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Aggregation Operators for Decision Making Based on q-Rung Orthopair Fuzzy Hypersoft Sets: An Application in Real Estate Project

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ABSTRACT

In this paper, a decision-making problem with a q-rung orthopair fuzzy hypersoft environment is developed, and two operators of ordered weighted average and induced ordered weighted average are developed. Several fundamental features are also derived. The induced ordered weighted average operator is essential in a q-ROFH environment as the induced ordered aggregation operators are special cases of the existing aggregation operators that already exist in q-ROFH environments. The main function of these operators is to help decision-makers gain a complete understanding of uncertain facts. The proposed aggregation operator is applied to a decision-making problem, with the aim of selecting the most promising real estate project for investment.

KEYWORDS

Hypersoft set; q-rung orthopair fuzzy hypersoft set; induced aggregation operator; decision-making

1 Introduction

Decision making [1] is a crucial aspect of our daily lives. Due to the complexity and subjectivity of complex alternatives, decision-makers are frequently confronted with a considerable measure of ambiguity in decision-making problems. To address these problems, Zadeh [2] developed fuzzy set theory. Each element of such set is assigned a membership value, between 0 and 1. The fuzzy set theory has a significant impact on the decision-making process. The fuzzy set theory has numerous applications in numerous domains of engineering, humanities, health, and physical sciences. In recent years, fuzzy set theory has also been utilized in the fields of energy, medicine, materials, economics, and nearly every other scientific discipline. There must be uncertainty if the decision-makers do not have sufficient knowledge to deal with the data necessary to express their preference for the object. Experts mostly consider membership and non-membership values in the decision-making process



that the fuzzy set cannot handle. Using the framework of fuzzy multisets, Du et al. [3] developed a multicriteria group decision-making method in 2021, which accounts for varying degrees of risk associated with each criterion.

To deal with this type of circumstance, Atanassov [4,5] proposed intuitionistic fuzzy sets, whose primary components are intuitionistic fuzzy numbers (IFN). Intuitionistic fuzzy numbers are defined by a membership function and a non-membership function, with the condition that the sum of the membership and non-membership grades is less than or equal to one. When the sum is greater than 1, the intuitionistic fuzzy sets do not work under specific conditions. Several operations on intuitionistic fuzzy sets were developed by De et al. [6,7] and incorporated into Sanchez's method for medical diagnosis. The distance between intuitionistic fuzzy sets was discussed by Szmidt et al. [8]. Using intuitionistic fuzzy sets as a theme, Gulzar et al. [9] introduced the t-intuitionistic fuzzy subgroup with basic algebraic properties and complex intuitionistic fuzzy sets in group theory and its applications. In some instances, when the sum of the degrees of MM and NMM is greater than 1, their sum of squares may be less than or equal to 1. Intuitionistic fuzzy sets fail in this situation.

To address these problems, Yager [10] proposed a novel concept known as the Pythagorean fuzzy set, a generalization of the intuitionistic fuzzy sets. The quadratic sum of membership and non-membership is less than or equal to 1, distinguishing it from other entities. Intuitionistic fuzzy sets and Pythagorean fuzzy sets are incredibly valuable for characterising fuzzy data. It is obvious that the Pythagorean fuzzy set can represent more decision-making information and is more inclusive than the intuitionistic fuzzy set. However, the Pythagorean fuzzy set frequently fails when the decision-maker is able to specify the degree of the membership and non-membership in a particular case where their sum is greater than 1. Some findings for Pythagorean fuzzy sets were developed in 2015 [11]. Pythagorean fuzzy sets, as well as a more refined approach to linguistic, Garg [12] utilized Pythagorean fuzzy sets for MCDM problems.

Yager [13] came up with q-rung orthopair fuzzy sets (q-ROFS), the key feature of which is that the sum of the q^{th} power of the membership and non-membership grade cannot exceed 1. Because of this, q-ROFS is advantageous to both the intuitionistic fuzzy set and the Pythagorean fuzzy set when dealing with ambiguous data. Innovative applications of fuzzy sets include aggregation operators, information measures, and decision-making tools. Madasi et al. [14] recently proposed the topic of N-cubic q-rung orthopair fuzzy sets, along with their properties and the use of mobile apps in the education sector as an application.

In order to address the issue of making group decisions based on several criteria, Zulqarnain et al. [15] developed certain aggregation operators inside the framework of the Interval-Valued Pythagorean fuzzy soft set. After transforming the function F into a multi-valued function, Smarandache [16] introduced the hypersoft set. Hypersoft sets are a fundamental mathematical tool for dealing with ambiguity and uncertainty in problem-based decision-making. Zulqarnain et al. [17,18] developed the Pythagorean fuzzy hypersoft sets, along with fundamental operations and techniques. By combining these concepts, Khan et al. [19] constructed a new structure called q-rung orthopair fuzzy hypersoft sets (q-ROFHSs). In q-ROFHSs, the multi-parameter membership and non-membership grades are limited to the q^{th} power.

It is crucial to understand the aggregation operators in the q-rung orthopair fuzzy environment and other types of fuzzy sets for tackling problems involving decision-making. In order to solve the decision-making techniques, the induced intuitionistic fuzzy ordered weighted geometric (I-IFOWG) and induced interval-valued intuitionistic fuzzy ordered weighted geometric (I-IIFOWG) operators were developed by Wei [20]. The induced generalised intuitionistic fuzzy ordered weighted averaging

(I-GIFOWA) and induced generalised interval-valued intuitionistic fuzzy ordered weighted averaging (I-GIIFOWA) operators were introduced by Xu et al. [21]. For the sake of better decision-making, Zulqarnain et al. [22] also explored a variety of aggregation operators in a Pythagorean fuzzy hypersoft environment. Basic aggregation operators, such as the weighted average and weighted geometric operators were designed by Liu et al. [23], and they were applied in a scenario based on q-rung orthopair fuzzy information. Several fundamental operations and exponential aggregation operators were developed by Peng et al. [24] for the q-rung orthopair fuzzy set. Two aggregation operators the complex q-rung orthopair fuzzy weighted average (Cq-ROFWA) and the complex q-rung orthopair fuzzy weighted geometric (Cq-ROFWG) were introduced by Liu et al. [25]. For a complex q-rung orthopair fuzzy (Cq-ROF) environment, Garg et al. [26] developed power aggregation operations. The q-rung orthopair fuzzy hypersoft weighted average (q-ROFHWA) and q-rung orthopair fuzzy hypersoft weighted geometric (q-ROFHWG) aggregation operators were developed by Khan et al. [27] for use in decision-making problems.

Due to the inadequacies of such mathematical modelling problems, we proposed q-ROFHS based mathematical model of such daily life problems in decision-making. For these mathematical models in the q-ROFH context, some aggregation operators and decision-making approaches are required. Therefore, we developed ordered weighted average and induced ordered weighted average aggregation operators in the q-ROFH environment.

Here are some of the key aspects of this research:

- The q-ROFHS process expert data with multi-sub-attributes of uncertainty in order to make decisions.
- Important aggregation operators in the decision-making process include ordered weighted average and induced ordered weighted average.
- Using basic operational laws, properties of the ordered weighted average and induced ordered weighted average aggregation operator.
- A novel decision-making technique was presented by using the proposed aggregation operators.
- Research is conducted within a real company to determine the efficacy and viability of the proposed method. The interpretations demonstrate that the predefined model is more practical and reliable for understanding the facts.

