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ARTICLE

A Comparative Study of Metaheuristic Optimization Algorithms for Solving Real-World Engineering Design Problems

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ABSTRACT

Real-world engineering design problems with complex objective functions under some constraints are relatively difficult problems to solve. Such design problems are widely experienced in many engineering fields, such as industry, automotive, construction, machinery, and interdisciplinary research. However, there are established optimization techniques that have shown effectiveness in addressing these types of issues. This research paper gives a comparative study of the implementation of seventeen new metaheuristic methods in order to optimize twelve distinct engineering design issues. The algorithms used in the study are listed as: transient search optimization (TSO), equilibrium optimizer (EO), grey wolf optimizer (GWO), moth-flame optimization (MFO), whale optimization algorithm (WOA), slime mould algorithm (SMA), harris hawks optimization (HHO), chimp optimization algorithm (COA), coot optimization algorithm (COOT), multi-verse optimization (MVO), arithmetic optimization algorithm (AOA), aquila optimizer (AO), sine cosine algorithm (SCA), smell agent optimization (SAO), and seagull optimization algorithm (SOA), pelican optimization algorithm (POA), and coati optimization algorithm (CA). As far as we know, there is no comparative analysis of recent and popular methods against the concrete conditions of real-world engineering problems. Hence, a remarkable research guideline is presented in the study for researchers working in the fields of engineering and artificial intelligence, especially when applying the optimization methods that have emerged recently. Future research can rely on this work for a literature search on comparisons of metaheuristic optimization methods in real-world problems under similar conditions.

KEYWORDS

Metaheuristic optimization algorithms; real-world engineering design problems; multidisciplinary design; optimization problems

1 Introduction

Experts involved in the design, manufacturing, and repair processes of an engineering system must make managerial and technological decisions about the system under certain constraints. Optimization is an attempt to achieve the best result under existing constraints. The most important aim of the optimization process is to minimize the effort and time spent on a system or to obtain maximum efficiency. So, if the design cost of the system is expressed as a function, optimization can be defined as the attempts to reach the minimum or maximum value of this function under certain conditions [1].

Constraint optimization is a critical component of any engineering or industrial problem. Most real-world optimization problems include a variety of constraints that affect the overall search space. Over the last few decades, a diverse spectrum of metaheuristic approaches for solving constrained optimization problems have been developed and applied. Constrained optimization problems provide more challenges in comparison to unconstrained optimization problems, primarily because they include the association of many constraints from different types (such as equalities or inequalities) and the interdependence between the objective functions. Nonlinear objective functions and nonlinear constraints in such problem instances may exhibit characteristics of being continuous, mixed, or discrete. There are two general categories of optimization techniques for such problems or functions: mathematical programming and metaheuristic methods. To solve such problems, various mathematical programming methods have been used, such as linear programming, homogeneous linear programming, dynamic, integer, and nonlinear programming. These algorithms use gradient information to explore the solution space in the vicinity of an initial beginning point. Gradient-based algorithms converge more quickly and generate more accurate results than stochastic approaches while performing local searches. However, for these methods to be effective, the generators' variables and cost functions must be continuous. Furthermore, for these methods to be successful, a good starting point is required. Many optimization problems require the consideration of prohibited zones, non-smooth, and side limits or non-convex cost functions. As a result, traditional mathematical programming methods are unable to solve these non-convex optimization problems. Although mixed-integer nonlinear programming or dynamic programming as well as its variants provide a limited number of possibilities for solving non-convex problems, they are computationally expensive [2].

Metaheuristic optimization approaches have been used as a viable alternative to conventional mathematical procedures in order to achieve global or near-global optimal solutions [3]. The aforementioned approaches are very suitable for conducting global searches, as they possess the capability to effectively explore and identify potential places within the search space with a high accuracy degree and efficiency value. Additionally, these techniques eliminate the necessity for continuous cost functions and variables, which are frequently utilized in mathematical optimization. Although these are approximation approaches, their solutions are acceptable but not necessarily be optimal. They do not necessitate the objective function's derivatives or constraints, and they employ probabilistic rather than deterministic transition rules. As a result, the searchers concentrate on metaheuristic strategies that seek a good constructive answer in a reasonable amount of time [4]. Classic algorithms, on the other hand, desire derivatives for all nonlinear constraint functions to evaluate system performance. However, due to the system's high computational complexity, it is difficult to derive a real-world problem. These disadvantages of classical methods have prompted researchers to employ nature-inspired metaheuristic methods according to simulations to clarify engineering design problems. Metaheuristic optimization algorithms are well recognized as a prominent global optimization approach used to address complex search and optimization problems of significant size. They are frequently utilized to solve optimization problems of a broad variety [5]. Metaheuristic methods often work by combining rules and randomness to mimic events in nature and the behavior of animals. Due to their inherent inflexibility, these algorithms provide superior performance in optimization problems and several other problem domains compared to conventional approaches. Numerous studies have provided evidence to support the notion that nature-inspired algorithms, consisting of genetic algorithms (GA), particle swarm optimization (PSO), differential evaluation (DE), and evolution strategies (ES), possess inherent advantages due to their lack of reliance on mathematical assumptions in addressing optimization problems. Moreover, these algorithms exhibit superior global search capabilities compared to conventional optimization algorithms. These metaheuristic techniques have been widely used to accomplish constrained optimization problems in different fields, including structural design problems, engineering problems, decision-making, reliability optimization, and so on.

Even though there are several optimization techniques in the academic literature, no one method has been shown to universally provide the optimal answer for all optimization issues. The assertion is rationally substantiated by the "no free lunch theorem". The aforementioned theorem has inspired by several scholars by prompting them to develop novel algorithms. Therefore, several methodologies have been lately suggested. However, there are not many studies on which of these suggested methods perform well in which areas. In this study, seventeen recently proposed and popular methods are employed to twelve constrained design problems in engineering with different constraints, objective functions, and decision variables, and performance analysis has been performed. For the problems; speed reducer, tension-compression spring, pressure vessel design, welded beam design, three-bar truss design, multiple disc clutch brake design, himmelblau's function, cantilever beam, tubular column design, piston lever, robot gripper, and corrugated bulkhead design are investigated. The types of the problems differ considering the problem domains. The reason for this is to obtain better comparison results by comparing the optimization algorithms with each other according to their distinct capabilities in several types of problems. The algorithms used to solve these problems can be stated as; transient search optimization (TSO), equilibrium optimizer (EO), grey wolf optimizer (GWO), moth-flame optimization (MFO), whale optimization algorithm (WOA), slime mold algorithm (SMA), harris hawks optimization (HHO), chimp optimization algorithm (COA), coot optimization algorithm (COOT), multi verse optimization (MVO), arithmetic optimization algorithm (AOA), aquila optimizer (AO), sine cosine algorithm (SCA), smell agent optimization (SAO), and seagull optimization algorithm (SOA), pelican optimization algorithm (POA), and coati optimization algorithm (CA). Each algorithm's performance is evaluated in terms of solution quality, robustness, and convergence speed.

The subsequent sections in the paper are structured in the following manner. Section 2 introduces a comprehensive review of the optimization approaches used in the study to address the complex challenges faced. Section 3 of this paper describes and examines the practical engineering design challenges and experimental findings that were encountered. In conclusion, Section 4 provides a comprehensive overview of the obtained results and offers suggestions for further investigations.

2 Literature Review

Metaheuristic optimization methods can be examined in five general groups: physics-based, swarm-based, game-based, evolutionary-based, and human-based. Optimization algorithms based on swarm intelligence are the methods that emerged from examining the movements of animals living in swarms. Numerous methodologies grounded in swarm intelligence have been put forward. Some of these are Red fox optimization algorithm [6], Cat and mouse based optimizer [7], Siberian tiger optimization [8], Dwarf mongoose optimization algorithm [9], Chimp optimization algorithm [10], Dingo optimizer [11], Flamingo search algorithm [12], and Orca predation algorithm [13].

Evolutionary-based metaheuristic optimization approaches have been generated by using modeling ideas in genetics, the law of natural selection, biological concepts, and random operators. GA and DE are widely utilized evolutionary algorithms that simulate the reproductive process, natural selection, and Darwin's theory of evolution. These algorithms employ randomly selection, crossover, and mutation operators to optimize solutions. Physics-based metaheuristic optimization approaches draw inspiration from the fundamental principles of physics. Some of these are Henry gas solubility optimization [14], Archimedes optimization method [15], Multi verse optimization [16], Equilibrium optimizer [17], Transient search algorithm [18].

Game-based metaheuristic optimization methods have been created by mimicking the rules and circumstances that govern different games, as well as the behavior of the participants. Some of these are World cup optimization [19], League championship algorithm [20], Ring toss game based optimization algorithm [21], Darts game optimizer [22]. Human-based metaheuristic optimization methods have been created using mathematical models of human activities, behaviors, and interactions in both individual and societal settings. Some of these are Forensic-based investigation optimization [23], Political optimizer [24], Human urbanization algorithm [25], Teamwork optimization algorithm [26].

There are many algorithms that solve engineering problems with metaheuristic optimization algorithms. Table 1 presents a comprehensive overview of the methods used throughout the last decade. As seen in the literature, different optimization techniques have been effectively used in a variety of constrained optimization situations. When obtaining an ideal or near-optimal solution, the performance obtained, on the other hand, reveals a statistically significant difference. Due to this rationale, despite the existence of several optimization algorithms documented in scholarly literature, a single algorithm capable of finding the best solution to every optimization problem has yet to be discovered.

Table 1: An overview of some of the algorithms investigated

Reference	Method	Problem	Year	Publisher
[27]	Water cycle algorithm	Pressure vessel design Three-bar truss design Multiple disk clutch brake design Tension-compression spring design Speed reducer design Rolling element bearing design Welded beam design	2012	Elsevier
[28]	Cuckoo search algorithm	Three-bar truss design Speed reducer design Cantilever beam I-beam design Tubular column design Piston lever Corrugated bulkhead design Gear train	2013	Springer
[29]	Grey wolf optimizer	Pressure vessel design Tension-compression spring design Welded beam design	2014	Elsevier

(Continued)

Table 1 (continued)

Reference	Method	Problem	Year	Publisher
[30]	Whale optimization algorithm	Pressure vessel design Tension-compression spring design Welded beam design	2016	Elsevier
[31]	Crow search algorithm	Pressure vessel design Belleville spring design Three-bar truss design Welded beam design Gear train design Tension-compression spring design	2016	Elsevier
[16]	Multi-verse optimizer	Gear train design Three-bar truss design Welded beam design Cantilever beam design	2016	Springer
[2]	Hybrid PSO-GA	Pressure vessel design Gear train design Welded beam design	2016	Elsevier
[32]	Salp swarm algorithm	Cantilever beam design Welded beam design Three-bar truss design I-beam design Tension-compression spring design	2017	Elsevier
[33]	Spotted hyena optimizer	Multiple disk clutch brake design 25-bar truss design	2017	IEEE
[34]	Iterative topographical global optimization	Tension-compression spring design Welded beam design Three-bar truss design Pressure vessel design Multiple disk clutch brake design Speed reducer design Gear train design	2018	Elsevier

(Continued)

Table 1 (continued)

Reference	Method	Problem	Year	Publisher
[35]	Artificial algae	Gear train design Welded beam design Spring design Multi-plate disk brake design Speed reducer design	2018	Springer
[36]	Harris hawks optimization	Pressure vessel design Three-bar truss design Multi-plate disc clutch brake Tension-compression spring design Rolling element bearing design Welded beam design	2019	Elsevier
[37]	Seagull optimization algorithm	Optical buffer design Pressure vessel structure Speed reducer design Constraint handling Tension-compression spring design Welded beam design	2019	Elsevier
[38]	Butterfly optimization algorithm	Welded beam design Spring design Gear train design	2019	Springer
[39]	Marine predators algorithm	Welded beam design Tension-compression spring design Pressure vessel structure	2020	Elsevier
[40]	Slime mould algorithm	Welded beam structure Pressure vessel structure Cantilever structure problem I-beam structure	2020	Elsevier
[10]	Chimp optimization algorithm	Heat exchanger network design (case 1) Reactor network design Two-reactor problem	2020	Elsevier

(Continued)

Table 1 (continued)

Reference	Method	Problem	Year	Publisher
[41]	Hybrid grasshopper optimization	Car side crash problem Design of the robot gripper Rough grinding Multiple disc clutch brake	2021	Wiley
[17]	Equilibrium optimizer	Welded beam design Pressure vessel design Tension-compression spring design	2021	Elsevier
[42]	Arithmetic optimization algorithm	Tension-compression spring design Welded beam design 3-bar truss design problem Pressure vessel structure Speed reducer problem	2021	Elsevier
[43]	Aquila optimizer	Pressure vessel structure Tension-compression spring design Cantilever beam design Welded beam design 3-bar truss design Speed reducer Multiple disc clutch brake problem	2021	Elsevier
[44]	Enhanced grasshopper optimization	Three-bar truss Car side crash problem Welded beam design problem Hydrostatic thrust bearing Multiple clutch disc problem Cantilever beam problem	2021	Springer
[45]	Chaotic Lévy flight distribution optimization	Gear train design Coupling with a bolted rim design Belleville spring Pressure vessel structure Rolling element bearing	2022	Wiley

(Continued)

Table 1 (continued)

Reference	Method	Problem	Year	Publisher
[46]	Pelican optimization algorithm	Pressure vessel design problem Speed reducer design problem Welded beam design problem Tension-compression spring design	2022	MDPI
[13]	Orca predation algorithm	Welded beam design Pressure vessel design Speed reducer design Tension-compression spring design Three-bar truss design problem	2022	Elsevier
[47]	Coati optimization algorithm	Pressure vessel design problem Speed reducer design problem Welded beam design problem Tension-compression spring design	2023	Elsevier

3 Metaheuristic Optimization Algorithms

This section provides a brief description of each algorithm that is employed in this study. Only the most significant parts are described; accordingly, interested readers can get all of the information they need in the cited papers.

3.1 Transient Search Algorithm

As a physics-based metaheuristic approach, Qais et al. introduced the transient search algorithm (TSO) in 2020. The source of motivation for this study is derived from the transient dynamics seen in switched electrical circuits having storage components, i.e., capacitance and inductance [18].

Electrical circuits consist of many components capable of storing energy. The components in question may be classified as inductors (L), capacitors (C), or a hybrid configuration consisting of both (LC). Typically, an electrical circuit that incorporates a resistor (R), capacitor (C), or inductor (L) exhibits a transient response as well as a steady-state response. Circuits that include both an energy storage device and a resistor are categorized as first-order circuits. When two energy storage devices are positioned next to a resistor inside a circuit, the resulting configuration is referred to as a second-order circuit. The TSO method is introduced, drawing inspiration from the transient response shown by these circuits in the vicinity of 0.

3.2 Equilibrium Optimizer (EO)

In the year 2020, Faramarzi et al. developed an equilibrium optimizer (EO), a metaheuristic algorithm that simulates the fundamental well-mixed dynamic mass balance in a control volume [17]. It is used in this method to describe the concentration of the non-reactive component in a control volume as a result of different source and sink components, based on the mass balance equation. When the dynamic mass balance against the control volume is compared, this comparison serves as the motivation for the balance optimization process. The equation of mass balance is utilized for characterizing the concentration of a non-reactive component inside the control volume, taking into account the numerous sources and leakage mechanisms that exist within the control volume. The conservation of mass principle is satisfied in a control volume due to the positive nature of the mass balance equation.

