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ARTICLE

Stochastic Analysis for the Dynamics of a Poliovirus Epidemic Model

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ABSTRACT

Most developing countries such as Afghanistan, Pakistan, India, Bangladesh, and many more are still fighting against poliovirus. According to the World Health Organization, approximately eighteen million people have been infected with poliovirus in the last two decades. In Asia, still, some countries are suffering from the virus. The stochastic behavior of the poliovirus through the transition probabilities and non-parametric perturbation with fundamental properties are studied. Some basic properties of the deterministic model are studied, equilibria, local stability around the stead states, and reproduction number. Euler Maruyama, stochastic Euler, and stochastic Runge-Kutta study the behavior of complex stochastic differential equations. The main target of this study is to develop a nonstandard computational method that restores dynamical features like positivity, boundedness, and dynamical consistency. Unfortunately, the existing methods failed to fix the actual behavior of the disease. The comparison of the proposed approach with existing methods is investigated.

KEYWORDS

Poliovirus model; differential equations; methods; analysis

1 Introduction

Jenkins et al. in 2006 formulated a model in which he concluded that the use of monovalent is better than other vaccines. It provides outstanding outbreak control [1]. Haldar et al. [2] introduced the poliovirus vaccine in India. Kalkowska et al. in 2020 represent a differential equation-based

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stochastic model for poliovirus transmission. The model shows the poliovirus transmission for 2019 to 2023 with a strategic eradication plan [3]. Minor studied the types of polioviruses, vaccination, and eradication of the virus worldwide [4]. Thompson [5] investigated the transmission dynamics of the poliovirus in Nigeria. Duque-Marin et al. [6] studied two types of vaccines in the mathematical model. Denes et al. [7] presented a model which describes polio transmission in tropical regions. Cheng et al. [8] discussed a polio vaccination model in two different age classes. Alba et al. [9] addressed the correlation between climate and vaccination through a mathematical model. Shaghaghi et al. in 2018, studied that the OPV and IVPPvs vaccine was helpful for the eradication of the virus last few years [10]. Shimizu in 2014 [11] explained IPV is very effective against the poliovirus, and the author reviewed the introduction, development, and characterization of the OPV vaccine. In addition, his place in the world was told. Rafique et al. in 2020 presented a mathematical model in which they discovered the dynamics of poliovirus transmission using standard methods with vaccination [12]. Nidia et al. [13] in 2007, examined the effects of the poliovirus on human life and the steps taken to eradicate the virus and discussed what steps we could take in the future to get rid of it. Thompson et al. [14] presented polio outbreaks in the USA. Kalkowska et al. introduced a model to identify poliovirus and opportunities to increase population immunity [15]. Kim et al. [16] presented a model to examine the transmission of virulent circulating vaccine-derived polioviruses. Hillis [17] formulated a model in different regions before using artificial poliovirus vaccination. Mendrazitsky et al. [18] explained a disease of epidemic development model. The model analyzed other properties of polio and its non-equilibrium outbreak dynamics. Debanne et al. [19] presented a mathematical model of poliovirus in America. Naik et al. [20,21] studied the fractional modeling of cancer and HIV infection with the well-known results of stabilities.

The strategy of the paper is as follows. The first section goes to literature, and Section 2 goes to stochastic modeling of poliovirus and its fundamental properties. Section 3 goes to the proposed numerical method and its simulation with current approaches in the literature. Section 4 goes to the paper's conclusion and remarks.

2 Poliovirus Model

For any time t, S: represents the class that is influenced by infection, E: represents the class that is disclosed by infection, I: represents an infective class, V: represents immunization class, A: represents the constant immigration rate of the human population. β: is the per unit time probability of infection transmission by the infective population. r: is the reduction in the exposed class due to transmission of infection. v: represents the proportion of recruits in the susceptible class moving to the vaccinated class, v_1: is the number of vaccinated exposed populations, b: number of exposed populations moving to the infection class. μ: natural death of the human population, α: disease death rate. The first order, nonlinear, and coupled ordinary differential equations of the poliovirus epidemic model are assumed as follows:

$$\mathrm{dS} = (\mathrm{A} - \beta \mathrm{SI} - \mathrm{r}\beta \mathrm{SE} - (\mu + \mathrm{v})\,\mathrm{S})\,\mathrm{dt} + \sigma_1 S dB(t). \tag{1}$$

$$\mathrm{dE} = (\beta \mathrm{SI} + \mathrm{r}\beta \mathrm{SE} - (\mathrm{b} + \mu + \mathrm{v}_1)\,\mathrm{E})\,\mathrm{dt} + \sigma_2 E dB(t). \tag{2}$$

$$dI = ((\mathrm{b} + \mathrm{v}_1)\,\mathrm{E} - (\mu + \alpha)\,\mathrm{I})\,\mathrm{dt} + \sigma_3 I dB(t). \tag{3}$$

$$\mathrm{dV} = (\mathrm{vS} - \mu \mathrm{V})\,\mathrm{dt} + \sigma_4 V dB(t). \tag{4}$$

with initial condition S (0) ≥ 0; E (0) ≥ 0; I (0) ≥ 0; V (0) ≥ 0, and (σ_i: $i = 1, 2, 3, 4$) is the peutrbation term with $B(t)$ is the Brownian motion [22,23].

2.1 Properties [24]

This section studies the positivity and boundedness of the system ((1)–(4)). Let us consider the vector as follows:

$$U(t) = (S(t), E(t), I(t), V(t)). \tag{5}$$

And the norm $|U(t)| = \sqrt{S^2(t) + E^2(t) + I^2(t) + V^2(t)}$.

$$dU(t) = H(U, t)\,dt + K(U, t)\,dW(t). \tag{6}$$

As, $L = \frac{\partial}{\partial t} + \sum_{i=1}^{4} H_i(U, t)\frac{\partial}{\partial u_i} + \frac{1}{2}\sum_{i,j=1}^{4}\left(K^T(U, t)\,K(U, t)\right)_{i,j} \times \frac{\partial^2}{\partial U_i \partial U_j}$, where L is differential operator.

If L acts on a function $V_1 \in C^{2,1}(R^4 \times (0, \infty); R_+)$ then we denote

$$LV(U, t) = V_{1_t}(U, t) + V_{1U}(U, t)\,H(U, t) + \frac{1}{2}Trace\left(K^T(U, t)\,V_{UU}(U, t)\,K(U, t)\right).$$

where transportation is denoted by T.

Theorem 1: For model ((1)–(4)) and any given initial value $(S(0), E(0), I(0), V(0)) \in R_+^4$, there is a unique solution $(S(t), E(t), I(t), V(t))$ on $t \geq 0$ and will remain in R_+^4 with probability one.

Proof: By Ito's formula, the model ((1)–(4)) admits a positive solution in the unique local on $[0, \tau_e]$, and explosion time is denoted by τ_e. Because the local Lipschitz condition is satisfied by all the coefficients of the model as mentioned earlier.

Next, let us show that the given model ((1)–(4)) admits this solution in the global sense; that is, $\tau_e = \infty$ almost sure.

Let $m_o = 0$ be sufficiently large for $S(0), E(0), I(0)$, and $V(0)$ lying with the interval $\left[\frac{1}{m_o}, m_o\right]$. For each integer $m \geq m_o$, define a sequence that is so-called stopping times as

$$\tau_m = \inf\left\{t \epsilon [0, \tau_e]:\ S(t) \notin \left(\frac{1}{m}, m\right) or\ E(t) \notin \left(\frac{1}{m}, m\right) or\ I(t) \notin \left(\frac{1}{m}, m\right) or\ V(t) \notin \left(\frac{1}{m}, m\right)\right\}. \tag{7}$$

where we set $inf\,\phi = \infty$(ϕ represents the empty set). Since τ_m is non-decreasing as $m \to \infty$,

$$\tau_\infty = \lim_{m\to\infty} \tau_m. \tag{8}$$

Then $\tau_\infty \leq \tau_e$. To prove, $\tau_\infty = \infty$.