The remaining study of this article is organized as follows. In [Section 2](#), briefly some basic materials related to the proposed structure are introduced. [Section 3](#) consists of ordered weighted average and induced ordered weighted average operators with properties. In [Section 4](#), the algorithm of the proposed structure is presented based on aggregation operators. A real-life application is also presented in this section. [Section 5](#) describes the comparative analysis of the proposed structure with existing structures. [Section 6](#) presents the conclusion and future work.

2 Preliminaries

In this section, we compile some basic data that will be utilised to construct the structure of the article.

Definition 2.1 (16). The pair (H, A) is called a hypersoft set over Y , where H is the cartesian of n disjoint sets H_1, H_2, \dots, H_n having attribute values of n distinct attributes h_1, h_2, \dots, h_n , respectively, and $H : A \rightarrow P(Y)$.

Definition 2.2 (19). Let $Y = \{Y_1, Y_2, \dots, Y_n\}$ be a collection of disjoint attribute valued sets corresponding to n distinct attributes a_1, a_2, \dots, a_n , respectively, with the condition $A_i \cap A_j = \phi$, for $i \neq j$ and $i, j \in \{1, 2, \dots, n\}$. A q -rung orthopair fuzzy hypersoft set \S_A over Y is defined as

$$\S_A = \left\{ \left(\left(h, \psi_{A(h)}, \tilde{\psi}_{A(h)} \right) \mid h \in H \text{ and } q \geq 1 \right) \right\},$$

where

- (a) $q - ROF(Y)$ is the collection of all q -rung orthopair fuzzy sets over Y .
- (b) $H = Y_1 \times Y_2 \times \dots \times Y_n$
- (c) A is a q -rung orthopair fuzzy over Y with $\psi_A, \tilde{\psi}_A : H \rightarrow X$, and
- (d) $\delta_{A(h)}$ is a q -rung orthopair fuzzy set for $h \in H$ with $\delta_{A(h)} : H \rightarrow Q(Y)$.

Here $q - ROFHS^{(Y)}$ represents the collection of all $q - ROF$ subsets of Y , such that $0 \leq (\psi_{A(h)})^q + (\tilde{\psi}_{A(h)})^q \leq 1$.

Definition 2.3 (19). The score function of the q -ROFHNs is defined as $S_{A(h)} = \psi_{A(h)}^q - \tilde{\psi}_{A(h)}^q$

3 Ordered and Induced Ordered Weighted Average Aggregation Operators

In this section, we use the concept of ordered weighted averaging operator and develop the q -rung orthopair fuzzy hypersoft ordered weighted averaging operator (q -ROFHOWA) and the induced- q -rung orthopair fuzzy hypersoft ordered weighted averaging operator (I- q -ROFHOWA).

Definition 3.1. An ordered weighted averaging q -rung orthopair fuzzy hypersoft (q -ROFHOWA) operator is defined as:

$$I - q - ROFHOWA (E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) = \bigoplus_{j=1}^n v_j \left(\bigoplus_{i=1}^m w_i E_{\sigma_{a_{ij}}} \right)$$

where $w_i = \{1, 2, \dots, n\}$ and $v_j = \{1, 2, \dots, m\}$ are weight vectors for experts and sub-attributes of the selected parameters respectively, having the condition that, $w_i > 0, \sum_{i=1}^m w_i = 1$, and $v_j > 0, \sum_{j=1}^n v_j = 1$. Then the mapping for the q -ROFHOWA operator is defined as $q - ROFHOWA : \Delta^n \rightarrow \Delta$ is the collection of all q -ROFHNs.

Theorem 3.1. Let $E_{a_{ij}}, (i = 1, 2, \dots, n)(j = 1, 2, \dots, m)$ be the collection of q -ROFHNs, then their aggregated values by using the q -ROFHOWA operator are also q -rung orthopair fuzzy hypersoft values, and

$$q - ROFHOWA (E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) = \left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \varphi_{\sigma_{a_{ij}}}^q \right)^{w_i} \right)^{v_j}} \right),$$

$\prod_{j=1}^n \left(\prod_{i=1}^m \left(\tilde{\varphi}_{\sigma_{a_{ij}}} \right)^{w_i} \right)^{v_j}$ where $w_i = \{1, 2, \dots, n\}$ and $v_j = \{1, 2, \dots, m\}$ are weight vectors for experts and sub-attributes of selected parameters, respectively, having the condition that, $w_i > 0, \sum_{i=1}^m w_i = 1$, and $v_j > 0, \sum_{j=1}^n v_j = 1$, and σ_{ij} are permutations of $(i = 1, 2, \dots, m), (j = 1, 2, \dots, n)$ such that $E_{\sigma_{(i-1)j}} \geq E_{\sigma_{(i)j}}$ and $E_{\sigma_{i(j-1)}} \geq E_{\sigma_{i(j)}} \forall i, j$.

Proof. The q -ROFHOWA operator can be proved using the principle of mathematical induction as follows. For $m = 1$, we get $w_i = 1$. Then, we have,

$$\begin{aligned}
 & q - ROFHOWA (E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) \\
 &= \bigoplus_{j=1}^n v_j E_{\sigma a_{1j}} \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^n (1 - \varphi_{\sigma a_{1j}})^{v_j}}, \prod_{j=1}^n (\tilde{\varphi}_{\sigma a_{1j}})^{v_j} \right) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{l=1}^1 (1 - \varphi_{\sigma a_{1j}})^{w_l} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{l=1}^1 (\tilde{\varphi}_{\sigma a_{1j}})^{w_l} \right)^{v_j} \right) \tag{1}
 \end{aligned}$$

For $n = 1$, we get $v_j = 1$.

$$\begin{aligned}
 & q - ROFHOWA (E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{mm}}) \\
 &= \bigoplus_{i=1}^m w_i E_{\sigma a_{i1}} \\
 &= \left(\sqrt[q]{1 - \prod_{i=1}^m (1 - \varphi_{\sigma a_{i1}})^{w_i}}, \prod_{i=1}^m (\tilde{\varphi}_{\sigma a_{i1}})^{w_i} \right) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^1 \left(\prod_{i=1}^m (1 - \varphi_{\sigma a_{ij}})^{w_i} \right)^{v_j}}, \prod_{j=1}^1 \left(\prod_{i=1}^m (\tilde{\varphi}_{\sigma a_{ij}})^{w_i} \right)^{v_j} \right) \tag{2}
 \end{aligned}$$

So, Eq. (2) is true for all $m = 1$ and $n = 1$. Suppose that the equation holds for $m = k_2, n = k_1 + 1$, and $m = k_2 + 1, n = k_1$.

$$\begin{aligned}
 & \bigoplus_{j=1}^{k_1+1} v_j \left(\bigoplus_{i=1}^{k_2} w_i E_{\sigma a_{ij}} \right) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^{k_1+1} \left(\prod_{l=1}^{k_2} (1 - \varphi_{\sigma a_{ij}})^{w_l} \right)^{v_j}}, \prod_{j=1}^{k_1+1} \left(\prod_{l=1}^{k_2} (\tilde{\varphi}_{\sigma a_{ij}})^{w_l} \right)^{v_j} \right) \tag{3}
 \end{aligned}$$

$$\begin{aligned}
 &= \bigoplus_{j=1}^{k_1} v_j \left(\bigoplus_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}} \right) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^{k_1} \left(\prod_{l=1}^{k_2+1} (1 - \varphi_{\sigma a_{ij}})^{w_l} \right)^{v_j}}, \prod_{j=1}^{k_1} \left(\prod_{l=1}^{k_2+1} (\tilde{\varphi}_{\sigma a_{ij}})^{w_l} \right)^{v_j} \right) \tag{4}
 \end{aligned}$$

