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ARTICLE

Modified DS *np* Chart Using Generalized Multiple Dependent State Sampling under Time Truncated Life Test

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ABSTRACT

This study presents the design of a modified attributed control chart based on a double sampling (DS) *np* chart applied in combination with generalized multiple dependent state (GMDS) sampling to monitor the mean life of the product based on the time truncated life test employing the Weibull distribution. The control chart developed supports the examination of the mean lifespan variation for a particular product in the process of manufacturing. Three control limit levels are used: the warning control limit, inner control limit, and outer control limit. Together, they enhance the capability for variation detection. A genetic algorithm can be used for optimization during the in-control process, whereby the optimal parameters can be established for the proposed control chart. The control chart performance is assessed using the average run length, while the influence of the model parameters upon the control chart solution is assessed via sensitivity analysis based on an orthogonal experimental design with multiple linear regression. A comparative study was conducted based on the out-of-control average run length, in which the developed control chart offered greater sensitivity in the detection of process shifts while making use of smaller samples on average than is the case for existing control charts. Finally, to exhibit the utility of the developed control chart, this paper presents its application using simulated data with parameters drawn from the real set of data.

KEYWORDS

Modified DS *np* chart; generalized multiple dependent state sampling; time truncated life test; Weibull distribution; average run length; average sample size

1 Introduction

A control chart is a statistical analysis tool used to monitor processes through time. It can also identify changes or trends that could indicate a potential problem. Control charts are used to control quality in production to ensure consistency and help identify areas for improvement. The concept of statistical process control (SPC) was introduced by Walter A. Shewhart during the 1920s. One of his key contributions to SPC was the development of the control chart, which is a tool used to monitor and control a process over time. Shewhart's control chart revolutionized the field of quality control by providing a way to monitor processes in real-time and make data-driven decisions to improve quality according to Montgomery [1]. Presently, control charts have many applications in manufacturing and are also used in healthcare, finance, and other fields. They aim to monitor and control processes and ensure consistent quality over time. The two control chart types are variable and attribute control charts. The most important difference between the two control charts is the type of data used to monitor them. A variable control chart serves to monitor continuous or quantitative data that are measurable using a numerical scale, such as weight, length, temperature, or time. On the other hand, the number of defects or the proportion or percentage of defects in a sample of a process can be monitored using an attribute control chart.

The *np* control chart finds widespread use in industry because it offers a simple and effective way to monitor the stability of a process by tracking the number of non-conforming items in a sample. However, it is known that the standard *np* charts are not effective at detecting process shifts when the proportion of nonconforming items (p) is moderate or small. Therefore, some researchers have focused on improving the efficiency of the *np* chart to detect process shifts through various methods such as that of Gan [2], who proposed an optimized design for CUSUM *np* charts. Gan [3] developed the concept of the modified exponentially weighted moving average (EWMA) chart together with the *np* chart. Epprecht et al. [4] studied the properties of the *np* chart in cases where sample sizes varied between small and large. Luo et al. [5] designed optimal variable sample sizes and variable sampling intervals *np* charts in a steady-state mode. Double sampling (DS) was first presented by Croasdale [6], who adopted the idea from the acceptance sampling plan and used it to apply the technique to the $\overline{X}$ chart. Thereafter, several studies were conducted on double sampling with various control charts or methods [7–10]. Rodrigues et al. [11] first proposed a DS *np* chart generated by a combination of double sampling and Shewhart *np* control charts. According to these authors, the DS *np* chart performs better than the standard *np* chart based on average run length (ARL). As a result, the average sample size (ASS) is also decreased without affecting the ARL performance. Chong et al. [12] integrated the concept of the DS *np* chart from Rodrigues et al. [11] and the conforming run length (CRL) chart. A new control chart called a synthetic DS *np* chart is suggested to detect shifts in the proportion of nonconforming items p. Zhou et al. [13] combined the methods of DS and variable sampling intervals (VSI) to *np* charts based on multiple dependent state sampling (MDS). The proposed DS *np* chart offers enhanced performance in terms of a reduced time to signal in out-of-control processes and a decrease in expected cost per unit of time. A new method for designing the DS *np* chart with approximated process parameters was proposed by Lee et al. [14]. The results show that the approach allows for reducing the variation in average run length values. As a result, it is known that this method reduces the variation in average run length.

There are also different techniques in sampling plans proposed by many researchers to improve processes to be more efficient. One popular acceptance sampling technique is MDS sampling proposed by Wortham et al. [15]. Since the acceptance or rejection of current lots depends on previous and current lots, the MDS sampling plan is intended for a continuous production process whereby lots are sent for serial inspection, which reduces the sample size. Several researchers have adopted MDS sampling plans to develop a more efficient acceptance sampling plan [16–19]. Many researchers created designs to apply MDS sampling in the area of control charts, such as Aslam et al. [20] who provided the $\overline{X}$ chart using MDS sampling based on a double control limit. Aslam et al. [21] also designed a t-chart for exponential distributions using MDS sampling. Meanwhile, an *np* chart using MDS sampling was suggested by Aslam et al. [22]. They showed that the proposed control chart outperformed the existing *np* control chart in terms of performance. A new control chart for the gamma distribution using MDS sampling was proposed by Aslam et al. [23]. An adaptive control chart was created by Khan et al. [24]

for monitoring the mean using EWMA statistics under MDS sampling. Aslam et al. [25] created a novel t-chart using generalized multiple dependent state (GMDS) sampling and the presumption that the time between events followed an exponential distribution. Raza et al. [26] constructed a new control chart for monitoring multivariate Poisson count data under GMDS sampling. Both works claim that GMDS sampling is more flexible and efficient than MDS sampling in designing the control chart. Balamurali et al. [27] created an *np* control chart for considering the mean life of a product, which follows the Pareto distribution of the second kind. An *np* control chart under MDS sampling was designed by Balamurali et al. [28] based on a time-truncated life test. This control chart was economically designed using a variable sampling interval scheme. With a small sample size and low cost, the proposed chart was particularly useful in detecting process shifts. Aslam et al. [29] presented control charts for attribute and variable data using modified MDS sampling. Based on an accelerated life test, Aslam et al. [30] created an *np* chart using modified MDS sampling for monitoring the mean lifetime of the items under a Weibull distribution. According to Woodall et al. [31], the use of MDS sampling combined with control charts is equivalent to using control chart run rules. They proposed methods based on Markov chains for determining the performance of control charts with run rules. Currently, most data come from complex processes or uncertain environments, so some researchers have applied neutrosophic statistics to construct control charts. For example, Aslam et al. [32] proposed the $\overline{X}$ control chart using MDS under neutrosophic statistics. Khan et al. [33] also presented the enhanced $\overline{X}$ control chart using GMDS sampling under neutrosophic statistics. Many products are highly reliable, and for this reason, it is not possible to test the lifetime of the product until it fails. Accordingly, the inspection process requires the design of a control chart under the time truncated life test. As mentioned above, it was found that the work of Balamurali et al. [27], Balamurali et al. [28], and Aslam et al. [29] not only designed the control chart using MDS sampling but also studied the outcomes under time truncated life tests. Recently, references [34–37] designed a control chart under time-truncated life tests for different distributions. From the literature review, the DS *np* chart is more efficient than the existing *np* chart and also reduces the average sample size in the inspection process. In addition, designing control charts using GMDS sampling is more efficient than MDS sampling.

During this study, the modified attributed *np* chart will be designed through a combination of GMDS sampling with the DS *np* chart approach, based upon the time truncated life test where the product lifespan adheres to a Weibull distribution. Genetic algorithm optimization during the in-control process serves to establish the optimal parameters for the developed control chart. The performance of the chart was assessed using the average run length, while sensitivity analysis was investigated using an orthogonal experimental design with multiple linear regression. One objective was to determine the influence of the model parameters on the solution delivered by the developed control chart. Comparisons between the developed control chart and the existing control charts could be drawn using the out-of-control average run length. Simulated data drawn from the parameters of the real set of data are used to present an example of the developed control chart.

