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ARTICLE

A Study on the Transmission Dynamics of the Omicron Variant of COVID-19 Using Nonlinear Mathematical Models

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ABSTRACT

This research examines the transmission dynamics of the Omicron variant of COVID-19 using SEIQI_cRVW and SQIRV models, considering the delay in converting susceptible individuals into infected ones. The significant delays eventually resulted in the pandemic's containment. To ensure the safety of the host population, this concept integrates quarantine and the COVID-19 vaccine. We investigate the stability of the proposed models. The fundamental reproduction number influences stability conditions. According to our findings, asymptomatic cases considerably impact the prevalence of Omicron infection in the community. The real data of the Omicron variant from Chennai, Tamil Nadu, India, is used to validate the outputs.

KEYWORDS

Omicron; local stability; reproduction number; steady states; global stability

1 Introduction

Since COVID-19 is a newly discovered virus, little is known about how it spreads. As a result, health authorities must thoroughly understand the incubation and recovery periods to implement more efficient quarantine procedures for those suspected of carrying the virus. As of November 24, 2021, Omicron has been found in countries, and it continues to be the most popular variant all over the world. The transmission dynamics and the potential roles of various intervention strategies have been better understood by recent COVID-19 studies [1–7]. These methodologies incorporate relief and concealment to dial back the spread of the pandemic, decreasing pinnacle medical care to safeguard the people who are most in danger from contaminations, lessening the number of infective cases to a low level, implementing lockdown to a district of exceptionally infective cases, confining suspect cases at home, isolating those residing in a similar family at home. Some authors developed an Omicron variant model with variable population size [8–14].

After becoming infected, a strengthening of the immune system may cause a delay in entering the infectious stage, and a significant amount of delay may even result in the disease being stopped at the exposure level. As a result, the effect of time delay on studying the dynamics of disease spread is significant. In addition, the effect of quarantine on preventing disease spread and the transmission of infection from both the exposed and infected groups are taken into consideration. On the one hand, people who were exposed have the virus, but unlike an asymptomatic patient, they do not show any symptoms right away. There is a latency period before an exposed person becomes infected, and it can take up to 14 days for some people to become infected. By developing the integer model, the current study aims to investigate the effects of the latency period. Using the delay differential equations model, newly infected individuals are given some time before contracting the disease.

To prevent COVID-19 infection in the host population, some authors developed delay-type models. Liu et al. proposed a time delay model and utilised the methodology to analyse the COVID-19 pandemic in China [15]. A new form of disease model based on a time delay dynamics was developed in [16]. They fitted model parameters based on the total number of reported cases in Beijing and Wuhan, China. Using mathematical and statistical modelling, Sedighe et al. developed a model to determine the epidemic trend and forecast the number of patients hospitalised due to COVID-19 in Iran [17]. The SEIQR COVID-19 propagation model with two delays was investigated by Fangfung et al. in [18]. Their model took supply chain transmission and hierarchical quarantine rate into account. A modified SIR model which combines suitable delay parameters and generates a more reliable forecasts of COVID-19 real-time data was proposed in [19]. Where the authors compared the predictions of the recently constructed SIR model to actual data collected from Germany, Italy, Kuwait, and Oman. Shidong et al. created a delay SEIR model based on the feedback linearization technique to manage the effects of COVID-19 [20]. The authors in [21] proposed a SIRDV model to investigate the impact of vaccination campaigns during the pandemic in Israel and Great Britain. In [22], the authors introduced a time delay model considering the migration of individuals from susceptible to infected class. The Omicron model can be mathematically modeled in a way that is reasonably accurate to the occurrences that have been observed when delay factors are included in the system of differential equations.

In this paper, two delay mathematical models are proposed. The work is significant, because it contains the mathematical modeling with a real-data of the Omicron variant from Chennai, Tamil Nadu, India. In the form of sections, the delayed SQIRV model is proposed and its stability is examined in Section 2. The delayed SEIQI_c RVW model is proposed and steady-state solution existence is tested in Section 3. In order to confirm and strengthen our theoretical findings regarding Omicron B.1.1.529 SARS-Cov-2, computational simulations are carried out from the real data which is collected from Tamilnadu in Section 4. In Section 5, we summarise our findings.

2 Delayed SQIRV Mathematical Model

Here we define the delay-type version of the integer-order SQIRV model proposed in [23]. The disease is considered to have an incubation time of $\tau > 0$, because the virus moved from the susceptible phase to the incubation phase. The time between becoming susceptible to the virus and experiencing its symptoms is referred to as the incubation period. Based on the policy decisions made by the government, a set of parameters have been obtained to forecast the pandemic trend. Table 1 lists the non-negative parameters that are used in this model.

Table 1: Parameters and their descriptions

Parameters	Descriptions
Γ	Rate at which humans are recruited into the population
η_1	The natural death rate applicable to all compartments
η_2	Rate at which a certain fraction of susceptible individuals receives vaccination
η_3	Effective infectious contact rate between the susceptible and infected individual
η_4	The quarantine rate of the susceptible individuals
η_5	The rate at which the recovered compartment loses its immunities to treatment
η_6	The rate at which the vaccinated compartment loses its immunities to vaccination
η_7	The treatment rate of the infected class
η_8	The natural recovery rates due to quarantine
η_9	The contact rate between Quarantined and Vaccinated people
η_{10}	The death rate induced by infections of infected individuals
η_{11}	The rate at which recovered individual moves to vaccinated compartment
η_{12}	The natural recovery rates transfere from infected to recovered individuals

Considering the given aspects, the delay SQIRV mathematical model is derived as follows:

$$\frac{dS}{dt} = \Gamma - \eta_{21}S - \eta_3 S(t-\tau)I(t-\tau) + \eta_4 Q + \eta_5 R + \eta_6 V,$$

$$\frac{dQ}{dt} = \eta_7 I - \eta_{22} Q - \eta_9 QV,$$

$$\frac{dI}{dt} = \eta_3 S(t-\tau)I(t-\tau) - (\eta_{23})I(t),$$

$$\frac{dR}{dt} = \eta_{12} I(t) + \eta_8 Q - \eta_{24} R,$$

$$\frac{dV}{dt} = \eta_{11} R + \eta_2 S - \eta_{25} V + \eta_9 QV, \tag{1}$$

where $\eta_{21} = \eta_1 + \eta_2$, $\eta_{22} = \eta_1 + \eta_4 + \eta_8$, $\eta_{23} = \eta_1 + \eta_7 + \eta_{10} + \eta_{12}$, $\eta_{24} = \eta_1 + \eta_5 + \eta_{11}$, and $\eta_{25} = \eta_1 + \eta_6$

Subject to initial conditions $S(0) = S_0, Q(0) = Q_0, I(0) = I_0, R(0) = R_0^0, V(0) = V_0$.

As in the case of Omicron, a susceptible individual is assumed to interact with an infected individual in the equation system but does not enter the infected compartment until after a predetermined incubation period. The incubation period τ is just while moving from the powerless compartment to the contaminated compartment.

There are two steady-state solutions to the model under consideration. Time-independent solutions are obtained when the model system (1) is made static. The steady-state solution, $I = 0$, when there are no infections is given by

$$E^0 = (S, Q, I, R, V) = \left(\frac{\Gamma(\eta_1 + \eta_6)}{\eta_1(\eta_1 + \eta_2 + \eta_6)}, 0, 0, 0, \frac{\Gamma\eta_2}{\eta_1(\eta_1 + \eta_2 + \eta_6)}\right) \tag{2}$$

Also, the steady-state solution when infection is persistent i.e., $I \neq 0$ is given by

$$E^* = (S^*, Q^*, I^*, R^*, V^*)$$
$$= \left(\frac{\nu_{33}}{\eta_3}, \frac{\eta_7 I^*}{\eta_{22} + \eta_9 V^*}, \frac{\eta_3\Gamma - \eta_{21}\nu_{33} + \eta_6 C}{\nu_{33} - \eta_4 A - \eta_5 B - \eta_6 C^*}, \frac{\eta_8 Q^* + \eta_{12} I^*}{\eta_1 + \eta_5 + \eta_{11}}, \frac{\eta_2\eta_{11}R^* + \eta_2\eta_{23}}{\eta_2(\eta_1 + \eta_6 - \eta_9 Q^*)}\right) \tag{3}$$

where $A = \dfrac{\eta_7}{\eta_{22} + \eta_9 V^*}$, $B = \dfrac{\eta_{12}}{\eta_1 + \eta_5 + \eta_{11}} + \dfrac{\eta_7\eta_8}{(\eta_{22} + \eta_9 V^*)(\eta_1 + \eta_5 + \eta_{11})}$, $C = \dfrac{\eta_{11}B}{\eta_1 + \eta_6 - \eta_9 Q^*}$.

The fundamental reproduction number R_0 is calculated by using the next generation operator matrix as folows [24,25]:

R_0 is the largest eigenvalue of the spectral radius given by

$$R_0(FV^{-1}) = \eta_3\left(\frac{\Gamma(\eta_1 + \eta_6)}{\eta_1(\eta_1 + \eta_2 + \eta_6)}\right)\left(\frac{1}{(\eta_1 + \eta_{10} + \eta_{12} + \eta_7)}\right) \tag{4}$$

2.1 Stability Analysis of the Delayed SQIRV Model

The following theorem applies Rouche's theorem [26] to characterise the local stability of the SQIRV system (1) for the infection-free steady state solution (2). The output is determined by the reproduction number R_0.

Theorem 2.1. The infection free steady state E^0 is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$ for $\tau \geq 0$.

Proof. The characteristic equation of system (1), for the equilibrium point E^0, is given by

$$\Delta(\lambda) = \left|\lambda Id_{5\times 5} - J^1_{00} - J^1_{01}e^{-\tau\lambda}\right| \tag{5}$$

where J^1_{00}

$$= \begin{pmatrix} -(\eta_1 + \eta_2) & \eta_4 & 0 & \eta_5 & \eta_6 \\ 0 & -(\eta_1 + \eta_4 + \eta_8 + \eta_9 V) & \eta_7 & 0 & 0 \\ 0 & 0 & -(\eta_1 + \eta_7 + \eta_{10} + \eta_{12}) & 0 & 0 \\ 0 & \eta_8 & \eta_{12} & -(\eta_1 + \eta_5 + \eta_{11}) & 0 \\ \eta_2 & \eta_9 V & 0 & \eta_{11} & -(\eta_1 + \eta_6) \end{pmatrix}$$

and

$$J^1_{01} = \begin{pmatrix} 0 & 0 & -\eta_3 S e^{-\iota\tau} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_3 S e^{-\iota\tau} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$C(\lambda) = (\lambda + (\eta_1 + \eta_4 + \eta_7 + \eta_8))(\lambda + (\eta_1 + \eta_4 + \eta_{11}))(\lambda - \eta_3 S e^{-\lambda\tau} + (\eta_1 + \eta_7 + \eta_{10} + \eta_{12}))$$
$$\left(\lambda + \frac{1}{2}\left((\eta_1 + \eta_2) + (\eta_1 + \eta_6) \pm \sqrt{(\eta_1 + \eta_2)^2 + 4\eta_2\eta_6 - \eta_1\eta_2(\eta_1 + \eta_6) + (\eta_1 + \eta_6)^2}\right)\right). \quad (6)$$

Then from the Jacobian matrix, the eigen values are $-(\eta_1+\eta_4+\eta_7+\eta_8), -(\eta_1+\eta_4+\eta_{11}), \eta_3 S e^{-\lambda\tau} - (\eta_1+\eta_7+\eta_{10}+\eta_{12})$, and $\frac{1}{2}\left(-(\eta_1+\eta_2)-(\eta_1+\eta_6) \pm \sqrt{(\eta_1+\eta_2)^2+4\eta_2\eta_6-\eta_1\eta_2(\eta_1+\eta_6)+(\eta_1+\eta_6)^2}\right)$.

When $\tau = 0$, the System (1) is stable iff $\eta_3 S - (\eta_1 + \eta_7 + \eta_{10} + \eta_{12}) < 0$, and $\eta_1(\eta_1 + \eta_2 + \eta_6) > 1$.

Then clearly the infection free steady state E^0 (2) is locally asymptotically stable if $R_0 < 1$.

