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ARTICLE

Fractional Order Modeling of Predicting COVID-19 with Isolation and Vaccination Strategies in Morocco

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ABSTRACT

In this work, we present a model that uses the fractional order Caputo derivative for the novel Coronavirus disease 2019 (COVID-19) with different hospitalization strategies for severe and mild cases and incorporate an awareness program. We generalize the SEIR model of the spread of COVID-19 with a private focus on the transmissibility of people who are aware of the disease and follow preventative health measures and people who are ignorant of the disease and do not follow preventive health measures. Moreover, individuals with severe, mild symptoms and asymptomatically infected are also considered. The basic reproduction number ($\mathscr{R}_0$) and local stability of the disease-free equilibrium (DFE) in terms of $\mathscr{R}_0$ are investigated. Also, the uniqueness and existence of the solution are studied. Numerical simulations are performed by using some real values of parameters. Furthermore, the immunization of a sample of aware susceptible individuals in the proposed model to forecast the effect of the vaccination is also considered. Also, an investigation of the effect of public awareness on transmission dynamics is one of our aim in this work. Finally, a prediction about the evolution of COVID-19 in 1000 days is given. For the qualitative theory of the existence of a solution, we use some tools of nonlinear analysis, including Lipschitz criteria. Also, for the numerical interpretation, we use the Adams-Moulton-Bashforth procedure. All the numerical results are presented graphically.

KEYWORDS

Fractional calculus; caputo derivatives; COVID-19; reproduction number; future prediction

1 Introduction

COVID-19 is an infectious viral disease brought about by a virus known as severe acute syndrome Coronavirus [1]. This virus was first reported at the end of 2019 in Wuhan, China. Then, it was reported in other places in China surprisingly [2]. In three months, the disease was transmitted in many countries of the world. Therefore, in April 2020, the World Health Organization (WHO) announced it as an outbreak. Its symptoms involve breathing trouble, exhaustion, fever, dry hack, sleepiness, conjunctivitis, chest torment, loss of discourse, loose bowels, and painful throat. Furthermore, in serious cases, pneumonia, multiorgan letdown, intense respiratory pain condition, septic shock, abnormal heart rhythm, myocarditis, blood clumps, cardiovascular breakdown, encephalitis, stroke, and Guillain Barré disorder, reciprocal lung entrance have been reported [3,4]. Additionally, a few patients might experience the ill effects of looseness of the bowels, loss of craving, taste, or smell, with no indications of breathing problem [5–7].

Investigating infectious diseases through various procedures is an attractive area of research in recent times. One of the important areas in this regard is devoted to the mathematical modeling of infectious diseases. Mathematical modeling is a powerful tool to study infectious diseases for analysis of the dynamic of diseases and backing control systems [8]. Plenty of research work has been published by researchers. The mentioned work is devoted to investigating disease transmission dynamics and their control. In the same fashion to understand the dynamics of COVID-19 and the tracking down—of the reasonable result of the outbreak is helpful for public health initiatives and consciousness programs, for which significant work has been published, and we refer to a few such as [9–14].

Fractional calculus can more precisely describe natural phenomena as compared to ordinary calculus, where derivatives and integrations have integer-orders. Numerous scientists have given more attention to investigating the dynamics of fractional-order models regarding the COVID-19 pandemic [15–38]. There are many papers showing the effectiveness of fractional calculus (see [39–45]). Further, the dynamics of a stochastic epidemic model has been studied, for which we refer [46,47] and mathematical modeling of COVID-19 pandemic using the Caputo-Fabrizio fractional derivative (see [48]). Also, a model of COVID-19 disease has been investigated under the stochastic concept in [49].

In this research work, an SEIR-type model that concentrates on the transmission dynamics of the COVID-19 outbreak are considered. Here, with a special spotlight on the transmissibility of people with mild, severe, and without side effects like the existence of people who tested positive for sharp, mild, or asymptomatic manifestations, and separating infectious compartment into two fundamental compartments of hospitalized people with mild symptoms and those in dense care units are involved in our proposed model.

This work is organized as: Part 2 is devoted to the formulation of the proposed model for COVID-19. In Parts 3 and 4, we analyze the proposed model with fractional order. Some qualitative analysis, computation of $\mathscr{R}_0$, and local stability for the disease-free equilibrium (DFE) in terms of $\mathscr{R}_0$, of the proposed model are obtained in Part 5. The stability analysis is studied in Part 6. Part 7 is dedicated to data fit and explanation of our model via numerical simulation and comparison with real data for Morocco. The conclusion is given in the last section.

2 The Proposed Epidemic Model

Here in this section, the proposed model is formulated. The flowchart of the model is given in Fig. 1 to understand the evolution of the mentioned disease. Let the total human population at time ϑ be denoted by $\mathbf{N}(\vartheta)$. The proposed model divides the human population at time ϑ into eleven compartments which are described in Table 1. We generalize the SEIR model of the spread of COVID-19 with a specific focus on the transmissibility of people who are Individuals who follow precautionary health measures $\mathbf{S}_a$ and Individuals who do not follow precautionary health measures $\mathbf{S}_u$. We separate hospitalized cases into severe $\mathbf{H}_s$ and mild cases $\mathbf{H}_m$ and incorporate awareness programs. Moreover, individuals with severe $\mathbf{I}_{ss}$, mild symptoms $\mathbf{I}_{ms}$ and asymptomatically infected $\mathbf{I}_a$ are also considered. Furthermore, the immunization of a sample of aware susceptible individuals in the proposed model to forecast the effect of vaccination $\mathbf{V}$ is also considered.

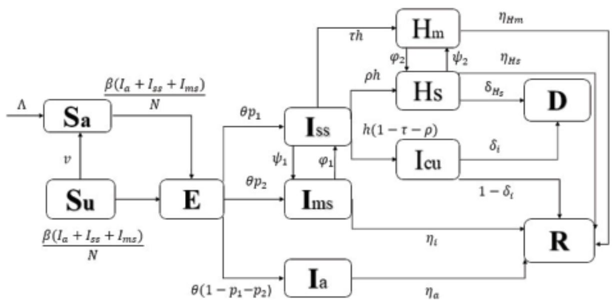

Figure 1: Flowchart of model (1)

Table 1: Compartments symbols and description

Symbol	Description
$\mathbf{S}_a$	Individuals follow precautionary health measures
$\mathbf{S}_u$	Individuals do not follow precautionary health measures
$\mathbf{E}$	Exposed class
$\mathbf{I}_{ss}$	Severe symptoms infectious individuals
$\mathbf{I}_{ms}$	Mild severe symptoms infectious individuals
$\mathbf{I}_a$	Infectious but asymptomatic individuals
$\mathbf{H}_m$	Hospitalized class
$\mathbf{H}_s$	Hospitalized individuals with sharp symptoms
$\mathbf{I}_{cu}$	Intensive cure unit class
$\mathbf{R}$	Recovery with immunity class
$\mathbf{D}$	Dead class
θ	The rate of conversion from exposed to infectious
β	The human-to-human transmission coefficient of each unit
p_1	The rate from exposed class to symptomatic infectious includes $\mathbf{I}_{ss}$
p_2	The rate of exposed class to infectious class $\mathbf{I}_{ms}$
$1 - p_1 - p_2$	Rate of exposed persons to asymptomatic class $\mathbf{I}_a$
h	The rate at which a person leaves the class $\mathbf{I}_{ss}$
τ	Hospitalization rates from $\mathbf{I}_{ss}$ to $\mathbf{H}_m$
ρ	Hospitalization rates from $\mathbf{I}_{ss}$ to $\mathbf{H}_s$
$\eta_k,\ k = a, i, \mathbf{H}_m,\ \mathbf{H}_s$	Recovery rates
$\delta_i,\ \delta_{\mathbf{H}_s}$	COVID-19 induced death rates
$\psi \mathbf{I}_1$	Rate at which $\mathbf{I}_{ss}$ to $\mathbf{I}_{ms}$
$\varphi \mathbf{I}_1$	Rate at which $\mathbf{I}_{ms}$ to $\mathbf{I}_{ss}$

(Continued)

Table 1 (continued)

Symbol	Description
$\psi \mathbf{I}_2$	The rate of hospitalized class from temperate to sharp isolation
$\varphi \mathbf{I}_2$	The rate of hospitalized class from sharp to temperate isolation
Λ	Requitement rate