2 Materials and Methods

2.1 Weibull Distribution

The Weibull distribution is often employed in statistical quality control studies [16–19,28]. Because of its flexibility and closed shape, the Weibull distribution serves as the most popular choice to model the data lifespan.

Table 1 shows that researchers applied the Weibull distribution to the attributed control chart to monitor the number of failures or mean life of products under a time truncated life test when the lifetime of the product follows a Weibull distribution. For the variable control chart, the Weibull distribution is often used to monitor variation in the manufacturing process because of the flexible selection of shape and scale parameters. Therefore, this research designs the modified DS *np* chart using GMDS sampling to monitor the mean life of the product based on the time truncated life test under the Weibull distribution. Let t represent the product lifespan under the Weibull distribution, so the cumulative distribution function can be expressed as follows:

$$F\left(t, \lambda, \delta\right) = 1 - e^{\left(-\left(\frac{t}{\lambda}\right)\right)^{\delta}}, \; t \geq 0, \lambda > 0, \delta > 0, \tag{1}$$

where λ is a scale parameter that is not known and δ is the known shape parameter. According to the Weibull distribution, the average product lifespan is as follows:

$$\mu = \left(\frac{\lambda}{\delta}\right) \Gamma \left(\frac{1}{\delta}\right) \tag{2}$$

Table 1: A literature survey of the control charts on the Weibull distribution

Authors/Year	Types of chart	Topics
Aslam et al. [34]/2015	*np* chart	Proposed control chart when the lifetime of the product follows a Weibull distribution based on the number of failure items in a truncated life test.
Akhundjanov et al. [38]/2015	Moving range EWMA chart	Presented a control chart for monitoring shifts in the Weibull shape parameters.
Faraz et al. [39]/2015	$\overline{Z}$ and S^2 chart	Proposed control charts for monitoring individual or joint shifts in the scale and shape parameters of a Weibull distributed process.
Aslam [40]/2016	Mixed EWMA-CUSUM chart	Proposed a mixed control chart combining CUSUM and EWMA statistics by assuming that the quality characteristic of interest follows a Weibull distribution.
Aslam et al. [41]/2017	*np* chart	Presented a control chart using accelerated hybrid censoring logic for the monitoring of defective items whose lifetime follows a Weibull distribution.

(Continued)

Table 1 (continued)

Authors/Year	Types of chart	Topics
Arif et al. [42]/2017	EWMA *np* chart	Designed the attribute control chart based on the number of failures under a time truncated life test when the lifetime of the product follows a Weibull distribution.
Balamurali et al. [28]/2019	*np* chart using MDS sampling	Designed a control chart using MDS sampling for monitoring the mean life of the products when the lifetime follows a Weibull distribution based on a time truncated life test.
Huwang et al. [43]/2020	new EWMA chart	Developed an EWMA chart for monitoring the shape parameters of a Weibull process.
Aslam et al. [44]/2021	*np* chart using modified MDS sampling	Designed a control chart for monitoring the mean lifetime of the products following a Weibull distribution under an accelerated life test.
Khan et al. [45]/2023	Moving average EWMA chart	Presented a control chart to monitor the number of defective counts before the specified time which follows a Weibull distribution.

Let $\Gamma(\cdot)$ be the complete gamma function. In Eq. (3), under the Weibull distribution, the probability of a given item failing before the experiment time t_0 is shown:

$$p = 1 - e^{\left(-\left(\frac{t_0}{\lambda}\right)\right)^{\delta}}. \tag{3}$$

The value of t_0 can be represented as $t_0 = a\mu_0$ for an experiment termination ratio of a using the specific mean lifetime μ_0. Consequently, the following can be used to rewrite Eq. (4):

$$p = 1 - e^{-a^{\delta}\left(\frac{\mu_0}{\mu}\right)^{\delta}\left(\frac{1}{\delta}\Gamma\left(\frac{1}{\delta}\right)\right)^{\delta}}. \tag{4}$$

If the process mean is the same as the target mean, or $\mu = \mu_0$, the process is said to be in-control. Eq. (5) thus becomes:

$$p_0 = 1 - e^{-a^{\delta}\left(\frac{1}{\delta}\Gamma\left(\frac{1}{\delta}\right)\right)^{\delta}}, \tag{5}$$

where p_0 represents the probability of a given item failing. If the process means changes from the target mean, this indicates that the process is out-of-control, shown as $\mu = f\mu_0$ where f is a shift constant. Next, p_1 represents the probability of a given item failing when a process is out-of-control, which has the following equation:

$$p_1 = 1 - e^{-a^\delta \left(\frac{1}{f}\right)^\delta \left(\frac{1}{\delta}\Gamma\left(\frac{1}{\delta}\right)\right)^\delta}, \tag{6}$$

From Eq. (6), we obtain the probability of a given item failing when there is a shift in process in terms of the specified values of δ, a and f under the Weibull distribution.

2.2 Design of the Modified DS np Chart Using GMDS Sampling

The following section presents the modified DS *np* control chart using GMDS sampling to monitor the mean life of the product created on the basis of time truncated life tests under the Weibull distribution. The developed control chart includes a pair of inspection stages. In Stage 1, two warning control limits are indicated by LWL and UWL, while the inner control limit is denoted as UCL_1. In Stage 2, there is a single outer control limit indicated as UCL_2. If a sample point for Stage 1 lies between LWL and UWL, the process can be considered in-control, whereas sample points found beyond the inner control limit will be indicative of a process that is out-of-control. If sample points are located between UWL and UCL_1, it is necessary to take a second sample from the same subgroup, whereupon the process can be considered in-control if these sample points fall within the outer control limit of Stage 2, while k of m previous subgroups were found to be in-control for Stage 1. If this is not the case, the process can be considered out-of-control. The operational process for this modified DS *np* chart with GMDS sampling based on the time-truncated life test under the Weibull distribution can be seen as follows:

1. Specify the limits indicated as LWL, UWL, UCL_1 and UCL_2.
2. The initial sample, of size n_1 should be taken for the production process from each subgroup. The lifespan of the item is tested, where t_0 is the experiment time, and the nonconforming items (d_1) prior to t_0 are counted.
3. In Stage 1 (see Fig. 1)
   3.1 If $LWL \le d_1 \le UWL$, the process can be considered in-control, then return to Step 2.
   3.2 If $d_1 > UCL_1$, the process can be considered out-of-control.
   3.3 If $UWL < d_1 \le UCL_1$, it is necessary to draw a second sample of size n_2. The nonconforming items (d_2) in the second sample prior to t_0 must be counted. Then go to Stage 2.
4. In Stage 2 (see Fig. 1), if $d_1 + d_2 \le UCL_2$ the process can be considered in-control where k of m previous subgroups were found to be in-control for Stage 1 ($LWL \le d_1 \le UWL$), then return to Step 2. If this is not the case, it can be determined that the process is out-of-control.

Let d_1 and d_2 be random variables with binomial distributions with parameters n_1, n_2 and p_0, where p_0 is the probability of a given item failing before t_0. We can summarize the above steps in a flow chart, as presented in Fig. 2. The modified DS *np* control chart using GMDS sampling is studied following assumptions and limitations:

1. The developed control chart monitors the mean life of products under a time-truncated life test when the lifetime of the product follows a Weibull distribution.

2. The developed control chart is constructed based on a double sampling *np* chart together with GMDS sampling where p_0 follows the Weibull distribution.
3. At the start of the process, the process is assumed to fit the in-control region, that is $\mu = \mu_0$.

The process mean may be shifted to the out-of-control region, that is $\mu = f\mu_0$.