Now we prove the equation for $n = k_1 + 1$ and $m = k_2 + 1$.

$$\begin{aligned}
 & \bigoplus_{j=1}^{k_1+1} v_j \left(\bigoplus_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}} \right) \\
 &= \bigoplus_{j=1}^{k_1+1} E_{\sigma a_j} \left(\bigoplus_{i=1}^{k_2} w_i E_{\sigma a_{ij}} \oplus w_{i+1} E_{\sigma a_{(k_2+1)j}} \right) \tag{5}
 \end{aligned}$$

$$\begin{aligned}
 &= \left(\bigoplus_{j=1}^{k_1+1} \bigoplus_{i=1}^{k_2} w_i v_j E_{\sigma a_j} \right) \left(\bigoplus_{j=1}^{k_1+1} v_j w_{i+1} E_{\sigma a_{(k_2+1)(j)}} \right) \\
 &= 2 \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \left(\tilde{\varphi}_{\sigma a_{(i)(j)}} \right)^{w_i} \right)^{v_j} \oplus 2 \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \left(\tilde{\varphi}_{\sigma a_{(k_2+1)(j)}} \right)^{w_{k_2+1}} \right)^{v_j} \\
 &= \sqrt[q]{1 - \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \left(1 - \varphi_{\sigma a_{ij}}^q \right)^{w_i} \right)^{v_j}} , 2 \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \left(\tilde{\varphi}_{\sigma a_{(i)(j)}} \right)^{w_i} \right)^{v_j} \\
 &= \bigoplus_{j=1}^{k_1+1} v_j \left(\bigoplus_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}} \right) \tag{6}
 \end{aligned}$$

So, it is valid for $n = k_1 + 1$ and $m = k_2 + 1$.

Remark 3.2. If the value of q is fixed, which is $q = 1$, the proposed q -ROFHWA operator reduces to the IFHOWA operator.

Remark 3.3. If the value of q is fixed, which is $q = 2$, the proposed q -ROFHWA operator reduces to the PFHOWA operator.

Remark 3.4. If there are single parameters, which is e_1, e_2, \dots, e_n , then the proposed q -ROFHWA operator reduces to the q -ROFSOWA operator.

Remark 3.5. From the above remarks, it is clear that IFHOWA, PFHOWA and q -ROFSOWA operators are the special cases of the proposed q -ROFHWA operator.

The I- q -ROFHSOWA operator is idempotent, bounded and monotonic. These can be proved by the following theorem:

(Idempotency) Let $E_{a_{ij}} = (\varphi_{a_{ij}}, \tilde{\varphi}_{a_{ij}})$ be a collection of q -ROFHNs, $(\forall i = 1, 2, \dots, n)(j = 1, 2, \dots, m)$. If $E_{\sigma a_{ij}} = E_{11}$ are mathematically identical, then q -ROFHWA $(E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) = E$.

Proof. As we know,

$$\begin{aligned}
 & q - ROFHWA (E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \left(1 - \varphi_{\sigma a_{ij}}^q \right)^{w_i} \right)^{v_j}} , 2 \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \left(\tilde{\varphi}_{\sigma a_{(i)(j)}} \right)^{w_i} \right)^{v_j} \right) \\
 & E_{\sigma a_{(i)(j)}} = E_{ij}, \text{ so} \\
 &= \left(\sqrt[q]{1 - \left(\left(1 - \varphi_{\sigma a_{ij}}^q \right)^{\sum_{i=1}^m w_i} \right)^{\sum_{j=1}^n v_j}} , 2 \left(\left(\tilde{\varphi}_{\sigma a_{(i)(j)}} \right)^{\sum_{i=1}^m w_i} \right)^{\sum_{j=1}^n v_j} \right) \\
 &= \left(\sqrt[q]{1 - \left(1 - \varphi_{\sigma a_{ij}}^q \right)} , 2 \left(\tilde{\varphi}_{\sigma a_{(i)(j)}} \right) \right) \\
 &= (\varphi_{a_{ij}}, \tilde{\varphi}_{a_{ij}}) = E
 \end{aligned}$$

(Boundedness) Let $E_{a_{ij}} = (\varphi_{a_{ij}}, \tilde{\varphi}_{a_{ij}})$ be a collection of q -ROFHNs, for all $i = 1, 2, \dots, m, j = 1, 2, \dots, n$. If $E_{\min \min} = \min \min E_{a_{(i)(j)}}$ and $E_{\max \max} = \max \max E_{a_{(i)(j)}}$, then $E_{\min \min} \leq q - ROFHWA$

$(E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{mm}}) \leq E_{\max \max}$, where w_i and v_j are weight vectors for expert's and sub attributes of selected parameters respectively, such that $w_i > 0$, $\sum_{i=1}^m w_i = 1$, and $v_j > 0$, $\sum_{j=1}^n v_j = 1$.

Proof. Let $f(x) = \sqrt[q]{1 - (1 - x^q)}$, $x \in [0, 1]$. Then,

$$\frac{d}{dx} (f(x)) = -qx^{q-1} \left(\sqrt[q]{\left(\frac{1-x^q}{q}\right)} - 1 \right) < 0,$$

so $f(x)$ is a decreasing function on $[0, 1]$. As $\varphi_{\min \min} \leq \varphi_{a_{ij}} \leq \varphi_{\max \max}$, $f(\varphi_{\min, \min_i}) \leq f(\varphi_{a_{ij}}) \leq f(\varphi_{\max, \max_i})$.

$$\begin{aligned} &\Leftrightarrow \sqrt[q]{1 - \max_j \max_i \varphi^q} \leq \sqrt[q]{1 - \varphi_{\sigma a_{ij}}^q} \leq \sqrt[q]{1 - \min_j \min_i \varphi^q} \\ &\Leftrightarrow \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \max_j \max_i \varphi^q \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \varphi_{\sigma a_{ij}}^q \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \min_j \min_i \varphi^q \right)^{w_i} \right)^{v_j}} \\ &\Leftrightarrow \sqrt[q]{\left(\left(1 - \max_j \max_i \varphi^q \right)^{\sum_{i=1}^m w_i} \right)^{\sum_{j=1}^n v_j}} \leq \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \varphi_{\sigma a_{ij}}^q \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{\left(\left(1 - \min_j \min_i \varphi^q \right)^{\sum_{i=1}^m w_i} \right)^{\sum_{j=1}^n v_j}} \\ &\Leftrightarrow \sqrt[q]{-1 + \min_j \min_i \varphi^q} \leq \sqrt[q]{- \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \varphi_{\sigma a_{ij}}^q \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{-1 + \max_j \max_i \varphi^q} \\ &\Leftrightarrow \min_j \min_i (\varphi) \leq \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \varphi_{\sigma a_{ij}}^q \right)^{w_i} \right)^{v_j}} \leq \max_j \max_i (\varphi) \end{aligned} \tag{7}$$