Thus, we have

$$\mathbf{N}(\vartheta) = \mathbf{S}_a(\vartheta) + \mathbf{S}_u(\vartheta) + \mathbf{E}(\vartheta) + \mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta) + \mathbf{H}_m(\vartheta) + \mathbf{H}_s(\vartheta) + \mathbf{I}_{cu}(\vartheta) + \mathbf{R}(\vartheta) + \mathbf{D}(\vartheta).$$

From Fig. 1, we formulate our proposed model as

$$\begin{cases} \mathbf{S}'_a(\vartheta) = \Lambda - \beta \dfrac{\mathbf{S}_a(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) + v\mathbf{S}_u(\vartheta), \\ \mathbf{S}'_u(\vartheta) = -\beta \dfrac{\mathbf{S}_u(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) - v\mathbf{S}_u(\vartheta), \\ \mathbf{E}'(\vartheta) = \beta \dfrac{\mathbf{S}_a(\vartheta) + \mathbf{S}_u(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) - \theta \mathbf{E}(\vartheta), \\ \mathbf{I}'_{ss}(\vartheta) = \theta p_1 \mathbf{E}(\vartheta) + \varphi_1 \mathbf{I}_{ms}(\vartheta) - (\psi_1 + h)\mathbf{I}_{ss}(\vartheta), \\ \mathbf{I}'_{ms}(\vartheta) = \theta p_2 \mathbf{E}(\vartheta) + \psi_1 \mathbf{I}_{ss}(\vartheta) - (\eta_i + \varphi_1)\mathbf{I}_{ms}(\vartheta), \\ \mathbf{I}'_a(\vartheta) = \theta(1 - p_1 - p_2)\mathbf{E}(\vartheta) - \eta_a \mathbf{I}_a(\vartheta), \\ \mathbf{H}'_m(\vartheta) = h\tau \mathbf{I}_{ss}(\vartheta) + \psi_2 \mathbf{H}_s(\vartheta) - (\varphi_2 + \eta_{\mathbf{H}_m})\mathbf{H}_m(\vartheta), \\ \mathbf{H}'_s(\vartheta) = h\rho \mathbf{I}_{ss}(\vartheta) + \varphi_2 \mathbf{H}_m(\vartheta) - (\psi_2 + \eta_{\mathbf{H}_s} + \delta_{\mathbf{H}_s})\mathbf{H}_s(\vartheta), \\ \mathbf{I}'_{cu}(\vartheta) = h(1 - \tau - \rho)\mathbf{I}_{ss}(\vartheta) - \mathbf{I}_{cu}(\vartheta), \\ R'(\vartheta) = \eta_i \mathbf{I}_{ms}(\vartheta) + \eta_a \mathbf{I}_a(\vartheta) + \eta_{\mathbf{H}_s}\mathbf{H}_s(\vartheta) + \eta_{\mathbf{H}_m}\mathbf{H}_m(\vartheta) + (1 - \delta_i)\mathbf{I}_{cu}(\vartheta), \\ \mathbf{D}'(\vartheta) = \delta_{\mathbf{H}_s}\mathbf{H}_s(\vartheta) + \delta_i \mathbf{I}_{cu}(\vartheta), \end{cases} \tag{1}$$

with initial conditions as

$$\begin{cases} \mathbf{S}_a(0) = \mathbf{S}_{a,0}, \mathbf{S}_u(0) = Su, 0, \\ \mathbf{E}(0) = \mathbf{E}_0, \mathbf{I}_{ss}(0) = \mathbf{I}_{ss,0}, \\ \mathbf{I}_{ms}(0) = \mathbf{I}_{ms,0}, \mathbf{I}_a(0) = \mathbf{I}_{a,0}, \\ \mathbf{H}_m(0) = \mathbf{H}_{m,0}, \mathbf{H}_s(0) = \mathbf{H}_{s,0}, \\ \mathbf{I}_{cu}(0) = \mathbf{I}_{cu,0}, \mathbf{D}(0) = \mathbf{D}_0, R(0) = \mathrm{R}_0. \end{cases}$$

3 Fractional Order Model

In this part, we recall some definitions from fractional calculus which are needed throughout the paper.

Definition 3.1. [50] The Caputo fractional derivative of f of order μ is described as

$$f^{(\mu)}(\vartheta) = \frac{1}{\Gamma(n-\mu)} \int_0^{\vartheta} \frac{f^{(n)}(s)}{(\vartheta - s)^{\mu-n+1}} ds, \tag{2}$$

where $n - 1 < \mu \le n$ and $\Gamma(x) = \int_0^{\infty} e^{-\vartheta} \vartheta^{x-1} d\vartheta$, which is called the Euler's Gamma function.

The fractional integral [51] having of order $\mu > 0$ is defined by

$$I^{\mu}\left(f\left(\vartheta\right)\right) = \frac{1}{\Gamma(\mu)} \int_{0}^{\vartheta} \frac{f\left(s\right)}{\left(\vartheta - s\right)^{1-\mu}} ds,$$

where $\vartheta > 0$, satisfies:

$$\begin{cases} (I^{\mu} f(\vartheta))^{(\mu)} = f(\vartheta), \\ I^{\mu}(f(\vartheta)^{(\mu)}) = f(\vartheta) - f(0). \end{cases} \tag{3}$$

Now, let us present the fractional order model of the COVID-19 by means of the Caputo derivative as follows:

$$\begin{cases} \mathbf{S}_a^{(\mu)}\left(\vartheta\right) = \Lambda - \beta \dfrac{\mathbf{S}_a(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}\left(\vartheta\right) + \mathbf{I}_{ms}\left(\vartheta\right) + \mathbf{I}_a\left(\vartheta\right)\right) + v\mathbf{S}_u\left(\vartheta\right), \\ \mathbf{S}_u^{(\mu)}\left(\vartheta\right) = -\beta \dfrac{\mathbf{S}_u(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}\left(\vartheta\right) + \mathbf{I}_{ms}\left(\vartheta\right) + \mathbf{I}_a\left(\vartheta\right)\right) - v\mathbf{S}_u\left(\vartheta\right), \\ \mathbf{E}^{(\mu)}\left(\vartheta\right) = \beta \dfrac{\mathbf{S}_a(\vartheta) + \mathbf{S}_u(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}\left(\vartheta\right) + \mathbf{I}_{ms}\left(\vartheta\right) + \mathbf{I}_a\left(\vartheta\right)\right) - \theta\mathbf{E}\left(\vartheta\right), \\ \mathbf{I}_{ss}^{(\mu)}(\vartheta) = \theta p_1\mathbf{E}(\vartheta) + \varphi_1\mathbf{I}_{ms}(\vartheta) - (\psi_1 + h)\mathbf{I}_{ss}(\vartheta), \\ \mathbf{I}_{ms}^{(\mu)}(\vartheta) = \theta p_2\mathbf{E}(\vartheta) + \psi_1\mathbf{I}_{ss}(\vartheta) - (\eta_i + \varphi_1)\mathbf{I}_{ms}(\vartheta), \\ \mathbf{I}_a^{(\mu)}(\vartheta) = \theta(1 - p_1 - p_2)\mathbf{E}(\vartheta) - \eta_a\mathbf{I}_a(\vartheta), \\ \mathbf{H}_m^{(\mu)}(\vartheta) = h\tau\mathbf{I}_{ss}(\vartheta) + \psi_2\mathbf{H}_s(\vartheta) - (\varphi_2 + \eta_{\mathbf{H}_m})\mathbf{H}_m(\vartheta), \\ \mathbf{H}_s^{(\mu)}(\vartheta) = h\rho\mathbf{I}_{ss}(\vartheta) + \varphi_2\mathbf{H}_m(\vartheta) - (\psi_2 + \eta_{\mathbf{H}_s} + \delta_{\mathbf{H}_s})\mathbf{H}_s(\vartheta), \\ \mathbf{I}_{cu}^{(\mu)}(\vartheta) = h(1 - \tau - \rho)\mathbf{I}_{ss}(\vartheta) - \mathbf{I}_{cu}(\vartheta), \\ R^{(\mu)}(\vartheta) = \eta_i\mathbf{I}_{ms}(\vartheta) + \eta_a\mathbf{I}_a(\vartheta) + \eta_{\mathbf{H}_s}\mathbf{H}_s(\vartheta) + \eta_{\mathbf{H}_m}\mathbf{H}_m(\vartheta) + (1 - \delta_i)\mathbf{I}_{cu}(\vartheta), \\ \mathbf{D}^{(\mu)}(\vartheta) = \delta_{\mathbf{H}_s}\mathbf{H}_s(\vartheta) + \delta_i\mathbf{I}_{cu}(\vartheta). \end{cases} \tag{4}$$

4 Existence Theory

In this part, we study the existence of a solution for the fractional order model Eq. (4). The existence theory is an important consequence of applied analysis and has many applications. It provides us with whether the model we study exists or not and if it exists whether it has a solution or not. If a problem has a solution, whether it will be unique or multiple. It also helps us in investigating the qualitative behavior of the solution to the problem.

