram = $\{p_{11} = 4GB, p_{12} = 8GB\}$ and ROM = $\{p_{21} = 32GB, p_{22} = 64GB\}$. Then $\Omega = \{t_1 = (4, 32), t_2 = (4, 64), t_3 = (8, 32), t_4 = (8, 64)\}$. Now $\Omega \times \Upsilon \times \Xi = \{\pi_1 = (t_1, \dot{e}_1, 1), \pi_2 = (t_1, \dot{e}_2, 1), \pi_3 = (t_2, \dot{e}_1, 1), \pi_4 = (t_2, \dot{e}_2, 1), \pi_5 = (t_3, \dot{e}_1, 1), \pi_6 = (t_3, \dot{e}_2, 1), \pi_7 = (t_4, \dot{e}_1, 1), \pi_8 = (t_4, \dot{e}_2, 1), \pi_9 = (t_1, \dot{e}_1, 0), \pi_{10} = (t_1, \dot{e}_2, 0), \pi_{11} = (t_2, \dot{e}_1, 0), \pi_{12} = (t_2, \dot{e}_2, 0), \pi_{13} = (t_3, \dot{e}_1, 0), \pi_{14} = (t_3, \dot{e}_2, 0), \pi_{15} = (t_4, \dot{e}_1, 0), \pi_{16} = (t_4, \dot{e}_2, 0)\}$. After the experts opinions, the sub-classes $\pi_1 = (t_1, \dot{e}_1, 1), \pi_7 = (t_4, \dot{e}_1, 1), \pi_{14} = (t_2, \dot{e}_2, 0), \pi_{18} = (t_4, \dot{e}_2, 0)\}$ are selected for the further process. Then the $\hat{\rho}$ -set can be described as

$$\psi \left(\frac{t_1}{0.3}, \dot{e}_1, 1 \right) = \left\{ \left(\frac{\alpha_1}{\langle 0.2, 0.4, 0.9 \rangle}, 0.3 \right), \left(\frac{\alpha_2}{\langle 0.3, 0.7, 0.8 \rangle}, 0.4 \right), \left(\frac{\alpha_3}{\langle 0.4, 0.5, 0.7 \rangle}, 0.6 \right), \left(\frac{\alpha_4}{\langle 0.5, 0.4, 0.7 \rangle}, 0.2 \right) \right\},$$

$$\psi \left(\frac{t_1}{0.4}, \dot{e}_1, 0 \right) = \left\{ \left(\frac{\alpha_1}{\langle 0.2, 0.3, 0.9 \rangle}, 0.7 \right), \left(\frac{\alpha_2}{\langle 0.3, 0.4, 0.8 \rangle}, 0.8 \right), \left(\frac{\alpha_3}{\langle 0.4, 0.3, 0.7 \rangle}, 0.5 \right), \left(\frac{\alpha_4}{\langle 0.5, 0.9, 0.7 \rangle}, 0.7 \right) \right\},$$

$$\psi \left(\frac{t_2}{0.5}, \dot{e}_2, 1 \right) = \left\{ \left(\frac{\alpha_1}{\langle 0.2, 0.8, 0.9 \rangle}, 0.2 \right), \left(\frac{\alpha_2}{\langle 0.3, 0.9, 0.8 \rangle}, 0.3 \right), \left(\frac{\alpha_3}{\langle 0.1, 0.5, 0.9 \rangle}, 0.7 \right), \left(\frac{\alpha_4}{\langle 0.1, 0.7, 0.9 \rangle}, 0.6 \right) \right\},$$

$$\psi \left(\frac{t_2}{0.6}, \dot{e}_2, 0 \right) = \left\{ \left(\frac{\alpha_1}{\langle 0.1, 0.4, 0.8 \rangle}, 0.1 \right), \left(\frac{\alpha_3}{\langle 0.1, 0.8, 0.2 \rangle}, 0.5 \right), \left(\frac{\alpha_3}{\langle 0.3, 0.4, 0.7 \rangle}, 0.5 \right), \left(\frac{\alpha_7}{\langle 0.5, 0.3, 0.8 \rangle}, 0.3 \right) \right\}.$$

$$\text{Or}\Lambda = \left\{ \begin{array}{l} \psi \left(\frac{t_1}{0.3}, \dot{e}_1, 1 \right) = \left\{ \left(\frac{\alpha_1}{\langle 0.2, 0.4, 0.9 \rangle}, 0.3 \right), \left(\frac{\alpha_2}{\langle 0.3, 0.7, 0.8 \rangle}, 0.4 \right), \right. \\ \left. \left(\frac{\alpha_3}{\langle 0.4, 0.5, 0.7 \rangle}, 0.6 \right), \left(\frac{\alpha_4}{\langle 0.5, 0.4, 0.7 \rangle}, 0.2 \right) \right\}, \\ \psi \left(\frac{t_1}{0.4}, \dot{e}_1, 0 \right) = \left\{ \left(\frac{\alpha_1}{\langle 0.2, 0.3, 0.9 \rangle}, 0.7 \right), \left(\frac{\alpha_2}{\langle 0.3, 0.4, 0.8 \rangle}, 0.8 \right), \right. \\ \left. \left(\frac{\alpha_3}{\langle 0.4, 0.3, 0.7 \rangle}, 0.5 \right), \left(\frac{\alpha_4}{\langle 0.5, 0.9, 0.7 \rangle}, 0.7 \right) \right\}, \\ \psi \left(\frac{t_2}{0.5}, \dot{e}_2, 1 \right) = \left\{ \left(\frac{\alpha_1}{\langle 0.2, 0.8, 0.9 \rangle}, 0.2 \right), \left(\frac{\alpha_2}{\langle 0.3, 0.9, 0.8 \rangle}, 0.3 \right), \right. \\ \left. \left(\frac{\alpha_3}{\langle 0.1, 0.5, 0.9 \rangle}, 0.7 \right), \left(\frac{\alpha_4}{\langle 0.1, 0.7, 0.9 \rangle}, 0.6 \right) \right\}, \\ \psi \left(\frac{t_2}{0.6}, \dot{e}_2, 0 \right) = \left\{ \left(\frac{\alpha_1}{\langle 0.1, 0.4, 0.8 \rangle}, 0.1 \right), \left(\frac{\alpha_3}{\langle 0.1, 0.8, 0.2 \rangle}, 0.5 \right), \right. \\ \left. \left(\frac{\alpha_3}{\langle 0.3, 0.4, 0.7 \rangle}, 0.5 \right), \left(\frac{\alpha_7}{\langle 0.5, 0.3, 0.8 \rangle}, 0.3 \right) \right\} \end{array} \right\}.$$