4. In this study, the genetic algorithm (GA) with the R program is used to find the optimal parameters.

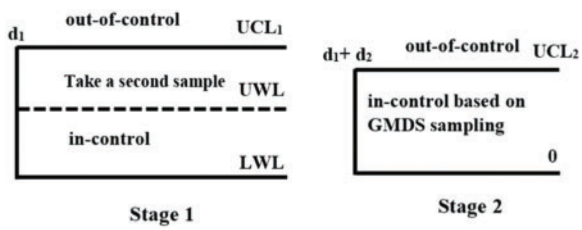

Figure 1: The modified DS *np* chart based on GMDS sampling procedure

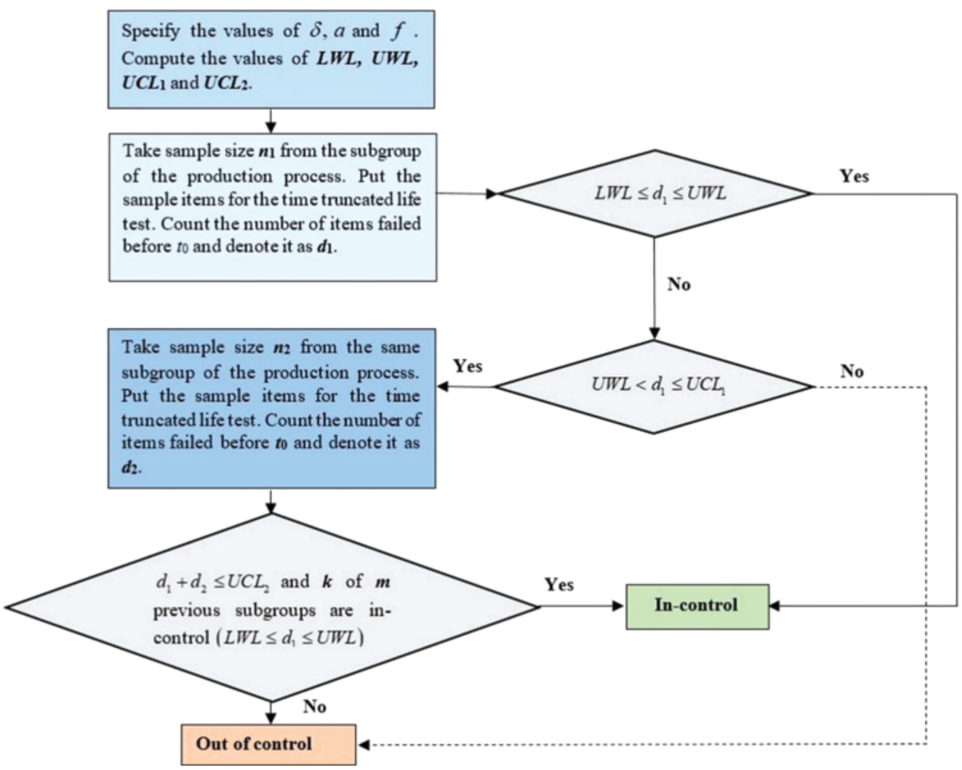

Figure 2: Flowchart of the inspection procedure for the modified DS *np* chart based on GMDS sampling

Therefore, the control limits for Stages 1 and 2 are shown as follows:

Stage 1

$$UWL = n_1p_0 + w\sqrt{n_1p_0(1-p_0)} \tag{7}$$

$$LWL = \max\left(0, n_1p_0 - w\sqrt{n_1p_0(1-p_0)}\right) \tag{8}$$

$$UCL_1 = n_1p_0 + L_1\sqrt{n_1p_0(1-p_0)} \tag{9}$$

Stage 2

$$UCL_2 = (n_1+n_2)p_0 + L_2\sqrt{(n_1+n_2)p_0(1-p_0)} \tag{10}$$

where w, L_1 and L_2 are control limit coefficients with $L_1 > w$ and $L_2 > 0$. The developed control chart becomes a DS *np* chart based on MDS sampling when $k = m$ occurs, while it reduces to a DS *np* chart if $k = m = 0$ occurs. Similarly, when $w = L_1 = L_2$ and $k = m = 0$ the developed control chart will reduce to the classical *np* chart.

Based on the developed control chart, the probability that the process will be considered in-control at Stage 1 is indicated by P_{S1} and provided as:

$$P_{S1}(p) = P(LWL \le d_1 \le UWL) = \begin{cases} \displaystyle\sum_{d_1=\lfloor LWL\rfloor+1}^{\lfloor UWL\rfloor}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1} & \text{if } LWL \text{ is not integer} \\ \displaystyle\sum_{d_1=LWL}^{\lfloor UWL\rfloor}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1} & \text{if } LWL \text{ is integer} \end{cases} \tag{11}$$

where $\lfloor . \rfloor$ is the largest integer that is either less than or equal to the argument. The probability that the second sample is taken from the same subgroup and the total number of nonconforming items in the two samples $(d_1 + d_2)$ is below the outer control limit is represented by P_D and expressed as:

$$P_D(p) = P(UWL < d_1 \le UCL_1) \times P(d_1 + d_2 \le UCL_2) = \left(\sum_{d_1=\lfloor UWL\rfloor+1}^{\lfloor UCL_1\rfloor}\binom{n_1}{d_1}p^{d_1}(1-p)^{n_1-d_1}\left(\sum_{d_2=0}^{\lfloor UCL_2\rfloor-d_1}\binom{n_2}{d_2}p^{d_2}(1-p)^{n_2-d_2}\right)\right) \tag{12}$$

The probability declares that the process is in-control at Stage 2 when given that k of m from the previous subgroup must be in-control at Stage 1, denoted by P_{S2} and defined as:

$$P_{S2}(p) = P_D \times \sum_{j=k}^{m}\binom{m}{j}(P_{S1})^j(1-P_{S1})^{m-j} \tag{13}$$

According to the modified DS *np* chart using GMDS sampling, the probability that the process was considered to be in-control is indicated as:

$$P_{in}(p) = P_{S1}(p) + P_{S2}(p) = P(LWL \le d_1 \le UWL) + P(UWL < d_1 \le UCL_1) \times P(d_1 + d_2 \le UCL_2) \times \sum_{j=k}^{m}\binom{m}{j}(P_{S1})^j(1-P_{S1})^{m-j} \tag{14}$$

The probability of declaring that a process is in-control when it is actually in-control $(p = p_0)$ can be shown as follows:

$$P_{in}(p_0) = P_{S1}(p_0) + P_{S2}(p_0) \tag{15}$$

Moreover, the probability of declaring that a process is in-control when it is actually out-of-control $(p = p_1)$ is obtained as follows:

$$P_{in}(p_1) = P_{S1}(p_1) + P_{S2}(p_1) \tag{16}$$

The ARL_0 indicating in-control average run length of the developed control chart is established by:

$$ARL_0 = \frac{1}{1 - P_{in}(p_0)}. \tag{17}$$

The ARL_1 indicating out-of-control average run length of the developed control chart is determined by:

$$ARL_1 = \frac{1}{1 - P_{in}(p_1)}. \tag{18}$$

Additionally, the average sample size (ASS) of the developed control chart for declaring that the process is in-control is provided by:

$$ASS(p_0) = n_1 + n_2 P(UWL < d_1 \leq UCL_1) \tag{19}$$

where $P(UWL < d_1 \leq UCL_1)$ is the probability of taking a second sample.