Consider the following subsystem:

$$\begin{cases} \mathbf{S}_u^{(\mu)}\left(\vartheta\right) = -\beta \dfrac{\mathbf{S}_u(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}\left(\vartheta\right) + \mathbf{I}_{ms}\left(\vartheta\right) + \mathbf{I}_a\left(\vartheta\right)\right) - v\mathbf{S}_u\left(\vartheta\right), \\ E^{(\mu)}\left(\vartheta\right) = \beta \dfrac{\mathbf{S}_a(\vartheta) + \mathbf{S}_u(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}\left(\vartheta\right) + \mathbf{I}_{ms}\left(\vartheta\right) + \mathbf{I}_a\left(\vartheta\right)\right) - \theta\mathbf{E}\left(\vartheta\right), \\ \mathbf{I}_{ss}^{(\mu)}(\vartheta) = \theta p_1\mathbf{E}(\vartheta) + \varphi_1\mathbf{I}_{ms}(\vartheta) - (\psi_1 + h)\mathbf{I}_{ss}(\vartheta), \\ \mathbf{I}_{ms}^{(\mu)}(\vartheta) = \theta p_2\mathbf{E}(\vartheta) + \psi_1\mathbf{I}_{ss}(\vartheta) - (\eta_i + \varphi_1)\mathbf{I}_{ms}(\vartheta), \\ \mathbf{I}_a^{(\mu)}(\vartheta) = \theta(1 - p_1 - p_2)\mathbf{E}(\vartheta) - \eta_a\mathbf{I}_a(\vartheta), \\ \mathbf{H}_m^{(\mu)}(\vartheta) = h\tau\mathbf{I}_{ss}(\vartheta) + \psi_2\mathbf{H}_s(\vartheta) - (\varphi_2 + \eta_{\mathbf{H}_m})\mathbf{H}_m(\vartheta), \\ \mathbf{H}_s^{(\mu)}(\vartheta) = h\rho\mathbf{I}_{ss}(\vartheta) + \varphi_2\mathbf{H}_m(\vartheta) - (\psi_2 + \eta_{\mathbf{H}_s} + \delta_{\mathbf{H}_s})\mathbf{H}_s(\vartheta), \\ \mathbf{I}_{cu}^{(\mu)}(\vartheta) = h(1 - \tau - \rho)\mathbf{I}_{ss}(\vartheta) - \mathbf{I}_{cu}(\vartheta). \end{cases} \tag{5}$$

Since

$$\begin{cases} \mathbf{S}_a^{(\mu)}(\vartheta) = \Lambda - \beta \dfrac{\mathbf{S}_a(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) + v\mathbf{S}_u(\vartheta), \\ R^{(\mu)}(\vartheta) = \eta_i \mathbf{I}_{ms}(\vartheta) + \eta_a \mathbf{I}_a(\vartheta) + \eta_{\mathbf{H}_s}\mathbf{H}_s(\vartheta) + \eta_{\mathbf{H}_m}\mathbf{H}_m(\vartheta) + (1-\delta_i)\mathbf{I}_{cu}(\vartheta), \\ \mathbf{D}^{(\mu)}(\vartheta) = \delta_{\mathbf{H}_s}\mathbf{H}_s(\vartheta) + \delta_i \mathbf{I}_{cu}(\vartheta). \end{cases} \tag{6}$$

Therefore, to study the existence and uniqueness of solutions of the system Eq. (4), we focus to study Eq. (5).

Assume that

$$\begin{cases} K_1(\vartheta, \mathbf{S}_u) = -\beta \dfrac{\mathbf{S}_u}{\mathbf{N}} \left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) - v\mathbf{S}_u, \\ K_2(\vartheta, \mathbf{E}) = \beta \dfrac{\mathbf{S}_a(\vartheta) + \mathbf{S}_u(\vartheta)}{\mathbf{N}} \left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) - \theta \mathbf{E}, \\ K_3(\vartheta, \mathbf{I}_{ss}) = \theta p_1 \mathbf{E}(\vartheta) + \varphi_1 \mathbf{I}_{ms}(\vartheta) - (\psi_1 + h)\mathbf{I}_{ss}, \\ K_4(\vartheta, \mathbf{I}_{ms}) = \theta p_2 \mathbf{E}(\vartheta) + \psi_1 \mathbf{I}_{ss}(\vartheta) - (\eta_i + \varphi_1)\mathbf{I}_{ms}, \\ K_5(\vartheta, \mathbf{I}_a) = \theta(1 - p_1 - p_2)\mathbf{E}(\vartheta) - \eta_a \mathbf{I}_a, \\ K_6(\vartheta, \mathbf{H}_m) = h\tau \mathbf{I}_{ss}(\vartheta) + \psi_2 \mathbf{H}_s(\vartheta) - (\varphi_2 + \eta_{\mathbf{H}_m})\mathbf{H}_m, \\ K_7(\vartheta, \mathbf{H}_s) = h\rho \mathbf{I}_{ss}(\vartheta) + \varphi_2 \mathbf{H}_m(\vartheta) - (\psi_2 + \eta_{\mathbf{H}_s} + \delta_{\mathbf{H}_s})\mathbf{H}_s \\ K_8(\vartheta, \mathbf{I}_{cu}) = h(1 - \tau - \rho)\mathbf{I}_{ss} - \mathbf{I}_{cu}. \end{cases} \tag{7}$$

Applying the fractional integral to Eq. (5), it follows from initial conditions that

$$\begin{cases} \mathbf{S}_u(\vartheta) = \mathbf{S}_{u,0} + \dfrac{1}{\Gamma(\mu)} \int_0^\vartheta (\vartheta - s)^{\mu-1} K_1(s, \mathbf{S}_u(s))\, ds, \\ \mathbf{E}(\vartheta) = \mathbf{E}_0 + \dfrac{1}{\Gamma(\mu)} \int_0^\vartheta (\vartheta - s)^{\mu-1} K_2(s, \mathbf{E}(s))\, ds, \\ \mathbf{I}_{ss}(\vartheta) = \mathbf{I}_{ss,0} + \dfrac{1}{\Gamma(\mu)} \int_0^\vartheta (\vartheta - s)^{\mu-1} K_3(s, \mathbf{I}_{ss}(s))\, ds, \\ \mathbf{I}_{ms}(\vartheta) = \mathbf{I}_{ms,0} + \dfrac{1}{\Gamma(\mu)} \int_0^\vartheta (\vartheta - s)^{\mu-1} K_4(s, \mathbf{I}_{ms}(s))\, ds, \\ \mathbf{I}_a(\vartheta) = \mathbf{I}_{a,0} + \dfrac{1}{\Gamma(\mu)} \int_0^\vartheta (\vartheta - s)^{\mu-1} K_5(s, \mathbf{I}_a(s))\, ds, \\ \mathbf{H}_m(\vartheta) = \mathbf{H}_{m,0} + \dfrac{1}{\Gamma(\mu)} \int_0^\vartheta (\vartheta - s)^{\mu-1} K_6(s, \mathbf{H}_m(s))\, ds, \\ \mathbf{H}_s(\vartheta) = \mathbf{H}_{s,0} + \dfrac{1}{\Gamma(\mu} \int_0^\vartheta (\vartheta - s)^{\mu-1} K_7(s, \mathbf{H}_s(s))\, ds, \\ \mathbf{I}_{cu}(\vartheta) = \mathbf{I}_{cu,0} + \dfrac{1}{\Gamma(\mu)} \int_0^\vartheta (\vartheta - s)^{\mu-1} K_8(s, \mathbf{I}_{cu}(s))\, ds. \end{cases} \tag{8}$$

Assume that $\|\mathbf{E}(\vartheta)\| \le c_1$, $\|\mathbf{I}_{ss}(\vartheta\| \le c_2$, $\|\mathbf{I}_{ms}(\vartheta)\| \le c_3$, where $c_i, i = 1, \ldots, 3$, are some positive constants. Denote $L_1 = \dfrac{\beta}{\mathbf{N}}(c_1 + c_2 + c_3) + v, L_2 = \theta, L_3 = \psi_1 + h, L_4 = \varphi_1 + \eta_i, L_5 = \eta_a, L_6 = \varphi_1 + \eta_{\mathbf{H}_m}, L_7 = \psi_2 + \eta_{\mathbf{H}_s} + \delta_{\mathbf{H}_s}$.