Let $\tau > 0$. In this case, we will use Rouches s theorem to prove that all roots of the characteristic Eq. (5) cannot intersect the imaginary axis, i.e., the characteristic equation cannot have pure imaginary roots.

Suppose for the opposite, that is, suppose there exists $w \in \mathbb{R}$ such that $\lambda = wi$ is a solution of (19).

Consider the term $\eta_3 S e^{-iw\tau} - (\eta_1 + \eta_7 + \eta_{10} + \eta_{12}) = 0$

$\implies wi + (\eta_1 + \eta_7 + \eta_{10} + \eta_{12}) = \eta_3 S e^{-iw\tau}$

$\implies wi + (\eta_1 + \eta_7 + \eta_{10} + \eta_{12}) = \eta_3 S(\cos w\tau - i \sin w\tau)$

Equating the real and imaginary parts we get

$w = -i\eta_3 S \sin w\tau, (\eta_1 + \eta_7 + \eta_{10} + \eta_{12}) = \eta_3 S \cos w\tau$

Squaring and adding we get $\implies (w)^2 + (\eta_1 + \eta_7 + \eta_{10} + \eta_{12})^2 = \mu_s S^2$

$\implies w^2 = \mu_s S^2 - (\eta_1 + \eta_7 + \eta_{10} + \eta_{12})^2$

If $\mathrm{R}_0 < 1$, *then* $w^2 < 0$, which is a contradiction.

Thus the infection free consistent state E^0 is locally asymptotically stable if $\mathrm{R}_0 < 1$ for $\tau \geq 0$.

The Ruth-Hurwitz stability theory and Rouche's theorem are used in the following theorem to characterize the local stability of the SQIRV system (1) for the infectious persistent steady state solution (3). The consequence is determined by the reproduction number R_0.

Theorem 2.2. The infection persistent steady state solution E^* of (1) is locally asymptotically stable if $R_0 > 1$ for $\tau \geq 0$.

Proof.

The characteristic equation of system (1), for the equilibrium point E^*, is given by

$$\Delta(\lambda) = \left|\lambda Id_{5\times5} - J^1_{10} - J^1_{11} e^{-\tau\lambda}\right| \quad (7)$$

where $J^1_{10} =$

$$\begin{pmatrix} -(\eta_1+\eta_2) & \eta_4 & 0 & \eta_5 & \eta_6 \\ 0 & -(\eta_1+\eta_4+\eta_8+\eta_9 V^*) & \eta_7 & 0 & -\eta_9 Q^* \\ 0 & 0 & -(\eta_1+\eta_7+\eta_{10}+\eta_{12}) & 0 & 0 \\ 0 & \eta_8 & \eta_{12} & -(\eta_1+\eta_5+\eta_{11}) & 0 \\ \eta_2 & \eta_9 V^* & 0 & \eta_{11} & -(\eta_1+\eta_6)+\eta_9 Q^* \end{pmatrix}$$

and

$$J_{11}^{1} = \begin{pmatrix} -\eta_3 I^* e^{-\iota\tau} & 0 & -\eta_3 S^* e^{-\iota\tau} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \eta_3^* e^{-\iota\tau} & 0 & \eta_3 S^* e^{-\iota\tau} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial is

$$\lambda^5 + (e^{-\lambda\tau}\eta_3 I^* + F_0)\lambda^4 + (e^{-\lambda\tau}\eta_3 I^* F_1 + F_2)\lambda^3 + (e^{-\lambda\tau}\eta_3 I^* F_3 + F_4)\lambda^2 + (e^{-\lambda\tau}\eta_3 I^* F_5 + F_6)\lambda + (e^{-\lambda\tau}\eta_3 I^* F_7 + F_8) = 0 \tag{8}$$

where $F_0 = \eta_{21} - \eta_9 Q^* + \eta_{22} + \eta_9 V^* + \eta_{24} + \eta_{25}$, $F_1 = \eta_{22} + \eta_9 V^* + \eta_{24} + \eta_{25} - \eta_9 Q^*$, $F_2 = \eta_{21}(\eta_{22} + \eta_{24} + \eta_{25} + \eta_9 V^* - \eta_9 Q^*) - \eta_2\eta_6$,

$F_3 = (\eta_{24} + \eta_{25} + \eta_3 S^* - \eta_9 Q^*)(\eta_{22} + \eta_9 V^*) - \eta_4\eta_7 - \eta_3 S^*\eta_9 Q^* - \eta_5\eta_{12} - \eta_9 Q^*\eta_{24} + \eta_{24}\eta_{25} + \eta_9 Q^*\eta_9 V^* + \eta_3 S^*\eta_{24} + \eta_3 S^*\eta_{25}$,

$F_4 = (\eta_{21} - \eta_9 Q^*)(\eta_{22} + \eta_9 V^*)\eta_{24} - [\eta_9 Q^*((\eta_{21})(\eta_{22} + \eta_9 V^*) + \eta_{21}\eta_{24}) + \eta_2\eta_6((\eta_{22} + \eta_9 V^*) + \eta_{24})] + \eta_{21}((\eta_{22} + \eta_9 V^*)\eta_{25} + \eta_{24}\eta_{25} + \eta_9 Q^*\eta_9 V^*) + \eta_9 Q^*(\eta_2\eta_4 + \eta_8\eta_{11}) + \eta_{24}((\eta_{22} + \eta_9 V^*)\eta_{25} + \eta_9 Q^*\eta_9 V^*)$,

$F_5 = \eta_5\eta_9 Q^*\eta_{12} - \eta_3 S^*\eta_9 Q^*(\eta_{22} + \eta_9 V^* + \eta_{24}) + \eta_4\eta_7(\eta_{24} + \eta_{25}) + \eta_5\eta_{12}(\eta_{22} + \eta_9 V^* + \eta_{25}) + \eta_5(\eta_7\eta_8 + \eta_6\eta_9 V^*) + \eta_6\eta_{12}\eta_{11}$,

$F_5 = (\eta_9 Q^*(\eta_8\eta_{11} + \eta_{24}\eta_9 V^*) + (\eta_{22} + \eta_9 V^*)\eta_{24}(\eta_{25} - \eta_9 Q^*)) - (\eta_3 S^*\eta_9 Q^*(\eta_{22} + \eta_9 V^* + \eta_{24}) + \eta_4\eta_7(\eta_{24} + \eta_{25}) + \eta_5\eta_{12}(\eta_{22} + \eta_9 V^* + \eta_{25}) + \eta_5(\eta_7\eta_8 + \eta_6\eta_9 V^*) + \eta_6\eta_{12}\eta_{11})$,

$F_6 = (\eta_9 Q^*(\eta_8\eta_{11} + \eta_{24}\eta_9 V^*) + (\eta_{22} + \eta_9 V^*)\eta_{24}(\eta_{25} - \eta_9 Q^*)) + \eta_4\eta_9 Q^*(\eta_5\eta_7 + \eta_2\eta_{24}) + \eta_5\eta_9 Q^*(\eta_3 I^*\eta_{12} + \eta_2\eta_8)$,

$F_7 = \eta_5\eta_9 Q^*(\eta_7\eta_8 + \eta_{12}(\eta_{22} + \eta_9 V^*)) + \eta_4\eta_9 Q^*\eta_{12}\eta_{11}$,

$F_8 = (\eta_8\eta_9 V^* + \eta_{24}\eta_9 V^*)(\eta_3 S^*\eta_9 Q^* - \eta_6\eta_7) + \eta_{24}(\eta_9 Q^* + \eta_{25})(\eta_4\eta_7 - \eta_3 S^*(\eta_{22} + \eta_9 V^*)) - [\eta_{12}(\eta_{22} + \eta_9 V^*)(\eta_6\eta_{24} + \eta_5\eta_{25}) + \eta_5(\eta_7\eta_8\eta_{25} + \eta_9 Q^*\eta_{12}\eta_{11})]$

If $\tau = 0$, then by using the rule of Descartes of sign, we can get there are no positive real roots.

Also by Routh-Hurwitz stability criterion, the real parts of the complex roots are also negative if $\eta_3 I^*(F_i) + F_j > 0$ for $i = 1, 3, 5, 7; j = 0, 2, 4, 6, 8$, $(R_0 - 1) > 0, R_0 > 1$. Then the infection persistent steady state $(S^*, Q^*, I^*, R^*, V^*)$ is locally stable when $R_0 > 1$.

If $\tau > 0$, then by using Rouch's theorem, we have to prove that all roots of the characteristic Eq. (6) cannot have pure imaginary roots.

Suppose that there exists $w \in \mathbb{R}$ such that $\lambda = wi$ is a solution of (6).

Now Eq. (22) becomes

$$iw^5 + (e^{-iw\tau}\eta_3 I^* + F_0)w^4 - i(e^{-iw\tau}\eta_3 I^* F_1 + F_2)w^3 - (e^{-iw\tau}\eta_3 I^* F_3 + F_4)w^2 + i(e^{-iw\tau}\eta_3 I^* F_5 + F_6)w - (e^{-iw\tau}\eta_3 I^* F_7 + F_8) = 0 \tag{9}$$

Then

$$iw^5 + F_0 w^4 - iF_2 w^3 - F_4 w^2 + iF_6 w + F_8 = \eta_3 I^*(-w^4 + iF_1 w^3 + F_3 w^2 - iF_5 w - F_7)(cos\tau w - isin\tau w) \tag{10}$$

Equating the real and imaginary parts of (10) we get

$$F_0w^4 - F_4w^2 + F_8 = \eta_3 I^*(-w^4 + F_3w^2 - F_7)cos\tau w + (F_1w^3 - F_5w)sin\tau w \tag{11}$$

$$w^5 - F_2w^3 + F_6w = (F_1w^3 - F_5w)cos\tau w - \eta_3 I^*(-w^4 + F_3w^2 - F_7)sin\tau w \tag{12}$$

Squaring both Eqs. (12), (12) and adding we get

$$\begin{aligned} w^{10} &+ (F_0^2 - 2F_2 - \eta_3 I^{*^2})w^8 + [F_2^2 + 2F_6 - 2F_0F_4 - \eta_3 I^{*^2}(F_1^2 - 2F_3)]w^6 + [F_4^2 + 2F_0F_8 - 2F_0F_6 \\ &- \eta_3 I^{*^2}(F_3^2 - 2F_1F_5)]w^4 + [F_6^2 - 2F_7F_8 - \eta_3 I^{*^2}(F_5^2 - 2F_3F_7)]w^2 + (F_8^2 - \eta_3 I^{*^2}F_7^2) = 0. \end{aligned} \tag{13}$$

Let $z = w^2$ in (13)

$$\begin{aligned} F(z) = z^5 &+ (F_0^2 - 2F_2 - \eta_3 I^{*^2})z^4 + [F_2^2 + 2F_6 - 2F_0F_4 - \eta_3 I^{*^2}(F_1^2 - 2F_3)]z^3 + [F_4^2 + 2F_0F_8 - 2F_0F_6 \\ &- \eta_3 I^{*^2}(F_3^2 - 2F_1F_5)]z^2 + [F_6^2 - 2F_7F_8 - \eta_3 I^{*^2}(F_5^2 - 2F_3F_7)]z + (F_8^2 - \eta_3 I^{*^2}F_7^2) = 0. \end{aligned} \tag{14}$$

If $R_0 > 1$, then from Eq. (14) we can see that $F_8^2 - \eta_3 I^{*^2}F_7^2$ is strictly negtive which implies $F(0) > 0$. Thus we can get atleast one positive real root. Hence, if $R_0 > 1$ all the real parts of the roots of (8) are negative. Thus the equilibrium position E^* is stable when $R_0 > 1$ for $\tau \geq 0$.

3 Delayed SEIQI_cRVW Model Formulation

This section is focused on constructing delay SEIQI_cRVW model for our problem formulation. The delayed SEIQI_cRVW model can be formulated from the integer-order model form given in [27]. It is considered that the disease has an incubation time of the virus $\tau > 0$ transferred from susceptible period to an incubation period. The incubation period is the delay time that passes between being susceptible and showing symptoms of the virus. The suitable parameters are used to formulate the Omicron delayed SEIQI_cRVW Model, which are described in Table 2.