2.3 Optimal Design of the Modified DS np Chart Using GMDS Sampling Based on the Weibull Distribution

In this section, we used the following optimization problem to obtain the optimal parameters for constructing the modified DS *np* chart using GMDS sampling as follows:

$$\text{Minimize} \quad ASS(p_0) \tag{20}$$

$$\text{Subject to} \quad ARL_0 \geq r_0,\ n_1 < \overline{n}_0 < n_2 \text{ and } n_1 \leq n_2, \quad L_1 > w \text{ and } L_2 > 0, m > k \geq 1.$$

where $\overline{n}_0$ is the pre-determined in-control average sample size. For more details see [14]. The value of ARL was used to assess the performance of the developed control charts. The ASS when the process is in-control was also used to study the performance of the developed control chart. With optimal average sample size, the control charts are more sensitive to detecting process variations. In this study, the genetic algorithm (GA) with the R program is used to find the optimal parameters. The optimal parameters n_1, n_2, k, m, a and control limit coefficients w, L_1, L_2 were determined for the specified values of in-control ARL (r_0), the shape parameter (δ), and $\overline{n}_0$. For this purpose, we consider $r_0 = 200$, 370 and $\delta = 2$, 3, whereas $\overline{n}_0 = 50$, 100. All the computations are carried out in the R program. The developed control chart parameters along with ARL are obtained using the following algorithm:

1. Assign the values of r_0, δ and $\overline{n}_0$.
2. Find out the values of the optimal parameters n_1, n_2, k, m, a and control limit coefficients w, L_1, L_2 that result in minimum ASS for which $ARL_0 \geq r_0$ by running the R programming under Eq. (20).
3. Calculate the ARL_1 in Eq. (18) using the optimal parameters n_1, n_2, k, m, a and control limit coefficients w, L_1, L_2 from the steps above for various values of the shift constant (f).

The pseudocode of the modified DS *np* chart based on GMDS sampling is shown as follows:

Pseudocode of the modified DS *np* chart based on GMDS sampling under time truncated life test

BEGIN
Define the function f with input x
function f(x):
Extract variables from x
n_1 = round(x[1]), n2 = round(x[2]), w = x[3], L1 = x[4], L2 = x[5], a = x[6], m = x[7], k = x[8]
SET delta = 2, shift = 1
Calculate p_0, LWL, UWL, UCL_1, UCL_2, PS1, PD
p_0 = calculate_p_0(a, delta, shift) (Eqs. (5) or (6))
LWL = calculate_LWL(n_1, p_0, w) (Eq. (7)), UWL = calculate_UWL(n_1, p_0, w)) (Eq. (8))
UCL_1 = calculate_UCL_1(n_1, p_0, L_1) (Eq. (9)), UCL_2 = calculate_UCL_2(n_1, n_2, p_0, L_2) (Eq. (10))
PS1 = calculate_PS1(LWL, UWL, n_1, p_0) (Eq. (11))
Pa2upper = calculate_Pa2upper(UWL, UCL_1, n_1, p_0)
Calculate PD and PS2 using nested loops
PD = calculate_PD(UWL, UCL_1, UCL_2, n_1, p_0, n2) (Eq. (12))
PS2 = calculate_PS2(Pa2upper, PD, m, k) (Eq. (13))
Calculate ARL and ASS
ARL = calculate_ARL(PS1, PS2) (Eq. (17)), ASS = calculate_ASS(n_1, n_2, Pa2upper) (Eq. (19))
SET $f_1 = -$ARL, f_2 = ASS
Evaluate constraints
$g_1 = n_1 - n_2$, $g_2 = w - L_1$, $g_3 = 370 - ARL$
Check the feasibility and calculate the fitness value
if g_1 <= 0 and g_2 <= 0 and g_3 <= 0: fitness = f1 else: fitness = −100000
return fitness
end function
Genetic Algorithm (GA) setup and execution
GA = initialize_GA(f, lower_bound, upper_bound, popSize, maxiter)
results = run_GA(GA)
show_summary(results)
END

3 Results

3.1 Numerical Results

The following section presents the assessment of the performance achieved by the modified DS *np* chart with GMDS sampling through the use of ARL and ASS. The optimal parameters for the developed control chart (n_1, n_2, k, m, a) along with the relevant control limit coefficients $(w,\ L_1, L_2)$ can be seen in Tables 2 and 3. In establishing the optimal parameters, it is necessary for ARL_0 in Eq. (20) to approximate as closely as possible to $r_0 = 200$ and 370 for the fixed values of $\overline{n}_0 = 50$, 100, and $\delta = 2, 3$. The calculation of ARL_1 is made for various values of f under optimal parameters whereby f lies in the range of 1.0 to 0.1. The findings determined from Table 2 can be expressed as shown below:

1. When δ and $\overline{n}_0$ are fixed, an increase in r_0 was linked to a decline in ASS.
2. When r_0 and $\overline{n}_0$ are fixed, an increase in δ leads to an increase in ASS.

3. It was found that if δ and r_0 are fixed, the results show that if $\overline{n}_0$ increases, this will result in ASS also increasing.
4. As ASS increases, the developed control chart shows greater efficiency in the detection of process shifts. It is also apparent that ARL_1 declines to a greater extent in relation to the same shift size. These findings concur with the earlier reports of Adeoti et al. [46].
5. The findings also indicate a decrease in the values of ARL_1 in line with a decrease in shift size.

Table 2: ARL of the modified DS *np* chart using GMDS sampling under $r_0 = 200$, 370, and $\overline{n}_0 = 50$, 100

	$\overline{n}_0 = 50$			
	$r_0 = 200$		$r_0 = 370$	
	$\delta = 2$	$\delta = 3$	$\delta = 2$	$\delta = 3$
f	$n_1 = 9$, $n_2 = 60$ $w = 2.5439$ $L_1 = 4.4555$ $L_2 = 1.6637$ $k = 3$, $m = 6$ $a = 0.8263$, $ASS = 9.30$	$n_1 = 42$, $n_2 = 55$ $w = 2.8678$ $L_1 = 3.7793$ $L_2 = 3.6392$ $k = 6$, $m = 7$ $a = 0.9723$, $ASS = 42.10$	$n_1 = 8$, $n_2 = 57$ $w = 2.0321$ $L_1 = 2.5759$ $L_2 = 4.7152$ $k = 2$, $m = 4$ $a = 0.9093$, $ASS = 8.15$	$n_1 = 23$, $n_2 = 59$ $w = 3.0320$ $L_1 = 4.2571$ $L_2 = 3.4771$ $k = 5$, $m = 6$ $a = 0.9285$, $ASS = 23.04$
1.0	200.64	200.06	370.05	370.56
0.9	192.30	188.11	287.15	279.50
0.8	147.12	139.63	193.50	183.59
0.7	117.64	77.02	174.84	163.80
0.6	104.70	66.93	66.80	47.31
0.5	68.77	47.24	57.53	41.77
0.4	26.67	1.00	7.69	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00
	$\overline{n}_0 = 100$			
	$r_0 = 200$		$r_0 = 370$	
	$\delta = 2$	$\delta = 3$	$\delta = 2$	$\delta = 3$
f	$n_1 = 41$, $n_2 = 145$ $w = 2.9244$ $L_1 = 3.8604$ $L_2 = 2.1466$ $k = 5$, $m = 6$ $a = 0.9200$, $ASS = 41.17$	$n_1 = 80$, $n_2 = 118$ $w = 2.8101$ $L_1 = 4.5778$ $L_2 = 1.5150$ $k = 3$, $m = 5$ $a = 0.9346$, $ASS = 80.18$	$n_1 = 22$, $n_2 = 129$ $w = 3.0509$ $L_1 = 3.8044$ $L_2 = 3.2938$ $k = 4$, $m = 5$ $a = 0.9368$, $ASS = 22.05$	$n_1 = 55$, $n_2 = 117$ $w = 1.3775$ $L_1 = 2.9970$ $L_2 = 4.1485$ $k = 3$, $m = 4$ $a = 0.9483$, $ASS = 55.11$
1.0	200.99	200.00	370.60	370.65
0.9	189.59	182.50	229.10	366.45
0.8	162.19	145.83	223.50	279.75
0.7	150.78	120.10	191.75	269.52

(Continued)

Table 2 (continued)

0.6	119.16	66.54	97.44	190.36
0.5	79.90	24.00	91.63	14.63
0.4	37.82	1.00	28.55	1.00
0.3	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00

Table 3: ARL of the modified DS *np* chart using GMDS sampling under $\overline{n}_0 = 50$, $r_0 = 200$, 370 and $m = 4$