Theorem 4.1. The kernels K_i $(i = 1, \ldots, 8)$, satisfy the Lipschitz and contractive conditions if

$$0 \le L_i < 1 \tag{9}$$

hold.

Proof. Let $\mathbf{S}^*$ and $\mathbf{S}^{**}$ be two functions, then

$$\begin{aligned}\|K_1\left(\vartheta,\mathbf{S}^*\right)-K_1\left(\vartheta,\mathbf{S}^{**}\right)\| &= \left\|\left(-\frac{\beta}{\mathbf{N}}\left(\mathbf{I}_{ss}\left(\vartheta\right)+\mathbf{I}_{ms}\left(\vartheta\right)+\mathbf{I}_a\left(\vartheta\right)\right)-v\right)\left(\mathbf{S}^{**}-\mathbf{S}^*\right)\right\| \\ &\leq \left[\frac{\beta}{\mathbf{N}}\left(\|\mathbf{I}_{ss}\|+\|\mathbf{I}_{ms}\|+\|\mathbf{I}_a\|\right)+v\right]\|\mathbf{S}^*\left(\vartheta\right)-\mathbf{S}^{**}(\vartheta)\| \\ &\leq L_1\|\mathbf{S}^*(\vartheta)-\mathbf{S}^{**}(\vartheta)\|.\end{aligned}$$

Similarly

$$\|K_2(\vartheta,\mathbf{E}^*)-K_2(\vartheta,\mathbf{E}^{**})\|\leq L_2\|\mathbf{E}^*(\vartheta)-\mathbf{E}^{**}(\vartheta)\|,$$

$$\|K_3(\vartheta,\mathbf{I}_{ss}^*)-K_3(\vartheta,\mathbf{I}_{ss}^{**})\|\leq L_3\|\mathbf{I}_{ss}^*(\vartheta)-\mathbf{I}_{ss}^{**}(\vartheta)\|,$$

$$\|K_4(\vartheta,\mathbf{I}_{ms}^*)-K_4(\vartheta,\mathbf{I}_{ms}^{**})\|\leq L_4\|\mathbf{I}_{ms}^*(\vartheta)-\mathbf{I}_{ms}^{**}(\vartheta)\|,$$

$$\|K_5(\vartheta,\mathbf{I}_a^*)-K_5(\vartheta,\mathbf{I}_a^{**})\|\leq L_5\|\mathbf{I}_a^*(\vartheta)-\mathbf{I}_a^{**}(\vartheta)\|,$$

$$\|K_6(\vartheta,\mathbf{H}_m^*)-K_6(\vartheta,\mathbf{H}_m^{**})\|\leq L_6\|\mathbf{H}_m^*(\vartheta)-\mathbf{H}_m^{**}(\vartheta)\|,$$

$$\|K_7(\vartheta,\mathbf{H}_s^*)-K_7(\vartheta,\mathbf{H}_s^{**})\|\leq L_7\|\mathbf{H}_s^*(\vartheta)-\mathbf{H}_s^{**}(\vartheta)\|,$$

$$\|K_8(\vartheta,\mathbf{I}_{cu}^*)-K_8(\vartheta,\mathbf{I}_{cu}^{**})\|\leq L_8\|\mathbf{I}_{cu}^*(\vartheta)-\mathbf{I}_{cu}^{**}(\vartheta)\|.$$

Thus, K_i satisfy Lipschitz condition, for $i = 1,\ldots,8$. From Eq. (9), K_i $(i = 1,\ldots,8)$ are contraction.

Theorem 4.2. The model Eq. (4) has a unique solution provided

$$\frac{1}{\mu\Gamma(\mu)}r^{\mu}L_i<1,\ \text{for } i=1,2,\ldots,8,$$

when $r\in[0,\vartheta]$.

Proof. The proof for the above result is the same as the proof of Theorem 1 in [52].

5 Reproductive Number and Stability Analysis

5.1 Reproductive Number

We follow the references [53,54] to calculate $\mathscr{R}_0$. In order to calculate $\mathscr{R}_0$, we consider the subsystem Eq. (5).

The DFE point is given by $\mathscr{E}_0=(\mathbf{N},0,0,0,0,0,0,0,0,0,0)$. From the system (4), we can write

$$\mathbb{F}=\begin{pmatrix}0\\ \beta\dfrac{\mathbf{S}_a+\mathbf{S}_u}{\mathbf{N}}\left(\mathbf{I}_{ss}+\mathbf{I}_{ms}+\mathbf{I}_a\right)\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\end{pmatrix},$$

and $\mathbb{V}$ associated with the net rate outside of the corresponding compartments

$$v = \begin{pmatrix} -\beta\frac{\mathbf{S}_u}{\mathbf{N}}(\mathbf{I}_{ss} + \mathbf{I}_{ms} + \mathbf{I}_a) - v\mathbf{S}_u \\ -\theta\mathbf{E} \\ \theta p_1\mathbf{E} + \varphi_1\mathbf{I}_{ms} - (\psi_1 + h)\mathbf{I}_{ss} \\ \theta p_2\mathbf{E} + \psi_1\mathbf{I}_{ss} - (\eta_i + \varphi_1)\mathbf{I}_{ms} \\ \theta(1 - p_1 - p_2)\mathbf{E} - \eta_a\mathbf{I}_a \\ h\tau\mathbf{I}_{ss} + \psi_2\mathbf{H}_s - (\varphi_2 + \eta_{\mathbf{H}_m})\mathbf{H}_m \\ h\rho\mathbf{I}_{ss} + \varphi_2\mathbf{H}_m - (\psi_2 + \eta_{\mathbf{H}_s} + \delta_{\mathbf{H}_s})\mathbf{H}_s \\ h(1 - \tau - \rho)\mathbf{I}_{ss} - \mathbf{I}_{cu} \end{pmatrix}$$

It follows that the Jacobian matrices of $\mathbb{F}$ and $\mathbb{V}$ at $\mathscr{E}_0$ respectively are

$$\mathscr{F} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

and $\mathscr{V} =$

$$\begin{pmatrix} -v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta p_1 & -(h + \psi_1) & \varphi_1 & 0 & 0 & 0 & 0 \\ 0 & \theta p_2 & \psi_1 & -(\eta_i + \varphi_1) & 0 & 0 & 0 & 0 \\ 0 & \theta(1 - p_1 - p_2) & 0 & 0 & -\eta_a & 0 & 0 & 0 \\ 0 & 0 & h\tau & 0 & 0 & -(\varphi_2 + \eta_{\mathbf{H}_m}) & \psi_2 & 0 \\ 0 & 0 & h\rho & 0 & 0 & \varphi_2 & -(\psi_2 + \eta_{\mathbf{H}_m} + \delta_{\mathbf{H}_s}) & 0 \\ 0 & 0 & h(1 - \tau - \rho) & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

Thus, the $\mathscr{R}_0$ is obtained as the spectral radius of $-\mathscr{F}\mathscr{V}^{-1}$ as

$$\begin{aligned} \mathscr{R}_0 &= \max_{\lambda \in \mathbf{S}p(-\mathscr{F}\cdot\mathscr{V}^{-1})} |\lambda| \\ &= \mathbf{S}p(-\mathscr{F}\cdot\mathscr{V}^{-1}) \\ &= \frac{\beta(1 - p_1 - p_2)}{\eta_a} - \frac{\beta(p_2(h + \psi_1) + p_1\psi_1 + p_1(\eta_i + \varphi_1) + p_2\varphi_1)}{h\eta_i + h\varphi_1 + \psi_1\eta_i}. \end{aligned}$$

5.2 *Stability Analysis*

The reported result below is given in Theorem 2 of [54].

Theorem 5.1. The DFE $\mathscr{E}_0$ of model Eq. (4), is locally asymptotically stable if $\mathscr{R}_0 < 1$ and unstable if $\mathscr{R}_0 > 1$.