In above representation $\frac{\alpha_1}{\langle 0.2, 0.4, 0.9 \rangle}, 0.3$, the values 0.2, 0.4, 0.9 have the meaning of true, indeterminate, and false belongings of α_1 , and the whole representation $\left(\frac{\alpha_1}{\langle 0.2, 0.4, 0.9 \rangle}, 0.3 \right)$ gives the meaning of possibility value for the membership level for α_1 is the 0.3 (i.e., 30 percent).

3.2 The Choice of Attributes and Their Fuzzy Parameterized Grades (Values)

In any MADM problem, the procedure of choosing the characteristic is crucial. As a substitute, a smart technique for the selection of attributes may result in the dependability and flexibility of any MADM-based structure. A few well-known methods of gathering data on the choice of factors required for the estimation of the items to the bottom investigation include group surveys, surveys distributed to residents, in-depth analyses of previous research published in the literature, consultations with pertinent parties and specialists, etc. According to the acknowledged quantitative technique, i.e., the hypersoft expert setup, attributes, and the associated sub-attribute values are required to roughly estimate the assessment of some sites for solid waste governance. This is vital in terms of the significance of the research. The availability of adequate pertinent works makes the “literature review” the most appropriate and approachable source in this study for the choice of grades of parameters that go with them. The values for the qualities and sub-attributes are obtained from Jafar et al. [72] while keeping in mind the research’s slogan. Consequently, we have only considered those characteristics that were determined to be most applicable and essential for MADM-based solid waste governance through peer research to provide a basic structure for MADM-based solid waste administration. By doing this, the computational hassle that results from including the Cartesian product is avoided. The following is a description of the qualities and their operating features:

1. **Economic Values:** Numerous elements have a direct bearing on a project’s finance and other economic features. Without considering the project’s associated economic issues, its viability cannot be approved. These elements could consist of land costs, building costs, operational costs, building costs, etc.

2. **Air Quality Index (AQI):** It is a crucial measurement tool used to examine and evaluate the air quality in a specific location. Since SWST has a significant impact on air quality, when making decisions about the best SWST and locations for solid waste supervisors, it is crucial to take AQI into account. The AQI levels have been qualitatively categorized by specialists based on linguistic phrases that aid in comprehending the appropriateness of the SWST for solid waste supervisors. Fig. 2 displays the phrases that go along with them.
3. **Distance from Locality:** It is yet another crucial element that significantly influences the choice of the location for solid waste management. A distance of fewer than one kilometer is usually not considered acceptable; nevertheless, a gradual increase up to three kilometer raises the acceptability threshold. It has an equal impact on each SWST’s level of appropriateness. Owing to the air pollution caused by Southwest, the locations, which are situated less than a kilometer away from the populace, can facilitate the propagation of various illnesses. On the other hand, locations farther than 3 km away may result in higher carrier costs.
4. **Land Inclination:** The area’s slope must be evaluated for the site to manage solid waste. The best case scenario is for the region chosen to be free of slope, but because entirely level lands aren’t always available, inclination up to 9 percent may be compromised. For the majority of SWSTs, it is regarded as the most important geographical feature. Fig. 3 displays the language classifications for land inclination values.

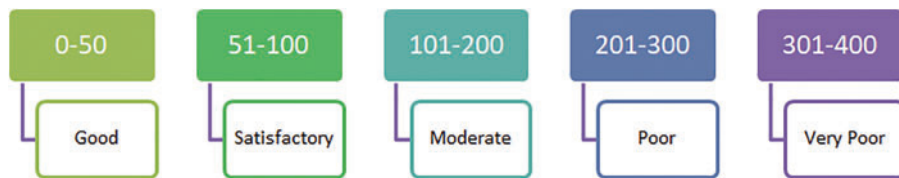


Figure 2: Categories of air quality index

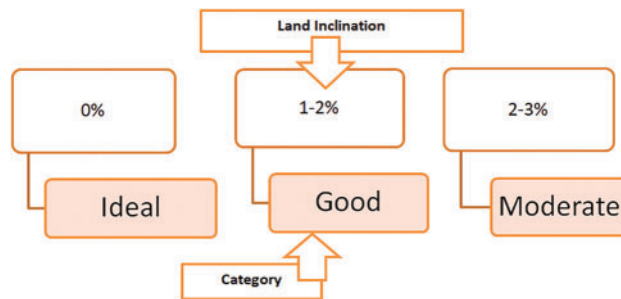


Figure 3: Categories of land inclination

3.3 Modification of Sanchez’s Method

Sanchez’s method, a decision-making strategy covered in [73], is used in this section, with some adjustments, to choose a suitable site for solid waste management using hypothetical values in a $\hat{\rho}$ -set setting. The idea of modified Sanchez’s method as discussed by Rahman et al. [74] is also followed due to hypersoft settings. The lack of a tool to link options with sub-characteristics scored pairs in any of the other decision-making approaches results in a computational challenge because complex operators for aggregation must be included. Sanchez’s approach, on the other hand, has less computing

complexity because of its simple phases. It connects options, decision-makers' perspectives, and sub-attribute pairs using fundamental principles that everyone can comprehend, regardless of their level of mathematics experience.