$r_0 = 200$						
	$\delta = 2$			$\delta = 3$		
	$k = 2$	$k = 3$	$k = m = 4$	$k = 2$	$k = 3$	$k = m = 4$
f	$n_1 = 12$, $n_2 = 86$ $w = 2.8402$ $L_1 = 4.9713$ $L_2 = 2.2273$ $a = 0.9105$, $ASS = 12.02$	$n_1 = 12$, $n_2 = 95$ $w = 2.7647$ $L_1 = 3.5293$ $L_2 = 2.9607$ $a = 0.9109$ $ASS = 12.01$	$n_1 = 14$, $n_2 = 92$ $w = 2.6747$ $L_1 = 3.7245$ $L_2 = 2.8176$ $a = 0.8792$ $ASS = 14.22$	$n_1 = 43$, $n_2 = 80$ $w = 2.8452$ $L_1 = 4.3429$ $L_2 = 1.3644$ $a = 0.8840$ $ASS = 43.10$	$n_1 = 34$, $n_2 = 94$ $w = 2.9069$ $L_1 = 4.5738$ $L_2 = 4.4251$ $a = 0.9039$ $ASS = 34.29$	$n_1 = 25$, $n_2 = 91$ $w = 2.7602$ $L_1 = 3.1215$ $L_2 = 2.1994$ $a = 0.8338$ $ASS = 25.13$
1.0	200.00	201.03	200.00	200.00	200.51	200.11
0.9	155.54	120.62	187.09	182.78	177.29	189.15
0.8	139.24	116.09	181.36	176.15	175.75	186.23
0.7	63.16	51.22	155.43	144.41	127.21	162.33
0.6	37.56	33.37	97.44	97.01	72.46	137.44
0.5	18.63	18.26	47.27	19.37	17.31	80.04
0.4	5.35	5.27	25.98	1.00	1.00	25.78
0.3	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00
$r_0 = 370$						
	$\delta = 2$			$\delta = 3$		
	$k = 2$	$k = 3$	$k = m = 4$	$k = 2$	$k = 3$	$k = m = 4$
f	$n_1 = 12$, $n_2 = 69$ $w = 2.7659$ $L_1 = 3.6984$ $L_2 = 3.0798$ $a = 0.8043$ $ASS = 12.19$	$n_1 = 12$, $n_2 = 72$ $w = 2.7743$ $L_1 = 3.2143$ $L_2 = 1.5167$ $a = 0.8042$ $ASS = 12.17$	$n_1 = 14$, $n_2 = 80$ $w = 2.8009$ $L_1 = 3.1383$ $L_2 = 2.0606$ $a = 0.7862$ $ASS = 14.18$	$n_1 = 26$, $n_2 = 62$ $w = 2.9583$ $L_1 = 4.2936$ $L_2 = 2.7951$ $a = 0.9338$ $ASS = 26.04$	$n_1 = 17$, $n_2 = 63$ $w = 2.8827$ $L_1 = 3.2071$ $L_2 = 3.7257$ $a = 0.9705$ $ASS = 17.04$	$n_1 = 20$, $n_2 = 59$ $w = 3.0649$ $L_1 = 4.4702$ $L_2 = 2.4693$ $a = 0.9158$ $ASS = 20.04$
1.0	370.14	370.32	370.08	370.00	370.00	370.01

(Continued)

Table 3 (continued)

0.9	209.53	129.45	323.20	252.02	222.94	299.01
0.8	174.31	111.80	292.22	153.04	131.05	198.91
0.7	161.51	104.96	165.76	125.55	121.13	145.51
0.6	80.82	80.66	139.42	52.39	20.79	55.90
0.5	72.74	72.59	107.90	38.36	11.22	38.63
0.4	11.46	5.74	36.76	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00

In Table 3, the ARL for the optimal parameters of the modified DS *np* chart using GMDS sampling can be seen in the context of values for $k = 2$, 3, 4 and $m = 4$, where $k = m$, the developed control chart applying GMDS sampling is reduced to MDS sampling. When the shift size is the same, it can be seen that $k = 3$ offers the greatest sensitivity in the detection of process shifts, with $k = 2$ rated slightly lower, while the poorest sensitivity was observed for $k = m = 4$, as may be observed from the ARL_1. It can be observed that the developed control chart with GMDS sampling exhibits the lowest ARL_1 when $k = m - 1$. No other value of k approaches this result, and the finding concurs with the reported results of [25,26]. This confirms the greater sensitivity in detecting process shifts achieved by the developed control chart with GMDS sampling in comparison to the results for the developed control chart with MDS sampling.

3.2 Sensitivity Analysis

In some cases, the parameters presented in Tables 2 and 3 showed no clear trends or strong correlations. The use of sensitivity analysis can, therefore, help to determine the extent of the influence of these parameters upon the solution from the developed control chart. An orthogonal-array experimental design is used along with multiple linear regression to conduct the sensitivity analysis. For independent variables, the model parameters (a, m, δ, r_0, and $\overline{n}_0$) are employed, while the role of the dependent variables is fulfilled by the six test parameters (n_1, n_2, w, L_1, L_2, and k) along with ASS and ARL_0. The sensitivity analysis tests five model parameters (a, m, δ, r_0, and $\overline{n}_0$), for which the corresponding level planning can be observed in Table 4.

Table 4: Planning for five model parameters at different levels

Model parameter	Level 1	Level 2	Model parameter	Level 1	Level 2
δ	2	3	r_0	200	370
a	0.4	0.8	$\overline{n}_0$	50	100
m	3	6			

The data shown in Table 5 represents the results from the use of an L_{32} orthogonal array in the experiment, whereby the five model parameters of the L_{32} array columns are defined. Accordingly, 32 experiments are required in the L_{32} orthogonal array experiment design. For the developed control chart, the best solution was produced in each of the trials via GA optimization, which can be seen in Table 5. Multiple linear regression analysis using Minitab 19.0 software was then performed to assess the impact of the various independent parameters on the control chart. ANOVA analysis and multiple linear regression findings for each of the dependent variables are presented in Tables 6–10, and these data can be employed to test statistical hypotheses. Stepwise regression is used to examine the relationships between all values at a significance level of 0.05. The results in Tables 6a–10a indicate that the values of $\hat{n}_2$, $\hat{w}$, $\hat{k}$, $A\hat{R}L_0$ and $A\hat{S}S$ are significantly influenced by at least one of the independent variables. Table 6b reveals that $\overline{n}_0$ influences $\hat{n}_2$. Where the coefficient of $\overline{n}_0$ is positive, this is indicative of a relationship whereby increasing $\overline{n}_0$ causes $\hat{n}_2$ to increase in turn. Therefore, $\overline{n}_0$ causes $\hat{n}_2$ to change by 90.20%.

Table 5: Assignment of model parameters to the L_{32} orthogonal array and the resulting solution