6 Fractional Order Model with Vaccination

In this part, we discuss the fractional order model with vaccination which is overseeing and controlling contagious diseases by giving security to powerless and susceptible individuals. We suppose that a specified proportion p of people in the mindful susceptible class are vaccinated. For this situation, inoculated people are moved to another compartment $\mathbf{V}(\vartheta)$. Due to the vaccine does not supply impunity to all vaccine recipients, vaccinated people might become infected yet a lower rate than unvaccinated. In this situation, let $\sigma_v \in [0, 1]$ such that $(1 - \sigma_v)$ be the vaccine efficacy. The flowchart of the fractional order model is as given in Fig. 2.

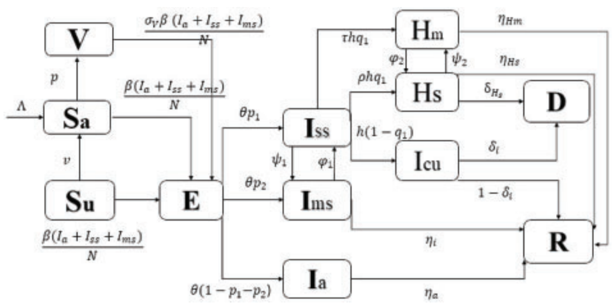

Figure 2: Flowchart of the fractional model (10)

We can interpret this diagram into fractional differential equations as follows:

$$\begin{cases} \mathbf{S}_a^{(\mu)}(\vartheta) = \Lambda - p\mathbf{S}_a(\vartheta) - \beta\dfrac{\mathbf{S}_a(\vartheta)}{\mathbf{N}}\left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) + v\mathbf{S}_u(\vartheta), \\ \mathbf{S}_u^{(\mu)}(\vartheta) = -\beta\dfrac{\mathbf{S}_u(\vartheta)}{\mathbf{N}}\left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) - v\mathbf{S}_u(\vartheta), \\ \mathbf{E}^{(\mu)}(\vartheta) = \beta\dfrac{\mathbf{S}_a(\vartheta) + \mathbf{S}_u(\vartheta)}{\mathbf{N}}\left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) + \dfrac{\sigma_V\beta\mathbf{V}(\vartheta)}{\mathbf{N}}\left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right) - \theta\mathbf{E}(\vartheta), \\ \mathbf{I}_{ss}^{(\mu)}(\vartheta) = \theta p_1\mathbf{E}(\vartheta) + \varphi_1\mathbf{I}_{ms}(\vartheta) - (\psi_1 + h)\mathbf{I}_{ss}(\vartheta), \\ \mathbf{I}_{ms}^{(\mu)}(\vartheta) = \theta p_2\mathbf{E}(\vartheta) + \psi_1\mathbf{I}_{ss}(\vartheta) - (\eta_i + \varphi_1)\mathbf{I}_{ms}(\vartheta), \\ \mathbf{I}_a^{(\mu)}(\vartheta) = \theta(1 - p_1 - p_2)\mathbf{E}(\vartheta) - \eta_a\mathbf{I}_a(\vartheta), \\ \mathbf{H}_m^{(\mu)}(\vartheta) = h\tau\mathbf{I}_{ss}(\vartheta) + \psi_2\mathbf{H}_s(\vartheta) - (\varphi_2 + \eta_{\mathbf{H}_m})\mathbf{H}_m(\vartheta), \\ \mathbf{H}_s^{(\mu)}(\vartheta) = h\rho\mathbf{I}_{ss}(\vartheta) + \varphi_2\mathbf{H}_m(\vartheta) - (\psi_2 + \eta_{\mathbf{H}_s} + \delta_{\mathbf{H}_s})\mathbf{H}_s(\vartheta), \\ \mathbf{I}_{cu}^{(\mu)}(\vartheta) = h(1 - \tau - \rho)\mathbf{I}_{ss}(\vartheta) - \mathbf{I}_{cu}(\vartheta), \\ \mathbf{R}^{(\mu)}(\vartheta) = \eta_i\mathbf{I}_{ms}(\vartheta) + \eta_a\mathbf{I}_a(\vartheta) + \eta_{\mathbf{H}_s}\mathbf{H}_s(\vartheta) + \eta_{\mathbf{H}_m}\mathbf{H}_m(\vartheta) + (1 - \delta_i)\mathbf{I}_{cu}(\vartheta), \\ \mathbf{D}^{(\mu)}(\vartheta) = \delta_{\mathbf{H}_s}\mathbf{H}_s(\vartheta) + \delta_i\mathbf{I}_{cu}(\vartheta), \\ \mathbf{V}^{(\mu)}(\vartheta) = p\mathbf{S}_a(\vartheta) - \dfrac{\sigma_{\mathbf{V}}\beta\mathbf{V}(\vartheta)}{\mathbf{N}}\left(\mathbf{I}_{ss}(\vartheta) + \mathbf{I}_{ms}(\vartheta) + \mathbf{I}_a(\vartheta)\right), \end{cases} \tag{10}$$

with $\mathbf{S}_a(0) = (1 - p)\mathbf{S}_{a,0}, \mathbf{S}_u(0) = (1 - p)\mathbf{S}_{u,0}, \mathbf{E}(0) = \mathbf{E}_0, \mathbf{I}_{ss}(0) = \mathbf{I}_{ss,0}, \mathbf{I}_{ms}(0) = \mathbf{I}_{ms,0}, \mathbf{I}_a(0) = \mathbf{I}_{a,0}$, $\mathbf{H}_m(0) = \mathbf{H}_{m,0}, \mathbf{H}_s(0) = \mathbf{H}_{s,0}, \mathbf{I}_{cu}(0) = \mathbf{I}_{cu,0}, \mathbf{D}(0) = \mathbf{D}_0, \mathbf{R}(0) = \mathbf{R}_0, \mathbf{V}(0) = p\mathbf{N}$. Then, the fractional order model (10) has one equilibrium point

$$\mathscr{E}_v = ((1 - p)\mathbf{N}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, p\mathbf{N}).$$

The components of infection in this model are, $\mathbf{S}_u$, $\mathbf{E}$, $\mathbf{I}_{ss}$, $\mathbf{I}_{ms}$, $\mathbf{I}_a$, $\mathbf{H}_m$, $\mathbf{H}_s$, and $\mathbf{I}_{cu}$.

Theorem 6.1. The reproduction number $\mathscr{R}_v$ for the vaccinated fractional order model (10) is given by

$\mathscr{R}_v = (1-(1-\sigma_v)p)\mathscr{R}_0.$

Proof. Let $\mathbb{F}$ and $\mathbb{V}$ be given by

$$\mathbb{F} = \begin{pmatrix} 0 \\ \beta \dfrac{\mathbf{S}_a + \mathbf{S}_u}{\mathbf{N}} \left(\mathbf{I}_{ss} + \mathbf{I}_{\mathrm{ms}} + \mathbf{I}_a\right) + \beta\sigma_v \dfrac{\mathbf{V}}{\mathbf{N}} \left(\mathbf{I}_{ss} + \mathbf{I}_{ms} + \mathbf{I}_a\right) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

and

$$\mathbb{V} = \begin{pmatrix} -\beta \dfrac{\mathbf{S}_u}{\mathbf{N}} \left(\mathbf{I}_{ss} + \mathbf{I}_{ms} + \mathbf{I}_a\right) - v\mathbf{S}_u \\ -\theta\mathbf{E} \\ \theta p_1 \mathbf{E} + \varphi \mathbf{I}_1 \mathbf{I}_{ms} - (\psi \mathbf{I}_1 + h)\mathbf{I}_{ss} \\ \theta p_2 \mathbf{E} + \psi \mathbf{I}_1 \mathbf{I}_{ss} - (\eta_i + \varphi \mathbf{I}_1)\mathbf{I}_{ms} \\ \theta(1 - p_1 - p_2)\mathbf{E} - \eta_a \mathbf{I}_a \\ h\tau \mathbf{I}_{ss} + \psi \mathbf{I}_2 \mathbf{H}_s - (\varphi \mathbf{I}_2 + \eta_{\mathbf{H}_m})\mathbf{H}_m \\ h\rho \mathbf{I}_{ss} + \varphi \mathbf{I}_2 \mathbf{H}_m - (\psi \mathbf{I}_2 + \eta_{\mathbf{H}_s} + \delta_{\mathbf{H}_s})\mathbf{H}_s \\ h(1 - \tau - \rho)\mathbf{I}_{ss} - \mathbf{I}_{cu} \end{pmatrix}.$$