Table 2: Parameters and their descriptions

Parameters	Descriptions
P	The rate of human recruitment into the community
μ_s	High seductive contact rate between the susceptible and sick people
α_v	The rate at which a given part of the recovering population is vaccinated
δ_n	The average death rate across all classes
ρ_r	The rate of losing medication immunity by recovered class
ρ_v	The value at which the immune system of the vaccinated compartment deteriorates
δ_e	The death rate caused by infected people's contamination
δ_c	The rate at which exposed people progress from unconfirmed to confirmed class
ν_r	Because of several components, the regular recovery rates

(Continued)

Table 2 (continued)

Parameters	Descriptions
ζ	The diseased class's treatment rate
ζ_q	Between confirmed and isolated people, there is a high rate of effective contact
η_v	Rate of getting vaccination of a specific part of isolated individuals
γ_c	The rate at which a particular subsection of confirmed people transitions to the recovered class
γ_i	The rate at which a particular subsection of exposed people becomes infected
γ_r	Contact rate between infected and recovered classes
ζ_w	Contact rate between the Confirmed people and reservoir
ω_c	Contact rate between the infected people and reservoir
k	Rate at which exposed people move to isolated class

Considering the given aspects, the SEIQI_cRVW delay mathematical model is derived as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= P - \delta_n S - \mu_s S(t-\tau)I(t-\tau) + \rho_r R + \rho_v V, \\
\frac{dE}{dt} &= \mu_s S(t-\tau)I(t-\tau) - (\xi_1)E, \\
\frac{dI}{dt} &= \gamma_i E - (\xi_2)I, \\
\frac{dQ}{dt} &= kE + \zeta I + \zeta_q I_c - (\xi_3)Q, \\
\frac{dI_c}{dt} &= \delta_c E - (\xi_4)I_c, \\
\frac{dR}{dt} &= \nu_r I + \gamma_r Q + \gamma_c I_c - (\xi_5)R, \\
\frac{dV}{dt} &= \eta_v Q + \alpha_v R - (\xi_6)V \\
\frac{dW}{dt} &= \omega_c I + \zeta_w I_c - \delta_n W,
\end{aligned}
\tag{15}
$$

where $\xi_1 = \delta_n + k + \gamma_i + \delta_c$, $\xi_2 = \delta_n + \delta_e + \nu_r + \zeta + \omega_c$, $\xi_3 = \gamma_r + \eta_v + \delta_n$, $\xi_4 = \zeta_q + \gamma_c + \zeta_w + \delta_n$, $\xi_5 = \delta_n + \rho_r + \alpha_v$ and $\xi_6 = \delta_n + \rho_v$.

Subject to initial conditions: $S(0) = S_0, E(0) = E_0, I(0) = I_0, Q(0) = Q_0, I_c(0) = I_{c_0}, R(0) = R_0^0, V(0) = V_0$.

3.1 Steady State Solutions the Delayed SEIQI$_c$RVW Model

The system (15) is found static, i.e., the solutions of time independent are obtained. The steady state solutions in the infection free state, when $I = 0$ is given by

$$E_q^0 = (S^0, E^0, I^0, Q^0, I_c^0, R^0, V^0, W^0) = \left(\frac{P}{\delta_n}, 0, 0, 0, 0, 0, 0, 0\right). \tag{16}$$

Also, when infection is persistent the steady state solutions, i.e., $I \neq 0$ is given by

$$E_q^* = (S^*, E^*, I^*, Q^*, I_c^*, R^*, V^*, W^*) \tag{17}$$

where

$$S^* = \frac{\xi_2}{\gamma_i}\left(\frac{P\mu_s\gamma_i - \delta_n\xi_1\xi_2}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)}\right), E^* = \frac{\xi_1\xi_2}{\mu_s\gamma_i}\left(\frac{P\mu_s\gamma_i - \delta_n\xi_1\xi_2}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)}\right)$$

$$I^* = \frac{P\mu_s\gamma_i - \delta_n\xi_1\xi_2}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)} = \frac{\delta_n\xi_1\xi_2(R_0 - 1)}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)}$$

$$Q^* = \frac{\xi_2\xi_4 + \zeta\xi_4\gamma_i + \zeta_q\delta_c\xi_2}{\xi_3\xi_4\gamma_i}\left(\frac{P\mu_s\gamma_i - \delta_n\xi_1\xi_2}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)}\right)$$

$$I_c^* = \frac{\delta_c\xi_2}{\xi_4\gamma_i}\left(\frac{P\mu_s\gamma_i - \delta_n\xi_1\xi_2}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)}\right)$$

$$R^* = \frac{\nu_r\gamma_i\xi_3\xi_4 + \gamma_r(\xi_2\xi_4 + \zeta\xi_4\gamma_i + \zeta_q\delta_c\xi_2) + \gamma_i\delta_c\xi_2\xi_3}{\gamma_i\xi_3\xi_4\xi_5}\left(\frac{P\mu_s\gamma_i - \delta_n\xi_1\xi_2}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)}\right)$$

$$V^* = \frac{\eta_v\gamma_i\xi_5(\xi_2\xi_4 + \zeta\xi_4\gamma_i + \zeta_q\delta_c\xi_2)}{\xi_3\xi_4\gamma_i\xi_5} + \frac{\alpha_v[\nu_r\gamma_i\xi_3\xi_4 + \gamma_r(\xi_2\xi_4 + \zeta\xi_4\gamma_i + \zeta_q\delta_c\xi_2) + \gamma_i\delta_c\xi_2\xi_3]}{\xi_3\xi_4\gamma_i\xi_5}\left(\frac{P\mu_s\gamma_i - \delta_n\xi_1\xi_2}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)}\right)$$

$$W^* = \frac{\gamma_i\omega_c\xi_4 + \zeta_w\delta_c\xi_2}{\delta_n\gamma_i\xi_4}\left(\frac{P\mu_s\gamma_i - \delta_n\xi_1\xi_2}{\mu_s(\xi_1\xi_2 - \rho_v\gamma_i J - \gamma_i G)}\right)$$

with

$$J = \frac{\nu_r\gamma_i\xi_3\xi_4 + \gamma_r(\xi_2\xi_4 + \zeta\xi_4\gamma_i + \zeta_q\delta_c\xi_2) + \gamma_i\delta_c\xi_2\xi_3}{\gamma_i\xi_3\xi_4\xi_5},$$

$$G = \frac{\eta_v\gamma_i\xi_5(\xi_2\xi_4 + \zeta\xi_4\gamma_i + \zeta_q\delta_c\xi_2)}{\xi_3\xi_4\gamma_i\xi_5} + \frac{\alpha_v[\nu_r\gamma_i\xi_3\xi_4 + \gamma_r(\xi_2\xi_4 + \zeta\xi_4\gamma_i + \zeta_q\delta_c\xi_2) + \gamma_i\delta_c\xi_2\xi_3]}{\xi_3\xi_4\gamma_i\xi_5}.$$

The basic reproduction number R_0 is

$$R_0(GV^{-1}) = \frac{P\mu_s\gamma_i}{\delta_n(\delta_n + k + \gamma_i + \delta_c)(\delta_n + \delta_e + \nu_r + \zeta + \omega_c)}. \quad (18)$$

3.2 Stability Analysis of the Delayed SEIQI$_c$RVW Model

The local stability of the SEIQI$_c$RVW system (15) for the infection-free steady state solution (16) is examined in the next theorem applying Rouche's theorem. The reproduction number R_0 determines the result.

Theorem 3.1. The infection free consistent state E^0 (16) is locally asymptotically stable if $\mathrm{R}_0 < 1$ and unstable if $R_0 > 1$ for the time delay $\tau \geq 0$.

Proof. The characteristic equation of system 15, for the equilibrium point E^0, is given by

$$\Delta(\lambda) = \left|\lambda Id_{8\times 8} - J_{00} - J_{01}e^{-\tau\lambda}\right| \quad (19)$$

where

$$J_{00} = \begin{pmatrix} -\delta_n & 0 & 0 & 0 & 0 & \rho_r & \rho_v & 0 \\ 0 & -\xi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_i & \xi_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & k & \zeta & -\xi_3 & \zeta_q & 0 & 0 & 0 \\ 0 & \delta_c & 0 & 0 & -\xi_4 & 0 & 0 & 0 \\ 0 & 0 & \nu_r & \gamma_r & \gamma_c & -\xi_5 & 0 & 0 \\ 0 & 0 & 0 & \eta_v & 0 & \alpha_v & -\xi_6 & 0 \\ 0 & 0 & \omega_c & 0 & \zeta_w & 0 & 0 & -\delta_n \end{pmatrix},$$

and

$$J_{01} = \begin{pmatrix} 0 & 0 & -\mu_s S & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_s S & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\begin{aligned} C_1(\lambda) &= (\lambda + \delta_n)(\lambda + \delta_n)\left(\lambda + \xi_1 + \xi_2 + \sqrt{\xi_1^2 + 4\mu_s S\gamma_i - 2\xi_1\xi_2 + \xi_2^2}\right)(\lambda + \xi_1 + \xi_2 \\ &\quad - \sqrt{\xi_1^2 + 4\mu_s S\gamma_i - 2\xi_1\xi_2 + \xi_2^2})(\lambda + \xi_3)(\lambda + \xi_4)(\lambda + \xi_5)(\lambda + \xi_6). \end{aligned} \quad (20)$$

When $\tau = 0$, the eigenvalues are $-\delta_n$, $-\delta_n$, $\frac{1}{2}\big(-\xi_1 - \xi_2 - \sqrt{\xi_1^2 + 4\mu_s S\gamma_i - 2\xi_1\xi_2 + \xi_2^2}\big)$, $\frac{1}{2}\big(-\xi_1 - \xi_2 + \sqrt{\xi_1^2 + 4\mu_s S\gamma_i - 2\xi_1\xi_2 + \xi_2^2}\big)$, $-\xi_3$, $-\xi_4$, $-\xi_5$, $-\xi_6$.

The given system (15) is stable when $-\xi_1 - \xi_2 + \sqrt{\xi_1^2 + 4\mu_s S\gamma_i - 2\xi_1\xi_2 + \xi_2^2} < 0$
or $\sqrt{\xi_1^2 + 4\mu_s S\gamma_i - 2\xi_1\xi_2 + \xi_2^2} < (\xi_1 + \xi_2)$ or $\xi_1^2 + 4\mu_s S\gamma_i - 2\xi_1\xi_2 + \xi_2^2 < (\xi_1 + \xi_2)^2$
or $\mu_s S\gamma_i < \xi_1\xi_2$. i.e., $\frac{\mu_s S\gamma_i}{\xi_1\xi_2} < 1$.

That is $R_0 < 1$. Clearly infection free steady state E^0 is locally asymptotically stable if $R_0 < 1$ when $\tau = 0$.

Let $\tau > 0$. In this case, we will use Rouche's theorem to prove that all roots of the characteristic Eq. (19) cannot intersect the imaginary axis, i.e., the characteristic equation cannot have pure imaginary roots.

Suppose for the opposite, that is, suppose there exists $w \in \mathbb{R}$ such that $\lambda = wi$ is a solution of (19).

Consider the term $wi + \xi_1 + \xi_2 - \sqrt{\xi_1^2 + 4\mu_s S e^{-\tau wi}\gamma_i - 2\xi_1\xi_2 + \xi_2^2} = 0$

$\implies wi + \xi_1 + \xi_2 = \sqrt{\xi_1^2 + 4\mu_s S e^{-\tau wi}\gamma_i - 2\xi_1\xi_2 + \xi_2^2} = 0$

$\implies (wi + \xi_1 + \xi_2)^2 = \xi_1^2 + 4\mu_s S e^{-\tau wi}\gamma_i - 2\xi_1\xi_2 + \xi_2^2 = 0$

$\implies (wi)^2 + (\xi_1 + \xi_2)^2 + wi(\xi_1 + \xi_2) - \xi_1^2 + 2\xi_1\xi_2 - \xi_2^2 = 4\mu_s S(\cos\tau w - i\sin\tau w)\gamma_i$

$\implies -w^2 + wi(\xi_1 + \xi_2) + 4\xi_1\xi_2 = 4\mu_s S(\cos\tau w - i\sin\tau w)\gamma_i$

By equating the real and imaginary part, we get

$4\xi_1\xi_2 - w^2 = 4\mu_s\gamma_i S\cos\tau w,\ w(\xi_1 + \xi_2) = -4\mu_s\gamma_i S\sin\tau w$

If $\mathrm{R}_0 < 1,\ then\ \mu_s S\gamma_i - \xi_1\xi_2 > 0$. Hence $w^2 < 0$, which is a contradiction.