As $\tilde{\varphi}_{\min \min} \leq \tilde{\varphi}_{a_{ij}} \leq \tilde{\varphi}_{\max \max}$, $f(\tilde{\varphi}_{\min, \min_i}) \leq f(\tilde{\varphi}_{a_{ij}}) \leq f(\tilde{\varphi}_{\max, \max_i})$.

$$\begin{aligned} &\Leftrightarrow \sqrt[q]{\min_j \min_i \tilde{\varphi}} \leq \sqrt[q]{\tilde{\varphi}_{\sigma a_{ij}}} \leq \sqrt[q]{\max_j \max_i \tilde{\varphi}} \\ &\Leftrightarrow \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(\min_j \min_i \tilde{\varphi} \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(\tilde{\varphi}_{\sigma a_{ij}} \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(\max_j \max_i \tilde{\varphi} \right)^{w_i} \right)^{v_j}} \\ &\Leftrightarrow \sqrt[q]{\left(\left(\min_j \min_i \tilde{\varphi} \right)^{\sum_{i=1}^m w_i} \right)^{\sum_{j=1}^n v_j}} \leq \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(\tilde{\varphi}_{\sigma a_{ij}} \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{\left(\left(\max_j \max_i \tilde{\varphi} \right)^{\sum_{i=1}^m w_i} \right)^{\sum_{j=1}^n v_j}} \\ &\Leftrightarrow \sqrt[q]{\min_j \min_i \tilde{\varphi}} \leq \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(\tilde{\varphi}_{\sigma a_{ij}} \right)^{w_i} \right)^{v_j}} \leq \sqrt[q]{\max_j \max_i \tilde{\varphi}} \\ &\Leftrightarrow \min_j \min_i (\tilde{\varphi}) \leq \sqrt[q]{\prod_{j=1}^n \left(\prod_{i=1}^m \left(\tilde{\varphi}_{\sigma a_{ij}} \right)^{w_i} \right)^{v_j}} \leq \max_j \max_i (\tilde{\varphi}) \end{aligned} \tag{8}$$

Let $q - ROFHOWA(E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) = E$. Then Eqs. (7) and (8) can be written as $\min_j \min_i (\varphi^q) \leq \varphi^q_{\sigma_{a_{ij}}} \leq \max_j \max_i (\varphi^q)$ and $\min_j \min_i (\tilde{\varphi}) \leq \tilde{\varphi}_{\sigma_{a_{ij}}} \leq \max_j \max_i (\tilde{\varphi})$. Thus,

$$S(H) = \varphi^q - \tilde{\varphi}^q \leq \varphi^q_{\max_j \max_i} - \varphi^q_{\min_j \min_i} = S(H_{\max}) \tag{9}$$

$$S(H) = \varphi^q - \tilde{\varphi}^q \geq \varphi^q_{\min_j \min_i} - \varphi^q_{\max_j \max_i} = S(H_{\min}) \tag{10}$$

If $S(H) < S(H_{\max})$ and $S(H) > S(H_{\min})$, then we have,

$$H_{\min} < qROFHOWA(E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) < H_{\max} \tag{11}$$

If $S(H) = S(H_{\max})$, then we have $\varphi^q = \varphi^q_{\max_j \max_i}$ and $\tilde{\varphi}^q = \tilde{\varphi}^q_{\max_j \max_i}$. Thus $A(H) = \varphi^q + \tilde{\varphi}^q$. Therefore,
 $q - ROFHOWA(E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) < H_{\max}$ (12)

If $S(H) = S(H_{\min})$, then we have $\varphi^q - \tilde{\varphi}^q = \varphi^q_{\min_j \min_i}$ and $\varphi^q = \varphi^q_{\min_j \min_i}$ and $\tilde{\varphi}^q = \tilde{\varphi}^q_{\min_j \min_i}$. Thus
 $A(H) = \varphi^q + \tilde{\varphi}^q = \varphi^q_{\min_j \min_i} + \tilde{\varphi}^q_{\min_j \min_i}$. Therefore,

$$q - ROFHOWA(E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) < H_{\min} \tag{13}$$

So by Eqs. (14) and (15), we get

$$H_{\min} \leq q - ROFHOWA(E_{a_{11}}, E_{a_{12}}, \dots, E_{a_{nm}}) \leq H_{\max} \tag{14}$$

Definition 3.2. An induced q-rung orthopair fuzzy hypersoft ordered weighted averaging (I-q-ROFHSOWA) operator is defined as:

$$\begin{aligned} & I - q - ROFHSOWA(\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}} \rangle) \\ &= \bigoplus_{j=1}^n v_j \left(\bigoplus_{i=1}^m w_i E_{\sigma_{a_{ij}}} \right) \\ &= \left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{l=1}^m (1 - \varphi^q_{\sigma_{a_{ij}}})^{w_l} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{l=1}^m (\tilde{\varphi}_{\sigma_{a_{ij}}})^{w_l} \right)^{v_j} \right) \end{aligned} \tag{15}$$

where $w_i = \{1, 2, \dots, n\}$ and $v_j = \{1, 2, \dots, m\}$ are weight vectors for experts and sub-attributes of selected parameters respectively, having the condition that, $w_i > 0, \sum_{i=1}^m w_i = 1$, and $v_j > 0, \sum_{j=1}^n v_j = 1$. Also $\sigma_{a_{ij}}$ is the σ_{ij} value of the q-ROFHSOWA pair $\langle E_{a_{ij}}, u_{a_{ij}} \rangle$ having the i th and j th largest $u_{a_{ij}}$, and $u_{a_{ij}}$ in $\langle E_{a_{ij}}, u_{a_{ij}} \rangle$ being referred to as the order inducing variable and $E_{a_{ij}}$ as the q-rung orthopair fuzzy hypersoft argument variable.

Theorem 3.6. Let $\langle E_{a_{ij}}, u_{a_{ij}} \rangle (i = 1, 2, \dots, n)(j = 1, 2, \dots, m)$ be a collection of 2 tuples, then their aggregated values by using the I-q-ROFHOWA operator is also a q-rung orthopair fuzzy hypersoft value and

$$\begin{aligned} & I - q - ROFHOWA(\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}} \rangle) \\ &= \left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{l=1}^m (1 - \varphi^q_{\sigma_{a_{ij}}})^{w_l} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{l=1}^m (\tilde{\varphi}_{\sigma_{a_{ij}}})^{w_l} \right)^{v_j} \right) \end{aligned} \tag{16}$$

where $w_i = \{1, 2, \dots, n\}$ and $v_j = \{1, 2, \dots, m\}$ are weight vectors for experts and sub-attributes of selected parameters respectively, having the condition that, $w_i > 0, \sum_{i=1}^m w_i = 1$, and $v_j > 0, \sum_{j=1}^n v_j = 1$.