Consider the set of sites as a universe of discourse denoted by as $\ddot{H} = \{s_1, s_2, s_3, \dots, s_r\}$ with two more sets of experts Υ and their opinions $\Xi = \{1 = \text{agree}, 0 = \text{disagree}\}$. If $\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3, \dots, \hat{\omega}_n$, are sub-attribute-valued non-overlapping collections corresponding n different attributes and let Ω be the cartesian product of $\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3, \dots, \hat{\omega}_n$, and $\Pi = \Omega \times \Upsilon \times \Xi = \{k_1, k_2, k_3, \dots, k_n\}$ be the set of triplets. Let a group of decision-makers be selected for the decision of problem and denoted by $\hat{D} = \{d_1, d_2, d_3, \dots, d_m\}$ and the set of SWSTs is represented by $\hat{\Lambda} = \{\hat{\Psi}_1, \hat{\Psi}_2, \hat{\Psi}_3, \dots, \hat{\Psi}_k\}$. The phases of this revised technique are as follows, starting with the core sets' suppositions:

1. Construction of two $\hat{\rho}$ -sets and their representation in two different matrices $\hat{M}_1 = [c_{ij}]_{r \times n}$ and $\hat{M}_2 = [d_{ij}]_{r \times k}$. The first matrix \hat{M}_1 is chosen for values of $\hat{\rho}$ -set and \hat{M}_2 is selected for the opinions of decision-makers.
2. First change the values of possibility single-valued neutrosophic hypersoft expert sets (PSV-NHSES) into reduced fuzzy grades by using any proper numerical method and then put them in two new different matrices \hat{M}_3, \hat{M}_4 .
3. Multiplication of fuzzy parameterized grades with each row or column and obtain two new matrices \hat{M}_5 , and \hat{M}_6 .
4. Find the simple product (product of matrix) of two matrices \hat{M}_5 , and \hat{M}_6 to get the decision-matrix \hat{M}_7 .

3.4 Criteria for Fuzzy Parameterization

Rahman et al. [74] presented a formula for the calculation of fuzzy parameterized degrees. In this section, we followed that formula with partial modification. Let the decision-makers assigned possibility grades to the approximation of substitutes be

$\partial = \{\lambda_1(\sigma), \lambda_2(\sigma), \lambda_3(\sigma), \dots, \lambda_m(\sigma)\}$. Then the fuzzy parameterized value of each element σ of $\Pi = \Omega \times \Upsilon \times \Xi$ can be calculated by using the suitable criteria

$$\chi(\sigma) = \frac{\sum_{i=1}^m \lambda_i(\sigma)}{|\partial|} = \frac{\lambda_1(\sigma) + \lambda_2(\sigma) + \dots + \lambda_m(\sigma)}{m}. \quad (1)$$

3.5 Problem Declaration

The environmental problems brought on by the fast rise in population of a particular city X within its authority are of great concern to both the Ministry of Environment and Municipal Corporations. The administration of the Ministry of Environment calls for a combined meeting of the officers from both departments. A committee is established which is comprised of four experts $\mathbb{E}_i, i = 1, 2, 3, 4$ from both departments with \mathbb{E}_1 as convener of the committee. The committee has decided on a few guidelines and sub-guidelines for carrying out the given assignment. Following the attributes and sub-attributes selected, the individuals of the committee are directed to visit different parts of the city to choose a few areas for closer inspection. They are advised to provide their thoughts about the sites and SWSTs together. The \mathbb{E}_1 has the power to determine potential grades for the evaluation of the level of acceptance of submitted viewpoints.

3.6 Suggested Algorithm

Now, a recommended algorithm is made to carry out the committee's work. This algorithm is the modified form of an algorithm proposed by Rahman et al. [74] and it incorporates the MADM, the modified Sanchez's approach, and the $\hat{\rho}$ -set configuration.

Algorithm 3.1: Evaluation of sites for solid waste management

1. Input Stage:

1.1. Let \wedge , Ψ , and Ω be the three different sets used for the set of experts, attributes, and opinions of experts, respectively. Find the Cartesian product of the first set of attributes with sub-attribute tuples, followed by the Cartesian products of the other two sets the set of experts with their opinions, and the Cartesian product of the attributes that has already been calculated.

2. Construction Stage:

2.1. Construct two matrices \hat{M}_1 and \hat{M}_2 based on the possibility grades got from associated authority and the opinions of decision-makers that have used in two $\hat{\rho}$ -sets $\hat{\Phi}_1$ and $\hat{\Phi}_2$, then separate them in agree and disagree- $\hat{\rho}$ -sets by putting them in two different matrices.
 2.2. By using the formula of Eq. (1), First change the values of the attributes into fuzzy parameterized grades, and then represent all these values in two different matrices \hat{M}_3 , \hat{M}_4 .