Thus the infection free consistent state E^0 is locally asymptotically stable if $\mathrm{R}_0 < 1$ for $\tau \geq 0$.

Now suppose that $R_0 > 1$ from the characteristic polynomial (20), it is enough to consider the term $(\lambda + \xi_1 + \xi_2 - \sqrt{\xi_1^2 + 4\mu_s S\gamma_i - 2\xi_1\xi_2 + \xi_2^2})$. It is easy to see that $C_1(0) < 0$. On the other hand, $\lim_{\lambda\to+\infty} C_1(\lambda) = +\infty$. Therefore, by continuity of $C_1(\lambda)$, there is at least one positive root of the characteristic Eq. (20). Hence, we conclude that Σ_1 is unstable, for any $\tau \geq 0$.

The local stability of the SEIQI$_c$RVW system (15) for the infection's persistent steady state solution (17) is determined using Rouche's theorem and the Routh-Hurwitz technique in the next theorem. The result is governed by the reproduction number R_0.

Theorem 3.2. If $\mathrm{R}_0 > 1$, then the endemic equilibrium point E^* is locally asymptotically stable for $\tau \geq 0$.

Proof. The characteristic equation of system 15, for the equilibrium point E^* 17 is given by

$$\Delta(\lambda) = \left|\lambda Id_{8\times 8} - J_{10} - J_{11}e^{-\tau\lambda}\right|. \tag{21}$$

Where the Jacobian matrices of the model at infection persistent steady state solution are

$$J(10) = \begin{pmatrix} -\delta_n & 0 & 0 & 0 & 0 & \rho_r & \rho_v & 0 \\ 0 & -\xi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_i & \xi_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & k & \zeta & -\xi_3 & \zeta_q & 0 & 0 & 0 \\ 0 & \delta_c & 0 & 0 & -\xi_4 & 0 & 0 & 0 \\ 0 & 0 & \nu_r & \gamma_r & \gamma_c & -\xi_5 & 0 & 0 \\ 0 & 0 & 0 & \eta_v & 0 & \alpha_v & -\xi_6 & 0 \\ 0 & 0 & \omega_c & 0 & \zeta_w & 0 & 0 & -\delta_n \end{pmatrix}.$$

and

$$J(11) = \begin{pmatrix} -\mu_s I^* & 0 & -\mu_s S^* & 0 & 0 & 0 & 0 & 0 \\ \mu_s I^* & 0 & \mu_s S^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The characteristic equation is

$(-\delta_n-\lambda)(\rho_v\mu_s I^* e^{-\lambda\tau}[(\xi_2\xi_4 k\xi_5\eta_v+\xi_2\zeta_q\delta_c\xi_5\eta_v+\xi_4\eta\gamma_i\xi_5\eta_v+\xi_3\xi_4\gamma_i\nu_r s+\xi_2\xi_4 k\gamma_r\alpha_v+\xi_2\zeta_q\delta_c\gamma_r\alpha_v+\xi_4\eta\gamma_i\gamma_r\alpha_v+\xi_3\xi_2\delta_c\gamma_c\alpha_v)+(\xi_2\xi_4 k\eta_v+\xi_2\zeta_q\delta_c\eta_v+\xi_4\eta\gamma_i\eta_v+\xi_2 k\xi_5\eta_v+\xi_4 k\xi_5\eta_v+\zeta_q\delta_c\xi_5\eta_v+\eta\gamma_i\xi_5\eta_v+\xi_3\gamma_i\nu_r\alpha_v+\xi_4\gamma_i\nu_r\alpha_v+\xi_2 k\gamma_r\alpha_v+\xi_4 k\gamma_r\alpha_v+\zeta_q\delta_c\gamma_r\alpha_v+\eta\gamma_i\gamma_r\alpha_v+\xi_3\delta_c\gamma_c\alpha_v+\xi_2\delta_c\gamma_c\alpha_v)\lambda+(\xi_2 k\eta_v+\xi_4 k\eta_v+\zeta_q\delta_c\eta_v+\eta\gamma_i\eta_v+k\xi_5\eta_v+\gamma_i\nu_r\alpha_v+k\gamma_r\alpha_v+\delta_c\gamma_c\alpha_v)\lambda^2+k\eta_v\lambda^3]+(-\xi_6-\lambda)(-\rho_r\mu_s I^* e^{-\lambda\tau}[(\xi_3\xi_4\gamma_i\nu_r+\xi_2\xi_4 k\gamma_r+\xi_2\zeta_q\delta_c\gamma_r+\xi_4\eta\gamma_i\gamma_r+\xi_3\xi_2\delta_c\gamma_c)+(\xi_3\gamma_i\nu_r+\xi_4\gamma_i\nu_r+\xi_2 k\gamma_r+\xi_4 k\gamma_r+\zeta_q\delta_c\gamma_r+\eta\gamma_i\gamma_r+\xi_3\delta_c\gamma_c+\xi_2\delta_c\gamma_c)\lambda+(\gamma_i\nu_r+k\gamma_r+\delta_c\gamma_c)\lambda^2]+(-\xi_3-\lambda)(-\xi_4-\lambda)(-\xi_5-\lambda)(-\gamma_i\mu_s S^* e^{-\lambda\tau}(-\delta_n-\lambda)+(-\xi_2-\lambda)(\delta_n\xi_1+\xi_1\mu_s I^* e^{-\lambda\tau}+(\delta_n+\xi_1+\mu_s I^* e^{-\lambda\tau})x+\lambda^2)))))=0.$

To check about the stability, consider the second term of the above characteristic equation

$$\begin{aligned} &\lambda^7+(e^{-\lambda\tau}\mu_s I^*+D_0)\lambda^6+(e^{-\lambda\tau}[\mu_s I^* D_1-\mu_s S^* D_2]+D_3)\lambda^5+(e^{-\lambda\tau}[\mu_s I^* D_4-\mu_s S^* D_5]+D_6)\lambda^4\\ &\quad+(e^{-\lambda\tau}[\mu_s I^* D_7-\mu_s S^* D_8]+D_9)\lambda^3+(e^{-\lambda\tau}[\mu_s I^* D_{10}-\mu_s S^* D_{11}]+D_{12})\lambda^2\\ &\quad+(e^{-\lambda\tau}[\mu_s I^* D_{13}-\mu_s S^* D_{14}]+D_{15})\lambda+(e^{-\lambda\tau}[\mu_s I^* D_{16}-\mu_s S^* D_{17}]+D_{18})=0 \end{aligned} \tag{22}$$

where $D_0=\delta_n+\xi_1+\xi_2+\xi_3+\xi_4+\xi_5+\xi_6$, $D_1=\xi_1+\xi_2+\xi_3+\xi_4+\xi_5+\xi_6$, $D_2=\gamma_i$, $D_3=\delta_n\xi_3+\delta_n\xi_1+\xi_3\xi_1+\delta_n\xi_2+\xi_3\xi_2+\xi_1\xi_2+\delta_n\xi_4+\xi_3\xi_4+\xi_1\xi_4+\xi_2\xi_4+\delta_n\xi_5+\xi_3\xi_5+\xi_1\xi_5+\xi_2\xi_5+\xi_4\xi_5+\delta_n\xi_6+\xi_3\xi_6+\xi_1\xi_6+\xi_2\xi_6+\xi_4\xi_6+\xi_5\xi_6$,

$D_4=\xi_1\xi_2+\xi_1\xi_3+\xi_1\xi_4+\xi_1\xi_5+\xi_1\xi_6+\xi_2\xi_3+\xi_2\xi_4+\xi_2\xi_5+\xi_2\xi_6+\xi_3\xi_4+\xi_3\xi_5+\xi_3\xi_6+\xi_4\xi_5+\xi_4\xi_6+\xi_5\xi_6$, $D_5=\gamma_i[\delta_n+\xi_3+\xi_4+\xi_5+\xi_6]$, $D_6=\delta_n[\xi_3\xi_1+\xi_3\xi_2+\xi_1\xi_2+\xi_3\xi_4+\xi_1\xi_4+\xi_2\xi_4+\xi_3\xi_5+\xi_1\xi_5+\xi_2\xi_5+\xi_4\xi_5+\xi_2\xi_6+\xi_3\xi_6+\xi_1\xi_6+\xi_4\xi_6+\xi_5\xi_6]+\xi_3\xi_4\xi_5+\xi_1\xi_4\xi_5+\xi_2\xi_4\xi_5+\xi_3\xi_1\xi_2+\xi_3\xi_1\xi_6+\xi_3\xi_2\xi_6+\xi_3\xi_2\xi_4+\xi_3\xi_2\xi_5+\xi_1\xi_2\xi_5+\xi_1\xi_2\xi_4+\xi_3\xi_1\xi_4+\xi_1\xi_2\xi_6+\xi_3\xi_1\xi_5+\xi_3\xi_4\xi_6+\xi_1\xi_4\xi_6+\xi_2\xi_4\xi_6+\xi_3\xi_5\xi_6+\xi_1\xi_5\xi_6+\xi_2\xi_5\xi_6+\xi_4\xi_5\xi_6$,

$D_7=\xi_3\xi_1\xi_2+\xi_3\xi_1\xi_2+\xi_3\xi_2\xi_4+\xi_1\xi_2\xi_4+\xi_3\xi_1\xi_5+\xi_3\xi_2\xi_5+\xi_1\xi_2\xi_5+\xi_3\xi_4\xi_5+\xi_1\xi_4\xi_5+\xi_2\xi_4\xi_5+\xi_3\xi_1\xi_6+\xi_3\xi_2\xi_6+\xi_1\xi_2\xi_6+\xi_3\xi_4\xi_6+\xi_1\xi_4\xi_6+\xi_2\xi_4\xi_6+\xi_3\xi_5\xi_6+\xi_1\xi_5\xi_6+\xi_2\xi_5\xi_6+\xi_4\xi_5\xi_6-\rho_r\gamma_i\nu_r-\rho_r k\gamma_r-\rho_r\delta_c\gamma_c-\rho_v k\eta_v$, $D_8=\gamma_i[\xi_3\xi_4+\delta_n\xi_5+\xi_3\xi_5+\delta_n\xi_6+\xi_3\xi_6+\xi_4\xi_6+\xi_5\xi_6+\delta_n\xi_3+\xi_4\xi_5+\delta_n\xi_4]$, $D_9=\xi_3\xi_1\xi_2\xi_4+\delta_n\xi_3\xi_1\xi_2+\delta_n\xi_3\xi_1\xi_4+\delta_n\xi_3\xi_2\xi_4+\delta_n\xi_1\xi_2\xi_4+\delta_n\xi_3\xi_1\xi_5+\delta_n\xi_3\xi_2\xi_5+\delta_n\xi_1\xi_2\xi_5+\xi_3\xi_1\xi_2\xi_5+\delta_n\xi_3\xi_4\xi_5+\delta_n\xi_1\xi_4\xi_5+\xi_3\xi_1\xi_4\xi_5+\delta_n\xi_2\xi_4\xi_5+\xi_3\xi_2\xi_4\xi_5+\xi_1\xi_2\xi_4\xi_5+\delta_n\xi_3\xi_1\xi_6+\delta_n\xi_3\xi_2\xi_6+\delta_n\xi_1\xi_2\xi_6+\xi_3\xi_1\xi_2\xi_6+\delta_n\xi_3\xi_4\xi_6+\delta_n\xi_1\xi_4\xi_6+\xi_3\xi_1\xi_4\xi_6+\delta_n\xi_2\xi_4\xi_6+\xi_3\xi_2\xi_4\xi_6+\xi_1\xi_2\xi_4\xi_6+\delta_n\xi_3\xi_5\xi_6+\delta_n\xi_1\xi_5\xi_6+\xi_3\xi_1\xi_5\xi_6+\delta_n\xi_2\xi_5\xi_6+\xi_3\xi_2\xi_5\xi_6+\xi_1\xi_2\xi_5\xi_6+\delta_n\xi_4\xi_5\xi_6+\xi_3\xi_4\xi_5\xi_6+\xi_1\xi_4\xi_5\xi_6+\xi_2\xi_4\xi_5\xi_6$,