Proof. The I-q-ROFHWA operator can be proved using the principle of mathematical induction as follows. For $m = 1$, we get $w_i = 1$. Then we have,

$$\begin{aligned}
 & I - q - ROFHWA \left(\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}} \rangle \right) \\
 &= \bigoplus_{j=1}^n v_j E_{\sigma a_{1j}} \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^n (1 - \varphi_{\sigma a_{1j}}^q)^{v_j}}, \prod_{j=1}^n (\tilde{\varphi}_{\sigma a_{1j}})^{v_j} \right) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{l=1}^1 (1 - \varphi_{\sigma a_{1j}}^q)^{w_l} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{l=1}^1 (\tilde{\varphi}_{\sigma a_{1j}})^{w_l} \right)^{v_j} \right) \tag{17}
 \end{aligned}$$

For $n = 1$, we get $v_j = 1$.

$$\begin{aligned}
 & I - q - ROFHWA \left(\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}} \rangle \right) \\
 &= \bigoplus_{i=1}^m w_i E_{\sigma a_{i1}} \\
 &= \left(\sqrt[q]{1 - \prod_{i=1}^m (1 - \varphi_{\sigma a_{i1}}^q)^{w_i}}, \prod_{i=1}^m (\tilde{\varphi}_{\sigma a_{i1}})^{w_i} \right) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^1 \left(\prod_{i=1}^m (1 - \varphi_{\sigma a_{ij}}^q)^{w_i} \right)^{v_j}}, \prod_{j=1}^1 \left(\prod_{i=1}^m (\tilde{\varphi}_{\sigma a_{ij}})^{w_i} \right)^{v_j} \right) \tag{18}
 \end{aligned}$$

So, Eq. (2) is true for $m = 1$ and $n = 1$. Suppose that the equation holds for $m = k_2, n = k_1 + 1$, and $m = k_2 + 1, n = k_1$.

$$\begin{aligned}
 &= \bigoplus_{j=1}^{k_1+1} v_j \left(\bigoplus_{i=1}^{k_2} w_i E_{\sigma a_{ij}} \right) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^{k_1+1} \left(\prod_{l=1}^{k_2} (1 - \varphi_{\sigma a_{ij}}^q)^{w_l} \right)^{v_j}}, \prod_{j=1}^{k_1+1} \left(\prod_{l=1}^{k_2} (\tilde{\varphi}_{\sigma a_{ij}})^{w_l} \right)^{v_j} \right) \tag{19}
 \end{aligned}$$

$$\begin{aligned}
 &= \bigoplus_{j=1}^{k_1} v_j \left(\bigoplus_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}} \right) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^{k_1} \left(\prod_{l=1}^{k_2+1} (1 - \varphi_{\sigma a_{ij}}^q)^{w_l} \right)^{v_j}}, \prod_{j=1}^{k_1} \left(\prod_{l=1}^{k_2+1} (\tilde{\varphi}_{\sigma a_{ij}})^{w_l} \right)^{v_j} \right) \tag{20}
 \end{aligned}$$

Now we prove the equation for $n = k_1 + 1$ and $m = k_2 + 1$.

$$\begin{aligned}
 & \bigoplus_{j=1}^{k_1+1} v_j \left(\bigoplus_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}} \right) \\
 &= \bigoplus_{j=1}^{k_1+1} E_{\sigma a_j} \left(\bigoplus_{i=1}^{k_2} w_i E_{\sigma a_{ij}} \bigoplus_{i=1}^{k_2} w_{i+1} E_{\sigma a_{(k_2+1)j}} \right) \\
 &= \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} (\tilde{\varphi}_{\sigma a_{ij}})^{w_i} \right)^{v_j} \oplus \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} (\tilde{\varphi}_{\sigma a_{(k_2+1)j}})^{w_{k_2+1}} \right)^{v_j} \tag{21} \\
 &= \sqrt[q]{1 - \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} (1 - \varphi_{\sigma a_{ij}}^q)^{w_i} \right)^{v_j}, \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} (\tilde{\varphi}_{\sigma a_{ij}})^{w_i} \right)^{v_j}} \\
 &= \bigoplus_{j=1}^{k_1+1} v_j \left(\bigoplus_{i=1}^{k_2+1} w_i E_{\sigma a_{ij}} \right)
 \end{aligned}$$

So, it is valid for $n = k_1 + 1$ and $m = k_2 + 1$.

Example 3.7. Let Y be the set of decision makers to decide the best laptop given as $Y = \{y_1, y_2, y_3\}$. Also consider the set of attributes as P_1 and P_2 , where P_1 represents the laptop type and P_2 represents the laptop RAM. Then their corresponding attributive sets can be $P_1 = \{a_{11} = HP, a_{12} = Dell\}$, and $P_2 = \{a_{21} = 8 \text{ GB}, a_{22} = 16 \text{ GB}, a_{23} = 32 \text{ GB}\}$. Suppose $A_1 = \{a_{11}, a_{12}\}$, $A_2 = \{a_{21}, a_{22}\}$ and, $\bar{a}_{11} = (a_{11}, a_{21})$, $\bar{a}_{12} = (a_{11}, a_{22})$, $\bar{a}_{21} = (a_{12}, a_{21})$, $\bar{a}_{22} = (a_{12}, a_{22})$. Then $\bar{A} = A_1 \times A_2$ will have 2-tuple elements, and we suppose $q = 8$. Then the I-q-ROFHS set (H, \bar{A}) can be written as $\langle E_{\bar{a}_{11}}, u_{\bar{a}_{11}} \rangle = \langle .3, (.4, .2) \rangle$, $\langle E_{\bar{a}_{12}}, u_{\bar{a}_{12}} \rangle = \langle .4, (.5, .3) \rangle$, $\langle E_{\bar{a}_{21}}, u_{\bar{a}_{21}} \rangle = \langle .6, (.4, .7) \rangle$, $\langle E_{\bar{a}_{22}}, u_{\bar{a}_{22}} \rangle = \langle .5, (.6, .3) \rangle$. Performing the ordering of q-ROFHS pairs with respect to the first component we have $\langle E_{\bar{a}_{21}}, u_{\bar{a}_{21}} \rangle = \langle .6, (.4, .7) \rangle$, $\langle E_{\bar{a}_{22}}, u_{\bar{a}_{22}} \rangle = \langle .5, (.6, .3) \rangle$, $\langle E_{\bar{a}_{12}}, u_{\bar{a}_{12}} \rangle = \langle .4, (.5, .3) \rangle$, $\langle E_{\bar{a}_{11}}, u_{\bar{a}_{11}} \rangle = \langle .3, (.4, .2) \rangle$. This order includes the order of the q-ROFHS arguments: $\langle E_{\sigma \bar{a}_{11}}, u_{\sigma \bar{a}_{11}} \rangle = \langle .6, (.4, .7) \rangle$, $\langle E_{\sigma \bar{a}_{12}}, u_{\sigma \bar{a}_{12}} \rangle = \langle .5, (.6, .3) \rangle$, $\langle E_{\sigma \bar{a}_{21}}, u_{\sigma \bar{a}_{21}} \rangle = \langle .4, (.5, .3) \rangle$, $\langle E_{\sigma \bar{a}_{22}}, u_{\sigma \bar{a}_{22}} \rangle = \langle .3, (.4, .2) \rangle$. Let $w_i = (.2, .1, .4, .3)$, $v_j = (.6, .1, .3)$, and $q = 8$.