Trial	Model parameters					Solution							
	a	m	δ	r_0	$\overline{n}_0$	n_1	n_2	w	L_1	L_2	k	ASS	ARL_0
1	0.4	3	2	200	50	24	51	2.8725	4.7133	1.0547	2	24.24	207.40
2	0.4	3	2	200	100	45	130	3.0150	3.3530	1.0102	2	45.43	201.86
3	0.4	3	2	370	50	42	60	3.0074	3.7802	3.1968	2	42.11	375.52
4	0.4	3	2	370	100	42	128	2.9870	3.2854	2.2429	2	42.00	375.08
5	0.4	3	3	200	50	36	60	3.2161	4.3097	2.3222	2	36.23	208.29
6	0.4	3	3	200	100	60	129	3.3091	4.8395	3.8636	2	60.64	200.50
7	0.4	3	3	370	50	7	57	3.3711	4.9495	1.9035	2	7.15	370.50
8	0.4	3	3	370	100	7	113	3.3815	3.6287	2.7589	1	7.00	370.02
9	0.4	6	2	200	50	29	65	2.8335	3.7658	1.2450	5	29.24	205.39
10	0.4	6	2	200	100	45	118	3.0755	3.9633	1.1358	5	45.53	201.95
11	0.4	6	2	370	50	22	65	3.1862	4.2079	1.1871	4	22.14	384.23
12	0.4	6	2	370	100	22	150	3.0091	4.8920	1.2810	3	22.39	384.56
13	0.4	6	3	200	50	36	72	3.0444	4.4546	1.1038	4	36.34	208.10
14	0.4	6	3	200	100	36	111	2.9094	3.3812	2.9678	3	36.00	207.97
15	0.4	6	3	370	50	7	61	3.9567	4.9899	2.2981	5	7.16	370.67
16	0.4	6	3	370	100	79	143	3.3983	4.1036	1.2079	4	79.35	375.52
17	0.8	3	2	200	50	17	66	2.8058	3.3188	1.1593	2	17.12	219.23
18	0.8	3	2	200	100	44	122	2.8691	3.2644	1.0168	2	44.21	205.15
19	0.8	3	2	370	50	18	68	2.9345	3.9863	1.0471	2	18.07	383.49
20	0.8	3	2	370	100	24	126	3.0376	3.3569	1.1195	2	24.18	376.90
21	0.8	3	3	200	50	11	72	2.8742	3.6765	1.0103	2	11.30	205.84
22	0.8	3	3	200	100	11	109	2.8671	3.4283	1.0517	2	11.46	205.89
23	0.8	3	3	370	50	39	89	3.0340	4.2022	1.1309	1	39.18	372.60
24	0.8	3	3	370	100	64	133	3.0573	3.7718	1.0453	2	64.25	372.17
25	0.8	6	2	200	50	17	62	2.8258	3.5193	1.0720	4	17.11	219.23
26	0.8	6	2	200	100	71	116	2.8970	4.9059	1.4949	4	71.35	203.28
27	0.8	6	2	370	50	35	69	2.9698	4.2856	1.1803	4	35.11	372.27
28	0.8	6	2	370	100	86	120	3.0262	3.1524	1.0841	5	86.10	370.53
29	0.8	6	3	200	50	11	60	3.0147	3.7385	1.1034	3	11.29	205.89
30	0.8	6	3	200	100	35	134	2.8122	4.2819	1.5467	4	35.39	203.67

(Continued)

Table 5 (continued)

Trial	Model parameters					Solution							
	a	m	δ	r_0	$\bar{n}_0$	n_1	n_2	w	L_1	L_2	k	ASS	ARL_0
31	0.8	6	3	370	50	5	63	2.9077	4.5687	3.2154	3	5.17	376.70
32	0.8	6	3	370	100	37	118	2.9835	3.5130	1.5543	2	37.26	376.36

Table 6: Minitab output for the second sample size $(\hat{n}_2)$

(a) Table of ANOVA

Source	DF	SS	MS	F-value	*p*-value
Regression	1	28800	28800.0	286.28	0.000
Residual	30	3018	100.6		
Total	31	31818			

(b) Table of regression coefficients

Independent variable	Coefficients	Std. error	T-value	*p*-value	VIF
Constant	5.00	5.61	0.89	0.380	
$\bar{n}_0$	1.20	0.0709	16.92	0.000	1.00

Adjusted $R^2 = 90.20\%$, Durbin–Watson statistic = 2.1740

Table 7: Minitab output for control limit coefficients of warning limit $(\hat{w})$

(a) Table of ANOVA

Source	DF	SS	MS	F-value	*p*-value
Regression	3	0.9427	0.31422	11.46	0.000
Residual	28	0.7678	0.02742		
Total	31	1.7105			

(b) Table of regression coefficients

Independent variable	Coefficients	Std. error	T-value	*p*-value	VIF
Constant	2.639	0.199	13.26	0.000	
a	−0.571	0.146	−3.90	0.001	1.00
δ	0.1741	0.0585	2.97	0.006	1.00
r_0	0.001105	0.000344	3.21	0.003	1.00

Adjusted $R^2 = 50.30\%$, Durbin–Watson Statistic = 1.9418

In Table 7b, it can be observed that $\hat{w}$ is influenced by a, δ and r_0. Where the coefficients of δ and r_0 are positive, this confirms that increasing δ and r_0 leads to an increase in $\hat{w}$, whereas a negative coefficient for a shows that increasing a causes $\hat{w}$ to decrease. It is therefore apparent that the effect of a, δ and r_0 is to change $\hat{w}$ by 50.30%. The data in Table 8b confirm that $\hat{k}$ is affected by m and δ. Where m has a positive coefficient, this indicates that increasing m results in an increase in $\hat{k}$, whereas a negative coefficient for δ shows that increasing δ causes $\hat{k}$ to decline. It is thus possible to conclude that changes in m and δ will influence $\hat{k}$ by 72.98%. In Table 9b, it is revealed that r_0, $\bar{n}_0$ and w have an influence upon $A\hat{R}L_0$. Where the coefficients of r_0 are positive, this confirms that increasing r_0 leads to an increase in $A\hat{R}L_0$, while negative coefficients for $\bar{n}_0$ and w confirm that when $\bar{n}_0$ and w increase there will be a corresponding decline in $A\hat{R}L_0$. From the data, it is apparent that r_0, $\bar{n}_0$ and w have the effect of changing $A\hat{R}L_0$ by 99.72%. Meanwhile, Table 10b reveals that r_0, n_1, n_2 and L_1 can influence $A\hat{S}S$. Where n_1, n_2 and L_1 have positive coefficients, an increase in n_1, n_2 and L_1 causes $A\hat{S}S$ to rise, whereas, in contrast, a negative coefficient for r_0 shows that an increase in r_0 causes a decline in $A\hat{S}S$. The data show that r_0,n_1 , n_2 and L_1 have the effect of changing $A\hat{S}S$ by 100.00%.

Table 8: Minitab output for $\hat{k}$

(a) Table of ANOVA					
Source	DF	SS	MS	F-value	*p*-value
Regression	2	34.000	17.0000	42.87	0.000
Residual	29	11.500	0.3966		
Total	31	45.500			
(b) Table of regression coefficients					
Independent variable	Coefficients	Std. error	T-value	*p*-value	VIF
Constant	1.125	0.659	1.71	0.098	
m	0.6667	0.0742	8.98	0.000	1.00
δ	−0.500	0.223	−2.25	0.033	1.00
Adjusted $R^2 = 72.98\%$, Durbin–Watson statistic = 1.9783					

Table 9: Minitab output for in-control average run length $\left(A\hat{R}L_0\right)$

(a) Table of ANOVA					
Source	DF	SS	MS	F-value	*p*-value
Regression	3	227599	75866	3664.48	0.000
Residual	28	580	21		
Total	31	228179			

(Continued)

Table 9 (continued)

(b) Table of regression coefficients					
Independent variable	Coefficients	Std. error	T-value	p-value	VIF
Constant	38.9	11.1	3.49	0.002	
r_0	1.0019	0.0104	96.73	0.000	1.20
$\overline{n}_0$	−0.0699	0.0322	−2.17	0.038	1.00
w	−9.19	3.81	−2.41	0.023	1.20
Adjusted $R^2 = 99.72\%$, Durbin–Watson statistic = 1.4151					

Table 10: Minitab output for in-control average sample size $\left(A\hat{S}S\right)$

(a) Table of ANOVA					
Source	DF	SS	MS	F-value	p-value
Regression	4	14189	3547.25	328444.48	0.000
Residual	27	0.3	0.01		
Total	31	14189.3			
(b) Table of regression coefficients					
Independent variable	Coefficients	Std. error	T-value	p-value	VIF
Constant	−0.235	0.162	−1.46	0.157	
r_0	−0.00099	0.000219	−4.54	0.000	1.02
n_1	1.00014	0.00106	942.38	0.000	1.47
n_2	0.00249	0.000726	3.43	0.002	1.55
L_1	0.1283	0.0335	3.83	0.001	1.07
Adjusted $R^2 = 100.00\%$, Durbin–Watson statistic = 1.7663					