$D_{10}=\xi_3\xi_1\xi_2\xi_4+\xi_3\xi_1\xi_2\xi_5+\xi_3\xi_1\xi_4\xi_5+\xi_3\xi_2\xi_4\xi_5+\xi_1\xi_2\xi_4\xi_5+\xi_3\xi_1\xi_2\xi_6+\xi_3\xi_1\xi_4\xi_6+\xi_3\xi_2\xi_4\xi_6+\xi_1\xi_2\xi_4\xi_6+\xi_3\xi_1\xi_5\xi_6+\xi_3\xi_2\xi_5\xi_6+\xi_1\xi_2\xi_5\xi_6+\xi_3\xi_4\xi_5\xi_6+\xi_1\xi_4\xi_5\xi_6+\xi_2\xi_4\xi_5\xi_6-\xi_3\rho_r\gamma_i\nu_r-\rho_r\xi_4\gamma_i\nu_r-\rho_r\xi_2 k\gamma_r-\rho_r\xi_2 k\gamma_r-\rho_r\zeta_q\delta_c\gamma_r-\rho_r\eta\gamma_i\gamma_r-\xi_3\rho_r\delta_c\gamma_c-\rho_r\xi_2\delta_c\gamma_c-\rho_v\xi_2 k\eta_v-\rho_v\xi_4 k\eta_v-\rho_v\zeta_q\delta_c\eta_v-\rho_v\eta\gamma_c\eta_v-\rho_v k\xi_5\eta_v-\rho_v\gamma_i\nu_r\alpha_v-\rho_v k\gamma_r\alpha_v-\rho_v\delta_c\gamma_c\alpha_v-\rho_r\gamma_i\nu_r\xi_6-\rho_r k\gamma_r\xi_6-\rho_r\delta_c\gamma_c\xi_6$, $D_{11}=\gamma_i[\delta_n\xi_3\xi_4+\delta_n\xi_3\xi_5+\delta_n\xi_4\xi_5+\xi_3\xi_4\xi_5+\delta_n\xi_3\xi_6+\delta_n\xi_4\xi_6+\xi_3\xi_4\xi_6+\delta_n\xi_5\xi_6+\xi_3\xi_5\xi_6+\xi_4\xi_5\xi_6]$, $D_{12}=\delta_n\xi_3\xi_1\xi_2\xi_4+\delta_n\xi_3\xi_1\xi_2\xi_5+\delta_n\xi_3\xi_1\xi_4\xi_5+\delta_n\xi_3\xi_2\xi_4\xi_5+\delta_n\xi_1\xi_2\xi_4\xi_5+\xi_3\xi_1\xi_2\xi_4\xi_5+\delta_n\xi_3\xi_1\xi_2\xi_6+\delta_n\xi_3\xi_1\xi_4\xi_6+\delta_n\xi_3\xi_2\xi_4\xi_6+\delta_n\xi_1\xi_2\xi_4\xi_6+\xi_3\xi_1\xi_2\xi_4\xi_6+\delta_n\xi_3\xi_1\xi_5\xi_6+\delta_n\xi_3\xi_2\xi_5\xi_6+\delta_n\xi_1\xi_2\xi_5\xi_6+\xi_3\xi_1\xi_2\xi_5\xi_6+\delta_n\xi_3\xi_4\xi_5\xi_6+\delta_n\xi_1\xi_4\xi_5\xi_6+\xi_3\xi_1\xi_4\xi_5\xi_6+\delta_n\xi_2\xi_4\xi_5\xi_6+\xi_3\xi_2\xi_4\xi_5\xi_6+\xi_1\xi_2\xi_4\xi_5\xi_6$,

$D_{13}=\xi_3\xi_1\xi_2\xi_4\xi_5+\xi_3\xi_1\xi_2\xi_4\xi_6+\xi_3\xi_1\xi_2\xi_5\xi_6+\xi_3\xi_1\xi_4\xi_5\xi_6+\xi_3\xi_2\xi_4\xi_5\xi_6+\xi_1\xi_2\xi_4\xi_5\xi_6-\xi_3\rho_r\xi_4\gamma_i\nu_r-\rho_r\xi_2\xi_4 k\gamma_r-\rho_r\xi_2\zeta_q\delta_c\gamma_r-\rho_r\xi_4\eta\gamma_i\gamma_r-\xi_3\rho_r\xi_2\delta_c\gamma_c-\rho_v\xi_2\xi_4 k\eta_v-\rho_v\xi_2\zeta_q\delta_c\eta_v-\rho_v\xi_4 h\gamma_i\eta_v-\rho_v\xi_2 k\xi_5\eta_v-\rho_v\xi_4 k\xi_5\eta_v-\rho_v\zeta_q\delta_c\xi_5\eta_v-$

$\rho_v\eta\gamma_i\xi_5\eta_v - \xi_3\rho_v\gamma_i\nu_r\alpha_v - \rho_v\xi_4\gamma_i\nu_r\alpha_v - \rho_v\xi_2 k\gamma_r\alpha_v - \rho_v\xi_4 k\gamma_r\alpha_v - \rho_v\zeta_q\delta_c\gamma_r\alpha_v - \rho_v\eta\gamma_i\gamma_r\alpha_v - \xi_3\rho_v\delta_c\gamma_c\alpha_v - \rho_v\xi_2\delta_c\gamma_c\alpha_v - \xi_3\rho_r\gamma_i\nu_r\xi_6 - \rho_r\xi_4\gamma_i\nu_r\xi_6 - \rho_r\xi_2 k\gamma_r\xi_6 - \rho_r\xi_4 k\gamma_r\xi_6 - \rho_r\zeta_q\delta_c\gamma_r\xi_6 - \rho_r\eta\gamma_i\gamma_r\xi_6 - \xi_3\rho_r\delta_c\gamma_c\xi_6 - \rho_r\xi_2\delta_c\gamma_c\xi_6$,

$D_{14} = \gamma_i[\delta_n\xi_3\xi_4\xi_5 + \delta_n\xi_3\xi_4\xi_6 + \delta_n\xi_3\xi_5\xi_6 + \delta_n\xi_4\xi_5\xi_6 + \xi_3\xi_4\xi_5\xi_6]$, $D_{15} = \delta_n\xi_3\xi_1\xi_2\xi_4\xi_5 + \delta_n\xi_3\xi_1\xi_2\xi_4\xi_6 + \delta_n\xi_3\xi_1\xi_2\xi_5\xi_6 + \delta_n\xi_3\xi_1\xi_4\xi_5\xi_6 + \delta_n\xi_3\xi_2\xi_4\xi_5\xi_6 + \delta_n\xi_1\xi_2\xi_4\xi_5\xi_6 + \xi_3\xi_1\xi_2\xi_4\xi_5\xi_6$,

$D_{16} = \rho_v\xi_2\xi_4 k\xi_5\eta_v + \xi_3\xi_1\xi_2\xi_4\xi_5\xi_6 - \rho_v\xi_2\zeta_q\delta_c\xi_5\eta_v - \rho_v\xi_4\eta\gamma_i\xi_5\eta_v - \xi_3\rho_v\xi_4\gamma_i\nu_r\alpha_v - \rho_v\xi_2\xi_4 k\gamma_r\alpha_v - \rho_v\xi_2\zeta_q\delta_c\gamma_r\alpha_v - \rho_v\xi_4\eta\gamma_i\gamma_r\alpha_v - \xi_3\rho_v\xi_2\delta_c\gamma_c\alpha_v - \xi_3\rho_r\xi_4\gamma_i\nu_r\xi_6 - \rho_r\xi_2\xi_4 k\gamma_r\xi_6 - \rho_r\xi_2\zeta_q\delta_c\gamma_r\xi_6 - \rho_r\xi_4\eta\gamma_i\gamma_r\xi_6 - \xi_3\rho_r\xi_2\delta_c\gamma_c\xi_6$, $D_{17} = \gamma_i\delta_n\xi_3\xi_4\xi_5\xi_6$, $D_{18} = \delta_n\xi_1\xi_2\xi_2\xi_4\xi_5\xi_6$.

If $\tau = 0$, then by using the rule of Descartes of sign, we can get there are no positive real roots.

Also by Routh-Hurwitz stability criterion, the real parts of the complex roots are also negative if $\mu_s I^*(\xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6) - \mu_s S^*\gamma_i + D_3 > 0, (1 - \mathrm{R}_0) < 0, \mathrm{R}_0 > 1$. Then the infection persistent steady state $(S^*, E^*, I^*, Q^*, I_c^*, R^*, V^*, W^*)$ is locally stable when $\mathrm{R}_0 < 1$.

If $\tau > 0$, then by using Rouche's theorem, we have to prove that all roots of the characteristic Eq. (22) cannot have pure imaginary roots. Suppose that there exists $w \in \mathbb{R}$ such that $\lambda = wi$ is a solution of (22). Now Eq. (22) becomes

$$-iw^7 - (e^{-iw\tau}\mu_s I^* + D_0)w^6 + i(e^{-iw\tau}[\mu_s I^* D_1 - \mu_s S^* D_2] + D_3)w^5 + (e^{-iw\tau}[\mu_s I^* D_4 - \mu_s S^* D_5] + D_6)w^4 - i(e^{-iw\tau}[\mu_s I^* D_7 - \mu_s S^* D_8] + D_9)w^3 - (e^{-iw\tau}[\mu_s I^* D_{10} - \mu_s S^* D_{11}] + D_{12})w^2 + i(e^{-iw\tau}[\mu_s I^* D_{13} - \mu_s S^* D_{14}] + D_{15})w + (e^{-iw\tau}[\mu_s I^* D_{16} - \mu_s S^* D_{17}] + D_{18}) = 0. \tag{23}$$

Then,

$$E_1 + iE_2 w - E_3 w^2 - iE_4 w^3 + E_5 w^4 + iE_6 w^5 - E_7 w^6 - iw^7 = (-E_1^* - iE_2^* w + E_3^* w^2 + iE_4^* w^3 - E_5^* w^4 - iE_6^* w^5 + E_7^* w^6)(\cos\tau w - i\sin\tau w) \tag{24}$$

where $E_1 = D_{18}, E_1^* = \mu_s I^* D_{16} - \mu_s S^* D_{17}, E_2 = D_{15}, E_2^* = \mu_s I^* D_{13} - \mu_s S^* D_{14}, E_3 = D_{12}, E_3^* = \mu_s I^* D_{10} - \mu_s S^* D_{11}, E_4 = D_9, E_4^* = \mu_s I^* D_7 - \mu_s S^* D_8, E_5 = D_6, E_5^* = \mu_s I^* D_4 - \mu_s S^* D_5, E_6 = D_3, E_6^* = \mu_s I^* D_1 - \mu_s S^* D_2, E_7 = D_0, E_7^* = \mu_s I^*$, Equating the real and imaginary parts of (24), we get

$$E_1 - E_3 w^2 + E_5 w^4 - E_7 w^6 = (-E_1^* + E_3^* w^2 - E_5^* w^4 + E_7^* w^6)\cos\tau w - (-E_2^* w + E_4^* w^3 - E_6^* w^5)\sin\tau w \tag{25}$$

$$E_2 w - E_4 w^3 + E_6 w^5 - w^7 = (-E_2^* w + E_4^* w^3 - E_6^* w^5)\cos\tau w + (-E_1^* + E_3^* w^2 - E_5^* w^4 + E_7^* w^6)\sin\tau w. \tag{26}$$

Squaring both Eqs. (26), (27) and adding, we get

$$w^{14} + (E_7^2 - E_6 - E_7^{*^2})w^{12} + (E_6^2 + E_2 E_6 + E_5^* E_7^* - E_4 - E_5 E_7 - E_2^* E_6^* - E_6^{*^2})w^{10} + (E_5^2 + E_3 E_7 + E_4^* E_6^* - E_2 - E_4 E_6 - E_3^* E_7^* - E_5^{*^2})w^8 + (E_4^2 + E_1^* E_7^* + E_3^* E_5^* - E_1 E_7 - E_3 E_5 - E_4^{*^2})w^6 + (E_3^2 + E_1 E_5 + E_2^* E_4^* - E_1^* E_5^* - E_2 E_4 - E_3^{*^2})w^4 + (E_2 - E_2^{*^2})w^2 + (E_1^2 - E_1^{*^2}) = 0. \tag{27}$$

Let $z = w^2$ in (27)

$$F(z) = z^7 + (E_7^2 - E_6 - E_7^{*^2})z^6 + (E_6^2 + E_2E_6 + E_5^*E_7^* - E_4 - E_5E_7 - E_2^*E_6^* - E_6^{*^2})z^5 + (E_5^2 + E_3E_7 + E_4^*E_6^* - E_2 - E_4E_6 - E_3^*E_7^* - E_5^{*^2})z^4 + (E_4^2 + E_1^*E_7^* + E_3^*E_5^* - E_1E_7 - E_3E_5 - E_4^{*^2})z^3 + (E_3^2 + E_1E_5 + E_2^*E_4^* - E_1^*E_5^* - E_2E_4 - E_3^{*^2})z^2 + (E_2 - E_2^{*^2})z + (E_1^2 - E_1^{*^2}) = 0. \tag{28}$$

If $R_0 > 1$, then from Eq. (28), we can see that $(E_1^{*^2} - E_1^2)$ is strictly positive which implies $F(0) < 0$. Thus, we can get atleast one positive real root. Hence, if $R_0 > 1$ all the real parts of the roots of (22) are negative. Thus, the equilibrium position E^* is stable when $R_0 > 1$ for $\tau \geq 0$.