3.3 Grey Wolf Optimizer (GWO)

Mirjalili et al. introduced the grey wolf algorithm (GWO) in 2014, which is an optimization algorithm influenced by the population of grey wolves, their natural leading capabilities, and hunting habits [29]. Grey wolves are classified into four major classes based on their social hierarchy as well as the abilities of each wolf in the group: Alpha, Beta, Delta, and Omega. The leader of the group is called as the alpha wolf and is in charge of making critical decisions such as sleeping location, hunting, waking time, and so on. The tracking, the encircling, and the attacking are the three steps in the grey wolf hunting technique.

3.4 Moth-Flame Optimization (MFO)

The moth-flame optimization (MFO) is a metaheuristic method affected by the population of moths and offered by Mirjalili in 2015. The MFO is based upon a simulation of a distinctive nocturnal navigation system used by moths. It begins the optimization procedure, like other meta-heuristics. In other words, it randomly generates a set of candidate solutions. When traveling at night, the moth uses a mechanism known as transverse orientation to navigate. In the MFO method, candidate solutions are postulated as moths, while the variables of a given issue are postulated as the locations of these moths inside the search space [48].

3.5 Whale Optimization Algorithm (WOA)

Mirjalili et al. [30] developed the whale optimization algorithm (WOA), another nature-inspired metaheuristic optimization algorithm that replicates the social behavior of humpback whales to tackle complicated optimization problems. Predators are able to recognize the location of humpback whales and cover them fully when they approach. During iterations of WOA, target prey is presumed to be the greatest available search tool, and humpback whales update their position by considering the best search tool as they progress through the game. However, despite their enormous size, these creatures are distinguished by their intellect and sophisticated methods of collaborative work throughout the hunting process. In addition to the initiation stage, the WOA consists of the surrounding hunt, the bubble-net hunting method, and the hunt for prey, among other activities.

3.6 Slime Mould Algorithm (SMA)

Li et al. proposed a new optimization algorithm inspired by the behavior of slime mould in obtaining the optimal way to bind foods [40]. Slime mould is a type of eukaryote that thrives in cold, moist environments. Plasmodium, in its active and dynamic phase, is the primary source of sustenance for the parasite. Additionally, this stage serves as the foundation for the SMA. Slime mould is on the lookout for food that contains an organic substance in this phase. After the slime mould has finished its hunt, it wraps itself around the meal and secretes enzymes to break it down. The front end enhances into a fan-shaped mesh during the migration phase. It then spreads into a network of interconnecting veins, allowing blood to flow in. Due to its distinctive patterns and structure, it is capable of forming a venous network for multiple foods at the same time. By grouping these negative *vs*. positive responses, the slime mould can construct the optimal food route to add food in a more meaningful way. Thus, SMA was modeled mathematically and applied to solve engineering problems [49].

3.7 Harris Hawks Optimization (HHO)

Heidari et al. [36] proposed a population-based metaheuristic optimization algorithm inspired by the behavior and hunting model of Harris hawks. Harris hawks optimization (HHO) is a stochastic algorithm that can be used to search for optimal solutions in large search spaces. The fundamental steps of HHO can be achieved at a variety of energy levels. The exploration phase replicates the technique by which the Harris hawk loses track of prey. Hawks take a break in this situation to trace and locate new prey. At each step of the HHO process, possible solutions are referred to as hawks, and the optimal solution is determined by hunting. Hawks randomly settle into different sites and wait for prey using two probability-based operators [36].

3.8 Chimp Optimization Algorithm (COA)

The chimp optimization algorithm (COA) was designed by Khishe et al. as a biology-based optimization algorithm originated by the individual intellect and sexual motives of chimps during group hunts [10]. There are some differences between it and other social carnivores. Four different stages are employed to model various intelligence in this methodology. The chaser, the driver, the attacker, and the barrier are all believed to be more familiar with the first option in this case. The four optimal solutions produced in the previous step are kept, and the other chimps are urged to change placements to the chimp's optimal locations.

3.9 Coot Optimization Algorithm (COOT)

The metaheuristic technique presented by Naruei et al. (2021) draws inspiration from the behavioral patterns shown by birds navigating on the surface of water. The behavior of the coot swarm on water includes three major movements [50]. These are irregular activity movement, synchronized movement, and chain movements on the water surface. Coots have different collective behaviors. There are four different water paddle movements on the water surface. These include acting randomly, chain movement, adjusting the position relative to the group leaders, and directing the group to the optimal area by the leaders. As in all optimization algorithms, the initial population is created first. After the initial population is created, the fitness value of the solution is calculated using the objective function.

3.10 Multi-Verse Optimization (MVO)

The notion of multi-verse optimization (MVO) was presented as a metaheuristic approach by Mirjalili et al. (2016), drawing inspiration from the field of cosmology. MVO explores search spaces with the concepts of black and white holes while exploiting search spaces with wormholes. Similar to other evolutionary algorithms, this method commences the optimization procedure by generating an initial population and endeavors to enhance these solutions via a predetermined number of iterations. Enhancement of individual performance inside each population may be attained via the utilization of this algorithm, which is grounded in one of the postulations about the presence of many universes. In the context of these theories, it is conceptualized that every solution to an optimization issue represents a distinct universe, whereby each constituent item is considered a variable within the specific problem at hand. In addition, they assign to each solution an inflation rate proportional to the value of the fitness function to which the solution corresponds [16].

3.11 Arithmetic Optimization Algorithm (AOA)

The arithmetic optimization algorithm (AOA), as presented by Abualigah et al. [42], is a meta-heuristic approach that leverages the distribution characteristics of fundamental arithmetic operators in mathematics, such as division, multiplication, addition, and subtraction. Four traditional arithmetic operators are modeled into the position update equations to search for the global optimization solution, as the name implies. Division and multiplication are employed for the exploration search, producing enormous steps in the search space due to the varied impacts of these four arithmetic operators. It is applied to execute exploitation searches, which can create small step sizes in the addition and subtraction search space [42].

3.12 Aquila Optimizer (AO)

The aquila optimizer (AO) is a population-based metaheuristic approach developed by Abualigah et al. [43]. It draws inspiration from the natural behavior of the Aquila bird while hunting its prey. The Aquila employs four distinct hunting techniques. The first technique used by the Aquila is vertically inclined high-flying, which enables the bird to capture avian prey while soaring at significant altitudes above the Earth's surface. The second method, known as contour flying with a brief glide attack, involves the Aquila ascending at a relatively low level from the surface. The third method involves using flying movement characterized by a gradual fall in order to execute an assault. This approach involves the Aquila descending to the ground and engaging in a slow pursuit of its prey. Additionally, the fourth way has the Aquila strolling on land and using tactics to capture its victim [43].

3.13 Sine Cosine Algorithm (SCA)

The sine cosine algorithm (SCA) is a recently developed meta-heuristic algorithm that is based on the properties of trigonometric sine and cosine functions [51]. SCA has gained a lot of attention from researchers since its debut by Mirjalili in 2016, and it is been extensively employed to achieve various optimization solutions of various fields. SCA employs a mathematical model depending on sine and cosine functions to create several beginning populations and afterwards select the optimal answer. A large number of random and adaptive variables are incorporated into the algorithm to ensure exploration and exploitation of the search space at various phases of the optimization process. The SCA optimization method begins with a random solution set, which is then refined. According to the method, the best solution produced is saved and assigned to a specific target point, after which all other solutions are updated in accordance with this solution. Meanwhile, as the number of iterations increases, the range of sine and cosine functions is updated to guarantee that they are exploitation. As a default, when the number of optimization iterations exceeds the maximum number of iterations, the optimization process is terminated by the algorithm [52].

3.14 Smell Agent Optimization (SAO)

The smell agent optimization (SAO) is a metaheuristic method responsible for implementing the relationships that exist between a smell agent and an object that evaporates a smell molecule [53]. The sniffing, trailing, and random modes are used to describe these relationships, and each mode has its own set of parameters. The sniffing mode simulates the capacity of an agent to perceive smells by causing the scent molecules to diffuse from a smell source toward the agent throughout the sniffing process. Using the trailing mode, the agent can simulate the ability to trace a portion of the scent molecules until the source of the smell molecules is determined. In contrast, the agent uses the random mode as a strategy to avoid becoming stuck in local minima.

3.15 Seagull Optimization Algorithm (SOA)

The seagull optimization algorithm (SOA) is an algorithm that pulls inspiration from the natural behavior of seagulls while migrating and attacking prey [37]. In this regard, these behaviors may be defined in a manner that is closely linked to the objective function that is to be improved. SOA, an optimization algorithm inspired by biology, starts the study with a randomly generated population. During position duplication operations, search agents are able to update their positions according to the best search agent. Migration represents exploration behavior and shows how a group of seagulls move from one location to another. There are three conditions that a seagull must meet at this stage. These are avoiding the collisions, moving towards best neighbor's direction, and remaining close to the best search agent. The attacking phase represents exploitation behavior. Hunting for seagulls aims to take advantage of all the experience and experience gained from the search processes in the past. During migration, seagulls can also change their attack angle from time to time, apart from their speed. However, they take advantage of their long wingspan and body weight to maintain their high altitude. Seagulls exhibit spiraling behavior in the air when attacking prey, they have identified.

3.16 Pelican Optimization Algorithm (POA)

The pelican optimization algorithm (POA) is a swarm-based optimization technique that draws inspiration from the hunting behavior and methods shown by pelicans. In POA, exploration agents are represented by pelicans that look for sustenance sources. POA is made up of two stages that are carried out consecutively in each iteration. In the first phase, there is a global objective to which all pelicans will migrate. This global target is chosen at random inside the issue space at the start of each cycle. The pelican has two options for mobility. If this goal is more desirable than the pelican's present position, the pelican will migrate toward it. Otherwise, the pelican will flee from this location. POA employs an acceptance-rejection method. The pelican will only relocate if the new place is superior to its present location. The pelican circles its present location throughout the second phase. Although this phrase is not always applicable, it might be thought of as a local or neighborhood search. During this step, a new location is chosen at random inside the local problem space of the pelican. With each iteration, the diameter of this local issue space rapidly decreases. It indicates that the local issue space is sufficiently large to begin with, and it may be seen as an exploration. This inquiry, on the other hand, progresses from investigation to exploitation with each repetition. In addition to the process of iteration, the current location of the agent has an impact on the problem space at a local level. In its initial form, the current positioning close to zero restricts the range of the local problem domain. Similar to the first phase, the pelican only advances toward the new location if the new location is superior to its current location [46].

3.17 Coati Optimization Algorithm (CA)

The coati optimization algorithm (CA) is a new metaheuristic method that imitates the coati's natural behavior when it encounters and flees from predators. The process of modifying the positions of candidate solutions in the CA is derived from the emulation of two distinct behaviors shown by coatis in nature. These behaviors include: (i) coatis' assault method on iguanas, and (ii) coatis' predator escape strategy. As a result, the CA population is updated in two stages. The first phase of enhancing the coati population in the designated region is shown via a simulation that models their strategies for targeting iguanas. In this particular strategy, a considerable number of coatis ascend the tree in order to closely approach an iguana and elicit a startled response. A group of coatis congregates under a tree, observing the descent of an iguana to the ground. Upon the iguana's descent, the coatis engage in aggressive behavior by launching an assault and pursuing the iguana. By using this approach, coatis demonstrate their ability to travel to different regions within the search area, hence highlighting the worldwide search capabilities of the COA within the realm of problem-solving. The subsequent phase of updating the locations of coatis inside the search space is formulated using mathematical modeling techniques, which take into account the coatis' inherent behavior while encountering and evading predators. When a predator initiates an attack on a coati, the coati promptly vacates its position. Coati's actions in this approach place it in a secure position near to its present location, indicating the CA's exploitation ability to utilize local search. The iteration of a CA concludes when all coordinates of the coatis in the solution space have been modified according to the results of the first and second phases. The best solution discovered across all rounds of the method is provided as the result after CA has finished running [47].

4 Engineering Design Problems and Experimental Results

In this section, the most prevalent design challenges in engineering are stated. To make the problems more understandable, the mathematical form and definition are provided. The following are the problems investigated in the study:

- Speed reducer problem
- Tension-compression spring design problem
- Pressure vessel design problem
- Welded beam design problem
- Three-bar truss design problem
- Multiple disk clutch brake design problem
- Himmelblau's function
- Cantilever beam problem
- Tubular column design problem
- Piston lever
- Robot gripper
- Corrugated bulkhead design problem

These engineering design issues are well recognized in practical applications. In order to identify the most favorable design, it is often necessary to use an active approach for determining the ideal parameters. In order to address each issue, some settings (variables) need adjustment. Furthermore, some limitations are included to guarantee that the variables' values stay within the designated range. The specifics of the optimization issue are presented below.

The evaluation of optimization approaches often involves selecting bound-constrained and common-constrained optimization problems. Each design vector must consistently provide a constrained solution to any engineering or optimization problem [54,55]:

$$lb_{(j)} \leq x_{(i,j)} \leq ub_{(j)},\ j = 1, 2, 3, \ldots, n \tag{1}$$

where $lb_{(j)}$ and $ub_{(j)}$ are the lower and upper bounds of the problem defined for position $x_{(i,j)}$, respectively, and n is vector size. In addition, the real-world constrained optimization problem is typically given as follows [56]:

$$Minimize, f(x), x = (x_1, x_2, \ldots, x_n) \tag{2}$$

$$Subject\, to: \; g_i(x) \leq 0, \; i = 1, \ldots, n$$

$$h_j(x) = 0, j = n+1, \ldots, m$$

The bound-constrained structure incorporates the cost function into the assessment of the selected optimization process to describe all the stated constrained issues in Eq. (2). The cost function associated with each infeasible option may be included into the goal function that is being employed. The determination of the cost function arises from its property of situational homogeneity. The use of a single helper cost function renders this approach applicable to a diverse array of challenges.

The above-mentioned real-world constrained problems are used to create a benchmark suite. The benchmark suite includes a total of 12 problems designed from the problems listed above. Table 2 summarizes the specifics of these issues. D denotes the entire decision variables in the problem, h indicates the number of equality constraints, g denotes the total of inequality constraints, and f_{min} represents the best known optimal objective function value.

Table 2: Details of real world constrained engineering design problems

Name	***D***	***g***	***h***	f_{min}
Speed reducer	7	11	0	2.9944244658E+03
Tension-compression spring design (case 1)	3	3	0	1.2665232788E-02
Pressure vessel design	4	4	0	5.8853327736E+03
Welded beam design	4	5	0	1.6702177263E+00
Three-bar truss design problem	2	3	0	2.6389584338E+02
Multiple disk clutch brake design problem	5	7	0	2.3524245790E-01
Himmelblau's function	5	6	0	−3.0665538672E+04
Cantilever beam	5	1	0	1.3399576
Tubular column design	2	6	0	26.486361473
Piston lever	4	4	0	8.41269832311
Robot gripper	7	7	0	2.5287918415E+00
Corrugated bulkhead design	4	6	0	6.8429580100808

The experiments are conducted using a computing system equipped with the Windows 11 operating system, 16 GB of RAM, and an Intel (R) Core (TM) i7-10750H CPU (2.60 GHz). The comparison methods are coded using MATLAB R2021a. The aforementioned issues are inherently constrained, therefore necessitating the implementation of an external penalty approach mechanism in order to address them. The maximum number of iterations for all problems is 1000, the number of populations is 30, and the number of evaluations is chosen as 30000. Algorithm parameters are default values found in the literature and are shown in Table 3. To facilitate the analysis of the convergence behavior of the algorithms under study, the convergence curves, which represent the best fitness values achieved for each issue, are graphically shown. Each algorithm is subjected to 30 separate experimental runs. This study compares the best, mean, worst, standard deviation (SD), and Friedman mean rank (FMR) values. The method that yields the best answer is emphasized in bold to enhance readability.

