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Longitudinal Vibration Analysis of Marine Riser During Installation and Hangoff in Ultra Deepwater

Yanbin Wang[1], **Deli Gao**[1], **Jun Fang**[1]

Abstract: This paper is concerned with the problem of longitudinal vibration of a 3,000 m long disconnected drilling riser during installation and hangoff operation. The natural frequency, amplitude of vibration displacement and vibration force of longitudinal vibration have been determined with consideration of the damping effect of the sea water. Results show the natural frequency of riser under these two situations are both decrease with the increase of water depth. The natural frequency of riser in installation operation is bigger than that in hangoff operation. The vibration displacement in hangoff configuration is bigger than that in installation while the vibration force in hangoff configuration is smaller than that in installation configuration. Under the condition of resonance, the vibration displacement increase along riser from top end to the bottom end while the vibration force increase along riser from riser bottom end to the top end.

Keywords: Marine riser; Installation; Hangoff; Longitudinal vibration

1 Introduction

As offshore drilling operations venture into deeper waters in regions with possible severe weather conditions, the requirements placed on the drilling riser become more and more severe. Installation and hangoff of drilling riser are two kinds of critical process in deepwater drilling. Riser installation is the foundation of deepwater drilling, which provides the wellbore in water for subsequent drilling operation. In the process of installation, riser is hung on the Rotary Kelly Bushing (RKB), and connection of riser and RKB can be regarded as a fixed end. When the marine environment goes too harsh to drilling, the riser will be separated with Blowout Preventers (BOPs) and Low Marine Riser Package (LMRP) from the subsea wellhead. Under this condition, the riser is suspended on the heave compensator

[1] MOE Key Laboratory of Petroleum Engineering, China University of Petroleum, Beijing102249, China, First and corresponding author: Yanbin Wang, Tel:+ 86 (10)89733702,
E-mail: wyb76219861@126.com.

and do heave movement under the action of floating drilling vessel. However, for the marine riser, these two kinds of operation, presented in Fig. 1, have a similar mechanical model and analytical method, so it can be discussed together in this paper.

Installation and hangoff of drilling risers in very deep water requires careful study of the mechanical behavior of the load carrying components, subjected to vibration stresses derived from the action of sea waves and vessel motion, combined with the static stresses derived from dead weight, internal fluids, etc. The American Petroleum Institute (API) recommends in RP 16Q that the maximum vibration tension amplitude be limited to 15 percent of the static tension on the suspended limited operating mode. Evaluation of operational limits for long disconnected risers must account for both axial and lateral problems. Prediction of the disconnected riser response and stresses due to expected sea states, currents, and ship response for various storm intensities becomes imperative to determine the potential storm survivability of the suspended riser so that informed operating decisions may be made based on a given weather forecast. Evaluation of the storm survivability limits for a particular riser, sea condition, and drillship requires careful consideration.

Long suspended risers are more susceptible to storm damage than short risers. A suspended riser can experience two types of operational 1imits. The first type is due to lateral loads from high currents. The current loads can cause excessive bending stresses in the top of the riser if no ball or flex joint is present. The use of an upper ball joint will help prevent high bending stresses, but will increase the 1ikelihood of impact between the riser and the ship moonpool structure. The second type of operational limit is due to the longitudinal excitation of the riser by the ship heave. So, a longitudinal vibration analysis of the initial design is required to ensure the vibration tension amplitude is smaller than the static tension.

Large vibration forces capable of inducing failure can develop from:

- Vibration force amplification due to excitation at or near the system's natural frequency.

- Downward acceleration of the vessel at the riser suspension point approaching or being greater than the free fall acceleration of the riser system.

The first cause of failure is of concern in deep water and when lower elastic modulus materials are used for riser construction, because the riser's natural frequency can approach the heave frequency. The second situation can arise when using very massive loads or risers, with low overall wet weight/wet mass ratio, or handling loads with large horizontal surfaces that produce large drag forces.