4 Numerical Analysis

During the second wave of the Corona virus, India experienced a high infection rate. We obtained data for this article from Tamilnadu, India. This current Omicron variant pandemic data of Tamilnadu, India is validated with our theoretical findings. The source of the data is specified by [28] and [23]. Tamil Nadu encountered its most memorable instance of the Omicron type of SARS-CoV-2 on December 15, 2021, as indicated by a traveller from another country. Three weeks after the first confirmed Omicron case was reported, Tamil Nadu was infected with the highly transmissible and rapidly spreading form of SARS-Cov-2. The data for this study is gathered from the state of Tamil Nadu (Chennai). As of March 11, 2022, there were 750606 positive cases, 750520 discharged cases, 48 deaths, 499 active cases, 42 positive cases on March 11, 2022, 86 recovered cases, and 3373 vaccinated cases. The state of Tamilnadu achieves a zero-death rate and a safe position against the spread of Omicron on March 11, 2022. We used Mathematica for plotting the solution. The values of the variables and parameters are listed in the Tables 3 and 4 below.

Table 3: Values of the variables (SQIRV)

Parameters	Values	Parameters	Values	Parameters	Values
$S(0)$	3233	η_1	0.0870	η_7	0.6707
$Q(0)$	744	η_2	0.0104	η_8	0.0580
$I(0)$	499	η_3	0.1543	η_9	0.2206
$R(0)$	86	η_4	0.2301	η_{10}	0.0002
$V(0)$	3373	η_5	0.0266	η_{11}	0.0255
Γ	5	η_6	0.0096	η_{12}	0.1723

Table 4: Values of the variables (SEIQI_cRVW)

Parameters	Values	Parameters	Values	Parameters	Values
$S(0)$	3233	μ_s	0.1543	ζ	0.6707
$E(0)$	3118	α_v	0.0255	ζ_q	0.0565
$I(0)$	499	δ_n	0.0870	η_v	0.2206
$Q(0)$	744	ρ_r	0.0266	γ_c	0.4884

(Continued)

Table 4 (continued)

Parameters	Values	Parameters	Values	Parameters	Values
$I_c(0)$	42	ρ_v	0.0096	γ_i	0.1600
$R(0)$	86	δ_e	0.0002	γ_r	0.0580
$V(0)$	3373	δ_c	0.0135	ζ_w	0.0005
$W(0)$	821	ν_r	0.1723	ω_c	0.0061
k	0.2386	R_0	0.6634	P	5

The susceptible individual curves for the systems SEIQI_cRVW and SQIRV are depicted in Figs. 1–3, respectively. For the system SEIQI_cRVW, we used the delay values (τ) 0.11, 0.14, and 0.16, and for the system SQIRV, we used the delay values (τ) 0.002, 0.003, and 0.004.

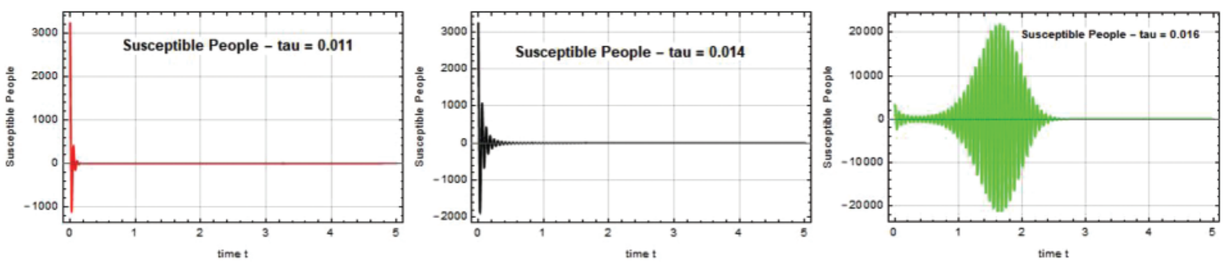


Figure 1: Susceptible people S(t) against time t with various τ for SEIQI$_c$RVW

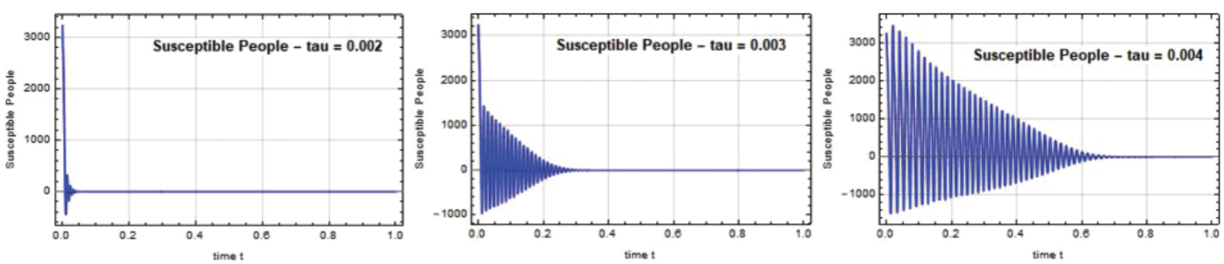
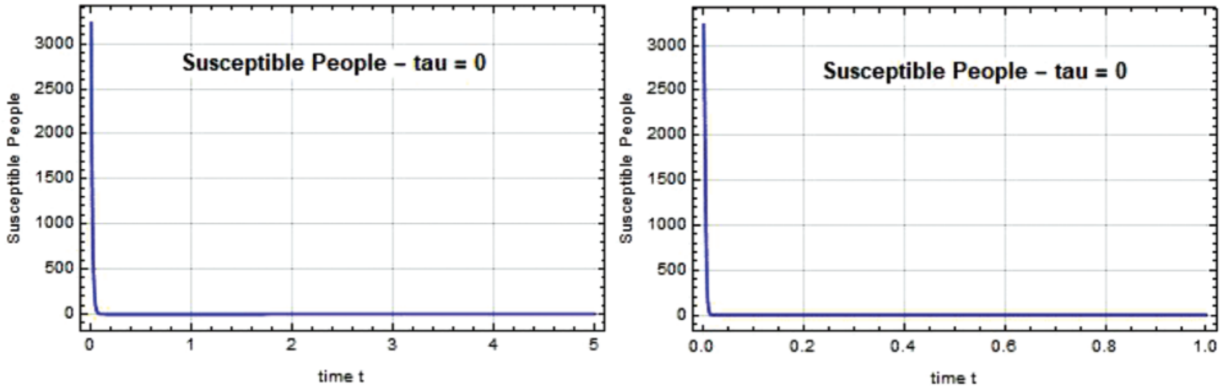


Figure 2: Susceptible people S(t) against time t with various τ for SQIRV

Fig. 4 demonstrates that when the exposed population decreases, the population of other compartments also decreases, while when the exposed population rises, the population of all related compartments rises.

Figs. 5–7 illustrate the possible reduction in the Omicron infection rate. Fig. 5 demonstrates that when the Omicron variant was first discovered, its spread was rapid, and that the variant's spread was reduced to a safe level when the government implemented quarantine and vaccination at a high rate. By adding more compartments from the models that came before it, the SEIQI_cRVW model is able to keep the increase in infected individuals under control at a moderate rate. The state of Tamilnadu discovered on March 11, 2022, that Omicron's death had not been caused by anyone. People were able to avoid contracting the SARS Cov-2 Omicron variant through vaccination against COVID-19.



(a) S(t) for SEIQIcRVW (b) S(t) for SQIRV

Figure 3: Susceptible people S(t) against time t with $\tau = 0$ for SEIQI$_c$RVW and SQIRV

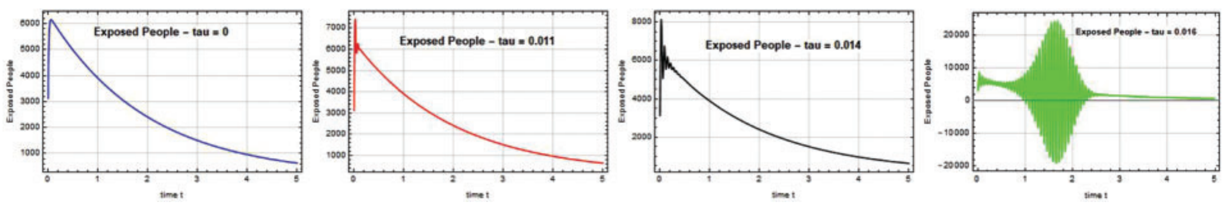


Figure 4: Exposed people E(t) against time t with various τ for SEIQI$_c$RVW

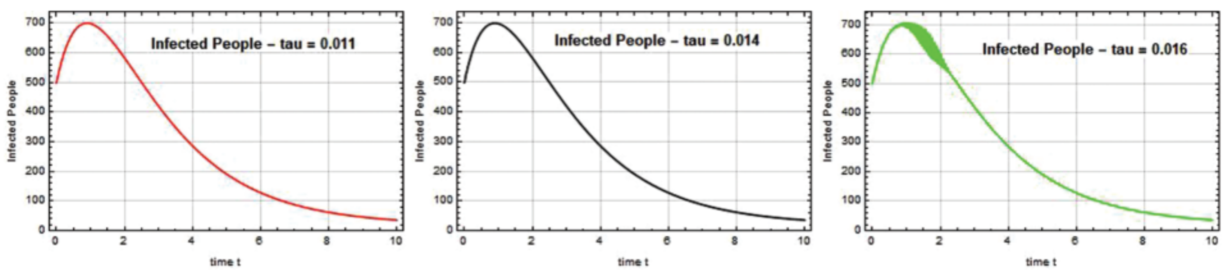


Figure 5: Infected people I(t) against time t with various τ for SEIQI$_c$RVW

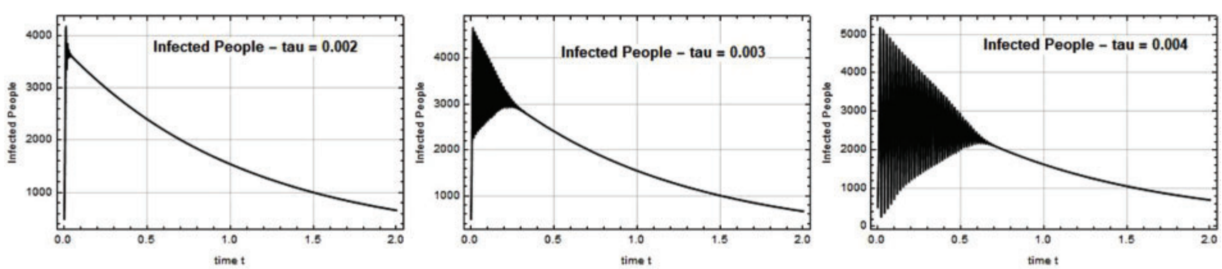
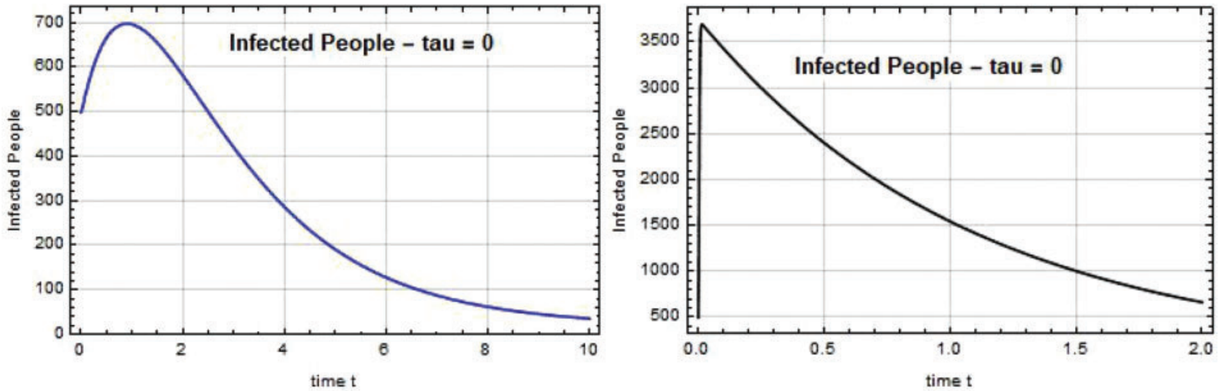