Table 3: Parameters of employed algorithms

Algorithm	Parameter settings
TSO	$k = 1, z = [0, 2]$
EO	$GP = 0.5, a_1 = 2, a_2 = 1$
GWO	$a =$ Linearly decreased from 2 to 0
MFO	$a =$ Linearly decreased from -1 to -2
WOA	$a =$ Linearly decreased from 2 to $a_2 =$ Linearly decreased from -1 to -2, $b = 1$
SMA	$z = 0.03$
HHO	$E_o = (-1, 1)$
COA	$f =$ Linearly decreased from 2 to 0
COOT	$R = [-1, 1]$
MVO	$WEP\_Max = 1, WEP\_Min = 0.2$
AOA	$MOP_{Max} = 1, MOP\_Min = 0.2, Alpha = 5, Mu = 0.499$
AO	$alpha = 0.1, delta = 0.1$
SCA	$a = 2$
SAO	$olf = 0.9, K = 0.6, T = 0.95, M = 0.9, Step = 0.02$
SOA	$f_c = 1$, $A =$ Linearly decreased from 2 to 0
POA	–
CA	–

4.1 Speed Reducer Problem

This problem is simply a gearbox problem that allows the aircraft engine to rotate at maximum efficiency [57]. In this problem, the minimum values of the seven decision variables must be optimized by finding the face width b (x_1), the tooth modulus m (x_2), the number of teeth on the pinion z (x_3), the length of the first shaft between the bearings $l_1(x_4)$, the length of the second shaft between the bearings $l_2(x_5)$, the diameter of the first shaft $d_1(x_6)$, and the diameter of the second shaft $d_2(x_7)$. The schematic representation of the speed reducer is given in Fig. 1. The aim of the design issue is discovering the speed reducer with the lowest cost weight. The issue is mathematically represented in the following manner:

Minimize:

$$f(x) = 0.7854x_2^2x_1\left(14.9334x_3 - 43.0934 + 3.3333x_3^2\right) + 0.7854\left(x_5x_7^2 + x_4x_6^2\right) - 1.508x_1\left(x_7^2 + x_6^2\right) + 7.477(x_7^3 + x_6^3) \quad (3)$$

Subject to:

$$g_1(x) = -x_1x_2^2x_3 + 27 \leq 0 \tag{4}$$

$$g_2(x) = -x_1x_2^2x_3^2 + 397.5 \leq 0 \tag{5}$$

$$g_3(x) = -x_2x_6^4x_3x_4^{-3} + 1.93 \leq 0 \tag{6}$$

$$g_4(x) = -x_2x_7^4x_3x_5^{-3} + 1.93 \leq 0 \tag{7}$$

$$g_5(x) = 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} - 1100 \leq 0 \tag{8}$$

$$g_6(x) = 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2} - 850 \leq 0 \tag{9}$$

$$g_7(x) = x_2x_3 - 40 \leq 0 \tag{10}$$

$$g_8(x) = -x_1x_2^{-1} + 5 \leq 0 \tag{11}$$

$$g_9(x) = x_1x_2^{-1} - 12 \leq 0 \tag{12}$$

$$g_{10}(x) = 1.5x_6 - x_4 + 1.9 \leq 0 \tag{13}$$

$$g_{11}(x) = 1.1x_7 - x_5 + 1.9 \leq 0 \tag{14}$$

with bounds:

$0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 2.6 \leq x_1 \leq 3.6, 5 \leq x_7 \leq 5.5, 7.3 \leq x_5, x_4 \leq 8.3, 2.9 \leq x_6 \leq 3.9$

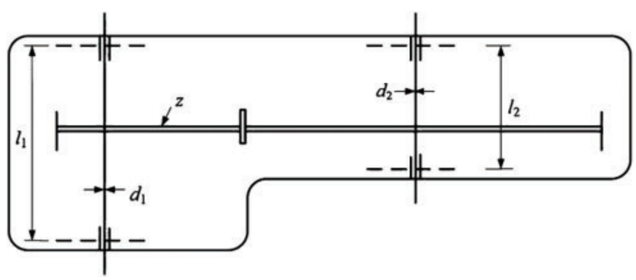

Figure 1: Speed reducer problem

Table 4 presents the comparative performance values of several approaches, including TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the speed reducer issue. Table 4 displays the optimal, mean, suboptimal, and standard deviation measurements of the used methodologies. Furthermore, Table 5 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 4, it becomes apparent that both EO and MFO exhibit superiority over other approaches in terms of the greatest value. Furthermore, when considering the average value, EO demonstrates greater success. Furthermore, Fig. 2 displays the convergence curve of the strategies used for the speed reducer issue.

Table 4: The statistical outcomes of the methods used for the speed reducer problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	3009.168854	399699.6461	1866150.765	597066.8458	15.37
EO	**2994.422564**	**2994.422564**	2994.422564	1.12621E-12	**1.52**
GWO	2999.884112	3006.149462	3014.184087	3.940870538	6.47
MFO	**2994.422564**	3003.320088	3043.034924	15.1867409	3.28
WOA	3002.230767	3107.084757	3612.31405	108.8572293	10.77
SMA	2994.422746	2994.424409	2994.428914	0.001656383	3.17
HHO	3007.417731	3425.055497	5080.047182	563.1360869	12.47
COA	3048.060533	3144.81531	3200.120506	40.11920307	12.60
COOT	2994.422579	2994.750943	3003.757984	1.673399569	3.37
MVO	3001.692079	3035.025167	3072.59999	16.33049716	8.93
AOA	3080.128866	3160.273461	3222.260728	40.2220024	13.07
AO	3048.786991	3850.139838	5143.275696	592.7613846	15.00
SCA	3061.137019	3114.63376	3199.643275	37.84743803	11.67
SAO	3252.352584	3915.597815	5763.29265	535.425852	15.63
SOA	3010.99658	3032.883533	3060.656495	13.93473393	8.93
POA	2994.424736	3000.035806	3007.515500	5.090249	5.30
COA	2994.555032	3001.492670	3016.723380	6.619445	5.47

Table 5: A comparative analysis of the best optimal solutions to the speed reducer problem

Algorithm	Parameters values							f_{min}
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
TSO	3.50151	0.7	17	7.3	8.073032	3.371746	5.288003	3009.168854
EO	3.49999	0.7	17	7.3	7.715319	3.350541	5.286654	**2994.422564**
GWO	3.500937	0.7	17	7.364591	7.809638	3.356307	5.288191	2999.884112
MFO	3.49999	0.7	17	7.3	7.715319	3.350541	5.286654	**2994.422564**
WOA	3.499959	0.7	17	7.443682	7.8541	3.351684	5.291651	3002.230767
SMA	3.499991	0.7	17	7.300002	7.715326	3.350541	5.286655	2994.422746
HHO	3.520425	0.7	17	7.38595	7.881842	3.352585	5.286711	3007.417731
COA	3.535505	0.7	17	7.3	8.3	3.428059	5.297065	3048.060533
COOT	3.49999	0.7	17	7.3	7.715319	3.350541	5.286654	2994.422579
MVO	3.500589	0.7	17	7.536276	7.764337	3.364298	5.287189	3001.692079
AOA	3.6	0.7	17	8.3	8.3	3.413136	5.299554	3080.128866
AO	3.531807	0.7	17	7.781691	8.162154	3.399743	5.310019	3048.786991
SCA	3.6	0.7	17	8.195417	8.046403	3.364929	5.300037	3061.137019
SAO	3.50601	0.7	17.19404	8.128777	8.174155	3.645772	5.469817	3252.352584
SOA	3.516004	0.7	17	7.689567	7.84431	3.356036	5.29075	3010.99658

(Continued)

Table 5 (continued)

Algorithm	Parameters values							f_{min}
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
POA	3.134689	0.773	18.56372	7.635233	7.529978	3.082312	5.296658	2994.424736
CA	2.779031	0.732381	23.68085	7.30957	7.508113	3.72635	5.404868	2994.555032

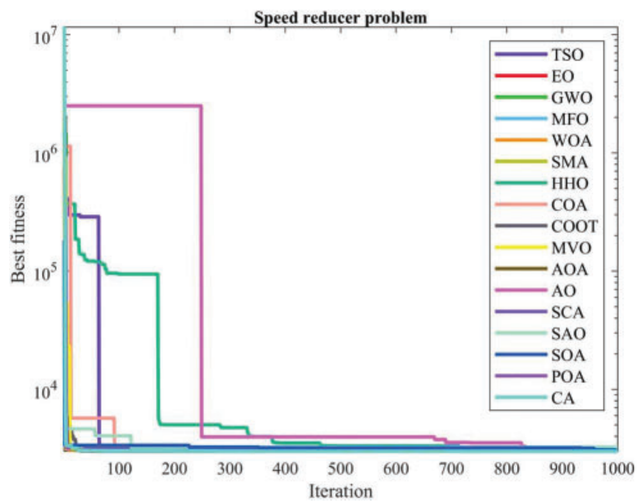

Figure 2: Convergence curve of the methods used on the speed reducer problem

4.2 Tension-Compression Spring Design Problem (Case 1)

The tension-compression spring design problem is a problem defined by Arora [58] which aims to create a spring design with the least amount of weight possible. This minimization problem, schematically illustrated in Fig. 3, has certain limitations such as cut-off voltage, ripple frequency and minimum deviation. The tension-compression spring problem has three decision variables: wire diameter d (x_1), average coil diameter D (x_2), and number of active coils N (x_3). This problem is mathematically represented as follows:

Minimize:

$$f(x) = (x_3 + 2)\, x_1^2 x_2 \tag{15}$$

Subject to:

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0 \tag{16}$$

$$g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3\_x_1^4)} + \frac{1}{5108 x_1^2} \leq 0 \tag{17}$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^3 x_3} \leq 0 \tag{18}$$

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0 \tag{19}$$

with bounds:

$0.05 \leq x_1 \leq 2.00, 0.25 \leq x_2 \leq 1.30, 2.00 \leq x_3 \leq 15.0$

Figure 3: Tension-compression spring design problem

Table 6 presents the comparative values of the performance results of several approaches, namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the tension-compression issue. Table 6 displays the optimal, mean, suboptimal, and standard deviation values of the used methodologies. Furthermore, Table 7 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 6, it becomes evident that CA outperforms other approaches in terms of the highest value, but POA has more performance when evaluated based on the average value. Furthermore, Fig. 4 illustrates the convergence graph of the approaches used for the tension-compression issue.

Table 6: The statistical outcomes of the methods used for the tension-compression spring design problem (case 1)

Algorithm	Best	Mean	Worst	SD	FMR
TSO	0.01280243	0.013419896	0.014640884	0.000546907	10.77
EO	0.012667047	0.012986249	0.014034056	0.000332122	7.07
GWO	0.012689887	0.012754846	0.013321233	0.000114098	4.80
MFO	0.012665686	0.013483807	0.017773158	0.001262609	8.67
WOA	0.012672465	0.013835231	0.015996637	0.000988024	10.97
SMA	0.012665885	0.013381332	0.015753456	0.000933324	8.03
HHO	0.012667145	0.013340624	0.016382034	0.000827094	9.00
COA	0.012735806	0.01340736	0.015791487	0.000755935	10.00
COOT	0.012665238	0.013331169	0.016001858	0.000906209	8.07
MVO	0.012803582	0.017131704	0.018147631	0.001737585	15.27
AOA	0.013142152	0.014388631	0.030631821	0.004325386	11.27
AO	0.013684295	0.016522416	0.025325732	0.002220195	15.33
SCA	0.012741025	0.013037944	0.013209119	0.000144589	8.50
SAO	0.013644472	0.018418919	0.026253801	0.002940077	15.93
SOA	0.01272481	0.012841598	0.014484334	0.000310579	6.03
POA	0.012665237	**0.012668098**	0.012690486	0.000005677	1.47
COA	**0.012665233**	0.012672088	0.012712224	0.000013390	1.83

Table 7: A comparative analysis of the best optimal solutions to the tension-compression spring design problem

Algorithm	Parameters values			f_{min}
	x_1	x_2	x_3	
TSO	0.054349629	0.424139632	8.218576613	0.01280243
EO	0.052005294	0.364373243	10.85386669	0.012667047
GWO	0.051729218	0.357405988	11.26857001	0.012689887
MFO	0.051846967	0.360528432	11.0690043	0.012665686
WOA	0.051063365	0.341850846	12.21692125	0.012672465
SMA	0.051519654	0.352654671	11.53131506	0.012665885
HHO	0.052013772	0.364579812	10.84249563	0.012667145
COA	0.05	0.317414002	14.04945763	0.012735806
COOT	0.05170472	0.357094576	11.26690843	**0.012665238**
MVO	0.05	0.316143553	14.199707	0.012803582
AOA	0.05	0.312114718	14.84271945	0.013142152
AO	0.053425204	0.395649521	10.11767744	0.013684295
SCA	0.052092338	0.365517089	10.84543941	0.012741025
SAO	0.050385213	0.316156139	15	0.013644472
SOA	0.05	0.317362481	14.03820349	0.01272481
POA	0.528153	0.934051	6.921844	0.012665237
CA	0.710522	1.026576	3.942698	**0.012665233**

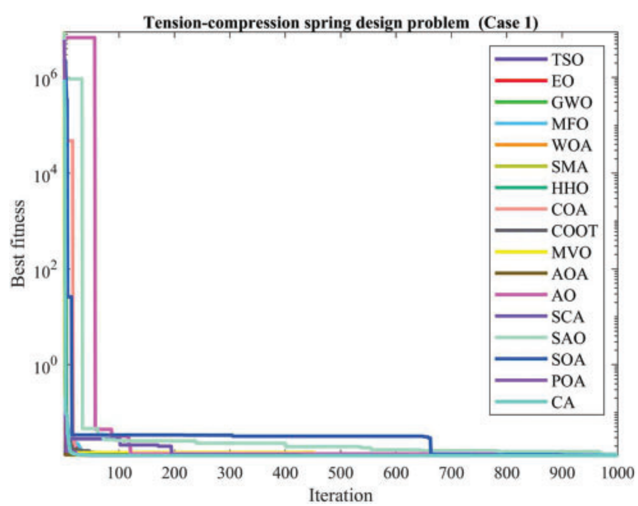

Figure 4: Convergence curve of the methods used on the tension-compression spring design problem

4.3 Pressure Vessel Design Problem

The primary aim of this challenge is to optimize the cost associated with welding, as well as the expenses related to materials and the creation process of a vessel [59]. This problem has four constraints that must be satisfied, and the objective function is calculated with respect to four variables: shell thickness (x_1), head thickness (x_2), inner radius (x_3), and length (x_4) without including vessel height. The schematic structure of the pressure vessel design problem is shown in Fig. 5. This problem is mathematically represented as follows:

Minimize:

$$f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3 \tag{20}$$

Subject to:

$$g_1(x) = -x_1 + 0.0193x_3 \le 0 \tag{21}$$

$$g_2(x) = -x_2 + 0.00954x_3 \le 0 \tag{22}$$

$$g_3(x) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0 \tag{23}$$

$$g_4(x) = x_4 - 240 \le 0 \tag{24}$$

with bounds:

$$0.51 \le x_1, x_2 \le 99.49, 10 \le x_3, x_4 \le 200$$

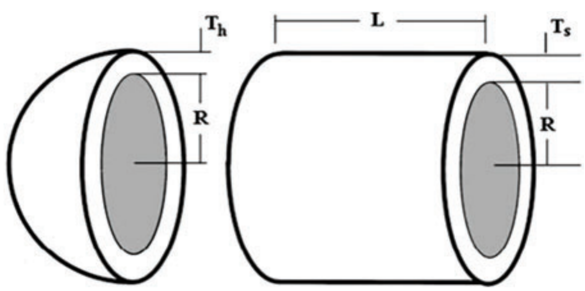

Figure 5: Pressure vessel

Table 8 presents the comparative values of the performances of several approaches, namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the pressure vessel design issue. Table 8 displays the optimal, mean, suboptimal, and standard deviation of the methodologies used. Furthermore, Table 9 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 8, it becomes evident that EO, POA, and CA exhibit superiority over other approaches in terms of the greatest value. Conversely, GWO has more success when evaluated based on the average value. Furthermore, Fig. 6 displays the convergence graph of the approaches used for the pressure vessel issue.