Over the past few decades, many researches have performed extensive studies regarding the riser longitudinal vibration analysis. Long, Harry, Steddum, and Young (1983) have evaluated the effects of venting the air can buoyancy modules on storm survivability of the 6000 foot riser as one method of increasing storm survivability of long disconnected risers. Azpiazu and Nguyen (1984) have presented some recommendations for the design of heave compensation equipment intended to limit the vibration force amplitude on riser-load systems hanging in deeper waters and harsher environments. Sattamini and Ferranti (1993) have studied and compared the riser axial behavior obtained using linear and non-linear formulations and finite element models, with particular regard to the hangoff condition. Nguyen, Thethi, and Lim (2006) have presented the feasibility of modifying the existing drilling riser by making the riser disconnectable closer to the surface and leave the long riser string below in a safe and freestanding mode to survive the storm and described the simple buoyancy system and additional components required. Ambrose, Grealish, and Whooley, (2001) have discussed the possibilities of using the soft hangoff option and compared the performance characteristics of the drilling riser in the soft hangoff and hard hangoff configurations, particularly for ultra deepwater applications down to 10,000 ft. Wu, Wang, Tian, Fu, Liu, Luo, Wang, Song, and Qi (2014) have constructed a vibrational model to simulate the vibration response of the riser in complex sea states with consideration of the wave and current effect, the platform motion and the large deformation of drilling riser in hard hangoff mode. Wang, and Fang (2014) have established the mechanical model with hard hangoff mode to analyze riser axial vibration characteristics during installation according to the basic principle of vibration mechanics and the influence of damping on vibration has been taken into account in the analysis model. Sun, Chen, Ju (2009) have studied the typhoon-avoidance management strategy of disconnected riser through analyzing the axial vibration characteristics of ultra deepwater risers in hard and soft hangoff configurations with ANSYS software. Zhang and Gao (2010) have derived the longitudinal vibration equation of riser regarded as a long homogeneous slender rod with same cross section and the inherent frequency of riser longitudinally vibrated is figured out by numerical method. Ju, Chang, Chen, and Chen (2012) have put forward a method to analyze operating riser based on the restraint criterion of riser hang-off operations and established a finite element model for the riser axial vibration windows for soft or hard hangoff modes without considering the influence of damping on riser mechanical behavior.

The purpose for this paper is to take longitudinal vibration analysis of a 3,000 m marine riser during installation operation, which can be regarded as the hard hangoff configuration, and hangoff operation, which can be regarded as the soft hangoff configuration in ultra deepwater with taking the influence of damping into consideration and compare the differences of riser axial mechanical behavior under the two hangoff configuration. The variations of water depth, riser size, weight of BOPs, amplitude and circular frequency of heave motion on natural frequency, amplitude of vibration displacement and vibration force of longitudinal vibration have also been discussed.

The remainder of the paper is organized as follows. The longitudinal vibration of riser in installation operation (hard hangoff configuration) is introduced in Section 2. The longitudinal vibration of riser in hangoff operation (soft hangoff configuration) is presented in Section 3. The natural frequency, amplitude of vibration displacement and vibration force of longitudinal vibration are discussed in Section 4. The conclusions of this paper are shown in Section 5.

2 Installation configurations

2.1 Without damping

After the conductor is installed, LMRP and BOPs are lowered together with the riser into the sea water to connect the subsea wellhead, as shown in **Figure1**. Here, we adopt an equivalent method: take the LMRP/BOPs as a virtual string with the same diameter as the riser, and the total length of riser system after equivalent is L', which is an undetermined value, as shown in **Figure 2-b**. Take the connected point of riser top end as the origin of coordinates, the positive direction of x axis is vertical downward along riser. The analysis model can be regarded as a beam located in the vertical plane.

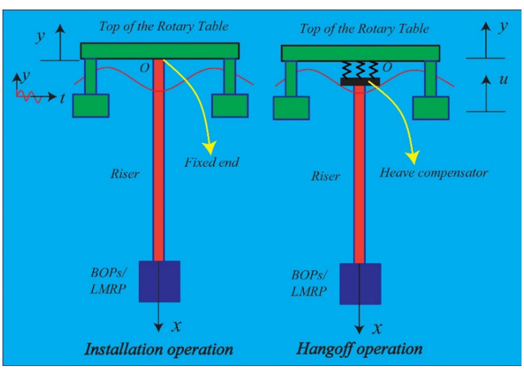

Figure 1: Schematic diagram of riser installation and hangoff.

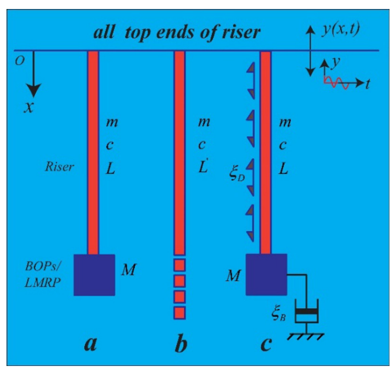

Figure 2: Equivalent processing of riser system.