Figure 6: Infected people I(t) against time t with various τ for SQIRV



(a) SEIQIcRVW (b) SQIRV

Figure 7: Infected people I(t) against time t with $\tau = 0$ for SEIQI$_c$RVW and SQIRV

The Quarantined individual level at time t is depicted in Figs. 8–10. When the government implemented the quarantine in Chennai at a high range, the spread of the disease was contained, and the situation in Chennai returned to normal.

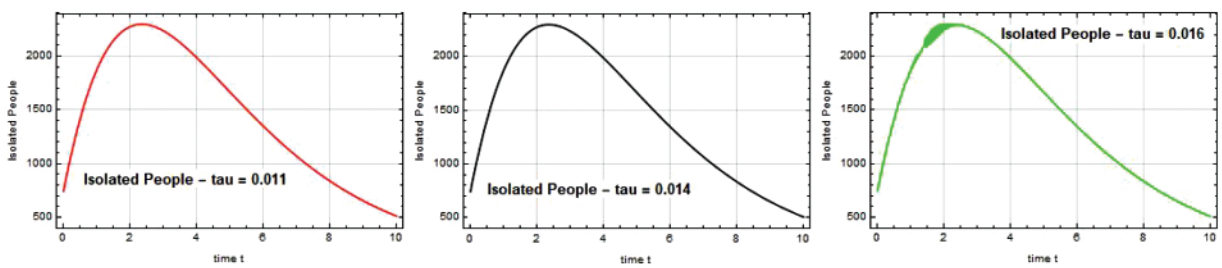


Figure 8: Quarantined people Q(t) against time t with various τ for SEIQI$_c$RVW

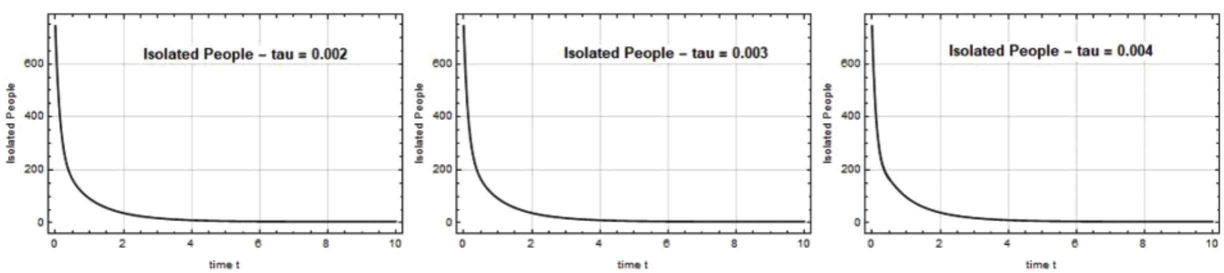
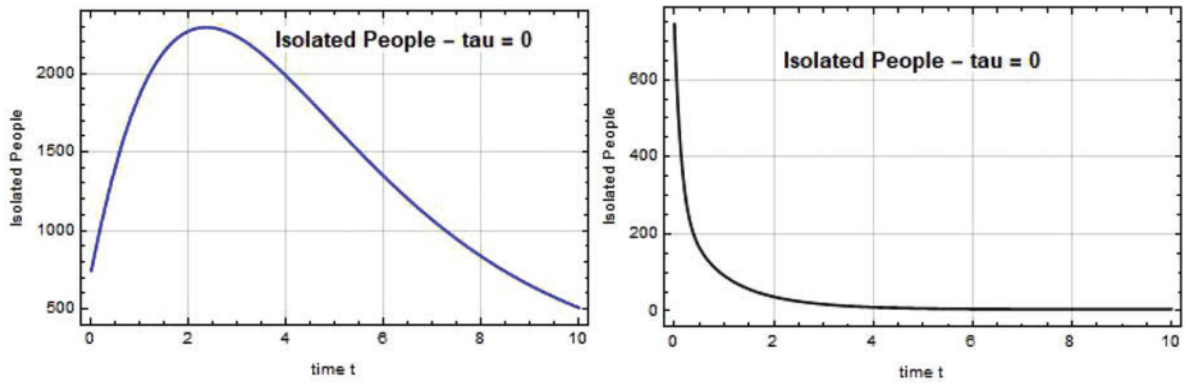


Figure 9: Quarantined people Q(t) against time t with various τ for SQIRV

According to Fig. 11, the population of these four districts experiences a high rate of illness during the Omicron period, which begins on December 25 and ends on March 11, 2022. The infection rate gradually decreased to a low level and there were no deaths when people were vaccinated in accordance with government instructions.



(a) SEIQIcRVW (b) SQIRV

Figure 10: Quarantined people Q(t) against time t with $\tau = 0$ for SEIQI$_c$RVW and SQIRV

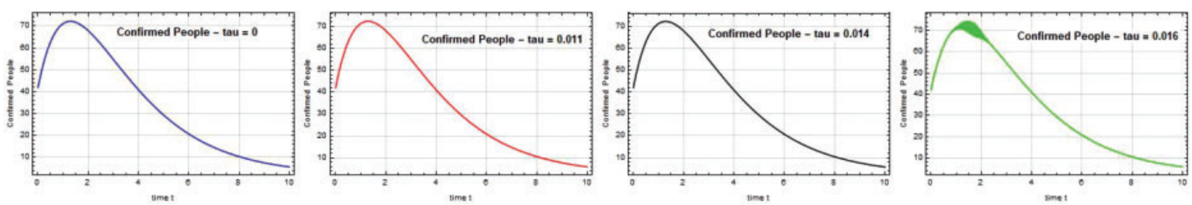
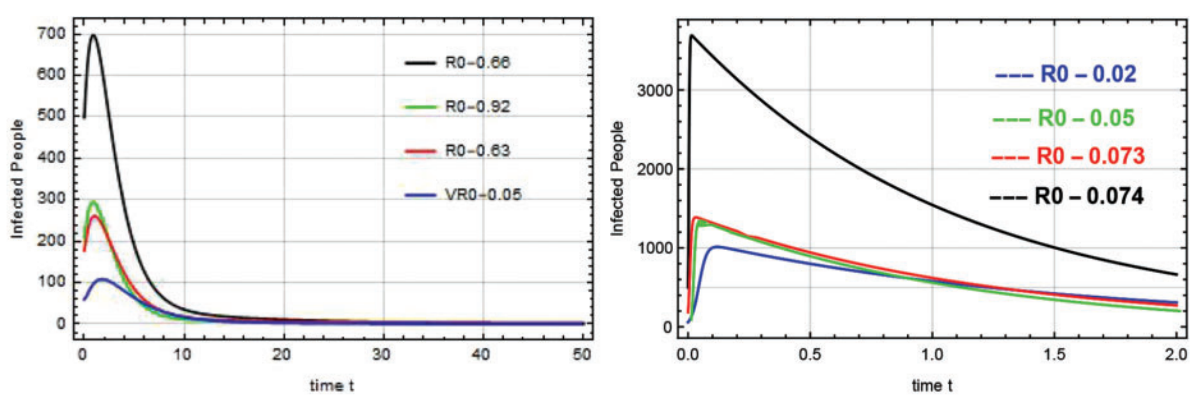


Figure 11: Confirmed people S(t) against time t with various τ for SEIQI$_c$RVW

Reproduction numbers of 0.66, 0.92, 0.63, and 0.06 for SEIQI_cRVW and 0.02, 0.05, 0.073, and 0.074 for SQIRV are shown in Fig. 12. Contaminations are being eliminated from the host population when $R_0 < 1$. However, if $R_0 > 1$, the contaminations cause harm and become endemic, necessitating appropriate clinical treatments to stop the spread of the disease. If the delay value $\tau = 0.014$ for the system SEIQI_cRVW and $\tau = 0.003$ for the system SQIRV, prominent oscillation is observed in the infected population. This could be interpreted as indicating that even though people recover over time, oscillations indicate that the exposed or asymptomatic population has a higher number of active cases than the infected population.



(a) SEIQIcRVW (b) SQIRV

Figure 12: Infected range about various reproduction numbers for the system SEIQI_cRVW and SQIRV

The rise in recovered rates for both systems in Chennai is depicted in Figs. 13–15. By balancing the recovered and infected rates with standard rates, the system SEIQI_cRVW achieves stability.

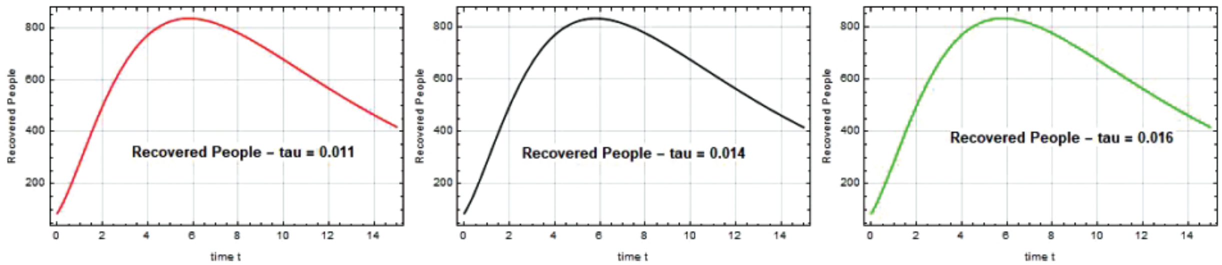


Figure 13: Recovered people R(t) against time t with various τ for SEIQI$_c$RVW

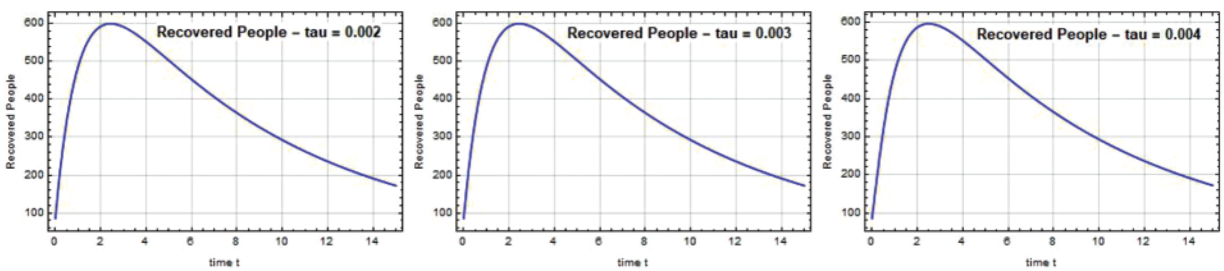
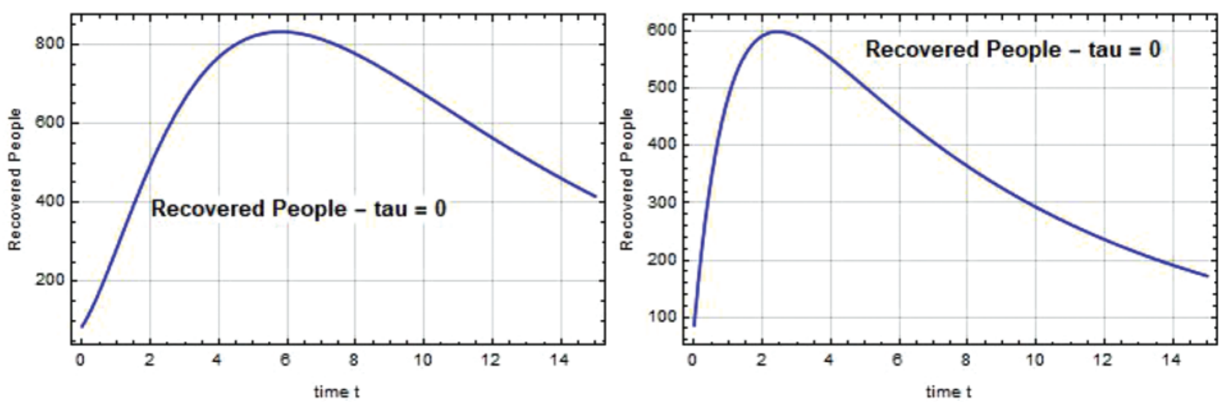