Table 8: The statistical outcomes of the methods used for the pressure vessel design problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	6879.866741	8704.301853	13069.25436	1582.300611	13.93
EO	**6058.71988**	6518.793086	7544.492518	512.0568854	6.68
GWO	6058.801906	**6219.805205**	7365.851023	356.5801053	3.63
MFO	6058.72346	6722.668915	7544.492518	508.6435023	8.25
WOA	6192.752799	8192.085505	11684.79247	1274.125452	13.47
SMA	6058.719892	6596.192699	7544.492519	549.8023233	7.10
HHO	6091.615788	6792.936176	7544.492518	365.7904697	9.30
COA	6340.732653	7846.678347	8548.44731	447.1938273	13.70
COOT	6058.720225	6406.088039	7352.612207	315.2009216	6.10
MVO	6091.584113	6741.27325	7419.29367	421.1743251	8.80
AOA	7177.685796	10134.5141	21033.80706	3059.849954	15.20
AO	6060.226707	6572.408065	7547.794432	474.7811046	7.63
SCA	6192.682512	6929.151724	8752.385245	588.2808671	9.70
SAO	8794.883762	13888.48817	21019.53969	3282.14935	16.73
SOA	6059.667071	6358.863021	7546.780769	466.1025508	5.50
POA	**6058.719878**	6224.114264	7273.278824	284.930973	3.33
CA	**6058.719878**	6249.526308	7273.278824	303.558052	3.93

Table 9: A comparative analysis of the best optimal solutions to the pressure vessel design problem

Algorithm	Parameters values				f_{min}
	x_1	x_2	x_3	x_4	
TSO	1.0625	0.5	50.60431861	93.67943093	6879.866741
EO	0.8125	0.4375	42.11479983	176.4340461	**6058.71988**
GWO	0.8125	0.4375	42.11775397	176.3995752	6058.801906
MFO	0.8125	0.4375	42.11575702	176.4221977	6058.72346
WOA	0.8125	0.4375	41.04114852	190.1938826	6192.752799
SMA	0.8125	0.4375	42.11511876	176.4300983	6058.719892
HHO	0.875	0.4375	45.32724166	140.3511195	6091.615788
COA	0.8125	0.4375	42.23592297	184.6334807	6340.732653
COOT	0.8125	0.4375	42.1150819	176.4305544	6058.720225
MVO	0.875	0.4375	45.33090975	140.3280126	6091.584113
AOA	0.8125	0.6875	41.09783997	200	7177.685796
AO	0.8125	0.4375	42.10333757	176.6195632	6060.226707
SCA	0.8125	0.4375	42.02127579	182.8885334	6192.682512
SAO	1.0625	0.875	52.572008	86.52438795	8794.883762
SOA	0.8125	0.4375	42.12719613	176.2965619	6059.667071

(Continued)

Table 9 (continued)

Algorithm	Parameters values				f_{min}
	x_1	x_2	x_3	x_4	
POA	3.4375	2.0625	193.1607	36.37869	**6058.719878**
CA	4.5625	0.25	67.12759	188.4114	**6058.719878**

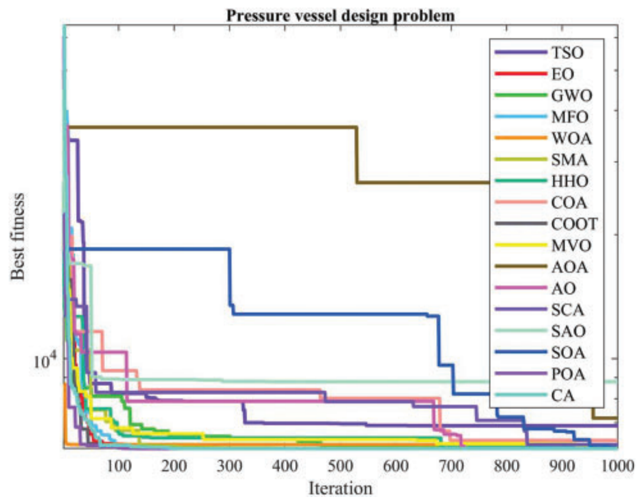

Figure 6: Convergence curve of the methods used on the pressure vessel design problem

4.4 Welded Beam Design Problem

The primary aim of the welded beam design challenge is to optimize the cost of producing a beam while adhering to certain limitations [5]. Fig. 7 depicts a welded beam structure comprised of beam A and the requisite weld to join this beam to object B. The problem includes four decision variables and five nonlinear inequality constraints. These design parameters are h (x_1), l (x_2), t (x_3) and b (x_4) representing the weld thickness, weld joint length, element width and element thickness, respectively. This problem is mathematically represented as follows:

Minimize:

$$f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2) \quad (25)$$

Subject to:

$$g_1(x) = \tau(\mathrm{x}) - \tau_{max} \leq 0 \quad (26)$$

$$g_2(x) = \sigma(x) - \sigma_{max} \leq 0 \quad (27)$$

$$g_3(x) = x_1 - x_4 \leq 0 \quad (28)$$

$$g_4(x) = 1.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0 \quad (29)$$

$$g_5(x) = 0.125 - x_1 \leq 0 \quad (30)$$

$$g_6(x) = \delta(x) - \delta_{max} \le 0 \tag{31}$$

$$g_7(x) = P - P_c(x) \le 0 \tag{32}$$

where,

$$\tau(x) = \sqrt{(\tau')^2 + (2\tau'\tau'')\frac{x_2}{2R} + (\tau'')^2}, \tau' = \frac{6000}{\sqrt{2}x_1x_2}, \tau'' = \frac{MR}{J}, M = 6000\left(14.0 + \frac{x_2}{2}\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

$$J = \left\{x_1x_2\sqrt{2}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}, \sigma(x) = \frac{504000}{x_4x_3^2}, \delta(x) = \frac{2.1952}{x_4x_3^3},$$

$$P_c(x) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1 - \frac{x_3\sqrt{\frac{E}{4G}}}{28}\right)$$

$$P_c(x) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1 - \frac{x_3\sqrt{\frac{E}{4G}}}{28}\right)$$

$\tau_{max} = 13600\ psi, \sigma_{max} = 30000\ psi, \delta_{max} = 0.25\ in, E = 30 \times 10^6\ psi, G = 12 \times 10^6\ psi$

with bounds:

$0.125 \le x_1 \le 2, 0.1 \le x_4 \le 2$ ve $0.1 \le x_2, x_3 \le 10$

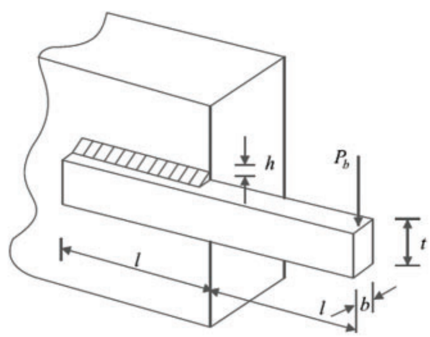

Figure 7: Schematic representation of welded beam

Table 10 presents the comparative values of the performances of several approaches, namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the welded beam design issue. Furthermore, Table 11 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 10, it becomes evident that EO and CA exhibit superiority over other ways in terms of greatest value, whereas POA demonstrates superiority over other methods in terms of average value. Furthermore, Fig. 8 illustrates the convergence graph of the approaches used for the welded beam design issue.

Table 10: The statistical outcomes of the methods used for the welded beam design problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	1.716054986	2.546468795	3.536807483	0.62365212	14.83
EO	**1.6702177**	1.670408299	1.675227413	0.000899033	1.93
GWO	1.671333735	1.673168037	1.678231819	0.001793212	5.37
MFO	1.670217701	1.722023637	1.975024976	0.073257707	6.80
WOA	1.73679486	2.170615908	4.197028463	0.530483963	13.63
SMA	1.670333603	1.674309181	1.726803703	0.010036131	5.03
HHO	1.690460887	1.891009449	2.227283322	0.13871246	12.00
COA	1.694398105	1.797886006	1.844566653	0.035111486	10.83
COOT	1.670482669	1.697798136	1.820191362	0.037171837	6.83
MVO	1.672676563	1.699051309	1.798001792	0.030387746	7.77
AOA	1.844129548	2.199481858	2.477850773	0.185303016	15.00
AO	1.725270134	1.953209489	2.220842457	0.122797404	13.30
SCA	1.726200239	1.804224071	1.874302512	0.040114991	11.00
SAO	1.918621628	3.204991211	6.074979783	0.876216532	16.47
SOA	1.679461577	1.690710091	1.754063757	0.014743169	7.63
POA	1.670217701	**1.670271663**	1.671045069	0.000156738	1.97
CA	**1.670217700**	1.670343953	1.671357022	0.000253049	2.60

Table 11: A comparative analysis of the best optimal solutions to the welded beam design problem

Algorithm	Parameters values				f_{min}
	x_1	x_2	x_3	x_4	
TSO	0.177884	3.819348	9.111658	0.202596	1.716054986
EO	0.198832	3.337364	9.192023	0.198832	1.6702177
GWO	0.198818	3.339296	9.191158	0.198966	1.671333735
MFO	0.198832	3.337364	9.192024	0.198832	1.670217701
WOA	0.181394	3.819648	9.424811	0.197769	1.73679486
SMA	0.198723	3.339432	9.192039	0.198833	1.670333603
HHO	0.185601	3.604042	9.199277	0.199369	1.690460887
COA	0.194345	3.404508	9.358266	0.198105	1.694398105
COOT	0.198834	3.33681	9.193963	0.198834	1.670482669
MVO	0.197168	3.372209	9.193376	0.198846	1.672676563
AOA	0.16434	4.163106	10	0.196826	1.844129548
AO	0.183829	3.589614	9.445074	0.199088	1.725270134
SCA	0.174972	3.942854	9.143209	0.201813	1.726200239
SAO	0.159253	4.713681	10	0.198437	1.918621628
SOA	0.193833	3.440986	9.192654	0.199217	1.679461577

(Continued)

Table 11 (continued)

Algorithm	Parameters values				f_{min}
	x_1	x_2	x_3	x_4	
POA	0.888043	6.126506	2.058919	0.910335	1.670217701
CA	0.749239	6.679802	8.02718	0.969918	**1.670217700**

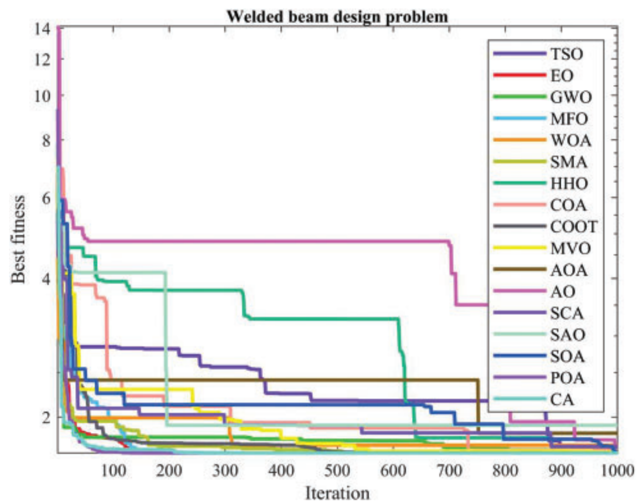

Figure 8: Convergence curve of the methods used on the welded beam design problem

4.5 Three-Bar Truss Design Problem

This problem is a structural optimization problem in civil engineering. The main objective of this problem introduced by Nowacki is to minimize the volume of the three-bar truss by adjusting the cross-sectional areas (x_1 and x_2), taking into account the stress (σ) on each of the truss members [27]. The value ranges that these values can take are $0 \le x_1, x_2 \le 1$. The mathematical definition of this problem, whose schematic representation is given in Fig. 9. This problem is mathematically represented as follows:

Minimize:

$$f(x) = l\left(x_2 + 2\sqrt{2x_1}\right) \tag{33}$$

Subject to:

$$g_1(x) = \frac{x_2}{2x_2x_1 + \sqrt{2x_1^2}}P - \sigma \le 0 \tag{34}$$

$$g_2(x) = \frac{x_2 + \sqrt{2x_1}}{2x_2x_1 + \sqrt{2x_1^2}}P - \sigma \le 0 \tag{35}$$

$$g_3(x) = \frac{1}{x_1 + \sqrt{2x_2}}P - \sigma \le 0 \tag{36}$$

where,

$l = 100, P = 2, and\ \sigma = 2$

with bounds:

$0 \leq x_1, x_2 \leq 1$

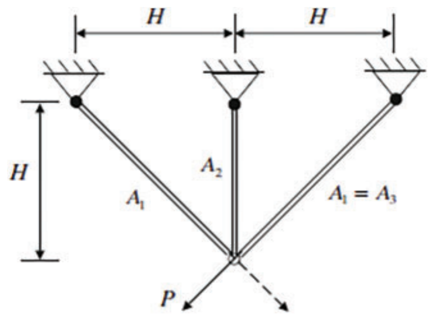

Figure 9: Three-bar truss design

Table 12 presents the comparative values of the performances of several approaches, including TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the three-bar truss design issue. Furthermore, Table 13 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 12, it becomes evident that EO, COOT, POA, and CA exhibit superiority over other approaches in terms of the greatest value. Furthermore, when considering the average value, POA and CA demonstrate greater success. Furthermore, Fig. 10 illustrates the convergence graph of the methodologies used in addressing the three-bar truss design issue.