$$\begin{aligned}
 & I - q - \text{ROFHWA} (\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \langle E_{a_{21}}, u_{a_{21}} \rangle, \langle E_{a_{22}}, u_{a_{22}} \rangle) \\
 &= \left(\sqrt[q]{1 - \prod_{j=1}^3 \left(\prod_{i=1}^4 (1 - \varphi_{\sigma a_{ij}}^q)^{w_i} \right)^{v_j}, \prod_{j=1}^3 \left(\prod_{i=1}^4 (\tilde{\varphi}_{\sigma a_{ij}})^{w_i} \right)^{v_j}} \right) \\
 &= \left(\sqrt[q]{\begin{aligned} & 1 - \left((1 - .7^8)^2 \right) \times \left((1 - .6^8)^1 \right) \times \left((1 - .5^8)^4 \right) \times \left((1 - .4^8)^3 \right)^{.6} \\ & \times \left((1 - .4^8)^2 \right) \times \left((1 - .6^8)^1 \right) \times \left((1 - .5^8)^4 \right) \times \left((1 - .4^8)^3 \right)^{.1} \\ & \times \left((1 - .7^8)^2 \right) \times \left((1 - .6^8)^1 \right) \times \left((1 - .5^8)^4 \right) \times \left((1 - .4^8)^3 \right)^{.3} \end{aligned}}, \begin{aligned} & (.7^2 \times .3^1 \times .3^4 \times .2^3)^{.6} \\ & \times (.7^2 \times .3^1 \times .3^4 \times .2^3)^{.1} \\ & \times (.7^2 \times .3^1 \times .3^4 \times .2^3)^{.3} \end{aligned}} \right) \\
 &= (.5099, .3149)
 \end{aligned}$$

The I-q-ROFHSA operator is idempotent, bounded and monotonic. These properties can be proved by the following theorems:

(Idempotency) Let $\langle E_{a_{ij}}, u_{a_{ij}} \rangle (i = 1, 2, \dots, n)(j = 1, 2, \dots, m)$ be a collection of 2-tuples I-q-ROFHSA. If $\langle E_{\sigma a_{ij}}, u_{\sigma a_{ij}} \rangle = \langle E_{\sigma a_{11}}, u_{\sigma a_{11}} \rangle$ are mathematically identical, then,

$$I - q - ROFHSA (\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}} \rangle) = \left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m (1 - \varphi_{\sigma a_{ij}}^q)^{w_i} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{i=1}^m (\tilde{\varphi}_{\sigma a_{ij}})^{w_i} \right)^{v_j} \right) \tag{22}$$

where $w_i = \{1, 2, \dots, n\}$ and $v_j = \{1, 2, \dots, m\}$ are weight vectors for experts and sub-attributes of selected parameters, respectively, having the condition that, $w_i > 0, \sum_{i=1}^m w_i = 1$, and $v_j > 0, \sum_{j=1}^n v_j = 1$.

- i. Let $E_{a_{ij}} = (\varphi_{a_{ij}}, \tilde{\varphi}_{a_{ij}})$ be a collection of I-q-ROFHSA, for all $i = 1, 2, \dots, m, j = 1, 2, \dots, n$. If $E_{\sigma a_{ij}} = E_{a_{11}}$ are mathematically identical, then for all $i = 1, 2, \dots, m, j = 1, 2, \dots, n$, then $I - q - ROFHSA (\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}} \rangle) = u$.

(Boundedness) Let $\langle E_{a_{ij}}, u_{a_{ij}} \rangle$ be a collection of I-q-ROFHSA, (for all $i = 1, 2, \dots, m, j = 1, 2, \dots, n$. If $E_{\min \min} = \min \min E_{a_{ij}}$ and $E_{\max \max} = \max \max E_{a_{ij}}$, then $E_{\min \min} \leq I - q - ROFHSA (\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}} \rangle) \leq E_{\max \max}$.

(Monotonicity) Let $\langle E_{a_{ij}}, u_{a_{ij}} \rangle, \langle E_{a_{ij}}, u_{a_{ij}}^* \rangle$ for all $i = 1, 2, \dots, m, j = 1, 2, \dots, n$ be two sets of 2-tuples. If $u_{a_{ij}} \leq u_{a_{ij}}^*$, then $I - q - OFHSA (\langle E_{a_{11}}, u_{a_{11}} \rangle, \langle E_{a_{12}}, u_{a_{12}} \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}} \rangle) \leq I - q - ROFHSA (\langle E_{a_{11}}, u_{a_{11}}^* \rangle, \langle E_{a_{12}}, u_{a_{12}}^* \rangle, \dots, \langle E_{a_{nm}}, u_{a_{nm}}^* \rangle)$.

4 Decision Making with q-ROFHSA I-q-ROFHSA Operator

Here we will analyze the applicability of the q-ROFHSA and I-q-ROFHSA operators and show their fundamental aspects. As mentioned earlier, the q-ROFHSA operator provides a parameterized family of aggregation functions, but the I-q-ROFHSA operator has the fundamental aspect of reordering the arguments to be aggregated, based on the order inducing variable. Also, the inducing order determines the generation of q-ROF measures. The following part provides an illustration of the above-mentioned innovative approaches. We analyze a group decision-making problem where an ordinary person studies the real-estate and decides which real estate developer's project is the best and worth investing. Also, they want to know about the soundness and reliability of real-estate developers. So, in this case, we want to reveal some of the attributes of the real estate before investing in a project. We choose four agents (alternatives) of different companies of real estate, which are represented as follows:

1. Agent 1 (A_1 , Estate Land Marketing)
2. Agent 2 (A_2 , Star Marketing)
3. Agent 3 (A_3 , ZEM builders)
4. Agent 4 (A_4 , Actual real estate)

After careful review of the information, the group of experts establishes the following information about investment in a real estate project. The following are some attributes of real estate from different companies agents:

$U = \{\text{Ability to deliver, Property location, Valuation of the property}\}$

We also consider their respective attributes, i.e.,

$U_1 = \text{Ability to deliver} = \{\text{Strong team } (a_{11}), \text{ Financial experts } (a_{12}), \text{ Incredible architects and urban planning engineers to design the project } (a_{13})\}$

$U_2 = \text{Property location} = \{\text{Closeness to the market } (a_{21}), \text{ transport hubs } (a_{22})\}$

$U_3 = \text{Valuation of the property} = \{\text{Investment analysis } (a_{31}), \text{ listing price } (a_{32})\}$

A committee of decision makers has a weight vector $(.32, .12, .36, 20)$, and thus $\acute{U} = U_1 \times U_2 \times U_3$ is a set of sub attributes having 3 tuples.

$$\acute{U} = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}\} \times \{a_{31}, a_{32}\} = \left\{ (a_{11}, a_{21}, a_{31}), (a_{11}, a_{22}, a_{31}), (a_{12}, a_{21}, a_{31}), (a_{12}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), (a_{11}, a_{21}, a_{32}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{32}) \right\}, \text{ and}$$

$\acute{U} = \{\acute{a}_1, \acute{a}_2, \acute{a}_3, \acute{a}_4, \acute{a}_5, \acute{a}_6, \acute{a}_7, \acute{a}_8\}$ is a set of all sub-attributes with the weights $(.18, .12, .15, .20, .16, .05, .10, .04)$, respectively. As the environment is very uncertain, the group of experts of the company needs to assess the available information with q-ROFHS numbers. In this problem, we will use the q-ROFHWA and I-q-ROFHWA operators. First, we construct a decision matrix of sub-attributes in Table 1, and then we show that the team of experts present their idea in the form of an induced q-rung orthopair fuzzy hypersoft number. They also use the attitudinal character of the expert team to show the order-inducing variables, which are shown in Tables 2–5. Then, in Table 7, we can make an aggregated value by using I-q-ROFHWA operator, which is shown in Table 6. In this example, we use the I-q-ROFHWA operator for choosing the best and the worth-ranking order of different alternatives. Finally, X_2 is the best alternative, and the ranking order is shown as: $X_2 > X_4 > X_3 > X_1$.