3. Computation Stage:

3.1. Convert the values used in the matrices \hat{M}_3 , \hat{M}_4 which are of PSV-NHSES in reduced form with the help of method $[(\varphi_i - \varphi_i - \varphi_f) + \lambda]/3$, then get two new matrices \hat{M}_5 , \hat{M}_6 .
 3.2. Use of matrix \hat{M}_5 by taking the product of fuzzy parameterized values and its columns and forming a new matrix \hat{M}_7 , and similarly for rows to obtain a new matrix called \hat{M}_8 .
 3.3. By using the ordinary product of two matrices \hat{M}_7 and \hat{M}_8 and get the new decision matrix \hat{M}_9 .

4. Out put Stage:

4.1. Determine which column in the choice matrix \hat{M}_9 has the highest value.

The flowchart depicting Algorithm's brief description is shown in Fig. 4.

3.7 Case Study: Explanation of Proposed Algorithm

This part uses the next example to illustrate each stage of the suggested algorithm. The case study discussed by Rahman et al. [74] has been followed for hypersoft expert settings.

Example 3.2. As mentioned in Section 3.6, the assessment committee is composed of four individuals (experts): \mathbb{E}_1 , \mathbb{E}_2 , \mathbb{E}_3 , and \mathbb{E}_4 . These individuals are arranged in the set $\{\mathbb{E}_1; \mathbb{E}_2; \mathbb{E}_3; \mathbb{E}_4\}$. The participants went to a lot of locations, but just four designated as $\{Site1; Site2; Site3; Site4\}$ were carefully examined. " T_1 ", " T_2 ", " T_3 ", and " T_4 " are the four SWST that are evaluated and added to the set. In the set $\{p_1, p_2, p_3, p_4\}$, let every member of the committee decide upon the variables: "Air quality index (p_1)", "Distance from the locality (p_2)", "Economical values (p_3)", and "Inclination of land (p_4).". After the careful study, the sub-classes values of attributes lying in the different sets $\hat{\omega}_1 = \{20, 30, 40\}$, $\hat{\omega}_2 = \{1, 1.5, 2 km\}$, and $\hat{\omega}_3 = \{less\ costly, costly, more\ costly\}$, $\hat{\omega}_4 = \{1\ percent, 2\ percent, 3\ percent\}$. Let $\Omega = \hat{\omega}_1 \times \hat{\omega}_2 \times \hat{\omega}_3 \times \hat{\omega}_4$ and then $\hat{\omega} \times \Upsilon \times \Xi = \{\tau_1, \tau_2, \tau_3, \dots, \tau_{641}\}$. If the different values like 20, 30 are selected in $\hat{\omega}_1$, 1, 1.5 km, in $\hat{\omega}_2$, more costly and 2 percent are selected in $\hat{\omega}_3$ and $\hat{\omega}_4$ respectively on the basis of preference. After a long discussion, four tuples $\tau_6, \tau_{15}, \tau_{35}, \tau_{44}$ have been selected for further process. Now, every decision-maker aside from the " \mathbb{E}_1 " shares their thoughts regarding sites, sub-attribute \hat{A} tuples, and SWSTs in terms of single-valued neutrosophic soft expert (SV-NHSES)

values. As the committee’s chairman, the “ \mathbb{E}_1 ” assigns appropriate possibility grades to the decision-makers SV-NHSES valued opinions. As a result, two PSV-NHSESs are created and displayed using matrix notations, \hat{M}_1 and \hat{M}_2 , which are found in Tables 1 and 2. Based on the possible grades that “ \mathbb{E}_1 ” offered for each of $\tau_6, \tau_{15}, \tau_{35}, \tau_{44}$, their corresponding fuzzy parameterized grades are now determined using Eq. (1), which is listed in Table 3 along with a detailed explanation of the computations. Now, two $\hat{\rho}$ -sets, \hat{M}_1 and \hat{M}_2 , are formed using these computed fuzzy parameterized grades. This is done by substituting fuzzy parameterized sub-parametric tuples for the sub-parametric tuples $\tau_6/0.471, \tau_{15}/0.601, \tau_{35}/0.425, \tau_{44}/0.489$. Then, these sets are reported in Tables 4 and 5, respectively, and expressed in matrix notations \hat{M}_3 and \hat{M}_4 . The mathematical criterion is applied to convert the PSV-NHSS entries of \hat{M}_3 and \hat{M}_4 into reduced fuzzy values $[(\varphi_i - \varphi_j - \varphi_r) + \lambda]/3$, which yields fuzzy parameterized fuzzy hypersoft expert sets (FP-FHSESs) Z_1 and Z_2 . Following that, these sets are shown in matrix notations, with \hat{M}_5 and \hat{M}_6 being tabulated in Tables 6 and 7, respectively. As per the algorithm’s Step 3(ii), the matrices \hat{M}_5 and \hat{M}_6 undergo transformations to become \hat{M}_7 and \hat{M}_8 , respectively. These new matrices are then summarised in Tables 8 and 9. The typical matrix product (also known as the decision matrix) of \hat{M}_7 and \hat{M}_8 is now found and given the designation \hat{M}_9 . Table 10 tabulates the data. The highest values found in every column of \hat{M}_9 are indicated to arrive at a distinct conclusion.