In case of violation of statement, then $T > 0$ and $\varepsilon\ \epsilon(0, 1)$ such that

$$P\{\tau_\infty \leq T\} > \varepsilon. \tag{9}$$

this, there is an integer $m_1 > m_0$ such that

$$P\{\tau_m \leq T\} \geq \varepsilon, \quad \forall m \geq m_1. \tag{10}$$

Define a C^4- function $V: R_+^4 \to R_+$ by

$$V_1(S,E,I,V) = (S - 1 - \ln S) + (E - 1 - \ln E) + (I - 1 - \ln I) + (V - 1 - \ln V). \quad (11)$$

By using Ito's formula, we calculate

$$dV_1(S,E,I,V) = \left(1 - \frac{1}{S}\right) dS + \left(1 - \frac{1}{E}\right) dE + \left(1 - \frac{1}{I}\right) dI + \left(1 - \frac{1}{V}\right) dV + \frac{\sigma^2}{2} dt.$$

$$\begin{aligned} dV_1(S,I,R) = &\left(1 - \frac{1}{S}\right)((\mathrm{A} - \beta \mathrm{SI} - \mathrm{r}\beta \mathrm{SE} - (\mu + \mathrm{v})\,\mathrm{S})\,\mathrm{dt} + \sigma_1 S dB(t)) \\ &+ \left(1 - \frac{1}{E}\right) \times ((\beta \mathrm{SI} + \mathrm{r}\beta \mathrm{SE} - (\mathrm{b} + \mu + \mathrm{v}_1)\,\mathrm{E})\,\mathrm{dt} + \sigma_2 E dB(t)) \\ &+ \left(1 - \frac{1}{I}\right) \times (((\mathrm{b} + \mathrm{v}_1)\mathrm{E} - (\mu + \alpha)\mathrm{I})\mathrm{dt} + \sigma_3 I dB(t)) \\ &+ \left(1 - \frac{1}{R}\right) [(\mathrm{vS} - \mu \mathrm{V})\mathrm{dt} + \sigma_4 V dB(t)] dt + \frac{\sigma^2}{2} dt. \end{aligned}$$

$$dV_1(S,E,I,V) \le \left[A + \mu + v + \alpha + \frac{\sigma^2}{2}\right] dt + \sigma_1 S dB(t) + +\sigma_2 E dB(t) + \sigma_3 I dB(t) + +\sigma_4 V dB(t). \quad (12)$$

Let, $N = A + \mu + v + \alpha + \frac{\sigma^2}{2}$, Then Eq. (12) could be written as

$$dV_1(S,E,I,V) \le N dt + \sigma_1 S dB(t) + +\sigma_2 E dB(t) + \sigma_3 I dB(t) + +\sigma_4 V dB(t). \quad (13)$$

By integrating from 0 to $\tau_m \wedge \tau$, we get

$$\int_0^{\tau m \wedge \tau} dV_1(S(s), E(s), I(s), V(s)) \le \int_0^{\tau m \wedge \tau} N ds + \int_0^{\tau m \wedge \tau} (\sigma_1 S + \sigma_2 E + \sigma_3 I + \sigma_4 V) dB(s). \quad (14)$$

where $\tau_m \wedge \tau = \min(\tau_m, T)$, the taking the expectations to lead to

$$EV_1(S(\tau_m \wedge \tau), I(\tau_m \wedge \tau), E(\tau_m \wedge \tau), V(\tau_m \wedge \tau)) \le V(S(0), E(0), I(0), V(0)) + NT \quad (15)$$

Set $\Omega_m = \{\tau_m \le T\}$ for $m > m_1$, and from (15), we have $P(\Omega_m \ge \varepsilon)$. For every $v \in \Omega_m$ there are some i such that $u_i(\tau_m, v)$ equals either m or $\frac{1}{m}$ for $i = 1, 2, 3$;

Hence, $V_1(S(\tau_m, v), E(\tau_m, v), I(\tau_m, v), V(\tau_m, v))$ is less than $\min\left\{m - 1 - \ln m, \frac{1}{m} - 1 - \ln \frac{1}{m}\right\}$.

Then we obtain

$$\begin{aligned} V_1(S(0), E(0), I(0), V(0)) + NT &\ge E\left(I_{\Omega m(v)} V_1(S(\tau_m), E(\tau_m), I(\tau_m), V(\tau_m))\right) \\ &\ge \left\{\min\left(m - 1 - \ln m, \frac{1}{m} - 1 - \ln \frac{1}{m}\right)\right\}. \end{aligned} \quad (16)$$

$I_{\Omega_{m(v)}}$ of Ω_m represents the indicator function. Letting $m \to \infty$ leads to the contradiction $\infty = V_1(S(0), E(0), I(0), V(0)) + NT < \infty$.

As desired.

2.2 Equilibria of Model

The disease-free equilibrium of the model is $K_0 = \left(\frac{A}{\mu + v}, 0, 0, \frac{v}{\mu} S\right)$.

The endemic equilibrium of the model is denoted by $K_1 = (S^*, E^*, I^*, V^*)$.

$S^* = \frac{(b + \mu + v_1)(\mu + \alpha)}{\beta(b + v_1) + r\beta(\mu + \alpha)}$, $E^* = \frac{A - (\mu + v) S^*}{r\beta S^* + \beta S^* \left(\frac{b+v_1}{\mu+\alpha}\right)}$, $I^* = \frac{b + v_1}{\mu + \alpha} E^*$, and $V^* = \frac{v}{\mu} S^*$.

2.3 Local Stability of Model

Theorem 2: The disease-free equilibrium $K_0 = \left(\frac{A}{\mu + v}, 0, 0, \frac{v}{\mu} S\right)$ is locally asymptotically stable if $R_0 < 1$; otherwise, unstable if $R_0 > 1$.

Proof: Considering the function from the system ((1)–(4)) as follows:

$F_1 = A - \beta SI - r\beta SE - (\mu + v) S.$

$F_2 = \beta SI + r\beta SE - (b + \mu + v_1) E.$

$F_3 = (b + v_1) E - (\mu + \alpha) I.$

$F_4 = vS - \mu V.$

The elements of the given Jacobean Matrix at K_O is as follows:

$$J(K_O) = \begin{bmatrix} -(\mu + v) & -r\beta \frac{A}{\mu + V} & -\beta \frac{A}{\mu + V} & 0 \\ 0 & r\beta \frac{A}{\mu + V} - (b + \mu + v_1) & \beta \frac{A}{\mu + V} & 0 \\ 0 & b + v_1 & -(\mu + \alpha) & 0 \\ V & 0 & 0 & -\mu \end{bmatrix}.$$

$$J(K_O - \lambda I) = \begin{vmatrix} -(\mu + v) - \lambda & -r\beta \frac{A}{\mu + V} & -\beta \frac{A}{\mu + V} & 0 \\ 0 & r\beta \frac{A}{\mu + V} - (b + \mu + v_1) - \lambda & \beta \frac{A}{\mu + V} & 0 \\ 0 & b + v_1 & -(\mu + \alpha) - \lambda & 0 \\ V & 0 & 0 & -\mu - \lambda \end{vmatrix} = 0.$$