According to Sparks, Cabillic, and Schawann (1982), the differential equation of riser longitudinal vibration with damping can be written as:

$$\frac{\partial}{\partial x}\left(EA\frac{\partial u}{\partial x}\right)-\xi_D\frac{\partial u}{\partial t}-m\frac{\partial^2 u}{\partial t^2}=0 \tag{1}$$

Where, $u=u(x,t)$ is the riser longitudinal vibration displacement, m; E is the Young's modulus, Pa; A is the riser sectional area of metal portion, m^2; ξ_D is the distributed linear damping of sea water, $N/(m\cdot m/s)$; m is the riser mass of unit length including buoyancy units and added mass, etc., kg/m.

Obviously the natural frequencies play a key roll in determining the vibration behavior of a suspended riser. According to the basic principle of vibration mechanics [Li (2012)], the natural frequency has nothing to do with the damping and only depends on the structure of the system itself. If get rid of the damping term in Eq. 1, the axial vibration control equation for a uniform riser is:

$$c^2\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial t^2}=0 \tag{2}$$

Where, $c=\sqrt{EA/m}$ is the propagation velocity of waves in riser, m/s, which is given in Table. 1 for various risers and riser materials. Once the riser material has been chosen, the propagation velocity can virtually only be modified by changing the mass of the buoyancy units, connectors and peripheral tubes.

Table 1: Propagation velocity of various risers and riser materials.

<table>
<tr><td rowspan="3"></td><td colspan="3">Steel 18 5/8″ O.D.
5/8″ W.T.</td><td colspan="3">Titanium
17 1/2″ O.D.
1/2″ W.T.</td><td colspan="2">Steel/Composite*
18 5/8″ O.D.
5/8″ W.T.</td></tr>
<tr><td rowspan="2">Bare metal</td><td colspan="2">Drilling riser</td><td rowspan="2">Bare metal</td><td colspan="2">Drilling riser</td><td colspan="2">Drilling riser</td></tr>
<tr><td>Without floaters</td><td>With floaters</td><td>Without floaters</td><td>With floaters</td><td>Without floaters</td><td>With floaters</td></tr>
<tr><td>Bending stiffness $EA(N)$</td><td colspan="3">4.8E9</td><td colspan="3">2.13E9</td><td colspan="2">4.8E9</td></tr>
<tr><td>Mass of unit length (including fluid) m(kg/m)</td><td>185</td><td>546</td><td>817</td><td>89</td><td>330</td><td>441</td><td>463</td><td>659</td></tr>
<tr><td>Propagation velocity $c=\sqrt{EA/m}$ (m/s)</td><td>5,100</td><td>2,965</td><td>2,425</td><td>4,900</td><td>2,540</td><td>2,198</td><td>3,220</td><td>2,700</td></tr>
</table>

*Steel riser with Carbon fibre kill and choke lines.

Assume that the movement of riser top end is sinusoidal vibration [Zhang, Gao, and Tang (2010)] with amplitude u_0 and circular frequency ω, which is:

$$y = u_0 \sin \omega t \tag{3}$$

Where, y is the vibration displacement of riser top end, m; u_0 is the vibration amplitude, m; ω is the circular frequency, rad/s.
For a uniform riser, Eq. 2 and Eq. 3 are satisfied by:

$$u = u_0 \left(A \cdot \sin \frac{\omega x}{c} + \cos \frac{\omega x}{c} \right) \cdot \sin \omega t \tag{4}$$

Where, $A = \tan \dfrac{\omega L'}{c}$ is an unknown parameter and L' is the riser total length after equivalent, which is also undetermined.
After substituting A into Eq. 4, one can obtain:

$$u = u_0 \left[\frac{\cos \frac{\omega}{c} (L' - x)}{\cos \frac{\omega L'}{c}} \right] \cdot \sin \omega t \tag{5}$$

From Eq. 5, we can see that if $\cos \dfrac{\omega L'}{c}$ equal to 0, the riser vibration displacement u will be go to infinity, that is to say the riser is resonance. So the natural frequency can be determined by:

$$\frac{\omega L'}{c} = (2n+1) \frac{\pi}{2} \tag{6}$$

Where, n is the mode number of longitudinal vibration (zero for the fundamental). The solution for L' is presented in **Appendix A**.

Table 2: Natural frequencies of the first five orders*.

	1^{th}	2^{th}	3^{th}	4^{th}	5^{th}
Hard hangoff	1.2838	4.8515	6.4192	8.9869	11.5546
Soft hangoff	0.2735	0.8147	1.3536	1.8892	2.4237

*The mass of LMRP/BOPs is 200 t.