$X_1 = (.7859, .4991), X_2 = (.8700, .5999), X_3 = (.8530, .6671), X_4 = (.8663, .5960)$

Table 1: Decision matrix of the experts

| | \acute{a}_1 | \acute{a}_2 | \acute{a}_3 | \acute{a}_4 | \acute{a}_5 | \acute{a}_6 | \acute{a}_7 | \acute{a}_8 |
|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| X_1 | (.99, .87) | (.89, .99) | (.94, .87) | (.93, .87) | (.99, .87) | (.95, .87) | (.84, .87) | (.94, .87) |
| X_2 | (.87, .89) | (.86, .88) | (.89, .79) | (.87, .85) | (.94, .79) | (.89, .99) | (.99, .79) | (.85, .76) |
| X_3 | (.79, .88) | (.89, .98) | (.85, .98) | (.92, .88) | (.86, .94) | (.86, .94) | (.83, .86) | (.93, .79) |
| X_4 | (.88, .97) | (.88, .99) | (.86, .92) | (.88, .93) | (.88, .94) | (.99, .90) | (.88, .96) | (.89, .77) |

Table 2: Decision matrix (A_1)

| | \acute{a}_1 | \acute{a}_2 | \acute{a}_3 | \acute{a}_4 | \acute{a}_5 | \acute{a}_6 | \acute{a}_7 | \acute{a}_8 |
|-------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| X_1 | $\langle .8, (.79, .87) \rangle$ | $\langle .9, (.89, .99) \rangle$ | $\langle .5, (.94, .87) \rangle$ | $\langle .7, (.95, .87) \rangle$ | $\langle .4, (.94, .81) \rangle$ | $\langle .8, (.91, .87) \rangle$ | $\langle .5, (.84, .84) \rangle$ | $\langle .9, (.94, .84) \rangle$ |
| X_2 | $\langle .5, (.77, .89) \rangle$ | $\langle .6, (.88, .88) \rangle$ | $\langle .4, (.89, .79) \rangle$ | $\langle .6, (.87, .85) \rangle$ | $\langle .3, (.94, .79) \rangle$ | $\langle .7, (.89, .92) \rangle$ | $\langle .4, (.92, .99) \rangle$ | $\langle .7, (.86, .76) \rangle$ |
| X_3 | $\langle .4, (.79, .87) \rangle$ | $\langle .2, (.89, .93) \rangle$ | $\langle .3, (.85, .98) \rangle$ | $\langle .5, (.90, .78) \rangle$ | $\langle .2, (.86, .94) \rangle$ | $\langle .5, (.89, .94) \rangle$ | $\langle .3, (.86, .86) \rangle$ | $\langle .6, (.90, .74) \rangle$ |
| X_4 | $\langle .2, (.88, .96) \rangle$ | $\langle .1, (.89, .79) \rangle$ | $\langle .2, (.86, .92) \rangle$ | $\langle .4, (.84, .90) \rangle$ | $\langle .1, (.88, .94) \rangle$ | $\langle .3, (.93, .90) \rangle$ | $\langle .2, (.88, .91) \rangle$ | $\langle .3, (.89, .77) \rangle$ |

Table 3: Decision matrix (A_2)

| | \acute{a}_1 | \acute{a}_2 | \acute{a}_3 | \acute{a}_4 | \acute{a}_5 | \acute{a}_6 | \acute{a}_7 | \acute{a}_8 |
|-------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| X_1 | $\langle .5, (.94, .87) \rangle$ | $\langle .6, (.84, .84) \rangle$ | $\langle .8, (.94, .81) \rangle$ | $\langle .6, (.95, .77) \rangle$ | $\langle .8, (.94, .81) \rangle$ | $\langle .5, (.94, .87) \rangle$ | $\langle .4, (.88, .84) \rangle$ | $\langle .7, (.94, .87) \rangle$ |
| X_2 | $\langle .4, (.89, .79) \rangle$ | $\langle .5, (.92, .99) \rangle$ | $\langle .6, (.94, .99) \rangle$ | $\langle .5, (.81, .95) \rangle$ | $\langle .7, (.94, .77) \rangle$ | $\langle .4, (.89, .79) \rangle$ | $\langle .3, (.90, .91) \rangle$ | $\langle .6, (.89, .79) \rangle$ |
| X_3 | $\langle .3, (.85, .98) \rangle$ | $\langle .4, (.86, .86) \rangle$ | $\langle .4, (.86, .94) \rangle$ | $\langle .4, (.79, .79) \rangle$ | $\langle .6, (.86, .74) \rangle$ | $\langle .3, (.85, .98) \rangle$ | $\langle .2, (.83, .87) \rangle$ | $\langle .5, (.85, .98) \rangle$ |
| X_4 | $\langle .1, (.96, .92) \rangle$ | $\langle .2, (.88, .91) \rangle$ | $\langle .2, (.87, .94) \rangle$ | $\langle .3, (.74, .91) \rangle$ | $\langle .2, (.88, .94) \rangle$ | $\langle .1, (.96, .92) \rangle$ | $\langle .1, (.81, .98) \rangle$ | $\langle .4, (.96, .92) \rangle$ |

Table 4: Decision matrix (A_3)

| | \acute{a}_1 | \acute{a}_2 | \acute{a}_3 | \acute{a}_4 | \acute{a}_5 | \acute{a}_6 | \acute{a}_7 | \acute{a}_8 |
|-------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| X_1 | $\langle .4, (.95, .87) \rangle$ | $\langle .9, (.84, .84) \rangle$ | $\langle .9, (.94, .87) \rangle$ | $\langle .7, (.95, .77) \rangle$ | $\langle .4, (.94, .81) \rangle$ | $\langle .5, (.71, .87) \rangle$ | $\langle .7, (.94, .86) \rangle$ | $\langle .7, (.89, .99) \rangle$ |
| X_2 | $\langle .3, (.87, .85) \rangle$ | $\langle .8, (.92, .99) \rangle$ | $\langle .8, (.89, .79) \rangle$ | $\langle .5, (.77, .83) \rangle$ | $\langle .3, (.94, .79) \rangle$ | $\langle .4, (.89, .90) \rangle$ | $\langle .6, (.79, .70) \rangle$ | $\langle .3, (.88, .88) \rangle$ |
| X_3 | $\langle .2, (.90, .78) \rangle$ | $\langle .7, (.86, .86) \rangle$ | $\langle .5, (.85, .98) \rangle$ | $\langle .4, (.93, .77) \rangle$ | $\langle .2, (.86, .94) \rangle$ | $\langle .3, (.99, .74) \rangle$ | $\langle .4, (.75, .90) \rangle$ | $\langle .2, (.89, .93) \rangle$ |
| X_4 | $\langle .1, (.84, .90) \rangle$ | $\langle .1, (.88, .91) \rangle$ | $\langle .4, (.86, .92) \rangle$ | $\langle .3, (.89, .79) \rangle$ | $\langle .1, (.88, .94) \rangle$ | $\langle .2, (.73, .91) \rangle$ | $\langle .3, (.96, .82) \rangle$ | $\langle .1, (.89, .79) \rangle$ |