$$-\left(-\mu-\lambda\right)\begin{vmatrix}-\left(\mu+v\right)-\lambda & -r\beta\dfrac{A}{\mu+V} & -\beta\dfrac{A}{\mu+V}\\ 0 & r\beta\dfrac{A}{\mu+V}-\left(b+\mu+v_1\right)-\lambda & \beta\dfrac{A}{\mu+V}\\ 0 & b+v_1 & -\left(\mu+\alpha\right)-\lambda\end{vmatrix}=0.$$

$$\lambda_1=-\mu<0,$$

$$\begin{vmatrix}-\left(\mu+v\right)-\lambda & -r\beta\dfrac{A}{\mu+V} & -\beta\dfrac{A}{\mu+V}\\ 0 & r\beta\dfrac{A}{\mu+V}-\left(b+\mu+v_1\right)-\lambda & \beta\dfrac{A}{\mu+V}\\ 0 & b+v_1 & -\left(\mu+\alpha\right)-\lambda\end{vmatrix}=0.$$

$$-\left(\mu+v\right)-\lambda\begin{vmatrix}r\beta\dfrac{A}{\mu+V}-\left(b+\mu+v_1\right)-\lambda & \beta\dfrac{A}{\mu+V}\\ b+v_1 & -\left(\mu+\alpha\right)-\lambda\end{vmatrix}=0.$$

$$\lambda_2=-\left(\mu+v\right)<0.$$

$$\lambda^2+\lambda\left[\left(\mu+\alpha\right)-\frac{r\beta A}{\mu+v}+\lambda^2\right]-\frac{r\beta A}{\mu+v}\left(\mu+\alpha\right)-\frac{\beta A}{\mu+v}\left(b+v_1\right)=0.$$

$$\lambda^2+\lambda\left[\left(\mu+\alpha\right)+\left(b+\mu+v_1\right)\left(1-R_2\right)\right]+\left(\mu+\alpha\right)\left(b+\mu+v_1\right)\left(1-R_0\right)=0.$$

We obtain the following results by applying Routh Hurwitz criteria for 2[nd] order.

$A_1>0, A_2>0$, if $R_0, R_2<1$, where $A_1=\left[\left(\mu+\alpha\right)+\left(b+\mu+v_1\right)\left(1-R_2\right)\right]$, $A_2=\left(\mu+\alpha\right)\left(b+\mu+v_1\right)\left(1-R_0\right)$.

Theorem 3: The endemic equilibrium $K_1=(S^*, E^*, I^*, V^*)$ is locally asymptotically stable if $R_o>1$.

Proof: The Jacobean matrix at $K_1=(S^*, E^*, I^*, V^*)$ is as follows:

$$J(K_1)=\begin{bmatrix}-\beta I^*-r\beta E^*-\left(\mu+v\right) & -r\beta S^* & -\beta S^* & 0\\ \beta I^*+r\beta E^* & r\beta S^*-\left(b+\mu+v_1\right) & \beta S^* & 0\\ 0 & b+v_1 & -\left(\mu+\alpha\right) & 0\\ V & 0 & 0 & -\mu\end{bmatrix}.$$

$$|J(K_1-\lambda\mathrm{I})|=\begin{vmatrix}-\beta I^*-r\beta E^*-\left(\mu+v\right)-\lambda & -r\beta S^* & -\beta S^* & 0\\ \beta I^*+r\beta E^* & r\beta S^*-\left(b+\mu+v_1\right)-\lambda & \beta S^* & 0\\ 0 & b+v_1 & -\left(\mu+\alpha\right)-\lambda & 0\\ V & 0 & 0 & -\mu-\lambda\end{vmatrix}=0.$$

$$-(-\mu-\lambda)\begin{vmatrix}-\beta I^*-r\beta E^*-(\mu+v)-\lambda & -r\beta S^* & -\beta S^* \\ \beta I^*+r\beta E^* & r\beta S^*-(b+\mu+v_1)-\lambda & \beta S^* \\ 0 & b+v_1 & -(\mu+\alpha)-\lambda\end{vmatrix}=0.$$

$\lambda_1=-\mu<0,$

$$\begin{vmatrix}-\beta I^*-r\beta E^*-(\mu+v)-\lambda & -r\beta S^* & -\beta S^* \\ \beta I^*+r\beta E^* & r\beta S^*-(b+\mu+v_1)-\lambda & \beta S^* \\ 0 & b+v_1 & -(\mu+\alpha)-\lambda\end{vmatrix}=0$$

$$-\beta I^*-r\beta E^*-(\mu+v)-\lambda\begin{vmatrix} r\beta S^*-(b+\mu+v_1)-\lambda & \beta S^* \\ b+v_1 & -(\mu+\alpha)-\lambda\end{vmatrix}$$

$$-(\beta I^*+r\beta E^*)\begin{vmatrix} -r\beta S^* & -\beta S^* \\ b+v_1 & -(\mu+\alpha)-\lambda\end{vmatrix}=0.$$

$$-\beta I^*-r\beta E^*-(\mu+v)-\lambda\left[(r\beta S^*-(b+\mu+v_1)-\lambda)(-(\mu+\alpha)-\lambda)-(\beta S^*)(b+v_1)\right]$$
$$-(\beta I^*+r\beta E^*)\left[(-r\beta S^*)(-(\mu+\alpha)-\lambda)-(-\beta S^*)(b+v_1)\right]=0.$$

$$-\beta I^*-r\beta E^*-(\mu+v)-\lambda\left[-r\beta S^*(\mu+\alpha)-r\beta S^*\lambda+(b+\mu+v_1)(\mu+\alpha)+(b+\mu+v_1)\lambda\right.$$
$$\left.+(\mu+\alpha)\lambda+\lambda^2-(\beta S^*)(b+v_1)\right]-\beta I^*-r\beta E^*\left[r\beta S^*(\mu+\alpha)+r\beta S^*\lambda+\beta S^*\ b+v_1\right]=0.$$

$$\begin{aligned}r\beta^2S^*I^*(\mu+\alpha)&+r\beta^2S^*I^*\lambda-\beta I^*(b+\mu+v_1)(\mu+\alpha)-\beta I^*(b+\mu+v_1)\lambda-\beta I^*(\mu+\alpha)\lambda-\beta I^*\lambda^2\\&+\beta^2S^*I^*(b+v_1)+r^2\beta^2E^*S^*(\mu+\alpha)+r^2\beta^2E^*S^*\lambda-r\beta E^*(b+\mu+v_1)(\mu+\alpha)\\&-r\beta E^*(b+\mu+v_1)\lambda-r\beta E^*(\mu+\alpha)\lambda-r\beta E^*\lambda^2+r\beta E^*S^*(b+v_1)+r\beta S^*(\mu+\alpha)(\mu+v)\\&+r\beta S^*(\mu+v)\lambda-(b+\mu+v_1)(\mu+\alpha)(\mu+v)-(\mu+v)(b+\mu+v_1)\lambda\\&-\lambda(\mu+\alpha)(\mu+v)-(\mu+v)\lambda^2+(\mu+v)\beta S^*(b+v_1)+r\beta S^*(\mu+\alpha)\lambda+r\beta S^*\lambda^2\\&-(b+\mu+v_1)(\mu+\alpha)\lambda-(b+\mu+v_1)\lambda^2-(\mu+\alpha)\lambda^2-\lambda^3+(\beta S^*)(b+v_1)\lambda\\&-r\beta^2S^*I^*(\mu+\alpha)-r\beta^2S^*I^*\lambda-\beta^2S^*I^*(b+v_1)-r^2\beta^2E^*S^*(\mu+\alpha)-r^2\beta^2E^*S^*\lambda\\&-r\beta^2S^*E^*(b+v_1)=0.\end{aligned}$$