The Eq. 6 shows that, for a uniform riser, the natural frequencies are determined by only two parameters: the equivalent length and the velocity with which longitudinal waves are transmitted along the riser. For a 3,000 m length, 18 5/8″ O.D. and 5/8″ W.T steel/composite hard hangoff drilling riser with floaters shown in **Table 1**, the natural frequency of the first five orders is presented in **Table 2**.

2.2 With damping

The discussion above is a uniform riser without damping, excited at non-resonant frequencies. When the excitation frequency is very close to a natural frequency, damping must be included. If taking damping into account, riser longitudinal vibration are shown in Eq. 1 and the vibration displacement is satisfied by Cheng, (1990):

$$u = u_0 \cdot \left[J \cdot e^{-jKx} + (1-J) \cdot e^{jKx}\right] \cdot e^{j\omega t} \tag{7}$$

Where, J and K are complex numbers.

Under this circumstance, the boundary condition of riser bottom end is:

$$M \cdot \left(\frac{\partial^2 u}{\partial t^2}\right)_L + mc^2 \cdot \left(\frac{\partial u}{\partial x}\right)_L + \xi_B \left(\frac{\partial u}{\partial t}\right)_L = 0 \tag{8}$$

Where, ξ_B is the linear damping at riser bottom end, $N/(m \cdot m/s)$. According to Eq. 1, Eq. 7 and Eq. 8, the expression for riser longitudinal displacements with damping can be represented by:

$$\begin{aligned} u = {} & u_o[e^{K_2x} + \alpha \sinh(K_2x)\cos(K_1x) + \beta \cosh(K_2x)\sin(K_1x)]\sin(\omega t) \\ & + u_o[e^{K_2x} + \alpha \cosh(K_2x)\sin(K_1x) - \beta \sinh(K_2x)\cos(K_1x)]\cos(\omega t) \end{aligned} \tag{9}$$

The riser axial vibration force can be calculated by $T = mc^2 \dfrac{\partial u}{\partial x}$, which is:

$$\begin{aligned} T = {} & mc^2 u_o\{\sin(\omega t)[K_2 e^{K_2x} + \alpha \cosh(K_2x)K_2\cos(K_1x) - \alpha \sinh(K_2x) \\ & \sin(K_1x)K_1 + \beta \sinh(K_2x)K_2\sin(K_1x) + \beta \cosh(K_2x)\cos(K_1x)K_1] \\ & + [K_2 e^{K_2x} + \alpha \sinh(K_2x)K_2\sin(K_1x) + \alpha \cosh(K_2x)\cos(K_1x)K_1 \\ & - \beta \cosh(K_2x)K_2\cos(K_1x)] + \beta \sinh(K_2x)\sin(K_1x)K_1]\cos(\omega t)\} \end{aligned} \tag{10}$$

The amplitudes of the displacements and the vibration tensions are eventually given by:

$$\left|\frac{u}{u_0}\right| = \sqrt{R_x - S_x} \tag{11}$$

$$\left|\frac{T}{mc\omega u_0}\right| = \sqrt{(R_x + S_x)\sec 2\varphi} \tag{12}$$

The specifics of Eq. 9, Eq. 10, Eq. 11 and Eq. 12 are shown in **Appendix B**.

3 Hangoff configurations

The establishment of coordinate system is the same with that in Section 2 and only difference between the installation configuration and hangoff configuration is that the riser top end connects the heave compensator in hangoff configuration. So, the Eq. 1 is satisfied under these two situations. As shown in Fig. 1, in the hangoff operation, suppose the vessel motion and relative and absolute displacements of riser are in phase and this gives a much simpler solution of the form:

The vessel motion:

$$y = y_0 \cdot \sin \omega t \tag{13}$$

Riser absolute displacement:

$$u = u_0 \cdot \sin \omega t \tag{14}$$

Riser relative displacement:

$$v = v_0 \cdot \sin \omega t \tag{15}$$

According to the basic principle of kinematics, one obtains:

$$u = y + v \tag{16}$$

The top and bottom boundary conditions in hangoff configuration are given by variational analysis. For the most general case of a riser hung off from a compensator system these are:

$$k_c\,(u-y)_{x=0} + \xi_c \left(\frac{\partial u}{\partial t} - \frac{\partial y}{\partial t}\right)_{x=0} - \left(EA\frac{\partial u}{\partial x}\right)_{x=0} = 0 \tag{17}$$

$$\left(M\frac{\partial^2 u}{\partial t^2}\right)_{x=L} + \left(EA\frac{\partial u}{\partial x}\right)_{x=L} + \xi_B \left(\frac{\partial u}{\partial t}\right)_{x=L} = 0 \tag{18}$$

Where, k_c is the compensator stiffness characteristic, N/m; ξ_c is the compensator damping characteristic, N/(m · m/s).