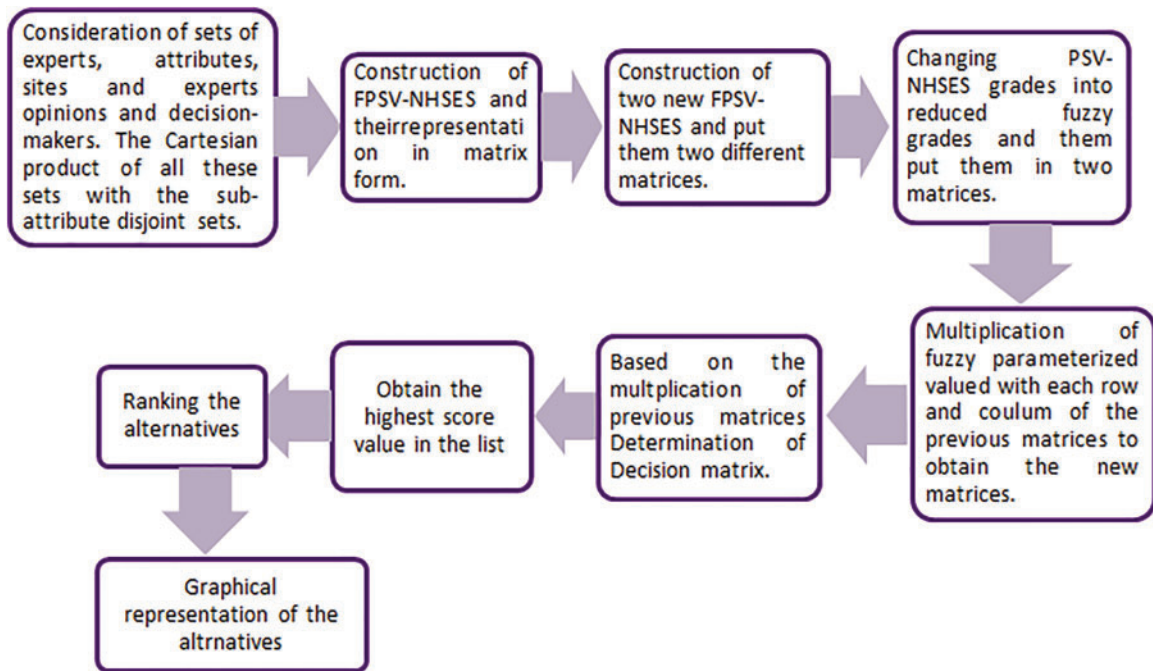


Figure 4: Flow chart of algorithm

Table 1: Matrix \hat{M}_1 tabular form

\hat{M}_1	τ_6	τ_{15}
Site 1	(< 0.60, 0.40, 0.50 >, 0.54)	(< 0.55, 0.45, 0.25 >, 0.25)

(Continued)

Table 1 (continued)

\hat{M}_1	τ_6	τ_{15}
Site 2	(< 0.70, 0.60, 0.52 >, 0.70)	(< 0.75, 0.60, 0.50 >, 0.62)
Site 3	(< 0.78, 0.52, 0.60 >, 0.43)	(< 0.50, 0.60, 0.70 >, 0.71)
Site 4	(< 0.60, 0.70, 0.80 >, 0.50)	(< 0.45, 0.50, 0.60 >, 0.69)
\hat{M}_1	τ_{35}	τ_{44}
Site 1	(< 0.46, 0.44, 0.37 >, 0.40)	(< 0.36, 0.53, 0.62 >, 0.39)
Site 2	(< 0.36, 0.50, 0.62 >, 0.63)	(< 0.89, 0.52, 0.75 >, 0.66)
Site 3	(< 0.80, 0.70, 0.61 >, 0.41)	(< 0.70, 0.73, 0.42 >, 0.40)
Site 4	(< 0.53, 0.61, 0.70 >, 0.80)	(< 0.58, 0.63, 0.42 >, 0.32)

Table 2: Matrix \hat{M}_2 tabular form

\hat{M}_2	\mathbf{T}_4	\mathbf{T}_2
Site 1	(< 0.80, 0.74, 0.47 >, 0.40)	(< 0.58, 0.52, 0.53 >, 0.51)
Site 2	(< 0.75, 0.70, 0.33 >, 0.69)	(< 0.54, 0.48, 0.49 >, 0.52)
Site 3	(< 0.74, 0.66, 0.39 >, 0.33)	(< 0.53, 0.44, 0.45 >, 0.51)
Site 4	(< 0.73, 0.82, 0.35 >, 0.29)	(< 0.52, 0.40, 0.41 >, 0.49)
\hat{M}_2	\mathbf{T}_3	\mathbf{T}_1
Site 1	(< 0.61, 0.59, 0.47 >, 0.39)	(< 0.41, 0.45, 0.52 >, 0.61)
Site 2	(< 0.59, 0.65, 0.53 >, 0.33)	(< 0.47, 0.51, 0.58 >, 0.66)
Site 3	(< 0.58, 0.51, 0.39 >, 0.31)	(< 0.36, 0.37, 0.44 >, 0.30)
Site 4	(< 0.57, 0.73, 0.35 >, 0.30)	(< 0.35, 0.33, 0.40 >, 0.52)

Table 3: The values of τ_i in terms of fuzzy parameterized

τ_i	Values of fuzzy parameterization
τ_6	$\left\{ \frac{0.54 + 0.25 + 0.37 + 0.70 + 0.40 + 0.51 + 0.39 + 0.61}{8} \right\} = 0.471$
τ_{15}	$\left\{ \frac{0.70 + 0.62 + 0.63 + 0.66 + 0.69 + 0.52 + 0.33 + 0.66}{8} \right\} = 0.601$
τ_{35}	$\left\{ \frac{0.43 + 0.71 + 0.41 + 0.40 + 0.33 + 0.51 + 0.31 + 0.30}{8} \right\} = 0.425$
τ_{44}	$\left\{ \frac{0.50 + 0.69 + 0.80 + 0.32 + 0.29 + 0.49 + 0.30 + 0.52}{8} \right\} = 0.489$