$$-\lambda^3-\lambda^2\left[\beta I^*+r\beta E^*+(\mu+v)-r\beta S^*+(b+\mu+v_1)+(\mu+\alpha)\right]$$
$$-\lambda\left[-r\beta^2S^*I^*+\beta I^*(b+\mu+v_1)+\beta I^*(\mu+\alpha)+r\beta E^*(b+\mu+v_1)\right.$$
$$+r\beta E^*(\mu+\alpha)-r\beta S^*(\mu+v)+(\mu+v)(b+\mu+v_1)+(\mu+\alpha)(\mu+v)-r\beta S^*(\mu+\alpha)$$

$$+\ (b+\mu+v_1)(\mu+\alpha)-(\beta S^*)(b+v_1)+r\beta^2S^*I^*\big]-\big[-r\beta^2S^*I^*(\mu+\alpha)+\beta I^*(b+\mu+v_1)(\mu+\alpha)$$
$$-r^2\beta^2E^*S^*(\mu+\alpha)+r\beta E^*(b+\mu+v_1)(\mu+\alpha)-r\beta E^*S^*(b+v_1)-r\beta S^*(\mu+\alpha)(\mu+v)$$
$$+(b+\mu+v_1)(\mu+\alpha)(\mu+v)-(\mu+v)\beta S^*(b+v_1)$$
$$+r\beta^2S^*I^*(\mu+\alpha)+r^2\beta^2E^*S^*(\mu+\alpha)+r\beta^2S^*E^*(b+v_1)\big]=0.$$

$$-\left[\lambda^3+\lambda^2A+\lambda C+D\right]=0.$$

$$\lambda^3+\lambda^2A+\lambda C+D=0.$$

where $A=\beta I^*+r\beta E^*+(\mu+v)-r\beta S^*+(b+\mu+v_1)+(\mu+\alpha)$,

$$B=-r\beta^2S^*I^*+\beta I^*(b+\mu+v_1)+\beta I^*(\mu+\alpha)+r\beta E^*(b+\mu+v_1)+r\beta E^*(\mu+\alpha)-r\beta S^*(\mu+v)$$
$$+(\mu+v)(b+\mu+v_1)+(\mu+\alpha)(\mu+v)-r\beta S^*(\mu+\alpha)+(b+\mu+v_1)(\mu+\alpha)-(\beta S^*)(b+v_1)$$
$$+r\beta^2S^*I^*,$$

$$C=-r\beta^2S^*I^*(\mu+\alpha)+\beta I^*(b+\mu+v_1)(\mu+\alpha)-r^2\beta^2E^*S^*(\mu+\alpha)+r\beta E^*(b+\mu+v_1)(\mu+\alpha)$$
$$-r\beta E^*S^*(b+v_1)-r\beta S^*(\mu+\alpha)(\mu+v)+(b+\mu+v_1)(\mu+\alpha)(\mu+v)-(\mu+v)\beta S^*(b+v_1)$$
$$+r\beta^2S^*I^*(\mu+\alpha)+r^2\beta^2E^*S^*(\mu+\alpha)+r\beta^2S^*E^*(b+v_1),$$

$$D=-r\beta^2S^*I^*(\mu+\alpha)+\beta I^*(b+\mu+v_1)(\mu+\alpha)-r^2\beta^2E^*S^*(\mu+\alpha)+r\beta E^*(b+\mu+v_1)(\mu+\alpha)$$
$$-r\beta E^*S^*(b+v_1)-r\beta S^*(\mu+\alpha)(\mu+v)+(b+\mu+v_1)(\mu+\alpha)(\mu+v)-(\mu+v)\beta S^*(b+v_1)$$
$$+r\beta^2S^*I^*(\mu+\alpha)+r^2\beta^2E^*S^*(\mu+\alpha)+r\beta^2S^*E^*(b+v_1).$$

Applying Routh-Hurwitz Criterion for 3rd order, $A>0, D>0$, and $AC>D$, if $R_0>1$.

Hence the given system is locally asymptotically stable.

2.4 Reproduction Number

The idea of reproduction number is presented in [25] by considering Eqs. (3) and (4), we get the following matrices:

$$\begin{bmatrix}E'\\I'\\V'\end{bmatrix}=\begin{bmatrix}rS\beta & \beta S & 0\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}E\\I\\V\end{bmatrix}-\begin{bmatrix}b+\mu+v_1 & 0 & 0\\-b-v_1 & \mu+\alpha & 0\\0&0&\mu\end{bmatrix}\begin{bmatrix}E\\I\\V\end{bmatrix}.$$

Here, F $=\begin{bmatrix}rS\beta & \beta S & 0\\0&0&0\\0&0&0\end{bmatrix}$ and G $=\begin{bmatrix}b+\mu+v_1 & 0 & 0\\-b-v_1 & \mu+\alpha & 0\\0&0&\mu\end{bmatrix}$

$$FG^{-1}=\begin{bmatrix}\dfrac{r\beta A}{(\mu+v)(b+\mu+v_1)}+\dfrac{\beta A(b+v_1)}{(\mu+v)(\mu+\alpha)(b+\mu+v_1)} & \dfrac{\beta A}{(\mu+v)(\mu+\alpha)} & 0\\0&0&0\\0&0&0\end{bmatrix}.$$

The spectral radius of the FG^{-1} is called the reproduction number is as follows:

$R_0 = R_1 + R_2$

$$R_0 = \frac{\beta A\,(b + v_1)}{(\mu + v)\,(\mu + \alpha)(b + \mu + v_1)} + \frac{r\beta A}{(\mu + v)\,(b + \mu + v_1)}$$

where $R_1 = \dfrac{\beta A\,(b + v_1)}{(\mu + v)\,(\mu + \alpha)(b + \mu + v_1)}$, $R_2 = \dfrac{r\beta A}{(\mu + v)\,(b + \mu + v_1)}$.

3 Stochastic Poliovirus Epidemic Model

Let us consider the vector $\mathrm{C} = [S, E, I, V]^T$ of stochastic differential equations (SDEs) of the poliovirus epidemic model ((1)–(4)). We want to calculate the expectation and variance (see Table 1).

Table 1: Transition probabilities of polio epidemic model

Transition	Probabilities
$T_1 = [1\ \ 0\ \ 0\ \ 0]^T$	$P_1 = A\Delta t$
$T_2 = [-1\ \ 1\ \ 0\ \ 0]^T$	$P_2 = \beta SI\Delta t$
$T_3 = [-1\ \ 0\ \ 0\ \ 0]^T$	$P_3 = r\beta SE\Delta t$
$T_4 = [-1\ \ 0\ \ 0\ \ 0]^T$	$P_4 = \mu S\Delta t$
$T_5 = [-1\ \ 0\ \ 0\ \ 1]^T$	$P_5 = VS\Delta t$
$T_6 = [0\ \ -1\ \ 1\ \ 0]^T$	$P_6 = bE\Delta t$
$T_7 = [0\ \ -1\ \ 1\ \ 0]^T$	$P_7 = \mu E\Delta t$

$$\mathrm{Drift} = \mathrm{G}\,(\mathrm{C}, \mathrm{t}) = \begin{bmatrix} A - \beta SI - r\beta SE - \mu S - VS \\ \beta SI + r\beta SE - bE - \mu E - v_1 E \\ bE + v_1 E - \mu I + \alpha I \\ A - \mu V \end{bmatrix} \Delta \mathrm{t}.$$