Table 12: The statistical outcomes of the methods used for the three-bar truss design problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	263.8962383	264.4652982	270.4725405	1.289643914	10.40
EO	**263.8954081**	263.8954132	263.8954469	8.56826E-06	3.70
GWO	263.8954858	263.8976257	263.9054651	0.002128135	5.97
MFO	263.8954304	263.9483466	264.4238545	0.10154114	8.30
WOA	263.8959272	264.9041197	268.7812754	1.309740127	12.37
SMA	265.2278409	269.8360947	272.5174213	1.924796679	16.03
HHO	263.8959527	263.981987	264.4710671	0.142168454	9.20
COA	263.9057513	264.0519785	264.3448934	0.115646492	10.80
COOT	**263.8954081**	263.8954747	263.8962154	0.00018314	3.35
MVO	263.8954144	263.8963591	263.8986298	0.000930608	5.57
AOA	263.9974295	265.4147112	282.8427125	3.29158132	14.15
AO	263.9107145	264.1310168	264.5574435	0.158136239	11.83
SCA	263.9012431	264.6511963	282.8426491	3.379030311	10.93

(Continued)

Table 12 (continued)

Algorithm	Best	Mean	Worst	SD	FMR
SAO	264.996317	274.5928341	308.6800351	10.15582073	16.17
SOA	263.8985813	268.9840652	282.8427125	8.357099664	11.22
POA	**263.8954081**	**263.8954081**	263.8954081	0.0000000	1.53
CA	**263.8954081**	**263.8954081**	263.8954081	0.0000000	1.48

Table 13: A comparative analysis of the best optimal solutions to the three-bar truss design problem

Algorithm	Parameters values		f_{min}
	x_1	x_2	
TSO	0.788658774	0.40827384	263.8962383
EO	0.788674018	0.408242743	**263.8954081**
GWO	0.788350591	0.409157785	263.8954858
MFO	0.788498637	0.408739011	263.8954304
WOA	0.787833323	0.410625778	263.8959272
SMA	0.81669173	0.342325367	265.2278409
HHO	0.787812977	0.410683581	263.8959527
COA	0.790361227	0.40357838	263.9057513
COOT	0.788672536	0.408246935	**263.8954081**
MVO	0.788580396	0.408507606	263.8954144
AOA	0.799935949	0.377413758	263.9974295
AO	0.792467196	0.397589981	263.9107145
SCA	0.79032088	0.403647416	263.9012431
SAO	0.799979814	0.387278564	264.996317
SOA	0.790744046	0.402416516	263.8985813
POA	0.613176	0.047329	**263.8954081**
CA	0.303088	0.936761	**263.8954081**

4.6 Multiple Disk Clutch Brake Design Problem

The primary aim of this topic is to decrease the bulk of a clutch braking system consisting of numerous disks. Inner radius (x_1), outer radius (x_2), disk thickness (x_3), force of actuators (x_4), and number of frictional surfaces (x_5) are the five integer choice variables employed in this problem [27]. There are nine nonlinear constraints in this problem. Fig. 11 depicts a schematic depiction of the issue. The issue is mathematically represented in the following manner:

Minimize:

$$f(x) = \pi * \left(x_2^2 - x_1^2\right) * x_3(x_5 + 1)\rho \quad (37)$$

Subject to:

$$g_1(x) = -P_{max} - P_{rz} \le 0 \tag{38}$$

$$g_2(x) = P_{rz}V_{sr} - V_{sr,max}P_{max} \le 0 \tag{39}$$

$$g_3(x) = \Delta R + x_1 - x_2 \le 0 \tag{40}$$

$$g_4(x) = -L_{max} + (x_5 + 1)(x_3 + \delta \le 0 \tag{41}$$

$$g_5(x) = sM_s - M_h \le 0 \tag{42}$$

$$g_6(x) = T \ge 0 \tag{43}$$

$$g_7(x) = -V_{sr,max} + V_{sr} \le 0 \tag{44}$$

$$g_8(x) = T - T_{max} \le 0 \tag{45}$$

where,

$$M_h = \frac{2}{3}\mu x_4 x_5 \frac{x_2^3 - x_1^3}{x_2^2 - x_1^2} N.mm, \omega = \frac{\frac{\pi n}{30} rad}{s}, A = \pi\left(x_2^2 - x_1^2\right) mm^2, P_{rz} = \frac{\frac{x_4}{A}N}{mm^2}, V_{sr} = \frac{\frac{\pi R_{sr} n}{30} mm}{s},$$

$$R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2 x_1^2} mm, T = \frac{I_z\omega}{M_h + M_f}$$

$\Delta R = 20\, mm, L_{max} = 30\, mm, \mu = 0.6, V_{sr,max} = 10\, m/s, \delta = 0.5\, mm, s = 1.5, T_{max} = 15\, s, n = 250\, rpm,$
$I_z = 55\, kg.m^2, M_s = 40\, Nm, M_f = 3\, Nm, and\ P_{max} = 1$

with bounds:

$60 \le x_1 \le 80, 90 \le x_2 \le 110, 1 \le x_3 \le 3, 0 \le x_4 \le 1000, 2 \le x_5 \le 9$

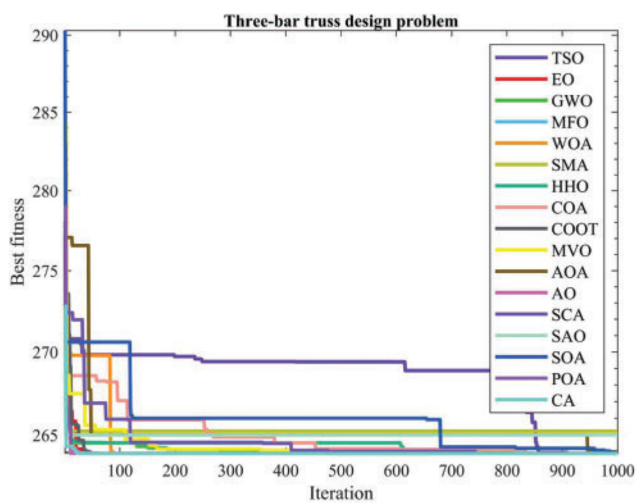

Figure 10: Convergence curve of the methods used on the three-bar truss design problem

Table 14 presents the comparative values of the performances of several approaches, namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the multiple disc clutch brake design issue. Furthermore, Table 15 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 14, it becomes evident that EO, MFO, WOA, SMA, HHO, COOT, POA, and CA exhibit superiority over other approaches in terms of the best value. Furthermore, when considering the average value, EO, HHO, POA, and CA demonstrate greater success. Furthermore, Fig. 12 displays the convergence graph of the strategies used for the multiple disc clutch brake design issue.

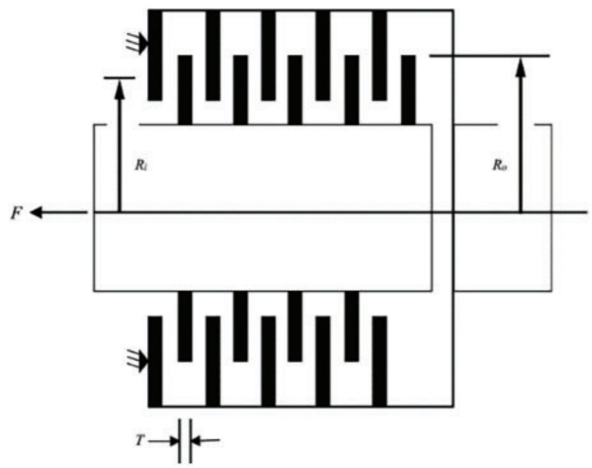

Figure 11: Multiple disk clutch brake design problem

Table 14: The statistical outcomes of the methods used for the multiple disk clutch brake design problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	0.235247188	0.235301639	0.235512629	5.92468E-05	10.63
EO	**0.235242458**	**0.235242458**	0.235242458	1.66533E-16	2.82
GWO	0.235243674	0.235269454	0.235367508	2.9044E-05	9.77
MFO	**0.235242458**	**0.235242458**	0.235242458	1.39792E-16	2.92
WOA	**0.235242458**	0.23524247	0.235242629	3.29813E-08	7.37
SMA	**0.235242458**	0.2352425	0.235242764	6.85923E-08	7.60
HHO	**0.235242458**	**0.235242458**	0.235242458	1.00841E-16	4.55
COA	0.235251931	0.235623716	0.236440761	0.000328328	12.73
COOT	**0.235242458**	0.235242459	0.235242485	4.79376E-09	5.12
MVO	0.235251002	0.23530991	0.235494147	5.82208E-05	10.90
AOA	0.235613936	0.239784504	0.253856183	0.006455082	15.20
AO	0.2355855	0.236635663	0.241295448	0.001083075	15.03
SCA	0.235257655	0.236260012	0.239379612	0.000916363	13.93
SAO	0.256485329	0.418898864	0.558433209	0.066573468	17.00

(Continued)

Table 14 (continued)

Algorithm	Best	Mean	Worst	SD	FMR
SOA	0.235247016	0.235459629	0.236194322	0.000233995	11.80
POA	**0.235242458**	**0.235242458**	0.235242458	0.000000000	2.82
CA	**0.235242458**	**0.235242458**	0.235242458	0.000000000	2.82

Table 15: A comparative analysis of the best optimal solutions to the multiple disk clutch brake design problem

Algorithm	Parameters values					f_{min}
	x_1	x_2	x_3	x_4	x_5	
TSO	69.99954044	90	1	871.2362442	2	0.235247188
EO	70	90	1	707.2368619	2	**0.235242458**
GWO	69.999836	90	1	152.2987899	2	0.235243674
MFO	70	90	1	958.6228265	2	**0.235242458**
WOA	69.99999994	90	1	1000	2	**0.235242458**
SMA	69.99999859	90	1	3.854620109	2	**0.235242458**
HHO	70	90	1	579.8999954	2	**0.235242458**
COA	69.99907953	90	1	1.103077777	2	0.235251931
COOT	70	90	1	859.8058151	2	**0.235242458**
MVO	69.99916981	90	1	999.8337472	2	0.235251002
AOA	70.01155738	90.0370556	1	1000	2	0.235613936
AO	69.96666064	90	1	668.7072249	2	0.2355855
SCA	69.99852333	90	1	1000	2	0.235257655
SAO	77.03512282	97.07407812	1	383.959645	2	0.256485329
SOA	69.99955715	90	1	965.9255451	2	0.235247016
POA	62.32327	94.44017	2.776293	659.5425	4.399259	**0.235242458**
CA	60.75571	108.0781	2.612441	321.4522	8.080696	**0.235242458**

4.7 Himmelblau's Function

The issue proposed by Himmelblau serves as a widely used benchmark problem for the analysis of nonlinear constrained optimization methods. This issue consists of a set of five variables and six nonlinear constraints. The issue is mathematically represented in the following manner.

Minimize:

$$f(x) = 5.3578547x_3^2 + 0.8356891x_1x_5 + 37.293239x_1 - 40792.141 \quad (46)$$

Subject to:

$$g_1(x) = -G1 \leq 0 \quad (47)$$

$$g_2(x) = G1 - 92 \leq 0 \tag{48}$$

$$g_3(x) = 90 - G2 \leq 0 \tag{49}$$

$$g_4(x) = G2 - 110 \leq 0 \tag{50}$$

$$g_5(x) = 20 - G3 \leq 0 \tag{51}$$

$$g_6(x) = G3 - 25 \leq 0 \tag{52}$$

where,

$G1 = 85.334407 + 0.0056858x_2x_5 + 0.0006262x_1x_4 - 0.0022053x_3x_5$

$G2 = 80.51249 + 0.0071317x_2x_5 + 0.0029955x_1x_2 - 0{:}\ 0021813x_3^2$

$G1 = 9.300961 + 0.0047026x_3x_5 + 0.00125447x_1x_3 - 0.0019085x_3x_4$

with bounds:

$78 \leq x_1 \leq 102, 33 \leq x_2 \leq 45, 27 \leq x_3, x_4, x_5 \leq 45$

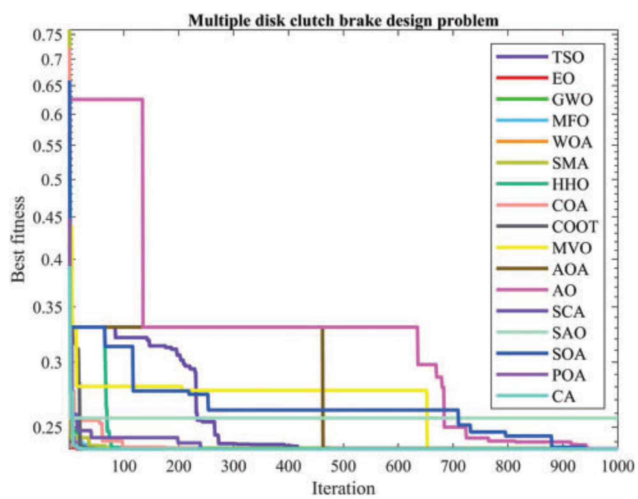

Figure 12: Convergence curve of the methods used on the multiple disk clutch design problem

Table 16 presents the comparative values of the performances of several approaches, namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on Himmelblau's function issue. Furthermore, Table 17 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 16, it becomes apparent that EO, MFO, SMA, COOT, POA, and CA exhibit superiority over other approaches in terms of the greatest value. Additionally, SMA demonstrates greater success when evaluated based on the average value. Furthermore, Fig. 13 displays the convergence graph of the strategies used for solving Himmelblau's function issue.

Table 16: The statistical outcomes of the methods used for the Himmelblau's function

Algorithm	Best	Mean	Worst	SD	FMR
TSO	−30632.46188	−30386.82957	−29753.31028	207.7208276	12.37
EO	**−30665.55912**	−30665.55912	−30665.55912	1.62288E-11	2.65
GWO	−30665.12066	−30660.90023	−30656.45398	2.430583688	7.00
MFO	**−30665.55912**	−30660.83703	−30560.13316	19.23734071	2.93
WOA	−30610.82885	−29938.37317	−29522.34584	232.4049417	15.30
SMA	**−30665.55912**	**−30665.55910**	−30665.55902	2.19388E-05	5.63
HHO	−30660.54681	−30570.18237	−30249.66843	123.1622639	9.93
COA	−30648.19157	−30443.71357	−30199.17052	104.3200577	12.43
COOT	**−30665.55912**	−30665.55742	−30665.52782	0.006229751	4.93
MVO	−30662.88669	−30532.84249	−30174.63609	114.3691665	10.87
AOA	−30627.87236	−29647.40327	−29130.41487	351.1225784	16.10
AO	−30646.13075	−30510.79641	−30254.06331	104.7187249	11.40
SCA	−30625.43612	−30487.51895	−30254.36315	90.87025306	11.70
SAO	−30224.24522	−29764.05291	−29434.89999	231.1897481	16.03
SOA	−30662.92388	−30630.3406	−30483.17315	33.77070124	8.63
POA	**−30665.55912**	−30665.55911	−30665.55894	0.00003	2.78
CA	**−30665.55912**	−30665.55912	−30665.55912	0.00000	2.30

Table 17: A comparative analysis of the best optimal solutions to the Himmelblau's function

Algorithm	Parameters values					f_{min}
	x_1	x_2	x_3	x_4	x_5	
TSO	78	33	30.03659921	44.19390758	37.07924883	−30632.46188
EO	78	33	29.99511433	45	36.77588429	**−30665.55912**
GWO	78	33	29.99728969	45	36.77219738	−30665.12066
MFO	78	33	29.99511434	45	36.77588429	**−30665.55912**
WOA	78	33	30.06458145	43.34405862	37.27272535	−30610.82885
SMA	78	33	29.99511546	45	36.77588291	**−30665.55912**
HHO	78	33	30.02118597	44.98433631	36.72447873	−30660.54681
COA	78	33	30.0732883	45	36.65666221	−30648.19157
COOT	78	33	29.99511433	45	36.77588429	**−30665.55912**
MVO	78.01538223	33.01207678	30.00686017	45	36.74166225	−30662.88669
AOA	78	33	30.22256747	45	36.22854345	−30627.87236
AO	78	33.02480065	30.08903458	44.8996988	36.61041061	−30646.13075
SCA	78	33.21663991	30.23023995	45	36.22779393	−30625.43612
SAO	79.20636734	33	31.68999069	42.44374862	33.74096769	−30224.24522
SOA	78	33	30.007539	45	36.75534739	−30662.92388

(Continued)

Table 17 (continued)

Algorithm	Parameters values					f_{min}
	x_1	x_2	x_3	x_4	x_5	
POA	87.47638	37.11719	30.29814	29.01444	34.208	**−30665.55912**
CA	85.56589	33.91255	30.06284	44.55554	37.89919	**−30665.55912**

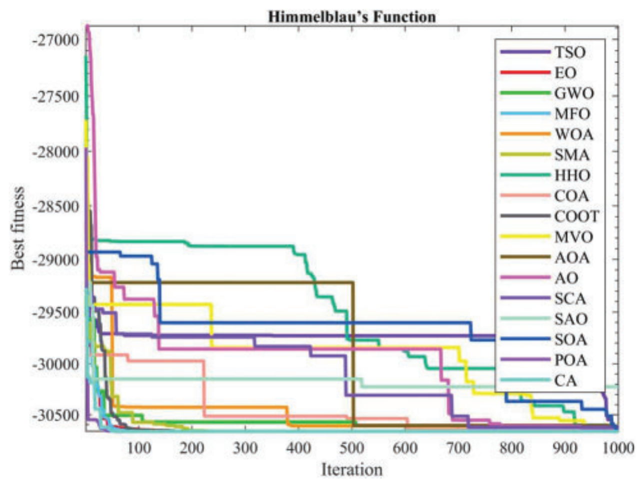

Figure 13: Convergence curve of the methods used on the Himmelblau's Function

4.8 Cantilever Beam Problem

A cantilever beam design problem is a common optimization problem faced in the area of engineering. In this problem, the minimum values of five choice variables x_1, x_2, x_3, x_4, x_5 must be found [28]. All variables provided are positive integers that belong to predetermined intervals. The schematic representation of the cantilever beam design problem is shown in Fig. 14. This design problem aims to find the minimum cost weight of the cantilever beam. The issue is mathematically represented in the following manner.