Figure 14: Recovered people R(t) against time t with various τ for SQIRV



(a) SEIQIcRVW (b) SQIRV

Figure 15: Recovered people R(t) against time t with $\tau = 0$ for SEIQI$_c$RVW and SQIRV

The rapid rise in the number of people being vaccinated is depicted in Figs. 16–18. As a result, the system's infection rate significantly decreased, and the system became stable. The significance of vaccination to the Omicron virus control strategy is demonstrated by these figures.

The effect of delayed SEIQI_cRVW model construction is depicted in Fig. 19 as a decrease in reservoir individuals over time t. Figs. 20 and 21 show the stability of the Omicron mathematical model for the Chennai district at various delay values. The infection rate decreases for both the SEIQI_cRVW and SQIRV systems following a rapid spread over a considerable period, as shown in Fig. 22. These systems control the infection and stop its spread within a few days.

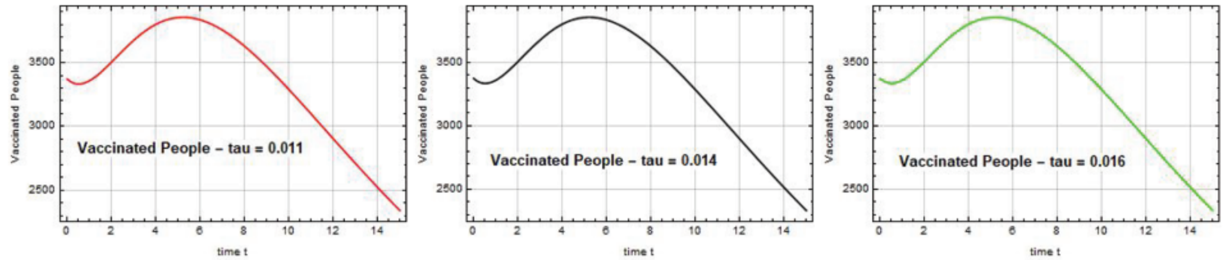


Figure 16: Vaccinated people V(t) against time t with various τ for SEIQI$_c$RVW

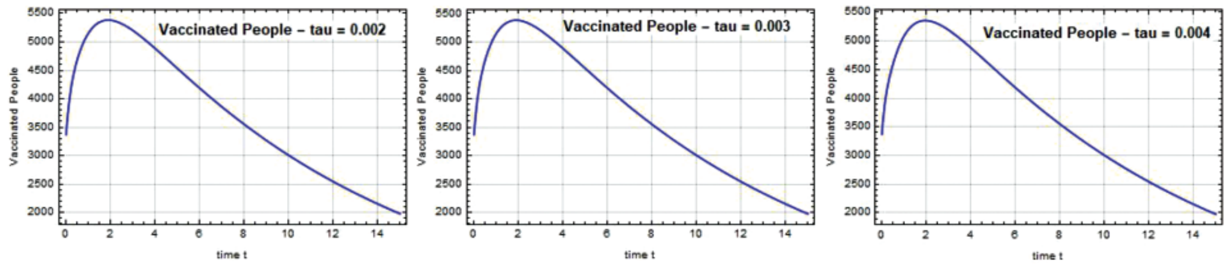
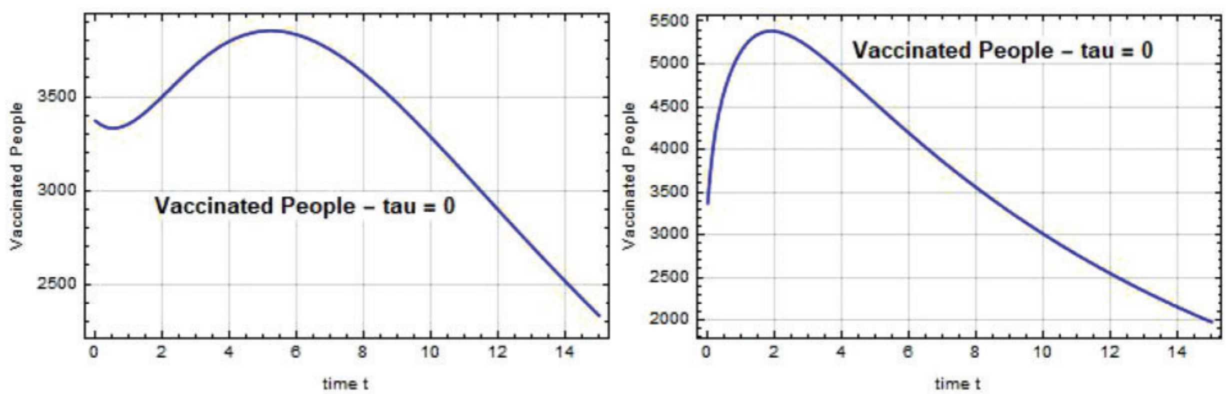


Figure 17: Vaccinated people V(t) against time t with various τ for SQIRV



(a) SEIQIcRVW (b) SQIRV

Figure 18: Vaccinated people V(t) against time t with $\tau = 0$ for SEIQI$_c$RVW and SQIRV

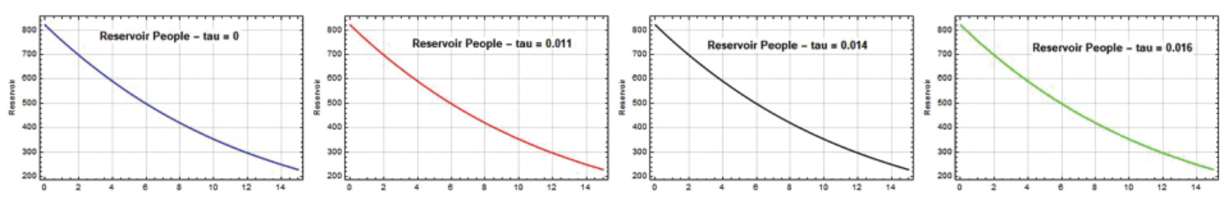


Figure 19: Reservoir people W(t) against time t with various τ for SEIQI$_c$RVW

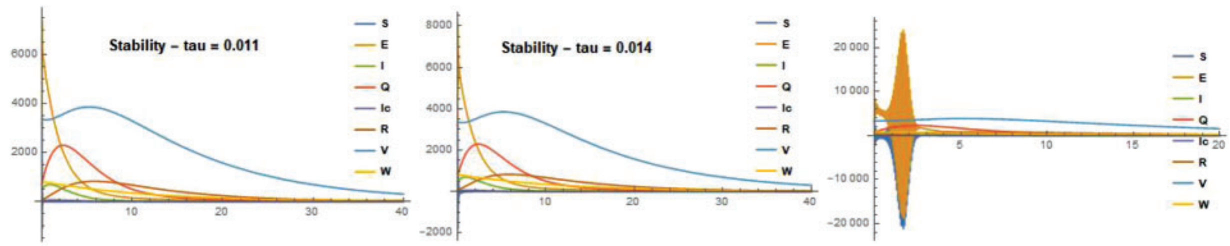


Figure 20: Stability of the system SEIQI$_c$RVW against time t with various τ

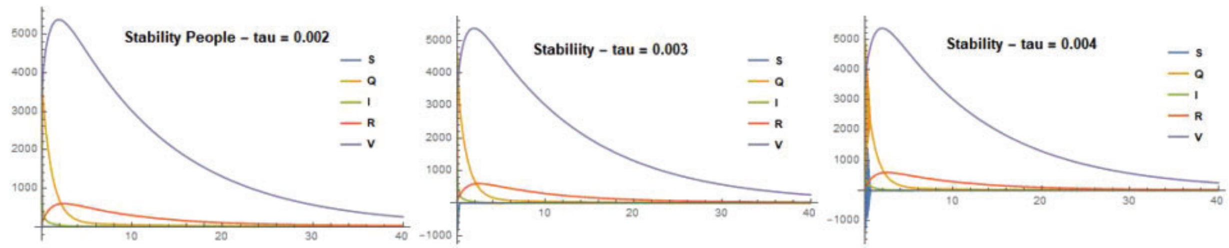
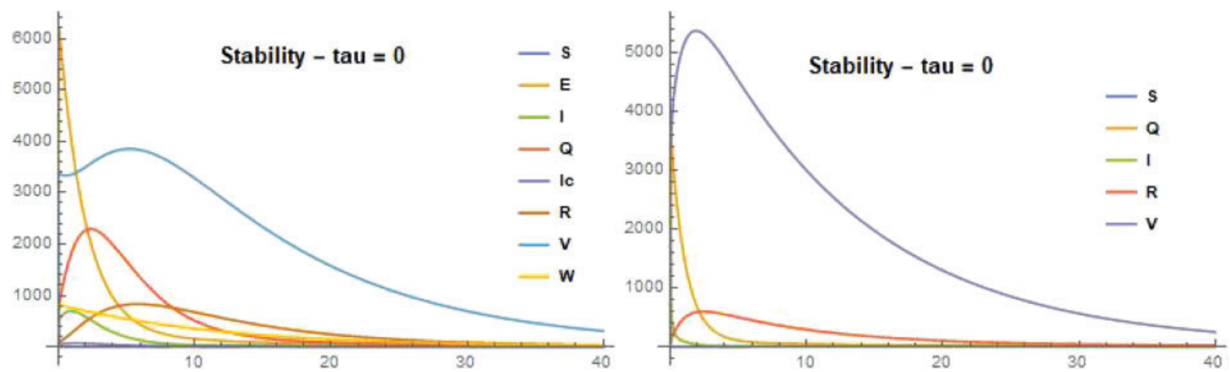


Figure 21: Stability of the system SQIRV against time t with various τ



(a) SEIQIcRVW (b) SQIRV

Figure 22: Stability of the systems with $\tau = 0$

5 Conclusions

Novel delayed mathematical models for the Omicron B.1.1.529 SARS-Cov-2 Variant were developed in this paper. The stability of the two models has been examined and validated, and the principles of reproduction number calculated with this model are an outbreak threshold that determined whether or not the disease would spread further in the district Chennai of Tamilnadu. As the figures show, infection-free steady-state solutions are locally asymptotically stable when $R_0 < 1$. The derived solutions show that the systems are locally unstable and will never become stable when $R_0 > 1$ for an infection-free steady state. From all the data, we can say that the host community will be safe from the Omicron variant if more people are isolated, recovered, and vaccinated. We also found that the second wave of SARS Cov-2 Omicron variant spreads less if the intercessions are strictly followed. Based on our mathematical models and the Chennai data, the Omicron variant infection appears to have stabilized after approximately 25 days. This study will be beneficial for scientists who are working in the medical field. This work can be further extended to generalize with different fractional derivative models.

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Availability of Data and Materials: Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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