By the differential of $\mathbb{F}$ and $\mathbb{V}$ with respect to $\mathbf{E}, \mathbf{I}_{ss}, \mathbf{I}_{ms}, \mathbf{I}_a, \mathbf{H}_m$, and $\mathbf{I}_{cu}$ and evaluating at the $\mathscr{E}_v$, respectively, we get

$$\begin{aligned} \mathscr{F} &:= J\mathbb{F}\left(\mathscr{E}_v\right) \\ &= (1-(1-\sigma_v)\,p) \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta & \beta & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

and $\mathscr{V} := J\mathbb{V}(\mathscr{E}_v) =$

$$\begin{pmatrix} -v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\theta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \theta p_1 & -(h+\psi \mathbf{I}_1) & \varphi \mathbf{I}_1 & 0 & 0 & 0 & 0 \\ 0 & \theta p_2 & \psi \mathbf{I}_1 & -(\eta_i + \varphi \mathbf{I}_1) & 0 & 0 & 0 & 0 \\ 0 & \theta(1-p_1-p_2) & 0 & 0 & -\eta_a & 0 & 0 & 0 \\ 0 & 0 & h\tau & 0 & 0 & -(\varphi \mathbf{I}_2 + \eta_{\mathbf{H}_m}) & \psi \mathbf{I}_2 & 0 \\ 0 & 0 & h\rho & 0 & 0 & \varphi \mathbf{I}_2 & -(\psi \mathbf{I}_2 + \eta_{\mathbf{H}_m} + \delta_{\mathbf{H}_s}) & 0 \\ 0 & 0 & h(1-\tau-\rho) & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

Thus, the $\mathscr{R}_v$ for the vaccinated fractional order model is:

$$\mathscr{R}_v = \beta\left(1-(1-\sigma_v)p\right)\left(\frac{(1-p_1-p_2)}{\eta_a} - \frac{(p_2(h+\psi \mathbf{I}_1) + p_1\psi \mathbf{I}_1 + p_1(\eta_i + \varphi \mathbf{I}_1) + p_2\varphi \mathbf{I}_1)}{h\eta_i + h\varphi \mathbf{I}_1 + \psi \mathbf{I}_1 \eta_i}\right)$$
$$= (1-(1-\sigma_v)p)\mathscr{R}_0.$$

Theorem 6.2. The DFE $\mathscr{E}_v$ of model Eq. (10) will be asymptotically stable if $\mathscr{R}_v < 1$, and unstable if $\mathscr{R}_v > 1$.

Proof. The proof is given in the Theorem 2 of [54].

The critical percentage of the population p_c needed to fulfill herd immunity is

$$p_c = \left(\frac{1}{1-\sigma_v}\right)\left(1 - \frac{1}{\mathscr{R}_0}\right).$$

p_c represents the rate for which $\mathscr{R}_0$ under the vaccination $\mathscr{R}_v$ is equal to 1.

7 Numerical Simulations

In the numerical analysis of traditional differential equations, various numerical methods have been established. Among various numerical procedures, Adams's methods represent one of the most used and studied classes of implicit (Adams-Moulton) and explicit (Adams-Bashforth) linear multistep methods. The aforementioned methods have gotten much popularity among researchers due to their good stability properties, reasonable computational cost, and simplicity of implementation. Therefore, several authors have extended the aforementioned methods. In particular, the mentioned procedures have also been extended to deal with fractional order differential equations numerically. A predictor-corrector algorithm has been extended to fractional differential equations with the corresponding Adams-Moulton-Bashforth procedure for traditional differential equations. The numerical method used to solve the model is the predictor-corrector method for fractional differential equations in [55]. Consider, the problem with fractional order

$$\begin{cases} y^{(\mu)}(\vartheta) = f(\vartheta, y(\vartheta)), 0 < \mu \leq 1, \\ y(\vartheta_0) = y_0, \end{cases} \tag{11}$$

where $f:[\vartheta_0, T] \times \mathbb{D} \rightarrow$ R, $\mathbb{D} \subset$ R. The proposed scheme for (11) can be written as

$$\begin{cases} y_n^p = y_0 + h^\mu \sum_{j=0}^{n-1} \Lambda_{n-j-1} f_j \\ y_n = y_0 + h^\mu \kappa_{n,0} f_0 + h^\mu \sum_{j=0}^{n-1} \Delta_{n-j} f_j + h^\mu \Delta_0 f\left(\vartheta_n, y_n^p\right), \end{cases} \tag{12}$$

where $\Lambda_n = (n+1)^\mu - \dfrac{n^\mu}{\Gamma(\mu+1)}$, $\kappa_{n,0} = (n-1)^{\mu-1} - \dfrac{n^\mu(n-\mu-1)}{\Gamma(\mu+2)}$ and

$$\Delta_n = \begin{cases} \dfrac{1}{\Gamma(\mu+2)}, & n = 0, \\ \dfrac{(n-1)^{\mu+1} - 2n^{\mu+1} + (n+1)^{\mu+1}}{\Gamma(\mu+2)}, & n = 1, 2, \ldots \end{cases}$$

We extend the afore given scheme in Eq. (12) for our considered model Eq. (4) to simulate the results. The numerical simulations for fractional dynamic of the considered model Eqs. (4) and (10) are performed as the case study of Morocco. The parameters values are given in the Table 2 for the numerical interpretation. Also we compute the value of the reproduction number as $\mathscr{R}_0 = 1.0575$ and $\mathscr{R}_v = 1.0574$.

Table 2: Values of the model parameters

Name	Value	Name	Value	Name	Value	Name	Value
$\mathbf{S}_{a,0}$	699205 [56]	$\mathbf{R}_0$	0 [56]	$\mathbf{I}_{a,0}$	3 [56]	$\mathbf{I}_{ss,0}$	100 [56]
$\mathbf{S}_{u,0}$	3000000 [56]	$\mathbf{D}_0$	0 [56]	$\mathbf{I}_{ms,0}$	100 [56]	$\mathbf{H}_{s,0}$	0 (Estimated)
$\mathbf{E}_0$	185 [56]	$\mathbf{I}_{cu,0}$	37 [56]	$\mathbf{H}_{m,0}$	5 (Estimated)	β	0.45 [57]
θ	1/5.1 [6]	η_a	0.4 [58]	η_i	0.4 [58]	τ	0.1259 [59]
ρ	0.13266 [59]	δ_i	0.6 [56]	$\eta_{\mathbf{H}_m}$	0.11624 [59]	$\eta_{\mathbf{H}_s}$	0.155 [59]
$\delta_{\mathbf{H}_s}$	$1 - \eta_{\mathbf{H}_m} - \eta_{\mathbf{H}_s}$	$\psi\mathbf{I}_1$	0.169055 [59]	$\phi\mathbf{I}_1$	0.0341 [59]	$\psi\mathbf{I}_2$	0.169055 (Estimated)
$\phi\mathbf{I}_2$	0.0041 (Estimated)	v	0.0173768 [59]	Λ	0.008 N (Estimated)	h	0.5 [56]
p_1	0.3 [56]	p_2	0.4 [56]	$(1-\sigma_v)$	0.9 [56]	p	4/1000 (Estimated)

In Fig. 3, the confirmed infected and death cases are described daily from July 01, 2020, to January 01, 2021, which is a total of 185 days. We implement numerical simulations to compare the results of our model with the real data in Fig. 3. The forecasted evolution of the outbreak of COVID-19 without and with vaccination in Morocco can be seen in Figs. 4 and 5, respectively.

From the analysis of the acquired graphs, we observe that the population of the infected class decreases significantly by decreasing the fractional order of the derivative and the endemic state of the disease goes to the disease-free state in all the cases deliberated. Besides, we demonstrate that our fractional order model well describes the actual data of daily confirmed, recovered, and death cases. In Fig. 5, we note that the plot is much flatter than that in Fig. 4. As well, the curve for asymptomatic individuals is almost identical to the x-axis, and this indicates the importance of the vaccination approach to overcome the pandemic.

In Fig. 6, we give the graphical results of the suggested model (4) to analyze the effect of the fractional order.

In Fig. 7, we give the graphical results of the suggested vaccination model (10) to analyze the influence of the fractional order.