$$\mathrm{Diffusion} = \sqrt{\begin{bmatrix} \mathrm{A} + \beta \mathrm{SI} + \mathrm{r}\beta \mathrm{SE} + \mu \mathrm{S} + \mathrm{vS} & -\beta \mathrm{SI} - \mathrm{r}\beta \mathrm{SE} & 0 & -\mathrm{vS} \\ -\beta \mathrm{SI} - \mathrm{r}\beta \mathrm{SE} & \beta \mathrm{SI} + \mathrm{r}\beta \mathrm{SE} + \mathrm{bE} + \mu \mathrm{E} + v_1 \mathrm{E} & -\mathrm{bE} - v_1 \mathrm{E} & 0 \\ 0 & -\mathrm{bE} - v_1 \mathrm{E} & b\mathrm{E} + v_1 \mathrm{E} + \mu \mathrm{I} + \alpha \mathrm{I} & 0 \\ -v\mathrm{S} & 0 & 0 & v\mathrm{S} + \mu \mathrm{V} \end{bmatrix}}$$

The equation of poliovirus epidemic model ((1)–(4)) can be written as

$$\mathrm{d}\begin{bmatrix} \mathrm{S} \\ E \\ I \\ V \end{bmatrix} = \begin{bmatrix} A - \beta SI - r\beta SE - \mu S - VS \\ \beta SI + r\beta SE - bE - \mu E - v_1 E \\ bE + v_1 E - \mu I + \alpha I \\ A - \mu V \end{bmatrix} \mathrm{dt}$$

$$+ \sqrt{\begin{bmatrix} \mathrm{A} + \beta \mathrm{SI} + \mathrm{r}\beta \mathrm{SE} + \mu \mathrm{S} + \mathrm{vS} & -\beta \mathrm{SI} - \mathrm{r}\beta \mathrm{SE} & 0 & -\mathrm{vS} \\ -\beta \mathrm{SI} - \mathrm{r}\beta \mathrm{SE} & \beta \mathrm{SI} + \mathrm{r}\beta \mathrm{SE} + \mathrm{bE} + \mu \mathrm{E} + v_1 \mathrm{E} & -\mathrm{bE} - v_1 \mathrm{E} & 0 \\ 0 & -\mathrm{bE} - v_1 \mathrm{E} & b\mathrm{E} + v_1 \mathrm{E} + \mu \mathrm{I} + \alpha \mathrm{I} & 0 \\ -v\mathrm{S} & 0 & 0 & v\mathrm{S} + \mu \mathrm{V} \end{bmatrix}}\,\mathrm{dB}. \tag{17}$$

The Euler Maruyama approach is cast-off to determine the numerical result of the Eq. (17) by using the values of the parameters given in Table 2 as follows:

Table 2: Values of parameter [26]

Parameters	DFE	EE
A	0.5	0.5
μ	0.5	0.5
v	0.6	0.6
α	0.0001	00.0001
v_1	0.001	0.001
b	0.9	0.9
r	0.5	0.5
σ_1	0.04	0.04
β	1.002	2.002

$C_{n+1} = C_n + f(C_n, t)\,\Delta t + L(C_n, t)\,dB.$

$$\begin{bmatrix} S^{n+1} \\ E^{n+1} \\ I^{n+1} \\ V^{n+1} \end{bmatrix} = \begin{bmatrix} S^n \\ E^n \\ I^n \\ V^n \end{bmatrix} + \left[\begin{bmatrix} A - \beta S^n I^n - r\beta S^n E^n - \mu S^n - vS^n \\ \beta S^n I^n + r\beta S^n E^n - bE^n - \mu E^n - v_1 E^n \\ bE^n + v_1 E^n - \mu I^n + \alpha I^n \\ A - \mu V^n \end{bmatrix}\right] \Delta t$$
$$+ \sqrt{\begin{bmatrix} A + \beta S^n I^n + r\beta S^n E^n + \mu S^n + vS^n & -\beta S^n I^n - r\beta S^n E^n & 0 & -vS^n \\ -\beta S^n I^n - r\beta S^n E^n & \beta S^n I^n + r\beta S^n E^n + bE^n + \mu E^n + v_1 E^n & -bE^n - v_1 E^n & 0 \\ 0 & -bE^n - v_1 E^n & bE^n + v_1 E^n + \mu I^n + \alpha I^n & 0 \\ -vS^n & 0 & 0 & vS^n + \mu V^n \end{bmatrix}} \Delta B_n \tag{18}$$

where $C(0) = C_o = [0.5, 0.3, 0.2, 0.1,]^T, 0 \le t \le C$ and Brownian motion is denoted as B.

3.1 Stochastic Nonstandard Finite Difference Method

The stochastic NSFD can be developed for the system ((1)–(4)) as

$$\frac{dS}{dt} = A - \beta SI - r\beta SE - (\mu + v_1) S.$$

The breakdown of the proposed method for the above equation.

$$S^{n+1} = S^n + h[A - \beta S^{n+1} I^n - r\beta S^{n+1} E^n - (\mu + v_1) S^{n+1} + \sigma_1 S^n \Delta B_1].$$

$$S^{n+1} = \frac{S^n + hA + h\sigma_1 S^n \Delta B_1}{1 + h\beta I^n + hr\beta E^n + h(\mu + v_1)}. \tag{19}$$

Similarly, we break the remaining system into a proposed method like (19), as follows:

$$E^{n+1} = \frac{E^n + h\beta S^n I^n + hr\beta S^n E^n + h\sigma_2 E^n \Delta B_2}{1 + h(b + \mu + v_1)}. \tag{20}$$

$$I^{n+1} = \frac{I^n + h((b + v_1)E^n + \sigma_3 I^n \Delta B_3}{1 + h(\mu + \alpha)}. \tag{21}$$

$$V^{n+1} = \frac{V^n + hv_1 s^n + \sigma_4 V^n \Delta B_4}{1 + h\mu}. \tag{22}$$

where, n = 0, 1, 2, . . . , and discretization gap is denoted by "h".

3.2 Stability Analysis

***Theorem 5*:** The stochastic NSFD method is stable if the eigenvalues of Eqs. (19)–(22) lie in the unit circle for any $n \geq 0$.

Proof: Let the functions L_1, L_2, L_3, L_4 by assuming $\Delta B_n = 0$, from the system ((19)–(22)) as follows:

$$L_1 = \frac{S + hA}{1 + h\beta I + hr\beta E + h(\mu + v_1)}, L_2 = \frac{E + h\beta SI + hr\beta SE}{1 + h(b + \mu + v_1)}, L_3 = \frac{I + h((b + v_1)E}{1 + h(\mu + \alpha)}, L_4 = \frac{V + hv_1 S}{1 + h\mu}.$$

The elements of the Jacobean matrix are given as

$$J(S, E, I, V) = \begin{bmatrix} \frac{\partial L_1}{\partial S} & \frac{\partial L_1}{\partial E} & \frac{\partial L_1}{\partial I} & \frac{\partial L_1}{\partial V} \\ \frac{\partial L_2}{\partial S} & \frac{\partial L_2}{\partial E} & \frac{\partial L_2}{\partial I} & \frac{\partial L_2}{\partial V} \\ \frac{\partial L_3}{\partial S} & \frac{\partial L_3}{\partial E} & \frac{\partial L_3}{\partial I} & \frac{\partial L_3}{\partial V} \\ \frac{\partial L_4}{\partial S} & \frac{\partial L_4}{\partial E} & \frac{\partial L_4}{\partial I} & \frac{\partial L_4}{\partial V} \end{bmatrix}.$$