Table 5: Decision matrix (A_4)

| | \acute{a}_1 | \acute{a}_2 | \acute{a}_3 | \acute{a}_4 | \acute{a}_5 | \acute{a}_6 | \acute{a}_7 | \acute{a}_8 |
|-------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|-----------------------------------|----------------------------------|----------------------------------|
| X_1 | $\langle .4, (.94, .97) \rangle$ | $\langle .8, (.74, .84) \rangle$ | $\langle .7, (.92, .81) \rangle$ | $\langle .9, (.95, .73) \rangle$ | $\langle .4, (.94, .81) \rangle$ | $\langle .8, (.91, .87) \rangle$ | $\langle .9, (.88, .85) \rangle$ | $\langle .6, (.94, .87) \rangle$ |
| X_2 | $\langle .3, (.89, .79) \rangle$ | $\langle .6, (.90, .99) \rangle$ | $\langle .4, (.97, .89) \rangle$ | $\langle .4, (.87, .95) \rangle$ | $\langle .3, (.93, .77) \rangle$ | $\langle .5, (.80, .79) \rangle$ | $\langle .8, (.95, .91) \rangle$ | $\langle .5, (.81, .79) \rangle$ |
| X_3 | $\langle .2, (.75, .98) \rangle$ | $\langle .5, (.86, .96) \rangle$ | $\langle .2, (.86, .74) \rangle$ | $\langle .3, (.99, .71) \rangle$ | $\langle .2, (.87, .74) \rangle$ | $\langle .3, (.88, .91) \rangle$ | $\langle .7, (.93, .77) \rangle$ | $\langle .4, (.95, .88) \rangle$ |
| X_4 | $\langle .1, (.96, .82) \rangle$ | $\langle .4, (.88, .71) \rangle$ | $\langle .1, (.87, .90) \rangle$ | $\langle .2, (.78, .91) \rangle$ | $\langle .1, (.82, .94) \rangle$ | $\langle .2, (.86, .072) \rangle$ | $\langle .5, (.86, .78) \rangle$ | $\langle .1, (.76, .90) \rangle$ |

Table 6: Aggregated values

| | \acute{a}_1 | \acute{a}_2 | \acute{a}_3 | \acute{a}_4 | \acute{a}_5 | \acute{a}_6 | \acute{a}_7 | \acute{a}_8 |
|-------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| X_1 | (0.979, .87) | (.922, .677) | (.840, .991) | (.569, .911) | (.692, .891) | (.785, .832) | (.764, .800) | (.881, .766) |
| X_2 | (.887, .689) | (.699, .976) | (.980, .681) | (.704, .775) | (.807, .753) | (.656, .900) | (.893, .712) | (.764, .569) |
| X_3 | (.979, .808) | (.854, .98) | (.776, .793) | (.599, .903) | (.920, .784) | (.744, .688) | (.745, .923) | (.549, .855) |
| X_4 | (.818, .907) | (.791, .695) | (.692, .852) | (.764, .835) | (.751, .813) | (.821, .671) | (.739, .787) | (.558, .801) |

Table 7: Comparison analysis

| Methods | Score values | Ranking |
|--------------|--|-------------------------------------|
| q-ROFWA | $x_1 = 0.3015, x_2 = 0.1184, x_3 = 0.1610, x_4 = 0.1791$ | $X_1 \succ X_4 \succ X_3 \succ X_2$ |
| q-ROFWG | $x_1 = 0.1701, x_2 = 0.0499, x_3 = 0.0164, x_4 = 0.1349$ | $X_1 \succ X_4 \succ X_3 \succ X_2$ |
| q-ROFSWA | $x_1 = 0.414877, x_2 = 0.46537, x_3 = 0.522354, x_4 = 0.484856$ | $X_3 \succ X_4 \succ X_2 \succ X_1$ |
| q-ROFSOWA | $x_1 = 0.426939, x_2 = 0.475573, x_3 = 0.521928, x_4 = 0.483572$ | $X_3 \succ X_4 \succ X_2 \succ X_1$ |
| I-q-ROFHSOWA | $x_1 = 0.268, x_2 = 0.420,$ $x_3 = 0.319, x_4 = 0.413$ | $X_2 \succ X_4 \succ X_3 \succ X_1$ |

Calculate the score value by using Definition 2.3, which is $X_1 = 0.268, X_2 = 0.420, X_3 = 0.319, X_4 = 0.413$.

5 Comparison Analysis

In this section, some current techniques under the existing structures, namely q-ROFS, q-ROFSS, and our proposed structure of q-ROFHSS have been compared in order to show the effectiveness of the proposed model. We have developed the MADM technique under the q-ROFHSOWA and I-q-ROFHSOWA operators. Our proposed technique can explain the most suitable results of DM problems. Since the proposed model is competent and adaptable, it can continue working hard despite any disruptions that may occur. The results obtained from the proposed techniques are different as compared to hybrid techniques. The results show that the operator of the proposed structure is more capable, reliable and effective. Fig. 1 shows a graphical ranking order of the alternatives to our proposed technique compared to the existing techniques.

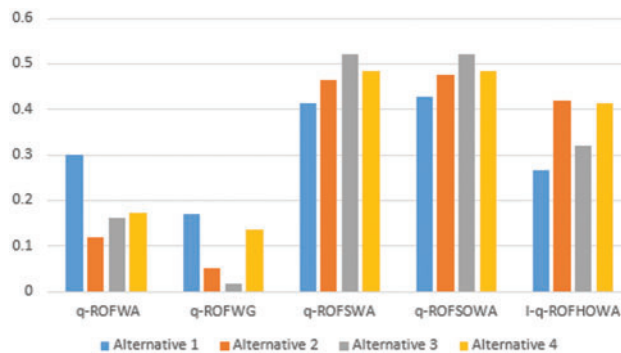


Figure 1: Graphical representation of existing studies with proposed study

6 Conclusion

In this study, we developed a new decision-making technique under the q-rung orthopair fuzzy hypersoft environment with the induced ordered weighted averaging operator. We have initiated two types of operators i.e., ordered weighted averaging operator and induced ordered weighted averaging operator. The induced ordered weighted averaging operator is very helpful in decision-making problems with q-rung orthopair fuzzy hypersoft numbers under uncertain conditions. We have also studied some of its basic results and properties. All attribute values are considered by the method described in this article for selecting a real estate project based on the computation of aggregation operators from considered data in the form of membership and non-membership. Finally, we developed an application in an MADM problem. In this problem, we discussed the selection of the best real-estate project for investment. The induced ordered weighted averaging operator is very useful for this problem because this operator represents the uncertain data in the best form under q-rung orthopair fuzzy hypersoft numbers. This decision-making technique proves the information in the best way because in this process the decision-maker uses the order-inducing strategy. The main feature of the suggested strategy is that it offers a useful and adaptable way to effectively help the DM in uncertain situations. In addition, the proposed method can be used both for q-rung orthopair fuzzy hypersoft numbers and q-rung orthopair fuzzy soft numbers, which accurately represent the uncertainty. We plan to use the aggregation operator to solve DM problems in the future when dealing with more ambiguous data, including interval-valued q-rung orthopair fuzzy hypersoft sets, and interval-valued q-rung orthopair fuzzy hypersoft linguistic term sets.

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