Terms in Eq. 17 represent forces on the riser top. The first term is the compensator force due to riser relative displacement, the second is the damping force due to relative speed and the third is the strain force on the riser top, all due to the relative movements of the vessel and the riser. The terms in Eq. 18 represent forces on the riser bottom and include the inertial force due to the load, the load damping force, and the strain force on the riser.

Eq. 1, Eq. 17 and Eq. 18 define the vibrations of the system in hangoff configuration but an exact solution can't be obtained because of the nonlinearity and coupling of the terms. The compensator spring coefficient has an exponentially hardening-softening characteristic due to the gas polytropic compression-expansion [Azpiazu, Thatcher, and Schwelm 1983]. The compensator damping has two components following power laws of 0 (Coulomb friction) and approximately 1.4 (turbulent, viscous fluid flow field). The damping on the riser and load are nonlinear and additionally, the damping effects are difficult to address, even in the linear case, by standard eigensolution techniques. So, the compensator spring constant needs to be decoupled and linearized. The real and assumed compensator hysteresis loops are shown in **Figure 3**.

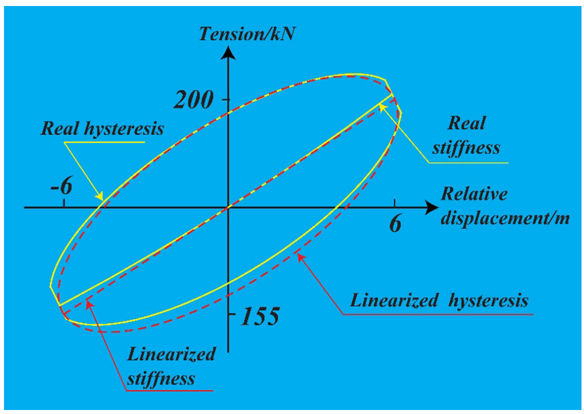

Figure 3: Compensator hysteresis loops.

Considering the riser to have a constant section, it is possible to find an expression for the mode shape of the form:

$$u = a \cdot \cos \eta x + b \cdot \sin \eta x \tag{19}$$

In which,

$$\eta = \omega \cdot \sqrt{\rho / E} \tag{20}$$

Where, ρ is the density of the riser, kg/m^3. This equation represents the depth dependent part of the solution to Eq. 1, when the damping due to the riser being surrounded by seawater is neglected.

Constants a and b vary with riser properties and end conditions. For the absolute displacement of a riser with boundary conditions Eq. 17 and Eq. 18, simplified by neglecting the damping, which are:

$$a = y_0 + \frac{EA\eta}{Mk_c} b \tag{21}$$

$$b = y_0 + \frac{\cos\eta L + (EA\eta/M\omega^2)\sin\eta L}{D} \tag{22}$$

Where,

$$D = EA\eta\left(\frac{1}{M\omega^2} - \frac{1}{k_c}\right)\cos\eta L - (1 + \frac{E^2 \cdot A^2 \cdot \eta^2}{k_c M\omega^2})\sin\eta L \tag{23}$$

From Eq. 19, 21, 22, and 23, we can see that if D equal to 0, the riser vibration displacement u will be go to infinity, that is to say the riser is resonance. So the natural frequency can be determined by solving the implicit equation:

$$\tan\eta L = EA\eta \frac{k_c - M\omega^2}{k_c M\omega^2 + E^2 \cdot A^2 \cdot \eta^2} \tag{24}$$

From Eq. 24 we see that the natural frequencies and general vibration response depend not only on riser and load properties ρ, E, A and M but also on the compensator spring rate k_c and by a proper choice of this value, the natural frequency of the riser system can be shifted away from the narrow band of exciting frequencies typical of the marine environment.

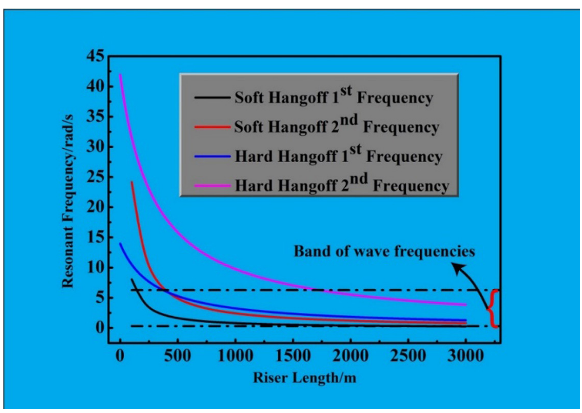

Figure 4: Riser resonant frequency variation with length.