The given Jacobean matrix at $K_0 = \left(\frac{A}{\mu + v}, 0, 0, \frac{v}{\mu} S\right)$ is as follows:

$$J(K_O) = \begin{bmatrix} \frac{1}{1 + h(\mu + v_1)} & \frac{-\left(\frac{A}{\mu + v} + hA\right) rh\beta}{[1 + h(\mu + v_1)]^2} & \frac{-h\beta\left(\frac{A}{\mu + v} + h\beta\right)}{[1 + h(\mu + v_1)]^2} & 0 \\ 0 & \frac{1}{1 - rh\beta\left(\frac{A}{\mu + v}\right) + h(b + \mu + v_1)} & \frac{h\beta\left(\frac{A}{\mu + v}\right)\left(1 + rh\beta\left(\frac{A}{\mu + v}\right) + h(b + \mu + v_1)\right)}{\left[1 - rh\beta\left(\frac{A}{\mu + v}\right) + h(b + \mu + v_1)\right]^2} & 0 \\ 0 & \frac{h(b + v_1)}{1 + h(\mu + \alpha)} & 0 & 0 \\ \frac{hv_1}{1 + h\mu} & 0 & 0 & \frac{1}{1 + h\mu} \end{bmatrix}.$$

$$\begin{vmatrix} \frac{1}{1+h(\mu+v_1)}-\lambda & \frac{-\left(\frac{A}{\mu+v}+hA\right)rh\beta}{[1+h(\mu+v_1)]^2} & \frac{-h\beta\left(\frac{A}{\mu+v}+h\beta\right)}{[1+h(\mu+v_1)]^2} & 0 \\ 0 & \frac{1}{1-rh\beta\left(\frac{A}{\mu+v}\right)+h(b+\mu+v_1)}-\lambda & \frac{h\beta\left(\frac{A}{\mu+v}\right)\left(1+rh\beta\left(\frac{A}{\mu+v}\right)+h(b+\mu+v_1)\right)}{\left[1-rh\beta\left(\frac{A}{\mu+v}\right)+h(b+\mu+v_1)\right]^2} & 0 \\ 0 & \frac{h(b+v_1)}{1+h(\mu+\alpha)} & -\lambda & 0 \\ \frac{hv_1}{1+h\mu} & 0 & 0 & \frac{1}{1+h\mu}-\lambda \end{vmatrix}=0.$$

$$\frac{1}{1+h\mu}-\lambda\begin{vmatrix} \frac{1}{1+h(\mu+v_1)}-\lambda & \frac{-(A+hA(\mu+v))rh\beta}{(\mu+v)[1+h(\mu+v_1)]^2} & \frac{-h\beta A-h\beta(\mu+v)}{(\mu+v)[1+h(\mu+v_1)]^2} \\ 0 & \frac{\mu+v}{(\mu+v)-rh\beta A+(\mu+v)h(b+\mu+v_1)}-\lambda & \frac{h\beta A((\mu+v)+rh\beta A+(\mu+v)h(b+\mu+v_1))}{[(\mu+v)-rh\beta A+(\mu+v)h(b+\mu+v_1)]^2} \\ 0 & \frac{h(b+v_1)}{1+h(\mu+\alpha)} & -\lambda \end{vmatrix}=0$$

$$\lambda_1=\frac{1}{1+h\mu}<1,$$

$$\begin{vmatrix} \frac{1}{1+h(\mu+v_1)}-\lambda & \frac{-(A+hA(\mu+v))rh\beta}{(\mu+v)[1+h(\mu+v_1)]^2} & \frac{-h\beta A-h\beta(\mu+v)}{(\mu+v)[1+h(\mu+v_1)]^2} \\ 0 & \frac{\mu+v}{(\mu+v)-rh\beta A+(\mu+v)h(b+\mu+v_1)}-\lambda & \frac{h\beta A((\mu+v)+rh\beta A+(\mu+v)h(b+\mu+v_1))}{[(\mu+v)-rh\beta A+(\mu+v)h(b+\mu+v_1)]^2} \\ 0 & \frac{h(b+v_1)}{1+h(\mu+\alpha)} & -\lambda \end{vmatrix}=0$$

$$\frac{1}{1+h(\mu+v_1)}-\lambda\left[\left(\frac{\mu+v}{(\mu+v)-rh\beta A+(\mu+v)h(b+\mu+v_1)}-\lambda\right)-(\lambda)\right.$$
$$\left.-\left(\frac{h\beta A((\mu+v)+rh\beta A+(\mu+v)h(b+\mu+v_1))}{[(\mu+v)-rh\beta A+(\mu+v)h(b+\mu+v_1)]^2}\right)\left(\frac{h(b+v_1)}{1+h(\mu+\alpha)}\right)\right]=0.$$

$$\lambda_2=\frac{1}{1+h(\mu+v_1)}<1.$$

$$\left[\left(\frac{\mu+v}{(\mu+v)-rh\beta A+(\mu+v)h(b+\mu+v_1)}-\lambda\right)(-\lambda)\right.$$
$$\left.-\left(\frac{h\beta A((\mu+v)+rh\beta A+(\mu+v)h(b+\mu+v_1))}{[(\mu+v)-rh\beta A+(\mu+v)h(b+\mu+v_1)]^2}\right)\left(\frac{h(b+v_1)}{1+h(\mu+\alpha)}\right)\right]=0.$$

$$\lambda^2 - \frac{\mu + v}{(\mu + v) - rh\beta A + (\mu + v)\, h\,(b + \mu + v_1)}\,(\lambda)$$

$$- \left(\frac{h\beta A\,((\mu + v) + rh\beta A + (\mu + v)\, h\,(b + \mu + v_1))}{[(\mu + v) - rh\beta A + (\mu + v)\, h\,(b + \mu + v_1)]^2}\right)\left(\frac{h\,(b + v_1)}{1 + h\,(\mu + \alpha)}\right) = 0.$$

Hence, by using the Mathematica software all the eigen values of the above Jacobean matrix lie in the unit circle if $R_0 < 1$. Thus, the system ((19)–(22)) is stable.

Now, for endemic equilibrium (EE) $K_1 = (S^*, E^*, I^*, V^*)$. The given Jacobean matrix is

$$J\,(K_1) = \begin{bmatrix} \frac{1}{1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)} & \frac{-\,(S^* + hA)\, rh\beta}{[1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)]^2} & \frac{-\,(S^* + hA)\, h\beta}{[1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)]^2} & 0 \\ \frac{h\beta I^*\,(1 - rh\beta S^* + h\,(b + \mu + v_1) + (E^* + h\beta S^* I^*)\, rh\beta)}{[1 - rh\beta S^* + h\,(b + \mu + v_1)]^2} & \frac{1}{1 - rh\beta S^* + h\,(b + \mu + v_1)} & \frac{h\beta S^*\,(1 - rh\beta S^* + h\,(b + \mu + v_1))}{[1 - rh\beta S^* + h\,(b + \mu + v_1)]^2} & 0 \\ 0 & \frac{h\,(b + v_1)}{1 + h\,(\mu + \alpha)} & 0 & 0 \\ \frac{hv_1}{1 + h\mu} & 0 & 0 & \frac{1}{1 + h\mu} \end{bmatrix}$$

$$\begin{vmatrix} \frac{1}{1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)} - \lambda & \frac{-\,(S^* + hA)\, rh\beta}{[1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)]^2} & \frac{-\,(S^* + hA)\, h\beta}{[1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)]^2} & 0 \\ \frac{h\beta I^*\,(1 - rh\beta S^* + h\,(b + \mu + v_1) + (E^* + h\beta S^* I^*)\, rh\beta)}{[1 - rh\beta S^* + h\,(b + \mu + v_1)]^2} & \frac{1}{1 - rh\beta S^* + h\,(b + \mu + v_1)} - \lambda & \frac{h\beta S^*\,(1 - rh\beta S^* + h\,(b + \mu + v_1))}{[1 - rh\beta S^* + h\,(b + \mu + v_1)]^2} & 0 \\ 0 & \frac{h\,(b + v_1)}{1 + h\,(\mu + \alpha)} & -\lambda & 0 \\ \frac{hv_1}{1 + h\mu} & 0 & 0 & \frac{1}{1 + h\mu} - \lambda \end{vmatrix} = 0.$$