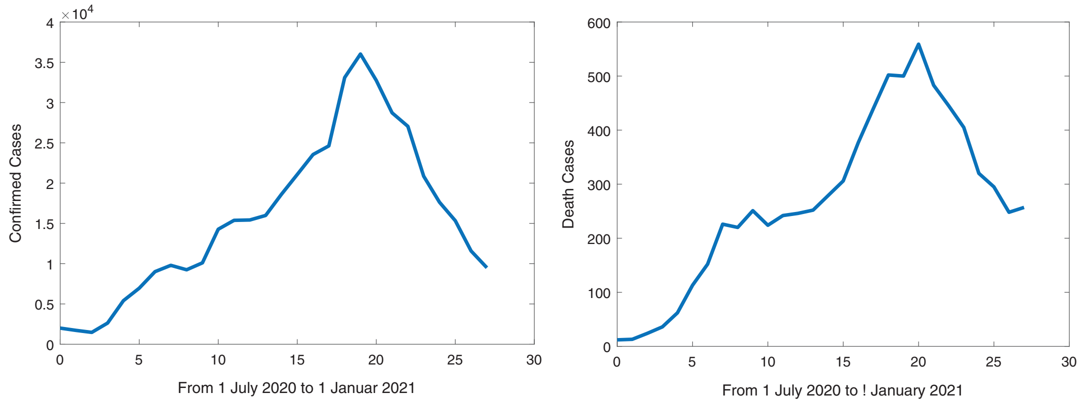

Figure 3: Evolution of COVID-19 in Morocco per day

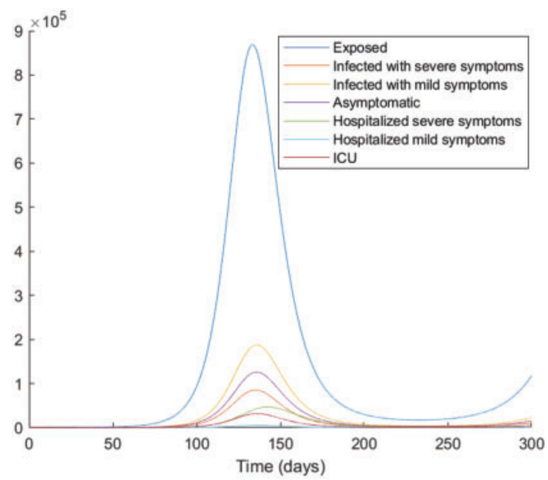

Figure 4: The evolution of the epidemic predicted by the fractional model (4) with $\mu = 1$

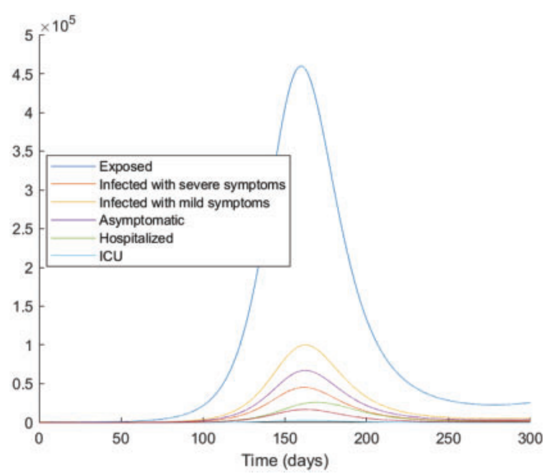

Figure 5: The evolution of the epidemic predicted by the fractional model vaccinated with $\mu = 1$

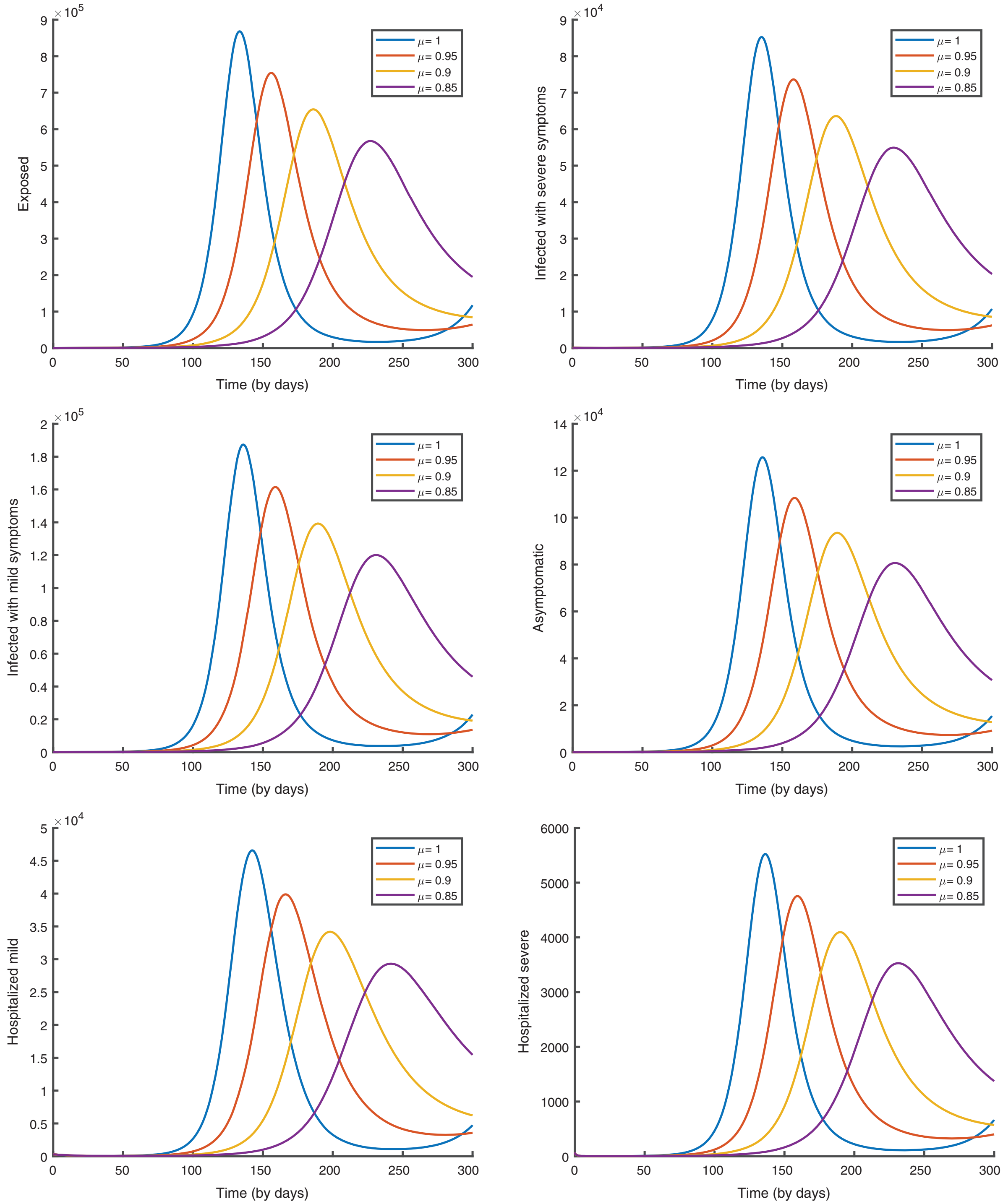

Figure 6: The evolution of the epidemic predicted by the model (4)