For a 3,000 m length, 18 5/8″ O.D. and 5/8″ W.T steel/composite hard hangoff drilling riser with floaters shown in **Table 1**, the natural frequency of the first five orders is presented in **Table 2**. The resonant frequency of riser under these two configurations variation with length is shown in **Figure 4**. The vibration tension in hangoff configuration can be figured out by substituting Eq. 19, Eq. 21, Eq. 22, Eq. 23, Eq. 24 into Eq. 9 and Eq. 10. The vibration displacement and vibration force of longitudinal vibration in installation operation and hangoff operation are shown in **Figure 5** and **Figure 6** respectively.

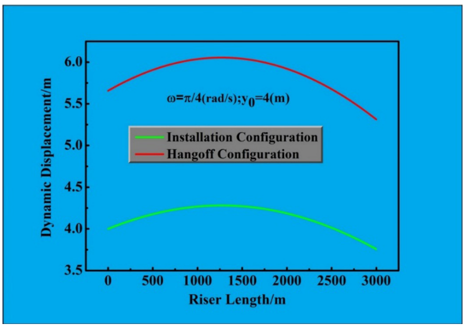

Figure 5: Vibration displacement.

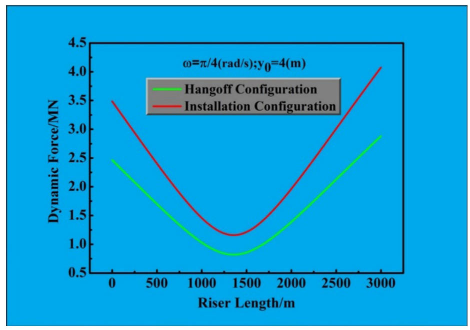

Figure 6: Vibration force.

4 Discussions

4.1 Resonant frequency

The first and second order natural frequency of riser in installation and hangoff configuration is presented in Fig. 4, from which we can see that the natural frequency of riser under these two situations are both decrease with the increase of water depth. The natural frequency in installation are higher than that in hangoff configuration, which shows that the characteristics of riser longitudinal variation have changed because of the existence of heave compensator fixed at the riser top end. For a typical sea wave, the wave period is between 1 s and 20 s and the corresponding wave circular frequency is between 0.3 rad/s and 6.2 rad/s, as shown in Fig. 4. When the riser length is beyond 400 m, riser under two kinds of configurations will both generated the first order resonance, and only the riser in hangoff configuration will generate the second order resonance. However, when the riser length is beyond 1800 m, riser under these two configurations will both generate the first and the second order resonance. What's more, the deeper the water is, the greater the chance of riser resonance occur is.

4.2 Vibration displacement

For a marine riser of 3000 m length, the variation displacement in hangoff configuration is bigger than that in installation. As shown in Fig. 5, when the wave circular frequency is 0.785 rad/s and the amplitude of vessel motion is 4 m, the variation displacement of riser top end is 4 m while that of riser bottom end is 3.75 m and the maximum variation displacement is 4.28 m in installation configuration. In the hangoff configuration, the variation displacement of riser top end is 5.65 m while that of riser bottom end is 5.31 m and the maximum variation displacement is 6.05 m. However, the maximum values of riser variation displacement both appear at $x = 1265$ m.

As shown in **Figure 7** and **Table 2**, when the wave circular frequency is between 0–7 rad/s, riser in installation configuration generate the first, the second and the third order resonance. What's more, as shown in **Figure 8**, the maximum value of vibration displacement locates at riser bottom end and the minimum value of that locates at riser top end. That is to say the vibration displacement increase along riser from top end to the bottom end.

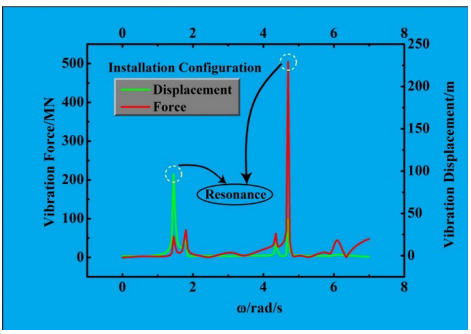

Figure 7: Vibration displacement and force variation withcircular frequency in installation configuration at $x = 3000$ m.