Minimize:

$$f(x) = 0.6224(x_1 + x_2 + x_3 + x_4 + x_5) \tag{53}$$

Subject to:

$$g(x) = \frac{60}{x_1^3} + \frac{27}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0 \tag{54}$$

with bounds:

$$0.01 \leq x_1, x_2, x_3, x_4, x_5 \leq 100$$

Table 18 presents the comparative values of the performances of several approaches, including TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the cantilever beam design issue. Furthermore, Table 19 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 18, it becomes evident that the CA technique exhibits superiority over other ways in terms of greatest value, while the POA approach demonstrates superiority over other methods in terms of average value. Furthermore, Fig. 15 displays the convergence curve of the strategies used in the cantilever beam design issue.

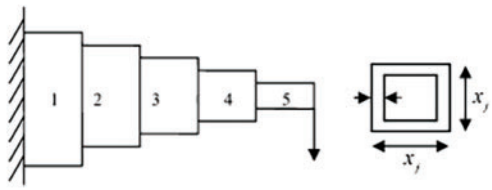

Figure 14: Cantilever beam

Table 18: The statistical outcomes of the methods used for the cantilever beam design problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	1.3808707	1.541428057	1.850529526	0.091666864	14.70
EO	1.339957787	1.339966064	1.339995913	8.27917E-06	1.47
GWO	1.339966023	1.340019779	1.340214936	4.85162E-05	3.63
MFO	1.340025322	1.340847969	1.342240497	0.000574436	7.83
WOA	1.355727617	1.48076134	1.744840151	0.101969482	14.10
SMA	1.339965208	1.34016411	1.340754168	0.000157703	5.17
HHO	1.340774991	1.342737909	1.346081888	0.001413135	10.27
COA	1.349655902	1.378232115	1.415968514	0.015606172	12.57
COOT	1.339993404	1.340515489	1.341698811	0.000404625	6.90
MVO	1.340107417	1.340557897	1.3412843	0.000348472	7.07
AOA	1.416490577	2.611250407	5.635836036	1.06543395	16.17
AO	1.340409794	1.343149904	1.347341023	0.00150102	10.33
SCA	1.354050532	1.389569289	1.427391273	0.02153387	12.77
SAO	1.560583449	5.787432723	10.70600435	2.740561266	16.70
SOA	1.340122039	1.340921536	1.342383254	0.000666399	8.10
POA	1.339958155	**1.339984347**	1.340036022	0.000024296	2.50
CA	**1.339957392**	1.339989731	1.340061639	0.000026726	2.73

Table 19: A comparative analysis of the best optimal solutions to the cantilever beam design problem

Algorithm	Parameters values					f_{min}
	x_1	x_2	x_3	x_4	x_5	
TSO	5.670155785	6.565312011	4.438323127	3.058677027	2.396870184	1.3808707

(Continued)

Table 19 (continued)

Algorithm	Parameters values					f_{min}
	x_1	x_2	x_3	x_4	x_5	
EO	6.009289686	5.31362623	4.495212338	3.502169327	2.153385515	1.339957787
GWO	6.021199168	5.308142494	4.490878061	3.500059078	2.153535662	1.339966023
MFO	5.982614347	5.350053873	4.500958815	3.497849413	2.143288223	1.340025322
WOA	5.714239494	5.117700403	4.757927854	3.465958349	2.670577941	1.355727617
SMA	6.007546137	5.307390778	4.507549834	3.503500314	2.14779716	1.339965208
HHO	6.072162327	5.412647819	4.49417035	3.423292247	2.084505887	1.340774991
COA	5.90969382	5.029028706	4.560689666	3.724153518	2.405535284	1.349655902
COOT	6.008751259	5.322951972	4.506063177	3.475725155	2.160761628	1.339993404
MVO	6.051023626	5.292876108	4.520661102	3.452340484	2.159179081	1.340107417
AOA	5.356286396	6.48053284	4.297025888	4.562178276	2.004146109	1.416490577
AO	6.123160098	5.244389814	4.453868547	3.527086297	2.132421426	1.340409794
SCA	6.634403914	5.010629598	4.321109865	3.539258349	2.194126028	1.354050532
SAO	4.999434962	5.003299123	4.999791919	5.000239438	5.006584696	1.560583449
SOA	6.066732579	5.287415377	4.487803249	3.481966957	2.152396558	1.340122039
POA	45.79272	74.26591	74.93475	69.57448	76.50655	1.339958155
CA	78.03137	23.95073	64.59176	63.55044	5.513643	**1.339957392**

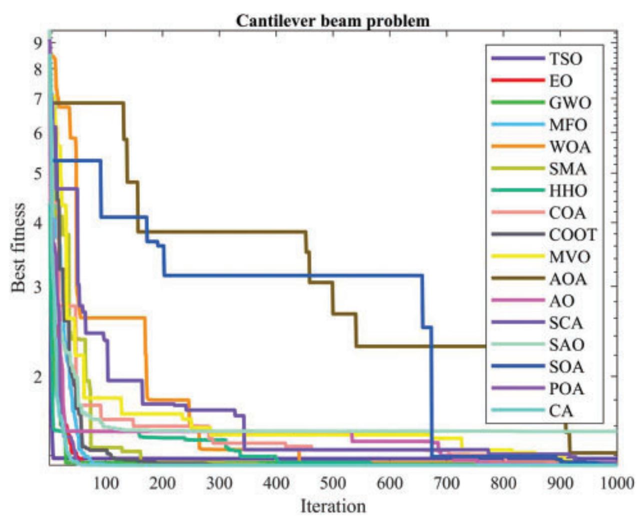

Figure 15: Convergence curve of the methods used on the cantilever beam problem

4.9 Tubular Column Design Problem

Fig. 16 shows an example of how to construct a uniform tubular column to handle a compressive load of $P = 2.500$ kgf for the lowest possible cost. The material used in the column has an elasticity modulus (E) of 0.85 × 106 kgf/cm^2, a yield stress (σ_y) of 500 kgf/cm^2, and a density (ρ) of 0.0025 kgf/cm^3. The column's length (L) is 250 cm. The stress in the column should be smaller than the yield stress (constraint g_1) and the buckling stress (constraint g_2). The column's mean diameter is limited between 2 and 14 cm (constraints g_3 and g_4), because thicknesses greater than 0.2–0.8 cm are not commercially accessible (constraint g_5 and g_6). The column's cost includes both construction costs and material [28]. It is assumed to be the objective function. The following is the optimization model for this problem:

Minimize:

$$f(d, t) = 9.8dt + 2d \tag{55}$$

Subject to:

$$g_1 = \frac{P}{\pi dt\sigma_y} - 1 \le 0 \tag{56}$$

$$g_2 = \frac{8PL^2}{\pi^3 Edt(d^2 + t^2)} - 1 \le 0 \tag{57}$$

$$g_3 = \frac{2.0}{d} - 1 \le 0 \tag{58}$$

$$g_4 = \frac{d}{14} - 1 \le 0 \tag{59}$$

$$g_5 = \frac{0.2}{t} - 1 \le 0 \tag{60}$$

$$g_6 = \frac{t}{0.8} - 1 \le 0 \tag{61}$$

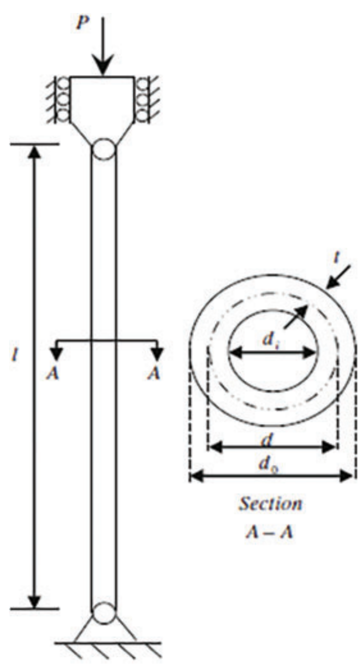

Figure 16: Tubular column design

Table 20 presents the comparative values of the performances of several approaches, namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, in relation to the tubular column design issue. Furthermore, Table 21 presents the choice factors that are contingent upon the optimal value obtained from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 20, it becomes evident that EO, MFO, COOT, POA, and CA exhibit superiority over other ways in terms of the best value. Furthermore, when evaluated based on the average value, EO, MFO, POA, and CA demonstrate greater success. Furthermore, Fig. 17 displays the convergence graph of the strategies used in solving the tubular column design issue.

Table 20: The statistical outcomes of the methods used for the tubular column design problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	26.48723538	26.52215446	26.71670925	0.047627405	10.03
EO	**26.48636047**	**26.48636047**	26.48636047	9.66443E-15	2.25
GWO	26.48669444	26.48898613	26.4923826	0.001406003	7.80
MFO	**26.48636047**	**26.48636047**	26.4863605	8.49963E-09	4.23
WOA	26.49142814	26.71389042	27.45916398	0.227930742	13.33
SMA	26.48636253	26.48644974	26.48677412	0.000106177	6.00
HHO	26.48817488	26.53351021	26.6437574	0.041042248	10.73
COA	26.51914471	26.61126269	26.73052204	0.064046766	13.07
COOT	**26.48636047**	26.4863627	26.48638274	4.88377E-06	4.67
MVO	26.48693664	26.48836793	26.49001498	0.000941322	7.40
AOA	26.97263461	27.88226718	28.71081678	0.540678369	16.87
AO	26.51558685	26.64085432	27.01857854	0.113082598	13.23
SCA	26.54809579	26.62667613	26.7792609	0.056449013	13.63
SAO	26.58472642	27.14467325	28.46629505	0.498784512	15.67
SOA	26.49154645	26.51134498	26.53977444	0.012814277	10.23
POA	**26.48636047**	**26.48636047**	26.48636047	0.00000000	1.97
CA	**26.48636047**	**26.48636047**	26.48636047	0.00000000	1.88

Table 21: A comparative analysis of the best optimal solutions to the tubular column design problem

Algorithm	Parameters values		f_{min}
	x_1	x_2	
TSO	5.452278337	0.291633911	26.48723538
EO	5.452181285	0.291626342	26.48636047
GWO	5.452331108	0.29161899	26.48669444
MFO	5.452181287	0.291626342	26.48636047
WOA	5.450793798	0.291847411	26.49142814

(Continued)

Table 21 (continued)

Algorithm	Parameters values		f_{min}
	x_1	x_2	
SMA	5.45218243	0.291626231	26.48636253
HHO	5.451684151	0.291705525	26.48817488
COA	5.453818274	0.292090964	26.51914471
COOT	5.452181285	0.291626342	26.48636047
MVO	5.452450386	0.291612678	26.48693664
AOA	5.32711782	0.312578548	26.97263461
AO	5.44908817	0.292455043	26.51558685
SCA	5.451693783	0.292826204	26.54809579
SAO	5.463051474	0.292477332	26.58472642
SOA	5.453665515	0.291588485	26.49154645
POA	7.706642	0.767859	**26.48636047**
CA	4.573024	0.633764	**26.48636047**

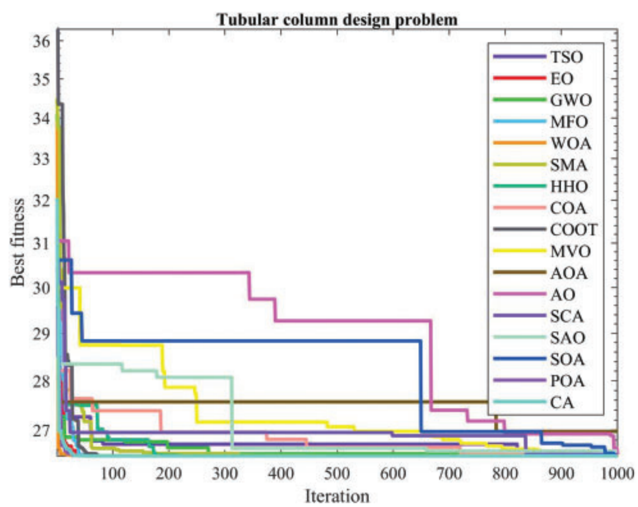

Figure 17: Convergence curve of the methods used on the tubular column design problem

4.10 Piston Lever

Piston lever problem was first raised by Vanderplaats [60]. When the piston lever is raised from 0 to 45 degrees, the main objective is to position the piston components $H(x_1)$, $B(x_2)$, $D(x_3)$, and $X(x_4)$ by limiting the oil volume, as shown in Fig. 18. The mathematical representation of this problem is as follows:

Minimize:

$$f(H, B, D, X) = \frac{1}{4}\pi D^2 (L_2 - L_1) \tag{62}$$

Subject to:

$$g1 = QLcos\theta - RF \leq 0 \; at \; \theta = 45^{\circ} \tag{63}$$

$$g2 = Q\,(L - X) - M_{max} \leq 0 \tag{64}$$

$$g1 = 1.2(L_2 - L_1) - L_1 \leq 0 \tag{65}$$

$$g1 = \frac{D}{2} - B \leq 0 \tag{66}$$

where,

$$R = \frac{|-X\,(Xsin\theta + H) + H(B - Xcos\theta)|}{\sqrt{(X - B)^2 + H^2}}, F = \pi PD^2/4, L_1 = \sqrt{(X - B)^2 + H^2},$$

$$L_2 = \sqrt{(Xsin45 + H)^2 + (B - Xcos45)^2}$$

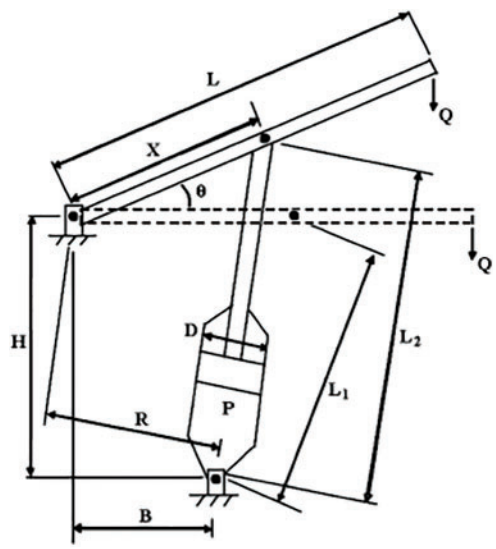

Figure 18: Piston lever

The payload is given as $P = 10.000$ *lbs*, the lever is given as $L = 240$ *in*, the maximum allowable bending moment of the lever is given as 6 max $M = 1.8 \times 10$ *lbs.in*, and the oil pressure is given as 1500 psi. Inequality limitations are enforced in a number of ways. Maximum bending moment of the lever, force equilibrium, geometrical conditions, and minimum piston stroke are all taken into account.