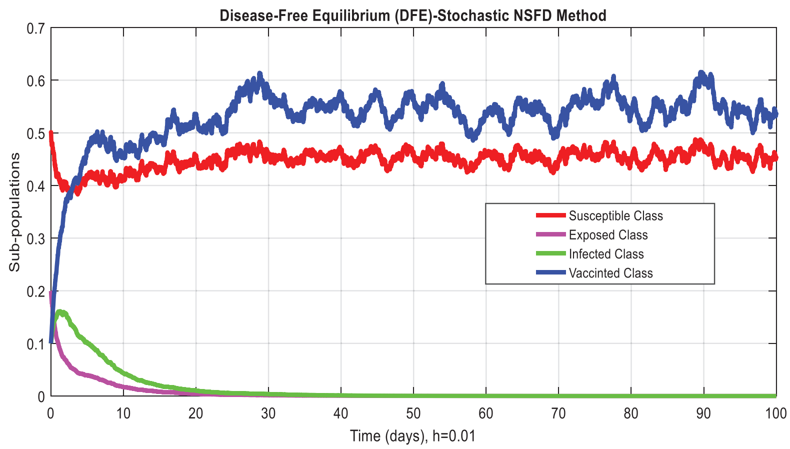
$$-\left(\frac{1}{1 + h\mu} - \lambda\right)$$

$$\begin{vmatrix} \frac{1}{1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)} - \lambda & \frac{-\,(S^* + hA)\, rh\beta}{[1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)]^2} & \frac{-\,(S^* + hA)\, h\beta}{[1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)]^2} \\ \frac{h\beta I^*\,(1 - rh\beta S^* + h\,(b + \mu + v_1) + (E^* + h\beta S^* I^*)\, rh\beta)}{[1 - rh\beta S^* + h\,(b + \mu + v_1)]^2} & \frac{1}{1 - rh\beta S^* + h\,(b + \mu + v_1)} - \lambda & \frac{h\beta S^*\,(1 - rh\beta S^* + h\,(b + \mu + v_1))}{[1 - rh\beta S^* + h\,(b + \mu + v_1)]^2} \\ 0 & \frac{h\,(b + v_1)}{1 + h\,(\mu + \alpha)} & -\lambda \end{vmatrix} = 0.$$

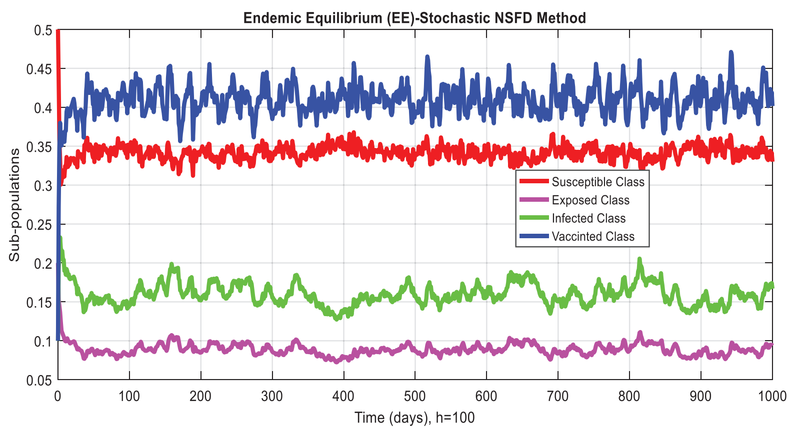
$$\lambda_1 = \frac{1}{1 + h\mu} < 1.$$

$$\begin{vmatrix} \frac{1}{1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)} - \lambda & \frac{-\,(S^* + hA)\, rh\beta}{[1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)]^2} & \frac{-\,(S^* + hA)\, h\beta}{[1 + h\beta I^* + rh\beta E^* + h\,(\mu + v_1)]^2} \\ \frac{h\beta I^*\,(1 - rh\beta S^* + h\,(b + \mu + v_1) + (E^* + h\beta S^* I^*)\, rh\beta)}{[1 - rh\beta S^* + h\,(b + \mu + v_1)]^2} & \frac{1}{1 - rh\beta S^* + h\,(b + \mu + v_1)} - \lambda & \frac{h\beta S^*\,(1 - rh\beta S^* + h\,(b + \mu + v_1))}{[1 - rh\beta S^* + h\,(b + \mu + v_1)]^2} \\ 0 & \frac{h\,(b + v_1)}{1 + h\,(\mu + \alpha)} & -\lambda \end{vmatrix} = 0$$

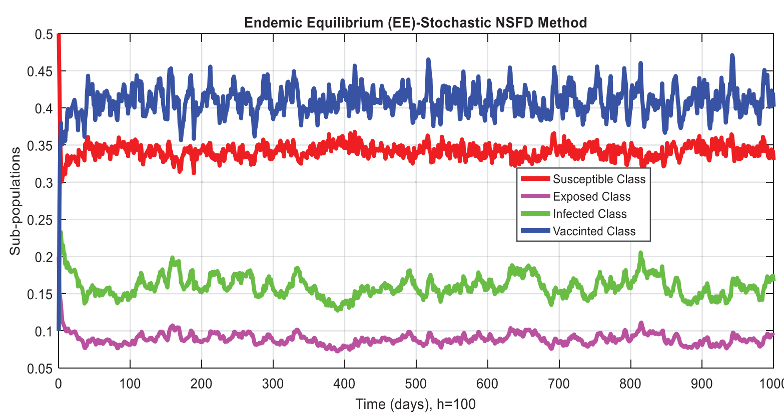
Using Mathematica software, the most many eigenvalues of the Jacobean is less than one when $R_0 > 1$. Thus, endemic equilibrium is stable.