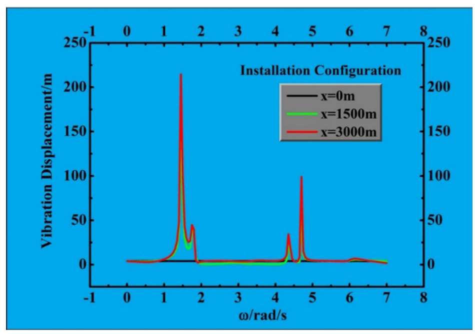

Figure 8: Vibration displacement at $x = 0$ m, $x = 1500$ m and $x = 3000$ m in installation configuration.

Similarly, as shown in **Figure 10** and **Table 2**, when the wave circular frequency is between 0–1.5rad/s, riser in hangoff configuration generate the first, the second and the third order resonance. What's more, as shown in Fig. 11, the maximum value of vibration displacement locates at riser bottom end and the minimum value of that locates at riser top end. That is to say the vibration displacement increase along riser from top end to the bottom end. However, according the **Figure 8** and **Figure 11**, the vibration displacement in hangoff configuration is bigger than that in installation configuration.

4.3 Vibration force

Riser longitudinal vibration characteristic have been changed by the heave compensator which results to the vibration displacement in hangoff configuration is bigger than that of installation configuration while the vibration force in hangoff is smaller than that in installation, which is shown in **Figure 6**. From this perspective, the heave compensator have also improved the riser stress state and enlarged the operation window of riser in hangoff.

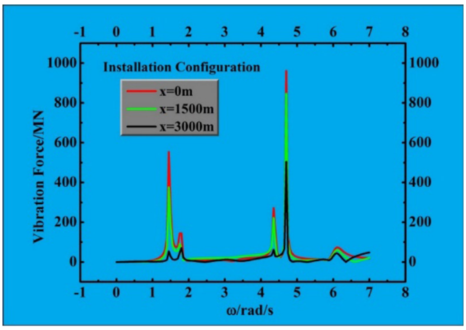

Figure 9: Vibration force at $x = 0$ m, $x = 1500$ m and $x = 3000$ m in installation configuration.

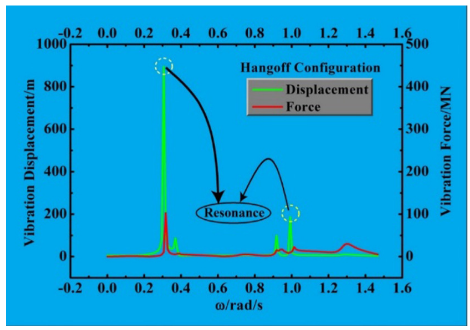

Figure 10: Vibration displacement and force variation with circular frequency in hangoff configuration at $x = 3000$ m.

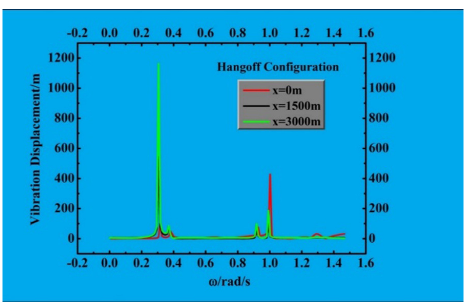

Figure 11: Vibration displacement at $x = 0$ m, $x = 1500$ m and $x = 3000$ m in hangoff configuration.

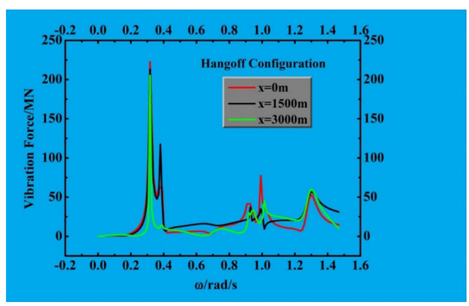

Figure 12: Vibration force at $x = 0$ m, $x = 1500$ m and $x = 3000$ m in hangoff configuration.

As shown in **Figure 7**, when the wave circular frequency is between 0–7 rad/s, riser vibration force suddenly increase three times under the condition of installation which means that the riser generate resonance at these frequency points, and the vibration force is largest when the second order resonance generates. What's more, as shown in **Figure 9**, the maximum value of vibration force locates at riser top end and the minimum value of that locates at riser bottom end in installation. That is to say the vibration force increase along riser from bottom end to the top end.

Similarly, as shown in **Figure 12**, when the wave circular frequency is between 0–1.5 rad/s, riser vibration force also suddenly increase three times under the condition of hangoff. What's more, the variation of vibration force with riser length in hangoff configuration is the same with that in installation which is the vibration force increase along riser from bottom end to the top end.