Table 22 presents the comparative performance values of several approaches, namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the piston lever issue. Furthermore, Table 23 presents the choice factors that are contingent upon the optimal value derived from the outcomes of 30 iterations conducted on this particular issue using the employed methodologies. Upon examination of Table 22, it becomes evident that EO, MFO, POA, and CA exhibit superiority over other approaches in terms of the greatest value. Conversely, COA demonstrates more success when evaluated based on the average value. Furthermore, Fig. 19 illustrates the convergence graph of the approaches used for the piston lever issue.

Table 22: The statistical outcomes of the methods used for the lever problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	17.42196241	397.5988085	2093.692333	373.4766691	14.37
EO	**8.412697953**	109.1507183	167.4727301	76.6501403	6.73
GWO	8.42233969	135.7896189	167.7832157	63.67587429	10.10
MFO	**8.412697953**	119.7547204	167.4727301	72.89046371	6.97
WOA	8.497928904	100.1318812	512.0065077	118.1165766	10.00
SMA	8.412698478	87.94276045	167.4732992	79.53005361	6.43
HHO	8.457048093	303.2021541	790.7522755	173.8110544	13.93
COA	8.534000505	**8.815060267**	9.21985017	0.167712838	5.67
COOT	8.412697954	123.8580549	203.1498737	76.06694022	9.10
MVO	8.472950733	140.9412925	284.7751907	117.3137109	10.23
AOA	185.0680395	325.5729621	501.3193251	100.46379	15.27
AO	8.444228253	20.49567107	190.9228266	44.62725153	4.87
SCA	8.625292195	9.313147737	10.70516073	0.404245037	6.97
SAO	283.1874112	15297.60183	77744.75245	22761.39193	16.60
SOA	8.433373601	19.26894305	170.2505714	40.15273808	4.47
PA	**8.412697953**	96.997720073	167.472730052	77.825586069	5.48
CA	**8.412697953**	100.753413785	167.472730052	76.533411694	5.82

Table 23: A comparative analysis of the best optimal solutions to the piston lever problem

Algorithm	Parameters values				f_{min}
	x_1	x_2	x_3	x_4	
TSO	0.071101082	4.304278939	4.05075328	119.9216191	17.42196241
EO	0.05	2.041513399	4.083027183	120	**8.412697953**
GWO	0.050132638	2.042506155	4.08410152	119.9542691	8.42233969
MFO	0.05	2.041513399	4.083027183	120	**8.412697953**
WOA	0.05	2.062664782	4.083411529	119.9550817	8.497928904
SMA	0.05	2.041513292	4.083027241	119.9999998	8.412698478
HHO	0.05	2.04610059	4.089278683	119.6218262	8.457048093
COA	0.05	2.065759358	4.089137843	120	8.534000505
COOT	0.05	2.041513382	4.083027184	120	8.412697954
MVO	0.05	2.046975471	4.092356386	119.9180173	8.472950733
AOA	500	500	2.302404367	61.12128883	185.0680395
AO	0.05	2.04729087	4.085132655	120	8.444228253
SCA	0.05010724	2.072171185	4.104600675	120	8.625292195
SAO	469.2131864	403.4320025	2.716971944	65.42688267	283.1874112

(Continued)

Table 23 (continued)

Algorithm	Parameters values				f_{min}
	x_1	x_2	x_3	x_4	
SOA	0.05	2.045182942	4.084523234	120	8.433373601
POA	376.3677	22.52272	347.7443	116.6528	**8.412697953**
COA	120.8287	67.23463	329.0836	14.40809	**8.412697953**

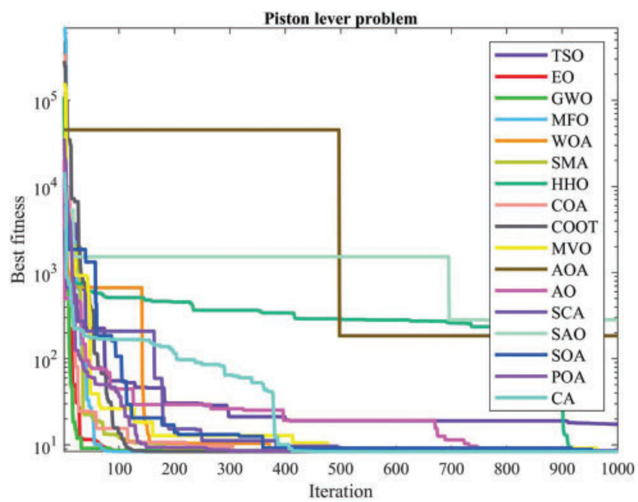

Figure 19: Convergence curve of the methods used on the piston lever

4.11 Robot Gripper

The difference between the robot gripper's minimum and maximum force is used as an objective function in this challenge. The robot is involved in this challenge, which has six nonlinear design constraints and seven design variables [41]. The schematic representation of the problem is shown in Fig. 20. This problem is mathematically defined as follows:

Minimize:

$$f(x) = |maxF_k(x,z) - minF_k(x,z)| \quad (67)$$

Subject to:

$$g_1(x) = y(x, z_{max}) - Y_{min} \leq 0 \quad (68)$$

$$g_2(x) = -y(x, z_{max}) \leq 0 \quad (69)$$

$$g_3(x) = Y_{max} - y(x, 0) \leq 0 \quad (70)$$

$$g_4(x) = y(x, 0) - Y_G \leq 0 \quad (71)$$

$$g_5(x) = l^2 + e^2 - (a+b)^2 \leq 0 \quad (72)$$

$$g_6(x) = b^2 - (a-e)^2 - (l - z_{max})^2 \leq 0 \quad (73)$$

$g_7(x) = z_{max} - l \leq 0$

where,

$$a = cos^{-1}\left(\frac{a^2 + g^2 - b^2}{2a \times g}\right) + \theta, g = \sqrt{e^2 + (z - l)^2}, \theta = tan^{-1}(\tfrac{e}{l-z}),$$

$$\beta = cos^{-1}\left(\frac{b^2 + g^2 - a^2}{2b \times g}\right) - \theta, y(x,z) = 2(f + e + c \times \sin(\beta + \delta))$$

$$F_k = \left(\frac{b \times p \times \sin(a + \beta)}{2c \times \cos(a)}\right), Y_{min} = 50, Y_{max} = 100, Y_G = 150, z_{max} = 100, P = 100$$

with bounds:

$10 \leq f, a, b \leq 150, 0 \leq e \leq 50, 100 \leq c \leq 200, 100 \leq l \leq 300, 1 \leq \beta \leq 3.14$

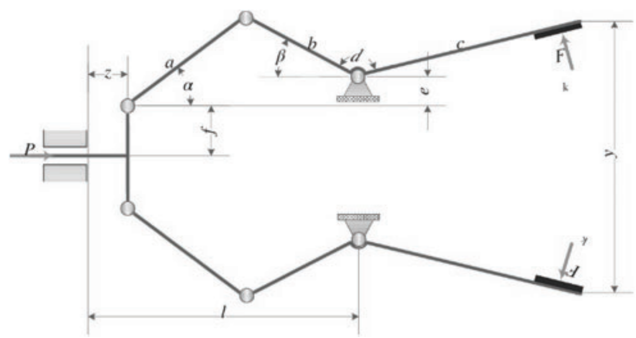

Figure 20: Robot gripper

Table 24 presents the comparative values of the performances of several approaches, namely TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA, on the robot gripper issue. Furthermore, Table 25 presents the decision variables associated with the optimal values obtained from the outcomes of 30 iterations conducted on this particular topic. Upon examination of Table 25, it becomes evident that the Simple Moving Average (SMA) approach outperforms other methods in terms of both the best value and the average value. Furthermore, Fig. 21 illustrates the convergence graph of the strategies used in addressing the robot gripper issue.

Table 24: The statistical outcomes of the methods used for the robot gripper problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	4.289551866	22872475335	2.26915E+11	4.289551866	14.77
EO	2.558265959	3.225935001	6.055703362	2.558265959	4.67
GWO	2.636395337	3.325110862	4.094105793	2.636395337	5.87
MFO	3.528008708	5.29421131	12.60060882	3.528008708	11.38
WOA	3.287423023	6.58577709	38.88747796	3.287423023	12.17
SMA	**2.545958371**	**2.715800832**	3.50251645	2.545958371	1.60
HHO	3.205121938	18.35609682	79.90636828	3.205121938	14.00

(Continued)

Table 24 (continued)

Algorithm	Best	Mean	Worst	SD	FMR
COA	2.795786758	4.064341655	4.289317141	2.795786758	9.13
COOT	2.680610563	3.503447897	4.590989339	2.680610563	6.70
MVO	2.695840012	3.197078896	4.574383286	2.695840012	4.93
AOA	3.690498919	4.998068535	10.2444732	3.690498919	11.43
AO	4.519607097	22.29788938	104.556152	4.519607097	15.17
SCA	4.123780873	4.447648925	7.017991237	4.123780873	10.62
SAO	5.190176925	6424961932	93324638481	5.190176925	16.07
SOA	2.621287418	3.38339899	4.289317141	2.621287418	5.43
POA	2.63467106	3.06246168	3.62569846	0.22417816	4.27
CA	2.63248350	3.12543950	3.89424622	0.24904286	4.80

Table 25: A comparative analysis of the best optimal solutions to the robot gripper problem

Algorithm	Parameters values							f_{min}
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
TSO	150	150	200	0	150	100.0027	2.404797	4.289551866
EO	149.7029	149.5393	199.9916	0.040805	149.9667	101.4237	2.334378	2.558265959
GWO	149.7173	149.5536	200	0	30.59937	105.1643	1.709293	2.636395337
MFO	150	145.711	200	0	150	165.1683	2.566817	3.528008708
WOA	148.0163	147.8544	160.7647	0	149.9996	104.385	2.601234	3.287423023
SMA	150	149.8805	200	0.001158	149.2655	101.0626	2.299768	**2.545958371**
HHO	150	149.3707	183.248	5.26E-05	148.6101	125.2552	2.539905	3.205121938
COA	150	149.7792	200	0	10	110.2619	1.623498	2.795786758
COOT	150	149.8301	195.8189	0.001801	149.7434	105.4282	2.379592	2.680610563
MVO	149.2102	142.5991	196.4037	6.458731	142.5376	103.1736	2.374035	2.695840012
AOA	150	122.1327	200	26.25289	150	139.686	2.671189	3.690498919
AO	149.2919	93.109	199.0805	49.46878	145.6248	160.2051	3.006883	4.519607097
SCA	150	147.3924	175.7737	0	109.8487	155.8737	2.374302	4.123780873
SAO	112.2475	96.3309	157.2066	15.29257	148.262	108.467	2.797333	5.190176925
SOA	150	149.8639	199.681	0	148.5837	103.2267	2.341468	2.621287418
POA	18.96377	117.8522	180.8935	27.38139	121.1049	176.5338	2.257163	2.63467106
CA	111.7183	65.50549	165.8307	3.049145	62.06039	230.6625	2.715952	2.63248350

4.12 Corrugated Bulkhead Design Problem

Corrugated bulkhead designs are frequently employed in chemical tankers and product tankers in order to aid in the efficient cleaning of cargo tanks at the loading dock [61]. This problem serves as an illustration of how to construct corrugated bulkheads for a tanker to be as light as possible while maintaining structural integrity. A tanker's corrugated bulkheads are designed to be as light as possible while maintaining their structural integrity. The problem has four design variables: width b (x_1), depth h (x_2), length l (x_3), and plate thickness t (x_4). The mathematical formula for the optimization problem is as follows:

Minimize:

$$f(b,h,l,t) = \frac{5.885t(b+l)}{b+\sqrt{(l^2-h^2)}} \tag{74}$$

Subject to:

$$g_1 = th\left(0.4b+\frac{1}{6}\right) - 8.94\left(b+\sqrt{(l^2-h^2)}\right) \geq 0 \tag{75}$$

$$g_2 = th^2\left(0.2b+\frac{1}{12}\right) - 2.2\left(8.94\left(b+\sqrt{(l^2-h^2)}\right)\right)^{\frac{4}{3}} \geq 0 \tag{76}$$

$$g_3 = t - 0.0156b - 0.15 \geq 0 \tag{77}$$

$$g_4 = t - 0.0156l - 0.15 \geq 0 \tag{78}$$

$$g_5 = t - 1.15 \geq 0 \tag{79}$$

$$g_6 = l - h \geq 0 \tag{80}$$

with bounds:

$0 \leq b, h, l \leq 100\, and\, 0 \leq t \leq 5$

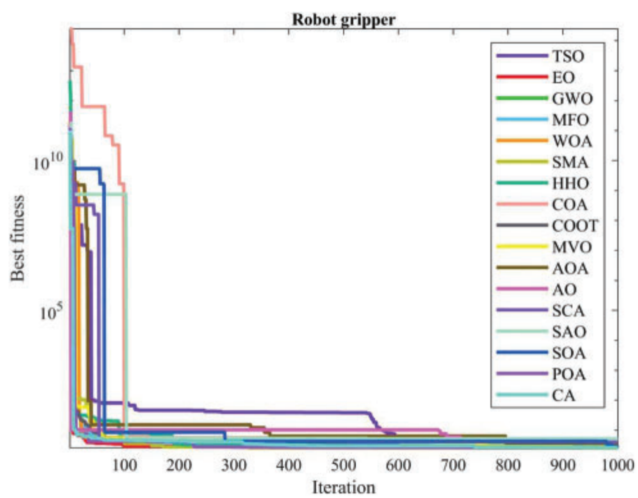

Figure 21: Convergence curve of the methods used on the robot gripper problem

Table 26 presents the comparative values of the performances of several approaches (TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA) on the corrugated bulkhead design issue. Furthermore, Table 27 presents the decision variables associated with the optimal values obtained from the outcomes of 30 iterations conducted on this particular topic. Upon examination of Table 26, it becomes evident that EO, MFO, and CA exhibit superiority over other ways in terms of the best value. Additionally, when considering the average value, EO emerges as the more effective approach. Furthermore, Fig. 22 illustrates the convergence graph of the strategies used in addressing the corrugated bulkhead design issue.