3.3 Comparative Study

Comparisons are drawn in this section between the performance of the modified DS *np* chart with GMDS sampling and the control charts of Balamurali et al. [28], Aslam et al. [34] and Arif et al. [42]. The work of Balamurali et al. [28] described an attribute *np* chart that uses MDS sampling, while the work of Aslam et al. [34] covered an attribute *np* chart based on single sampling. In addition, the work of Arif et al. [42] presented the design of an attribute EWMA *np* chart. In all cases, the product lifespan followed a Weibull distribution based on the time-truncated life test. Accordingly, comparisons of the control charts' performance can be shown using ARL_S with identical or similar values for specific parameters of the control charts, such as $\delta = 2$, $a = 0.9$, $m = 3$ (for MDS and GMDS sampling), a smoothing constant $\lambda = 0.5$ (for the EWMA *np* chart), $r_0 = 200$ and 370. Four control charts were simulated under the same conditions to compare the ARLs. In the case of the developed control chart proposed in this study, the optimization in Eq. (20) uses $\overline{n}_0 = 30$ and $k = 2$, and pseudocodes of the existing control charts of Balamurali et al. [28], Aslam et al. [34], and Arif et al. [42] are shown as follows:

Pseudocode of *np* chart using MDS sampling under time truncated life test [28]

BEGIN
Define the function f with input x
function f(x):
Extract variables from x
n = round(x[1]), k_1 = x[2], k_2 = x[3]
SET m = 3, delta = 2, shift = 1, a = 0.9
Calculate p_0, LWL, UWL, UCL1, UCL2, Pa1, Pa2upper, Pa2lower, Pa2, Pin
p_0 = calculate_p_0(a, delta, shift)
UCL1 = calculate_UCL(n, p_0, k_1), LCL1 = calculate_LCL(n, p_0, k_1)
UCL2 = calculate_UCL(n, p_0, k_2), LCL2 = calculate_LCL(n, p_0, k_2)
Pa1 = calculate_Pa1(LCL1, UCL1, n, p_0)
Pa2upper = calculate_Pa2upper(UCL1, UCL2, n, p_0)
Pa2lower = calculate_Pa2lower(LCL2, LCL1, n, p_0)
Pa2 = calculate_Pa2(Pa2upper, Pa2lower)
Pin = calculate_ Pin(Pa1, Pa2, m)
Calculate ARL and ASS
ARL = calculate_ARL(Pin), ASS = n
SET f_1 = −ARL, f2 = ASS
Evaluate constraints
g_1 = 370−ARL, g_2 = k1−k2
Check the feasibility and calculate the fitness value
if g_1 <= 0 and g_2 <= 0: fitness = f_1 else: fitness = −100000
return fitness
end function
Genetic Algorithm (GA) setup and execution
GA = initialize_GA(f, lower_bound, upper_bound, popSize, maxiter)
results = run_GA(GA)
show_summary(results)
END

Pseudocode of *np* chart under time truncated life test [34]

BEGIN
Define the function f with input x
function f(x):
Extract variables from x
n = round(x[1]), k = x[2]
SET delta = 2, shift = 1, a = 0.9
Calculate p_0, UCL, LCL, Pa1, Pin, and ASS
p_0 = calculate_ p_0 (a, delta, shift),
UCL = calculate_UCL(n, p_0, k), LCL = calculate_LCL(n, p_0, k)
Pa1 = calculate_Pa1(LCL, UCL, n, p_0), ARL = calculate_ARL(Pa1), ASS = n
SET f_1 = −ARL, f_2 = ASS

(Continued)

(continued)

Evaluate constraints
g = 370−ARL
Check the feasibility and calculate the fitness value
if g <= 0: fitness = f_1 else: fitness = −100000
return fitness
end function
Genetic Algorithm (GA) setup and execution
GA = initialize_GA(f, lower_bound, upper_bound, popSize, maxiter)
results = run_GA(GA)
show_summary(results)
END

Pseudocode of EWMA *np* chart under time truncated life test [42]

BEGIN
Define the function f with input x
function f(x):
Extract variables from x
n = round(x[1]), k = x[2]
SET delta = 2, lamda = 0.5, shift = 1, a = 0.9
Calculate p_0, p_1
p_0 = calculate_ p_0 (a, delta, shift), p_1 = calculate_p_1(a, delta, shift)
Calculate P_out and ARL
P_out = 1−calculate_A(n, p_0, k, lamda, p_1) + calculate_B(n, p_0, k, lamda, p_1)
ARL = calculate_ARL(P_out)
SET f_1 = −ARL, f_2 = n
Evaluate constraints: g = 200−ARL
Check the feasibility and calculate the fitness value
if g <= 0: fitness = f_1 else: fitness = −100000
return fitness
end function
Genetic Algorithm (GA) setup and execution
GA = initialize_GA(f, lower_bound, upper_bound, popSize, maxiter)
results = run_GA(GA)
show_summary(results)
END

Table 11 presents the optimal parameters at $r_0 = 200$ and 370. The results show that the developed control chart exhibits a smaller ARL_1 for every shift size than is the case for those charts presented by [28,34,42]. This confirms that the developed control chart offers greater sensitivity in the detection of small shifts during the process. For instance, when $f = 0.9$, $\delta = 2$ and $r_0 = 200$, the ARL_1 value for the developed control chart shown in Table 10 is just 25.13, whereas it is 28.52 for the chart of [28], 78.75 using the chart of [34] and 54.34 from the chart of [42]. Furthermore, the developed control chart made use of average sample sizes (ASS) of just 7.19 in each of the subgroups, whereas the sample size employed by [28] was 16, for [34] the sample size was 22 and the sample size of [42] was 8. In comparison to the existing control charts, the developed control chart can employ smaller sample sizes for each of the subgroups.

Table 11: Comparison in ARL of the developed control chart and existing charts by [28,34,42]

	$r_0 = 200$				$r_0 = 370$			
	Developed control chart	*np* chart using MDS sampling [28]	*np* chart [34]	EWMA *np* chart [42]	Developed control chart	*np* chart using MDS sampling [28]	*np* chart [34]	EWMA *np* chart [42]
f	$n_1 = 7$, $n_2 = 39$ $w = 2.2670$ $L_1 = 3.4100$ $L_2 = 1.1305$ $ASS = 7.19$	$n = 16$ $k_1 = 1.8880$ $k_2 = 3.8587$	$n = 22$ $k = 2.6086$	$n = 8$ $k = 2.8081$	$n_1 = 7$, $n_2 = 36$ $w = 2.1879$ $L_1 = 4.7016$ $L_2 = 2.1475$ $ASS = 7.10$	$n = 21$ $k_1 = 1.9856$ $k_2 = 2.6616$	$n = 26$ $k = 2.8321$	$n = 9$ $k = 2.9998$
1	205.04	200.51	201.64	200.73	370.09	370.75	377.84	370.19
0.9	25.13	28.52	78.75	54.34	14.25	48.61	79.85	79.37
0.8	3.25	4.58	12.11	10.10	3.12	5.72	13.78	12.10
0.7	1.25	2.05	3.08	2.69	1.13	1.85	3.12	2.82
0.6	1.00	1.58	1.94	1.21	1.00	1.32	1.56	1.20
0.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

4 The Application of the Developed Control Chart Using Real Data

The following section presents the implementation of a modified DS *np* chart using GMDS sampling in the context of real data concerning the times to failure for 20 aluminum reduction cells where the units are thousands of days [47].