Although the vibration displacement in hangoff configuration is bigger than that in installation, the vibration force in hangoff configuration is smaller than that in installation configuration, which shows that the main limiting factor in installation operation is the riser strength while that in hangoff operation is the stroke of heave compensator.

5 Conclusions

(1) The natural frequency, amplitude of vibration displacement and vibration force of longitudinal vibration have been determined with consideration of the damping effect of the sea water.

(2) The natural frequency of riser under these two situations are both decrease with the increase of water depth. The natural frequency of riser in installation operation is bigger than that in hangoff operation, which shows that the characteristics of riser longitudinal variation have changed because of the existence of heave compensator fixed at the riser top end. What's more, the deeper the water is, the greater the chance of riser resonance occur is.

(3) The vibration displacement in hangoff configuration is bigger than that in installation configuration. Under the condition of resonance, the maximum value of vibration displacement in these two configurations both locate at riser bottom end and the minimum value of that locate at riser top end. That is to say the vibration displacement increase along riser from top end to the bottom end.

(4) The vibration force in installation configuration is bigger than that in hangoff configuration. Under the condition of resonance, the maximum value of vibration force in these two configurations both locate at riser top end and the minimum value of that locate at riser bottom end. That is to say the vibration force increase along riser from bottom end to the top end.

Appendix A

As shown in **Figure 2-a** and **Figure 2-b**, the boundary condition of riser bottom end is:

$$M \cdot \left(\frac{\partial^2 u}{\partial t^2}\right)_L + mc^2 \left(\frac{\partial u}{\partial x}\right)_L = 0 \tag{A-1}$$

Where, M is the mass of LMRP/BOPs, kg; L is the riser total length before equivalent.

Substitute Eq. 5 into Eq. A-1, the unknown parameter L' can be determined by:

$$\frac{M}{mL}\left(\frac{\omega L}{c}\right) = \tan\frac{\omega}{c}\left(L' - L\right) \tag{A-2}$$

For a very long riser, where $\frac{M}{mL}$ is small, Eq. A-2 reduces to:

$$L' = L + \frac{M}{m} \tag{A-3}$$

Appendix B

The specifics of Eq. 9, Eq. 10, Eq. 11 and Eq. 12 are shown as follows:

$$K_1 = \frac{\omega}{c\sqrt{1 - \tan^2\varphi}}; \tag{B-1}$$

$$K_2 = \frac{\omega}{c\sqrt{\cot^2\varphi - 1}}; \tag{B-2}$$

$$\tan 2\varphi = \frac{\xi_D}{m\omega}; \tag{B-3}$$

$$\alpha = \frac{F_1}{F_3}; \tag{B-4}$$

$$\beta = \frac{F_2}{F_3}; \tag{B-5}$$

$$F_1 = G_1 - (P_1 + 1 - N_1)e^{2K_2L}; \tag{B-6}$$

$$F_2 = Q\cos 2K_1L + N\sin 2K_1L; \tag{B-7}$$

$$F_3 = (1 - N_1)\cosh 2K_2L + P_1\sinh 2K_2L - G_1; \tag{B-8}$$

$$G_1 = Q_1\sin 2K_1L - N_1\cos 2K_1L; \tag{B-9}$$

$$N_1 = \frac{1 - D_1^2 - D_2^2}{2}; \tag{B-10}$$

$$P_1 = D_1\cos\phi - D_2\sin\phi; \tag{B-11}$$

$$Q_1 = D_1 \sin\varphi + D_2 \cos\varphi; \tag{B-12}$$

$$D_1 = \frac{\xi_B}{mc}\sqrt{\cos 2\varphi}; \tag{B-13}$$

$$D_2 = \frac{M\omega}{mc}\sqrt{\cos 2\varphi} \tag{B-14}$$

$$R_x = \left[(1+\alpha)\cdot e^{2K_2x} + \left(\frac{\alpha^2+\beta^2}{2}\right)\cdot \cosh 2K_2x\right]; \tag{B-15}$$

$$S_x = \left[\left(\frac{\alpha^2+\beta^2+2\alpha}{2}\right)\cdot \cos 2K_1x - \beta\cdot \sin 2K_1x\right] \tag{B-16}$$

One the four dimensionless parameters ($\frac{\omega L}{c}, \frac{\xi_B}{mc}, \frac{\xi_D L}{mc}, \frac{M}{mL}$) have been defined, the constants φ, K_1L, K_2L, α and β can be figured out. The amplitudes of the vibration displacement and vibration forces can then be determined from Eq. 9 to Eq. 12, for values of x/L.

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