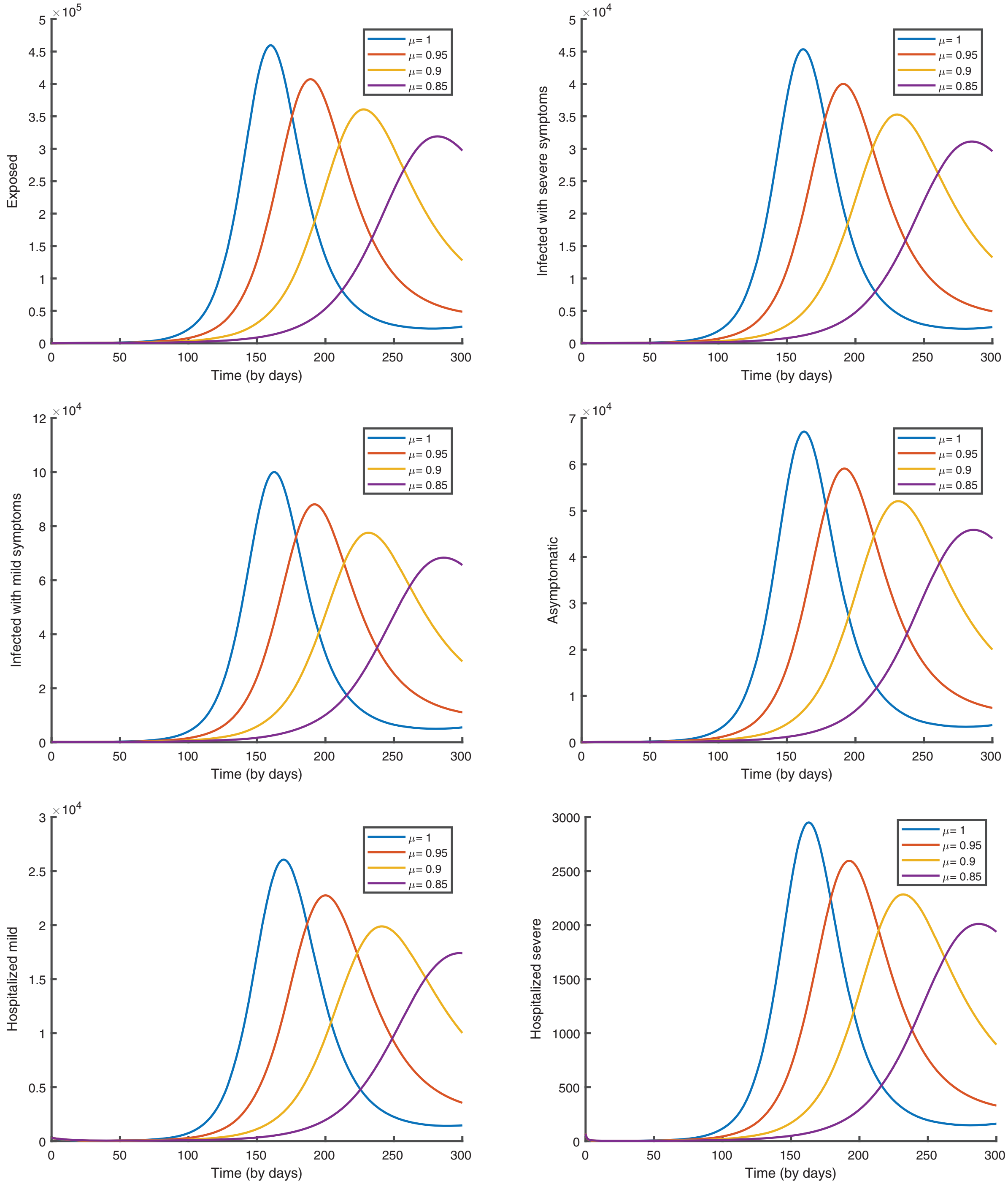

Figure 7: Epidemic evolution predicted by the fractional vaccination model (10)

We represent the spread of infection without vaccination in Fig. 8 and with vaccination in Fig. 9, for a period of 1000 days.

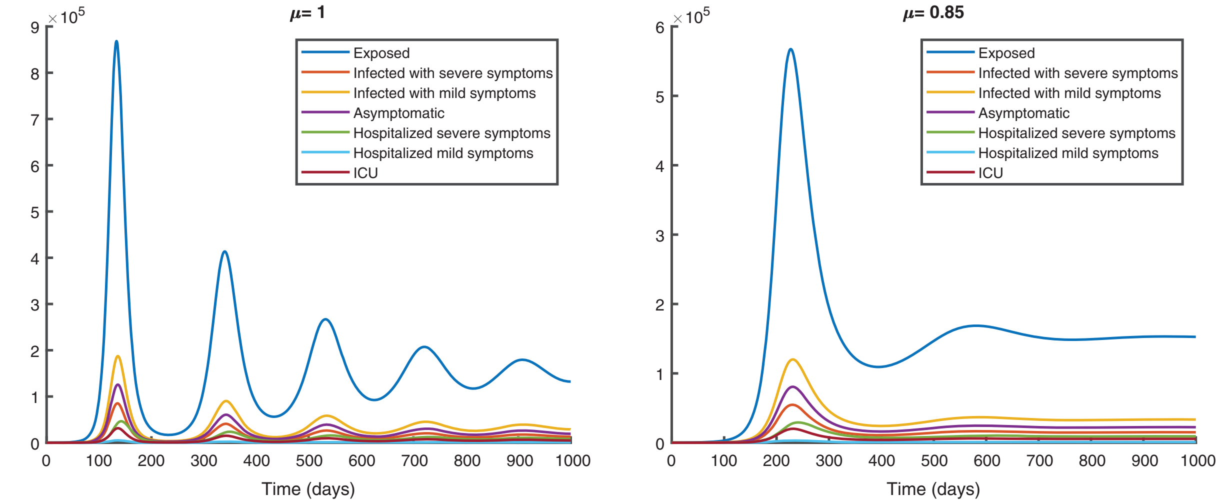

Figure 8: Epidemic evolution predicted by the fractional model

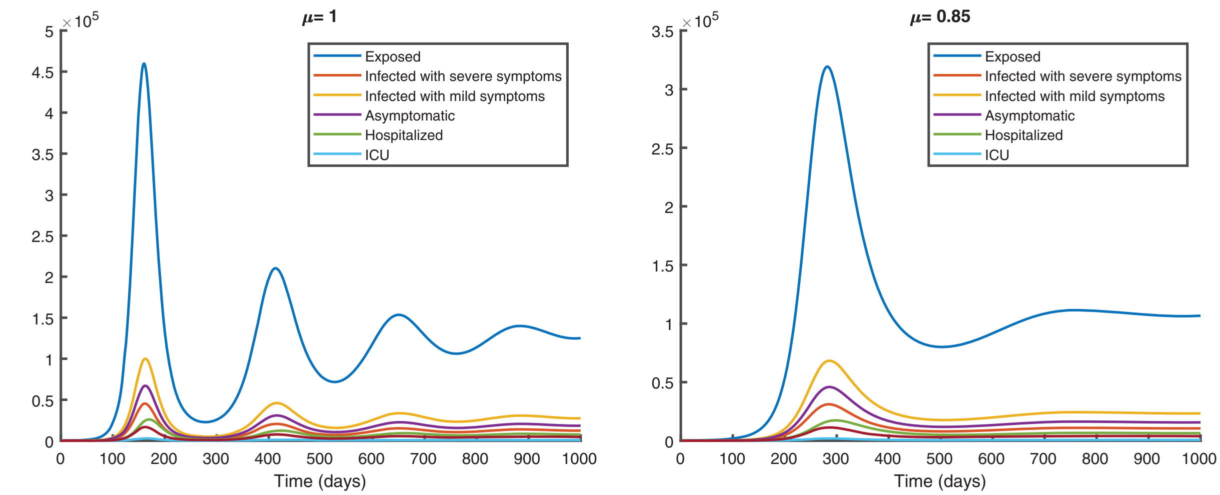

Figure 9: Epidemic evolution predicted by the fractional vaccination model

8 Conclusion

In this manuscript, we have given an analysis of the fractional order model of COVID-19, with theoretical calculations. We have established sufficient conditions for the existence theory using some tools of nonlinear analysis. The existence theory is an important area of research in recent times. Also, we have computed the basic reproductive number $\mathscr{R}_0$. From the number mentioned above, we have predicted the proposed model's stability. Further, we have extended the predictor-corrector method to numerically interpret the proposed model. We have given several graphical presentations using the numerical procedure. We have used some real data. It should be kept in mind that the model under consideration has described the evolution of COVID-19 in Morocco. We have given a graphical presentation of approximation solutions of various compartments in Figs. 3 and 4) and a simulation of this model under vaccination in Fig. 5). On another side, we have compared the numerical simulations with various values of the fractional-order μ. Moreover, our model gives a prediction about the evolution of COVID-19 in the next 1000 days. It seems from Figs. 6, and 7 that we must commit to preventive measures with the necessity of vaccination and adaptation to control and reduce the infection from further spreading in communities. In the future, the aforementioned analysis can be established for a more complex dynamical system of COVID-19 and the effect of vaccination on its reduction in the community. In our future work, we intend to study the model using the newly generalized partial derivative (see [60]), which overcomes some of the problems associated with conformational derivatives and some other fractional order derivatives.

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