0.468	0.725	0.838	0.853	0.965	1.554	1.658	1.764	1.776	1.139
1.990	1.142	2.010	1.304	1.317	2.224	2.279	1.427	2.244	2.286

First of all, the dataset must be examined to determine whether a Weibull distribution is applicable. To check the goodness of fit, the Kolmogorov–Smirnov (K-S) test was used, while the unknown parameters were estimated using the maximum likelihood method. The result for the K-S test is 0.11212, giving a p-value of 0.9391. It can thus be concluded that the data follow the Weibull distribution. Meanwhile, the shape parameter $\hat{\delta} = 3.0489 \approx 3$, while the scale parameter $\hat{\lambda} = 1.6813$. These values were estimated using the maximum likelihood estimate, leading to the value of $\hat{\mu} = 1.50$. For this research, it can be assumed that $\delta = 3$, $r_0 = 370$, $\overline{n}_0 = 50$ and $\mu_0 = 150$. The optimal parameters $(n_1, n_2, w, L_1, L_2, k, m) = (23,\ 59,\ 3.0320,\ 4.2571,\ 3.4771,\ 5,\ 6)$ are shown in Table 2, while $a = 0.9285$ and $ASS = 23.04$, resulting in the value of p_0 from Eq. (5) $= 0.4345$ while $t_0 = 1.3928$. Calculation of the control limits for the developed control chart with the optimal values resulted in $LWL = 2.79$, $UWL = 17.20$, $UCL_1 = 20.11$, and $UCL_2 = 51.23$. Generation of the initial 20 subgroups used the in-control process (based on a binomial distribution in which $p_0 = 0.4345$), whereupon the generation of the subsequent 20 subgroups relied upon an out-of-control process making use of the shifted size $f = 0.9$. Finally, the 20 subgroups can be generated based on a binomial distribution in which $p_1 = 0.5425$. In Stage 1, a binomial distribution with the parameters $(n_1, p_0) = (23,\ 0.4345)$ is employed for the in-control process while $(n_1, p_1) = (23,\ 0.5425)$ is used for the out-of-control process to provide a simulation for the number of nonconforming items d_1. Meanwhile, in Stage 2, a binomial distribution with the parameters $(n_2, p_0) = (59,\ 0.4345)$ is employed for the in-control process while $(n_2, p_1) = (59,\ 0.5425)$ is used for the out-of-control process to provide a simulation for the number of nonconforming items d_2. Table 12 presents the simulated data. The values of d_1 and d_2 are plotted on the developed control chart using GMDS sampling in Fig. 3. It can be seen from Fig. 3 that, at Stage 1, the first 20 subgroups are the in-control process as all the points lie within the warning control limits LWL and UWL. For the next 20 subgroups, the first example of size 23 is taken. Observe that the 35$^{\text{th}}$ subgroup has nonconforming items $d_1 = 19$ whereas the 39$^{\text{th}}$ subgroup has nonconforming items $d_1 = 18$. As $d_1 = 18$ and 19 falls in the interval $UWL < d_1 \le UCL_1$ then we go to Stage 2 and take the second sample of size $n_2 = 59$. For the 35$^{\text{th}}$ subgroup, the number of nonconforming $d_2 = 31$ is observed, and $d_1 + d_2 = 50 < UCL_2$ with the result that the process is considered in-control because it meets the requirement that $k = 5$ of the previous $m = 6$ subgroups are in-control processes. Moreover, in the 39$^{\text{th}}$ subgroup, it is shown that $d_1 = 18$, which falls in the interval $UWL < d_1 \le UCL_1$. At Stage 2, we take a second sample of size $n_2 = 59$ with $d_2 = 27$ and obtain $d_1 + d_2 = 45 < UCL_2$. Then this subgroup result reveals that the process is in-control because it is shown that 5 of the previous 6 subgroups are in-control processes. Fig. 3 clearly shows that the developed control chart declares the process to be in-control.

Table 12: Simulated dataset for the DS *np* chart using GMDS sampling at a fixed $k = 5$, $m = 6$, and $r_0 = 370$

Sub group	First sample ($n_1 = 23$) d_1	Sub group	First sample ($n_1 = 23$) d_1	Sub group	First sample ($n_1 = 23$) d_1	Sub group	First sample ($n_1 = 23$) d_1	Second sample ($n_2 = 59$) d_2	$d_1 + d_2$
1	10	11	12	21	12	31	11		
2	7	12	11	22	10	32	14		
3	10	13	4	23	11	33	14		
4	12	14	10	24	12	34	10		
5	16	15	14	25	13	35	19	31	50
6	11	16	16	26	10	36	15		
7	12	17	6	27	15	37	11		
8	7	18	10	28	11	38	12		
9	15	19	11	29	11	39	18	27	45
10	11	20	7	30	6	40	13		

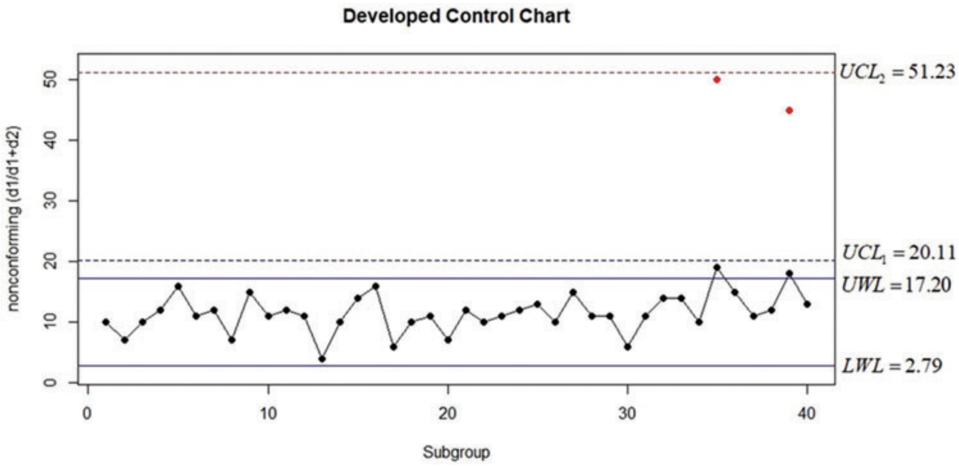

Figure 3: The modified DS *np* chart using GMDS sampling for simulated data

5 Conclusions

The design of a novel attributed control chart is achieved through the application of the DS *np* chart combined with GMDS sampling based on a time-truncated life test under the Weibull distribution. The optimal parameters (n_1, n_2, k, m, a) and control limit coefficients (w, L_1, L_2) were determined using a genetic algorithm with the R program when r_0, δ and $\overline{n}_0$ were fixed. The developed control chart performance was evaluated using the average run length, while the sensitivity analysis was based on an orthogonal experimental design with multiple linear regression. These techniques sought to investigate the influence of the model parameters upon the solution of the developed control chart. The findings revealed that an increased value for $\overline{n}_0$ served to increase $\hat{n}_2$. In the case of $\hat{w}$, higher values for δ and r_0 tend to increase $\hat{w}$ whereas a higher value for a leads to a decline in $\hat{w}$. For $\hat{k}$, increases in m will increase $\hat{k}$, while in contrast, higher values for δ cause $\hat{k}$ to decrease. Furthermore, increased values for r_0 will extend $A\hat{R}L_0$, whereas higher values for $\overline{n}_0$ and w lead to a decline in $A\hat{R}L_0$. It could also be seen that when the values of n_1, n_2 and L_1 rose, this led to a rise in $A\hat{S}S$, whereas an increase in r_0 caused reduced $A\hat{S}S$. The comparative study revealed that the developed chart offered greater sensitivity in terms of the detection of small process shifts than was the case for the previously existing control charts. Furthermore, the developed control chart appears superior in the detection of process shifts when ASS rises and the other parameters remain fixed. The implementation of the developed control chart in this study was based on simulated data which employed parameters taken from the real set of data, thus exhibiting a true measure of the chart's utility. In conclusion, it can be stated that the developed control chart, which is based on the time truncated lifetime test under a Weibull distribution, offers greater sensitivity in detecting small process shifts as well as utilizing smaller sample sizes than would be necessary with the existing control charts. For future research, a steady-state ARL will be used to evaluate the performance of the proposed control chart based on the Markov chain method for a more accurate assessment. Moreover, neutrosophic statistics will be applied to the proposed control chart when the data comes from a complex process or an uncertain environment.

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