Table 4: Matrix form of $\hat{\rho}$ -set

\hat{M}_3	$\tau_6/0.471$	$\tau_{15}/0.601$
Site 1	(< 0.60, 0.40, 0.50 >, 0.54)	(< 0.55, 0.45, 0.25 >, 0.25)
Site 2	(< 0.70, 0.60, 0.52 >, 0.70)	(< 0.75, 0.60, 0.50 >, 0.62)
Site 3	(< 0.78, 0.52, 0.60 >, 0.43)	(< 0.50, 0.60, 0.70 >, 0.71)
Site 4	(< 0.60, 0.70, 0.80 >, 0.50)	(< 0.45, 0.50, 0.60 >, 0.69)
\hat{M}_3	$\tau_{35}/0.425$	$\tau_{44}/0.489$
Site 1	(< 0.46, 0.44, 0.37 >, 0.40)	(< 0.36, 0.53, 0.62 >, 0.39)
Site 2	(< 0.36, 0.50, 0.62 >, 0.63)	(< 0.89, 0.52, 0.75 >, 0.66)
Site 3	(< 0.80, 0.70, 0.61 >, 0.41)	(< 0.70, 0.73, 0.42 >, 0.40)
Site 4	(< 0.53, 0.61, 0.70 >, 0.80)	(< 0.58, 0.63, 0.42 >, 0.32)

Table 5: Matrix form of $\hat{\rho}$ -set

\hat{M}_4	\mathbf{T}_4	\mathbf{T}_2
$\tau_6/0.471$	(< 0.80, 0.74, 0.47 >, 0.40)	(< 0.58, 0.52, 0.53 >, 0.51)
$\tau_{15}/0.601$	(< 0.75, 0.70, 0.33 >, 0.69)	(< 0.54, 0.48, 0.49 >, 0.52)
$\tau_{35}/0.425$	(< 0.74, 0.66, 0.39 >, 0.33)	(< 0.53, 0.44, 0.45 >, 0.51)
$\tau_{44}/0.489$	(< 0.73, 0.82, 0.35 >, 0.29)	(< 0.52, 0.40, 0.41 >, 0.49)
\hat{M}_4	\mathbf{T}_3	\mathbf{T}_1
$\tau_6/0.471$	(< 0.61, 0.59, 0.47 >, 0.39)	(< 0.41, 0.45, 0.52 >, 0.61)
$\tau_{15}/0.601$	(< 0.59, 0.65, 0.53 >, 0.33)	(< 0.47, 0.51, 0.58 >, 0.66)
$\tau_{35}/0.425$	(< 0.58, 0.51, 0.39 >, 0.31)	(< 0.36, 0.37, 0.44 >, 0.30)
$\tau_{44}/0.489$	(< 0.57, 0.73, 0.35 >, 0.30)	(< 0.35, 0.33, 0.40 >, 0.52)

Table 6: Matrix form of $\hat{\rho}$ -set Z_1 having reduced fuzzy values

\hat{M}_5	$\tau_6/0.471$	$\tau_{15}/0.601$	$\tau_{35}/0.425$	$\tau_{44}/0.489$
Site 1	0.08	0.03	0.02	0.13
Site 2	0.09	0.09	0.04	0.09
Site 3	0.03	0.03	0.02	0.03
Site 4	0.13	0.01	0.01	0.05

Table 7: Matrix form of $\hat{\rho}$ -set Z_2 having reduced fuzzy values

\hat{M}_6	\mathbf{T}_4	\mathbf{T}_2	\mathbf{T}_3	\mathbf{T}_1
$\tau_6/0.471$	0.03	0.01	0.02	0.02
$\tau_{15}/0.601$	0.14	0.03	0.09	0.02
$\tau_{35}/0.425$	0.01	0.05	0.01	0.05
$\tau_{44}/0.489$	0.05	0.07	0.07	0.05

Table 8: Matrix form of $\hat{\rho}$ -set Z_1 having reduced fuzzy values

\hat{M}_7	τ_6	τ_{15}	τ_{35}	τ_{44}
Site 1	0.0377	0.0180	0.0085	0.0636
Site 2	0.0424	0.0541	0.0072	0.0440
Site 3	0.0141	0.0180	0.0085	0.0147
Site 4	0.0612	0.0060	0.0043	0.0245

Table 9: Matrix form of $\hat{\rho}$ -set Z_2 having reduced fuzzy values

\hat{M}_8	\mathbf{T}_4	\mathbf{T}_2	\mathbf{T}_3	\mathbf{T}_1
τ_6	0.0141	0.0047	0.0094	0.0094
τ_{15}	0.0841	0.0180	0.0541	0.0120
τ_{35}	0.0043	0.0213	0.0043	0.0213
τ_{44}	0.0245	0.0342	0.0342	0.0245

Table 10: Matrix $\hat{M}_9 = \hat{M}_7 \times \hat{M}_8$ having reduced fuzzy values

\hat{M}_9	\mathbf{T}_4	\mathbf{T}_2	\mathbf{T}_3	\mathbf{T}_1	Site scores
Site 1	0.00296	0.00223	0.00354	0.00231	0.01104
Site 2	0.00654	0.00283	0.00486	0.00228	0.01651
Site 3	0.00245	0.00107	0.00165	0.00089	0.00106
Site 4	0.00199	0.00133	0.00176	0.00134	0.00443
Scores	0.01394	0.00746	0.01181	0.00682	

4 Result and Discussion

This section is meant to discuss the sensitivity analysis of the results obtained and their comparison with the existing models.