Table 26: The statistical outcomes of the methods used for the corrugated bulkhead design problem

Algorithm	Best	Mean	Worst	SD	FMR
TSO	6.855395898	7.515017746	10.98772723	0.892030186	11.07
EO	**6.842957472**	**6.842957472**	6.842957472	2.50569E-11	1.17
GWO	6.84594174	6.849404911	6.857978732	0.00329714	6.40
MFO	**6.842957472**	6.95886219	10.31538608	0.623291412	2.67
WOA	6.868197444	7.234972405	8.666867301	0.463809051	10.50
SMA	7.953976548	11.66989827	12.57187707	1.108933488	16.93
HHO	6.855228127	7.085261807	7.505850206	0.192900564	10.03
COA	6.18061627	7.46789485	8.743988275	0.657396042	11.23
COOT	6.842961718	6.84395948	6.855249262	0.002264306	4.53
MVO	6.844112205	6.854550423	6.891634775	0.009622446	6.70
AOA	7.147273011	7.94042808	10.50177315	0.889597343	13.67
AO	6.892071425	7.346390448	8.332439081	0.400497798	11.17
SCA	7.021747181	7.936506325	8.617725871	0.621047699	13.37
SAO	7.020186794	8.536171185	10.79430556	1.006701203	14.33
SOA	6.863150498	7.609158004	8.30725787	0.642814739	11.67
POA	6.842957474	6.843508225	6.846584381	0.001067412	3.83
CA	**6.842957472**	6.843177079	6.845076983	0.000488454	3.73

Table 27: A comparative analysis of the best optimal solutions to the corrugated bulkhead design problem

Algorithm	Parameters values				f_{min}
	x_1	x_2	x_3	x_4	
TSO	55.94375825	34.12553171	57.63213667	1.049990562	6.855395898
EO	57.69229443	34.14762159	57.69229626	1.049999774	**6.842957472**
GWO	57.40760601	34.14377986	57.63211944	1.050001441	6.84594174
MFO	57.6922948	34.1476216	57.6922968	1.04999978	**6.842957472**
WOA	53.75332544	34.11435394	57.67117989	1.050052281	6.868197444
SMA	38.6549421	37.15897337	66.97119063	1.194658396	7.953976548
HHO	57.64186443	34.33682599	57.57011798	1.049999674	6.855228127
COA	4.93434E-07	0	0	1.050199052	6.18061627
COOT	57.69158469	34.14761555	57.69229153	1.049999734	6.842961718
MVO	57.67291664	34.14655471	57.68941522	1.050156185	6.844112205
AOA	54.86760749	37.15326823	54.86760749	1.054086491	7.147273011

(Continued)

Table 27 (continued)

Algorithm	Parameters values				f_{min}
	x_1	x_2	x_3	x_4	
AO	55.60404503	34.45085913	56.91844174	1.050288305	6.892071425
SCA	55.21204025	35.58586585	58.65384066	1.067117361	7.021747181
SAO	54.15668556	34.76085576	58.12707556	1.070304388	7.020186794
SOA	57.70299259	34.16103858	57.92914589	1.053813338	6.863150498
POA	66.76194	8.339457	21.65966	4.846303	6.842957474
CA	68.80847	51.83041	73.10743	1.992981	**6.842957472**

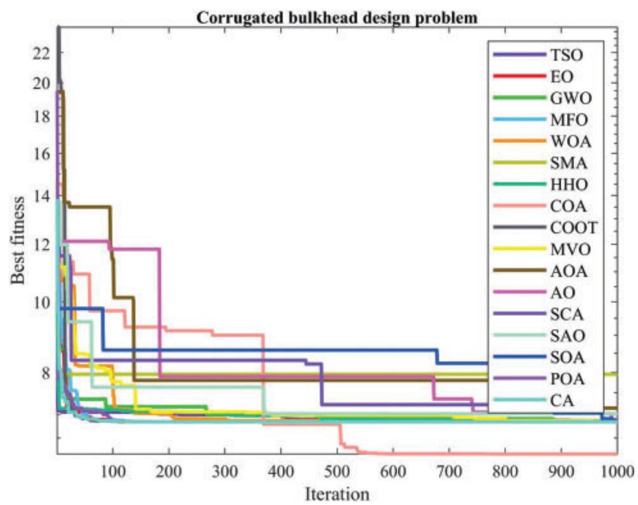

Figure 22: Convergence curve of the methods used on the corrugated bulkhead design problem

5 Discussions

This study introduces the investigation of 17 different metaheuristic optimization algorithms, which have been proposed in recent years and are popular in the literature, on 12 real-world engineering problems. In this study, an external penalty is imposed on algorithms that are used to cope with inequality and equality constraints when they are implemented. Although the use of such a method is relatively straightforward, determining the optimal values of penalty terms, particularly for optimization problems with a high number of constraints, proves to be a challenging optimization issue in and of itself.

According to the experimental results, CA produced the best optimum value in 10 problems, EO in 9 problems, and POA and MFO in 6 problems. Following these, COOT managed to produce the best optimum value in 4 problems and WOA in 3 problems. Although GWO could not find the best optimum value in any of the 12 different engineering problems, statistically, it showed the sixth-best performance among the methods. Furthermore, COOT could not find the mean value in any of the 12 different engineering problems; statistically, it showed the fourth-best performance among the methods. It is thought that statistical analysis has great importance, especially in the comprehensive examination of the performance of metaheuristic optimization methods.

Friedman statistical analysis is performed, and mean ranks are calculated to analyze the results of the investigation. According to these calculated values, the performances of metaheuristic optimization methods in all problems are presented in Table 28 by taking the average value. It is clear from the comparative results presented in the subheadings of Section 3 that the POA technique exhibits a distinct advantage over other algorithms. When the obtained results are examined in more detail, EO performed better in 5 of the 12 problems, CA in 4, and POA in 3 of them. It is also seen that the POA method is significantly superior to other methods statistically when Table 28 is examined. POA is followed by CA, EO, and COOT. SAO, AOA, and TSO are the methods with the worst performance among the methods compared. In light of the results obtained, it can be said that CA has the fastest convergence rate among the 17 different metaheuristic optimization methods used in the study and also has the best balance between exploration and exploitation stages.

Table 28: Mean of Friedman mean rank

Algorithm	Mean FMR	Mean manuel rank
TSO	12.77	15
EO	3.55	3
GWO	6.40	6
MFO	6.19	5
WOA	12.00	13
SMA	7.39	7
HHO	10.45	10
COA	11.23	12
COOT	5.81	4
MVO	8.70	9
AOA	14.45	16
AO	12.03	14
SCA	11.23	12
SAO	16.11	17
SOA	8.30	8
POA	3.10	1
CA	3.28	2

The research conducted a rigorous analysis by using the Wilcoxon signed-rank test, a non-parametric statistical test, to provide a robust comparison between the suggested and competing algorithms and to provide statistical validation for the findings obtained. The statistical analyses at a significance level of 5% are shown in Table 29 using the Wilcoxon signed-rank test and the objective function values. While performing the Wilcoxon signed rank test, statistical analyses were performed based on the method that was found to be the best according to the Friedman mean rank value. According to the FMR value, out of 12 different problems, EO in 4, POA in 3, CA in 3, SMA in 1, and SOA in 1 problem achieved the best results. In light of these results, the results obtained in real-world engineering problems according to the Wilcoxon signed rank test are given in Table 29. $\triangle$ indicates that the first method, based on the FMR value, is substantially superior to the other competitive methods, whereas ≈ indicates that its performance is negligible. - indicates the method that gives the best result according to the FMR value.

When Table 29 is examined, the EO method, which achieved the best result according to the FMR value in the speed reducer problem (Problem-1), provided a significant superiority to all methods compared to the Wilcoxon signed rank test. POA, which gave the best result according to the FMR value in the tension-compression spring design problem (Problem-2), could not provide a significant superiority to the CA method compared to the Wilcoxon signed rank test, but it outperformed all other methods. The POA method, which gave the best result according to the FMR value in the pressure vessel design problem (Problem-3), provided a significant superiority over the Wilcoxon signed rank test for all methods except GWO, SOA, and CA. The EO method, which gave the best result according to the FMR value in the welded beam design problem (Problem-4), provided a significant superiority to all other methods except POA compared to the Wilcoxon signed rank test. In the three-bar truss design problem (Problem-5), the CA method, which gave the best result according to the FMR value, provided a significant superiority to all other methods except POA compared to the Wilcoxon signed rank test. The POA method, which gives the best result according to FMR value in the multiple disc clutch brake design problem (Problem-6), provides a significant superiority over the Wilcoxon signed rank test over all other methods except EO, MFO, HHO, COOT, and CA. The CA method, which gave the best result according to the FMR value in Himmelblau's Function (Problem-7), provided a significant superiority over the Wilcoxon signed rank test over all other methods except EO, MFO, and POA. In the cantilever beam problem (Problem-8), the EO method, which gave the best result according to the FMR value, provided a significant superiority to all methods compared to the Wilcoxon signed rank test. In the tubular column design problem (Problem-9), the CA method, which gave the best result according to the FMR value, provided a significant superiority to all other methods except EO and POA compared to the Wilcoxon signed rank test. The SOA method, which gave the best result according to the FMR value in the piston lever (Problem-10), provided a significant superiority to all other methods except AO compared to the Wilcoxon signed rank test. The SMA method, which gave the best result according to the FMR value in the robot gripper (Problem-11), provided a significant superiority to all methods compared to the Wilcoxon signed rank test. The EO method, which gave the best result according to the FMR value in the corrugated bulkhead design problem (Problem-12), provided a significant superiority to all methods compared to the Wilcoxon signed rank test.

Table 29a: Wilcoxon signed ranks test results for all metaheuristic optimization algorithms

Metaheuristic optimization algorithms	Real-world engineering design problem					
	Problem-1	Problem-2	Problem-3	Problem-4	Problem-5	Problem-6
TSO	1.73E-06 △	1.73E-06 △	1.92E-06 △	1,73E-06 △	1.73E-06 △	1.73E-06 △
EO	– –	1.73E-06 △	4.20E-04 △	– –	1.73E-06 △	1.00E+00 ≈
GWO	1.73E-06 △	1.73E-06 △	4.53E-01 ≈	1,36E-05 △	1.73E-06 △	1.73E-06 △

(Continued)

Table 29a (continued)

Metaheuristic optimization algorithms	Real-world engineering design problem					
	Problem-1	Problem-2	Problem-3	Problem-4	Problem-5	Problem-6
MFO	1.95E-03 △	2.35E-06 △	2.05E-04 △	7,69E-06 △	1.73E-06 △	1.00E+00 ≈
WOA	1.73E-06 △	1.73E-06 △	1.73E-06 △	1,73E-06 △	1.73E-06 △	1.73E-06 △
SMA	1.73E-06 △	1.73E-06 △	1.48E-03 △	1,73E-06 △	1.73E-06 △	1.73E-06 △
HHO	1.73E-06 △	1.73E-06 △	3.41E-05 △	1,73E-06 △	1.73E-06 △	1.00E+00 ≈
COA	1.73E-06 △	1.73E-06 △	1.73E-06 △	1,73E-06 △	1.73E-06 △	1.73E-06 △
COOT	1.73E-06 △	3.52E-06 △	2.11E-03 △	1,73E-06 △	1.22E-05 △	1.25E-01 ≈
MVO	1.73E-06 △	1.73E-06 △	2.61E-04 △	1,73E-06 △	1.73E-06 △	1.73E-06 △
AOA	1.73E-06 △	1.73E-06 △	1.73E-06 △	1,73E-06 △	1.73E-06 △	1.72E-06 △
AO	1.73E-06 △	1.73E-06 △	2.41E-03 △	1,73E-06 △	1.73E-06 △	1.73E-06 △
SCA	1.73E-06 △	1.73E-06 △	1.13E-05 △	1,73E-06 △	1.73E-06 △	1.73E-06 △
SAO	1.73E-06 △	1.73E-06 △	1.73E-06 △	1,73E-06 △	1.73E-06 △	1.73E-06 △
SOA	1.73E-06 △	1.73E-06 △	1.92E-01 ≈	1,73E-06 △	1.70E-06 △	1.73E-06 △
POA	1.73E-06 △	– –	– –	8,13E-01 ≈	1.00E+00 ≈	– –
CA	1.73E-06 △	1.41E-01 ≈	6.06E-01 ≈	1,11E-02 △	– –	1.00E+00 ≈

Table 29b: Wilcoxon signed ranks test results for all metaheuristic optimization algorithms (Continued)

Metaheuristic optimization algorithms	Real-world engineering design problem					
	Problem-7	Problem-8	Problem-9	Problem-10	Problem-11	Problem-12
TSO	1.73E-06 △	1,73E-06 △	1.73E-06 △	1,73E-06 △	1,73E-06 △	1.73E-06 △
EO	1.00E+00 ≈	– –	1.00E+00 ≈	4,99E-03 △	4,07E-05 △	– –

(Continued)

Table 29b (continued)

Metaheuristic optimization algorithms	Real-world engineering design problem					
	Problem-7	Problem-8	Problem-9	Problem-10	Problem-11	Problem-12
GWO	1.73E-06 △	3,18E-06 △	1.73E-06 △	4,86E-05 △	2,88E-06 △	1.73E-06 △
MFO	2.50E-01 ≈	1,73E-06 △	1.73E-06 △	1,20E-03 △	1,73E-06 △	3.88E-06 △
WOA	1.73E-06 △	1,73E-06 △	1.73E-06 △	7,51E-05 △	1,73E-06 △	1.73E-06 △
SMA	1.73E-06 △	1,92E-06 △	1.73E-06 △	8,97E-02 △	– –	1.73E-06 △
HHO	1.73E-06 △	1,73E-06 △	1.73E-06 △	2,60E-06 △	1,73E-06 △	1.73E-06 △
COA	1.73E-06 △	1,73E-06 △	1.73E-06 △	2,11E-03 △	1,73E-06 △	1.06E-04 △
COOT	1.82E-05 △	1,73E-06 △	5.61E-06 △	2,61E-04 △	1,73E-06 △	1.73E-06 △
MVO	1.73E-06 △	1,73E-06 △	1.73E-06 △	9,32E-06 △	3,52E-06 △	1.73E-06 △
AOA	1.73E-06 △	1,73E-06 △	1.73E-06 △	1,73E-06 △	1,73E-06 △	1.73E-06 △
AO	1.73E-06 △	1,73E-06 △	1.73E-06 △	1,20E-01 ≈	1,73E-06 △	1.73E-06 △
SCA	1.73E-06 △	1,73E-06 △	1.73E-06 △	3,59E-04	1,73E-06 △	1.73E-06 △
SAO	1.73E-06 △	1,73E-06 △	1.73E-06 △	1,73E-06 △	1,73E-06 △	1.73E-06 △
SOA	1.73E-06 △	1,73E-06 △	1.73E-06 △	– –	4,86E-05 △	1.73E-06 △
POA	9.38E-02 ≈	4,53E-04 △	1.00E+00 ≈	2,18E-02 △	4,45E-05 △	1.73E-06 △
CA	– –	2,60E-05 △	– –	6,04E-03 △	2,37E-05 △	1.92E-06 △

As a result, in the study conducted, EO and POA in 7 different problems, CA in 6 different problems, SOA and MFO in 2 different problems, and GWO, HHO, COOT, and SMA in 1 different problem showed the most successful results, or the method showing the most successful results could not provide a significant superiority to these methods.

6 Conclusion

The efficient resolution of real-world engineering design optimization issues is widely acknowledged as a significant difficulty for any new metaheuristic algorithm presented to the market.

Moreover, these issues include several goals and diverse variables, including integers, continuous values, and discrete elements. Additionally, they involve a range of nonlinear restrictions related to kinematic conditions, performance parameters, operational situations, and manufacturing specifications, among others. The TSO, EO, GWO, MFO, WOA, SMA, HHO, COA, COOT, MVO, AOA, AO, SCA, SAO, SOA, POA, and CA algorithms are used to address design optimization of twelve real-world engineering issues. Accordingly, their performances are compared considering the quality of solution, robustness, and convergence speed of the solutions obtained by various approaches. The outcomes reveal that EO and POA produce better optimized results against other available techniques. However, the results of statistical comparisons show that EO and POA achieve more competitive and better performance outcomes among most of the constraint problems investigated. In addition, as a consequence of the statistical examination, it was reported that the CA approach is at a level that can compete with these two methods.

This study discusses the most important subjects in engineering and artificial intelligence disciplines. Future research confidently relies on this review to investigate metaheuristic optimization approaches and engineering design challenges in greater depth in the near future. Moreover, by examining the studies in question, the researchers can more easily identify a beginning point for future researchers.

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Availability of Data and Materials: All data generated and analyzed throughout the research process are given in the published article.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

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