(a)

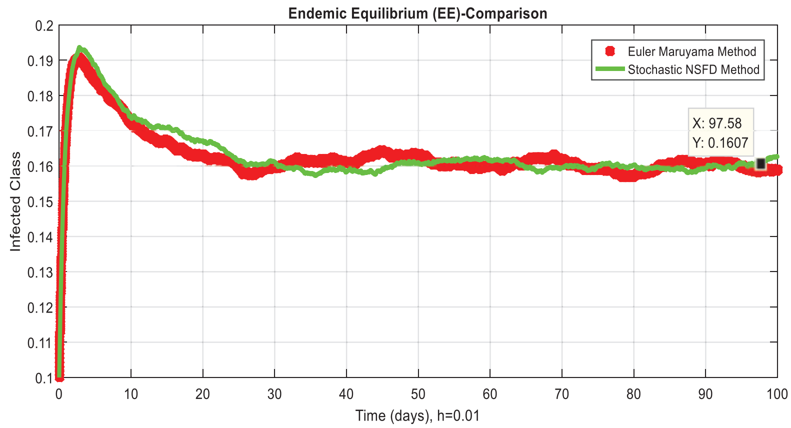


(b)

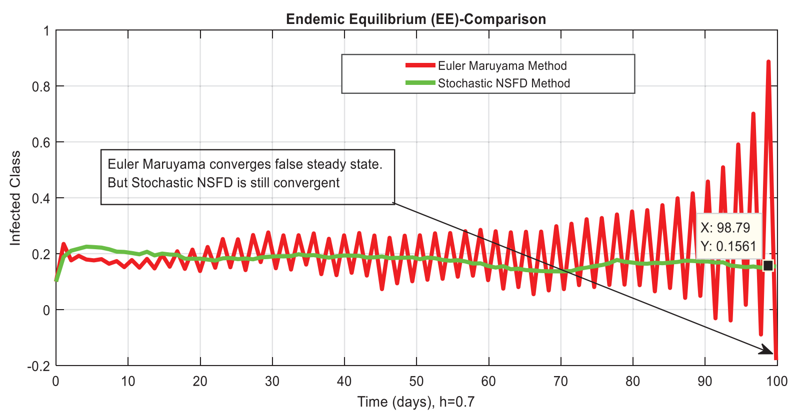


(c)

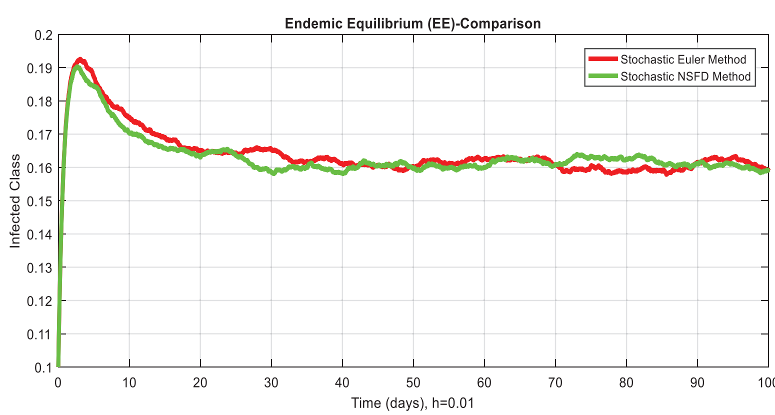
Figure 1: (Continued)



(d)

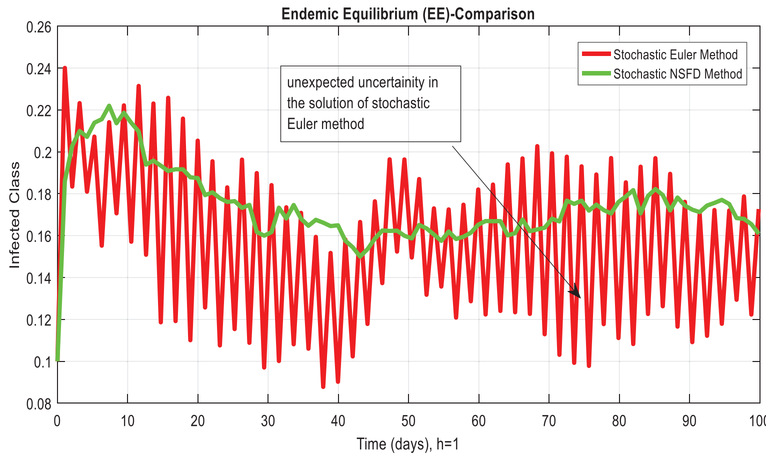


(e)

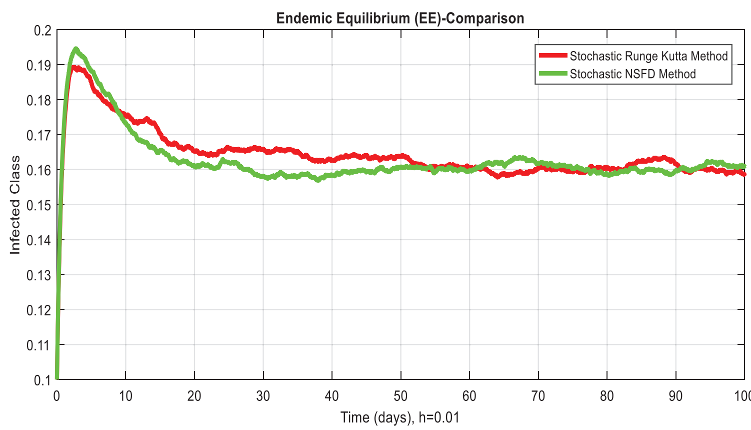


(f)

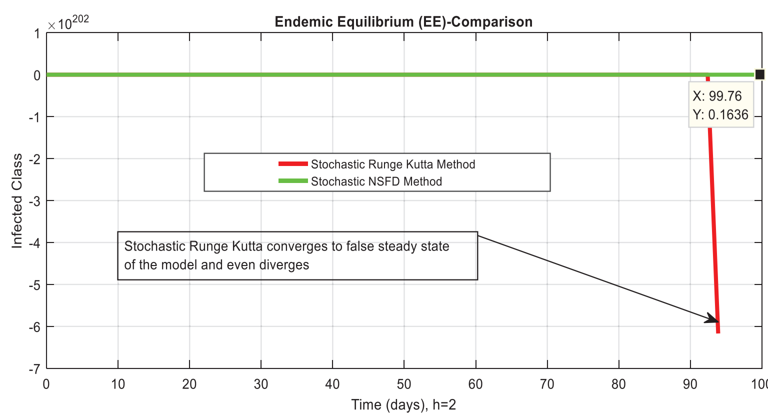
Figure 1: (Continued)



(g)



(h)



(i)

Figure 1: (a) Stochastic NSFD for DFE at h = 0.01 (b) Stochastic NSFD at h = 100 (c) stochastic NSFD for EE at h = 100 (d) Infected class (comparison) at h = 0.01 (e) Infected class (comparison) at h = 0.7 (f) Infected class (comparison) at h = 0.01 (g) Infected class (comparison) at h = 1 (h) Infected class (comparison-RK) of at h = 0.01 (i) Infected class (comparison-RK) at h = 2

4 Results and Conclusion

Fig. 1 admits the comparative analyses of the proposed approach with current methods in the sense of stochastic. The numerical experimentations can easily conclude that other stochastic numerical methods are conditionally convergent or diverge with larger time step values. The nature of biological properties is not consistent with existing literature methods. For this sake, the nonstandard finite difference is designed to restore the structure of continuous models. Computational methods like stochastic Euler, stochastic Runge Kutta, and Euler Maruyama are presented. Unfortunately, these methods are only applicable for the small step size. These methods diverge when we increase the time and do not obey the dynamical properties (positivity, stability, consistency, and boundedness). The stochastic nonstandard finite difference (SNSFD) method is appropriate for all complex and nonlinear stochastic epidemic models. The stochastic model is a reliable and efficient technique to handle highly nonlinear problems close to nature. The stochastic model is the extension of the deterministic model. We present the non-parametric perturbation technique for the said model. Our focus is to propose an always dynamically consistent, positive, and bounded scheme. That is why we investigate the nonstandard finite difference method in the sense of the stochastic. A comparison section is presented for the efficiency of the processes. Furthermore, we extend this idea to other types of models in the future, as shown in [27–31].

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