4.1 Comparison and Discussion

Regarding the effectiveness of the provided method, this study is an attempt by the authors to give the transdisciplinary solid waste management problem based on MADM a mathematical component. As such, it is imperative to utilize an appropriate mathematical method with little processing overhead. The proposed algorithm meets this criterion. Even people without a strong background in mathematics can easily understand it thanks to its simple and basic computation-based techniques. Initially, the computed outcomes of the proposed algorithm are analyzed, followed by a discussion of their comparison with the computed outcomes of other pertinent structures that have been established previously. Examining the computed values in the decision matrix allows one to make the following significant findings. Studies that are significant, pertinent, and closely related to the suggested subject are taken into consideration for comparative analysis. After looking into the minimum (maximum) operators, distance and similarity metrics, and neutrosophic HSES, they created an algorithm to deal with solid waste management issues. While it is commendable that efforts are being made to tackle a significant problem with the environment, the efforts have shortcomings that include:

1. Fuzzy parameterization is not explained as a practical method to assess the ambiguities that administrators need to take into account while choosing characteristics or sub-attributes.
2. Since experts are permitted to freely describe their professional views in a multidimensional triad-based membership operator, they may occasionally be unclear. However, it needs to be controlled to avoid this ambiguity. One of the decision-makers ought to be named the coordinator in this situation. This person will have the authority to assess the estimates and offer each a possible grade. Stated differently, no thought is given to a framework based on grades.
3. Relatively little is discussed regarding the correctness of sites and SWSTs for solid waste management. A quantitative overview is given in place of an explanation for the autonomous ordering determined by rating numbers, maintaining the interdependence of the sites and SWSTs.

The recommended approach is superior because it addresses the aforementioned constraints by employing a stable and consistent ranking system based on results as depicted in [Table 11](#). A few more evaluating elements are included in [Table 12](#) to further highlight to readers the beneficial potential of the suggested investigation. The calculated outcomes of the suggested methodology have low values of numerical. A numerical calculable condition states that a result is more accurate and dependable the smaller its numerical value. In light of this, the suggested structure has produced trustworthy results when compared to pertinent structures [72]. In a similar way [Table 12](#), presents a structural comparison of the proposed structure to evaluate its favorable characteristics and adjustability. This is done by taking some assessing features into consideration. Certain evaluating parameters have been used in [Table 12](#) for a comparison point of view. Such parameters are consideration of fuzzy parameterization, fuzzy membership values, consideration of possibility values, having a multi-argument approximate function, single-valued neutrosophic setting, solid waste site selection, and the usage of multi-decisive opinions.

Table 11: Final score values

Techniques	Appropriate SWST	Appropriate site
Jafar et al. [72]	T ₂ having score 0.9248	Site 2 having score 0.6592
Rahman et al. [74]	T ₄ having score 0.14486	Site 2 having score 0.12849
$\hat{\rho}$ -set	T ₄ having score 0.01394	Site 2 having score 0.01651
Techniques	Ranking of SWST	Ranking of site
Jafar et al. [72]	Not appropriate	Not appropriate
Rahman et al. [74]	T ₄ > T ₂ > T ₃ > T ₁	Site 2 > Site 1 > Site 4 > Site 3
$\hat{\rho}$ -set	T ₄ > T ₃ > T ₂ > T ₁	Site 2 > Site 1 > Site 4 > Site 3

Table 12: Comparison with some structures based on specific attributes

Authors	Consideration of fuzzy parameterization	Fuzzy membership values	Consideration of possibility values	Multi-argument approximate function	Single-valued neutrosophic setting	Solid waste site selection	Multi-decisive opinions
Alkhazaleh et al. [42]	Insufficient	Applicable	Applicable	Insufficient	Insufficient	Insufficient	Insufficient
Bashir et al. [43]	Insufficient	Applicable	Applicable	Insufficient	Insufficient	Insufficient	Insufficient
Karaaslan [44]	Insufficient	Applicable	Applicable	Insufficient	Applicable	Insufficient	Insufficient
Zhao et al. [45]	Insufficient	Applicable	Applicable	Applicable	Applicable	Insufficient	Insufficient
Rahman et al. [46]	Insufficient	Applicable	Applicable	Applicable	Insufficient	Insufficient	Insufficient
Çağman et al. [48]	Applicable	Applicable	Insufficient	Insufficient	Insufficient	Insufficient	Insufficient
Sulukan et al. [51]	Applicable	Applicable	Insufficient	Insufficient	Insufficient	Insufficient	Insufficient
Deli [52]	Applicable	Applicable	Insufficient	Applicable	Insufficient	Insufficient	Insufficient
Luo et al. [60]	Insufficient	Applicable	Insufficient	Insufficient	Insufficient	Applicable	Insufficient
Rahimi et al. [63]	Insufficient	Applicable	Insufficient	Insufficient	Insufficient	Applicable	Insufficient
Rahman et al. [74]	Applicable	Applicable	Insufficient	Applicable	Applicable	Applicable	Insufficient
Suggested approach	Applicable	Applicable	Applicable	Applicable	Applicable	Applicable	Applicable

4.2 Sensitivity Analysis

Sensitivity analysis for SWSTs based on $\hat{\rho}$ -sets includes determining how changes to input parameters and subparameters affect the outputs of the structure. This is carried out to assess the consistency and reliability of the results and to determine which elements have the most impact on the final result. By doing sensitivity analysis, decision-makers can determine which aspects are most crucial to the solid waste management program’s performance and adjust their plan accordingly. Sensitivity analysis results can also help decision-makers allocate funds and resources more prudently. Policymakers may opt to augment their funding for waste minimization and recycling programs. We use the idea of Pythagorean means to compute the scores of sites and SWSTs to evaluate the sensitivity of the generated scores as shown in Table 10. Consequently, we have the subsequent three cases:

1. **Case 1:** Table 10 computations are altered to produce Table 13 if the idea of the arithmetic mean is used to calculate the score values of sites and SWSTs.
2. **Case 2:** Table 10 calculations are altered to produce Table 14 if the geometric mean idea is used to calculate the score values of sites and SWSTs.
3. **Case 3:** Table 10 calculations are altered to produce Table 15 if the harmonic mean idea is used to calculate the score values of sites and SWSTs.

Table 13: Score values based on the calculation of arithmetic mean

\hat{M}_9	\mathbf{T}_4	\mathbf{T}_2	\mathbf{T}_3	\mathbf{T}_1	Site scores
Site 1	0.00296	0.00223	0.00354	0.00231	0.00276
Site 2	0.00654	0.00283	0.00486	0.00228	0.00413
Site 3	0.00245	0.00107	0.00165	0.00089	0.00152
Site 4	0.00199	0.00133	0.00176	0.00134	0.00161
Scores	0.00349	0.00187	0.00295	0.00171	

Table 14: Score values based on the calculation of geometric mean

\hat{M}_9	\mathbf{T}_4	\mathbf{T}_2	\mathbf{T}_3	\mathbf{T}_1	Site scores
Site 1	0.00296	0.00223	0.00354	0.00231	0.00271
Site 2	0.00654	0.00283	0.00486	0.00228	0.00378
Site 3	0.00245	0.00107	0.00165	0.00089	0.00140
Site 4	0.00199	0.00133	0.00176	0.00134	0.00158
Scores	0.00312	0.00173	0.00266	0.00158	

Table 15: Score values based on the calculation of harmonic mean

\hat{M}_9	\mathbf{T}_4	\mathbf{T}_2	\mathbf{T}_3	\mathbf{T}_1	Site scores
Site 1	0.00296	0.00223	0.00354	0.00231	0.0027
Site 2	0.00654	0.00283	0.00486	0.00228	0.0035
Site 3	0.00245	0.00107	0.00165	0.00089	0.0013
Site 4	0.00199	0.00133	0.00176	0.00134	0.0016
Scores	0.00285	0.00161	0.00241	0.00146	

Table 16 readily demonstrates the consistency of the ranking outcomes for both sites and SWSTs.

Table 16: Final score values

Samples	Associated tables	SWST ranking	Site ranking
1st case	Table 13	$\mathbf{T}_4 > \mathbf{T}_3 > \mathbf{T}_2 > \mathbf{T}_1$	Site 2 > Site 1 > Site 4 > Site 3
2nd case	Table 14	$\mathbf{T}_4 > \mathbf{T}_3 > \mathbf{T}_2 > \mathbf{T}_1$	Site 2 > Site 1 > Site 4 > Site 3
3rd case	Table 15	$\mathbf{T}_4 > \mathbf{T}_3 > \mathbf{T}_2 > \mathbf{T}_1$	Site 2 > Site 1 > Site 4 > Site 3
General case	Table 10	$\mathbf{T}_4 > \mathbf{T}_3 > \mathbf{T}_2 > \mathbf{T}_1$	Site 2 > Site 1 > Site 4 > Site 3

5 Conclusion

This study has provided a novel approach to the selection of suitable SWSTs and locations for the management of solid waste collected from multiple sources within a city. By defining the concepts of $\hat{\rho}$ -set, the anticipated ambiguities regarding the attributes, sub-characteristics valued pairs, expert advice based on approximations of substitutes regarding attributes, and sub-attribute grade pairs are addressed. To connect the SWSTs, sites, and sub-attribute valued pairs in the context of the $\hat{\rho}$ -set, the traditional method, also known as Sanchez's method, is modified. To make this adjustment, a $\hat{\rho}$ -set matrix representation is used. An explanation and simple example are provided for a method that combines the ideas of $\hat{\rho}$ -set and Sanchez's approach to find the best SWST and location for solid waste control. This approach takes into account the fact that the solid waste management problem is a MADM problem. Two different opinions about the outcomes that were obtained have been expressed: (a) The interdependencies between sites and SWSTs are taken into account when answering the question, "Which of the sites is the most appropriate for any kind of SWST?" (b) The rankings of the site and the SWST are analyzed as independent variables. By comparing the proposed strategy with the most relevant, previously developed structure, it is examined. It is concluded that because of the new approach's flexibility and reliability in effectively managing uncertainty, it has more positive aspects than previous research. Although this study uses pseudo data on expert opinions and other relevant variables to give a general impression, it can be successfully applied to some case study-based efforts using real data sets.

This work focuses on the fuzzy parameterized degree of sub-characteristic pairs to approximate alternatives; however, numerous other scenarios may require the use of neutrosophic, intuitionistic fuzzy, or picture fuzzy parameterizations of sub-characteristic pairs to evaluate their ambiguous and imprecise characteristics. Since the study employs Sanchez's technique and expert-graded ideas to measure and estimate SWST with sites, other decision-making methodologies that provide true, value-based feedback from experts through review are also appropriate for these kinds of analyses. Additionally, to give some basic context, researchers only considered the characteristics that the peer literature evaluation deemed most important for MADM-based solid waste development. This avoids mathematical difficulties with the Cartesian product in a hypersoft expert setting. The writers plan to build upon this basic framework in the future by taking into account sets of traits or criteria, making it more global in scope, and utilizing machine learning and other pertinent computer-assisted software. This is because adding more criteria creates numerical challenges that necessitate the use of artificial intelligence and machine learning. Applied mathematics and artificial intelligence are both included in this research.

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