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Adaptive Differentiators via Second Order Sliding Mode for a Fixed Wing Aircraft

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Abstract: Safety automation of complex mobile systems is a current topic issue in industry and research laboratories, especially in aeronautics. The dynamic models of these systems are nonlinear, Multi-Input Multi-Output (MIMO) and tightly coupled. The nonlinearity resides in the dynamic equations and also in the aerodynamic coefficients' variability.

This paper is devoted to developing the piloting law based on the combination of the robust differentiator with a dynamic adaptation of the gains and the robust controller via second order sliding mode, by using an aircraft in virtual simulated environments.

To deal with the design of an autopilot controller, we propose an environment framework based on a Software In the Loop (SIL) methodology and we use Microsoft Flight Simulator (FS-2004) as the environment for plane simulation.

The first order sliding mode control may be an appropriate solution to this piloting problem. However, its implementation generates a chattering phenomenon and a singularity problem. To overcome these problems, a new version of the adaptive differentiators for second order sliding modes is proposed and used for piloting.

For the sliding mode algorithm, higher gains values may be used to improve accuracy; however this leads to an amplification of noise in the estimated signals. A good tradeoff between these two criteria (accuracy, robustness to noise ratio) is difficult to achieve. On the one hand, these values must increase the gains in order to derive a signal sweeping of some frequency ranges. On the other hand, low gains values have to be imposed to reduce noise amplification. So, our goal is to develop a differentiation algorithm in order to have a good compromise between error and robustness to noise ratio. To fit this requirement, a new version of differentiators

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with a higher order sliding modes and a dynamic adaptation of the gains, is proposed: the first order differentiator for the control of longitudinal speed and the second order differentiator for the control of the Euler angles.

Keywords: Adaptive differentiators, second order sliding modes, Dynamic adaptation of the gains, Microsoft Flight Simulator.

1 Introduction

The control of dynamical systems in presence of uncertainties and disturbances is a common problem to deal with when considering real plants. The effect of these uncertainties on the dynamical systems should be carefully taken into account in the controller design phase since they can degrade the performance or even lead to system instability. For this reason, during recent years, the problem of controlling dynamical systems in presence of heavy uncertainty conditions has become an important research subject. As a result, considerable progress has been attained in robust control techniques, such as nonlinear adaptive control, model predictive control, backstepping, sliding model control and others [Harkegard (2001); Slotine (1991); Junkins, Subbarao and Verma, A. (2000); Chiroi, Munteanu, and Ursu (2011)].

These techniques are able to guarantee the attainment of the control objectives in spite of modeling errors and/or uncertainties on parameters that can affect the controlled plant. Sliding mode control is generally considered to be very robust and simple to implement, but the so-called chattering phenomenon (effects of the discontinuous nature of the control), and the high control activity, have originated a certain skepticism about such an approach.

The first order sliding mode control can be a solution for this piloting problem; however, its implementation generates the chattering phenomenon [Bandyopadhyay (2006); Sabanovic (2004); Perruquetti (2000)] and the singularity problem. In order to avoid them, a new version of the differentiators with a dynamic adaptation of the gains via second order sliding modes approach, is proposed and used for the piloting. These techniques ensure a good tradeoff between error and robustness to noise ratio and especially a good accuracy for a certain frequency range, regardless the gains setting of the algorithm. They have been used to estimate the successive derivatives of the mode sliding surface S(t) and transmit them to the control block, by using an aircraft in virtual simulated environments. It is real-time virtual simulation which is close to the real world situation.

The piloting technique proposed in this work is more robust and simpler to implement than the quaternion one. It only requires information about the sliding mode surface.

2 Problem statement

Through a methodology based on the confrontation of the real and the simulated worlds, the main objective of the present work is to design an autopilot based on robust controller to maintain the desired trajectory (Figure 1).

Figure 1: Real trajectory.

To achieve this objective, we use Flight Simulator FS2004 as simulated world environment coupled to a hardware and software development platform. It is developed by Microsoft, with several simulated aircrafts included in its airplane library.
We chose the Zlin-142 airplane which is used in various aeronautic schools (pilot training) because modify its electronics, actuators and sensors are essay to modify.


Figure 2: Aircraft and environment visualization.

3 Characteristics of the aircraft Zlin-142

Air Wrench tool gives access to flight dynamic characteristics (mudpond.org/Air Wrench/main.htm). This tool allows creating and tuning flight dynamics files description of simulated planes models. This software uses aerodynamics formulas and equations described on the Mudpond Flight Dynamics Workbook. It calculates aerodynamic coefficients based on the physical characteristics and performance of the aircraft (Table 1).

Table 1: FS2004 Aircraft simulated characteristics Zlin-142.

Dimensions	Constant speed propeller	Moments of inertia
Length : 7.42m	Prop diameter: 2.08m	Pitch : 2780.00
Wingspan: 9.27m	Prop gear ratio 1.00:	Roll : 4060.00
Wing surface area: 13.94m^2	Tip velocity: 0.834$mach$	Yaw : 2340.00
Wing root chord: 1.50m	Prop blades: 2	Cross : 0.00
Aspect ratio: 6.17	Beta fixed pitch: 20.00deg	
Taper ratio: 1.00	Prop efficiency: 0.870	
	Design altitude: 1524.0m	

4 Implementation of a real-time interface between Microsoft Flight Simulator and the module Real Time Windows Target of Simulink/Matlab

We design our Software to interface the simulated aircraft in Flight Simulator environment (read and write many sensors, actuators data and parameters).

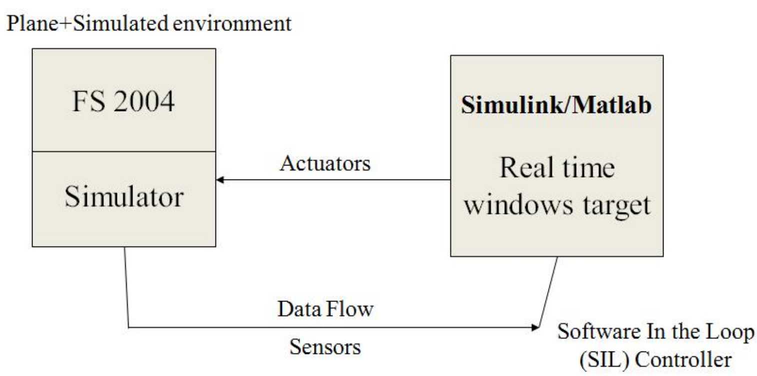

Figure 3: Block diagram of the software environment design.

We communicate with FS2004 by using a dynamic link library called FSUIPC.dll (Flight Simulator Universal Inter-Process Communication). This library created by Peter Dowson is downloadable from his website (www.schiratti:com/Dowson.html), and can be installed by copying the directory (module) of FS2004. It allows external applications to read and write in and from Microsoft Flight Simulator MSFS by exploiting a mechanism for IPC (Inter-Process Communication) using a buffer of 64 Ko. The organization of this buffer is explained in the documentation given with FSUIPC, from which the Figure 4 is taken.

To read or write a variable, we need to know its address in the table, its format and the necessary conversions. For example, the indicated air speed is read as a signed long S32 at the address 0x02BC.

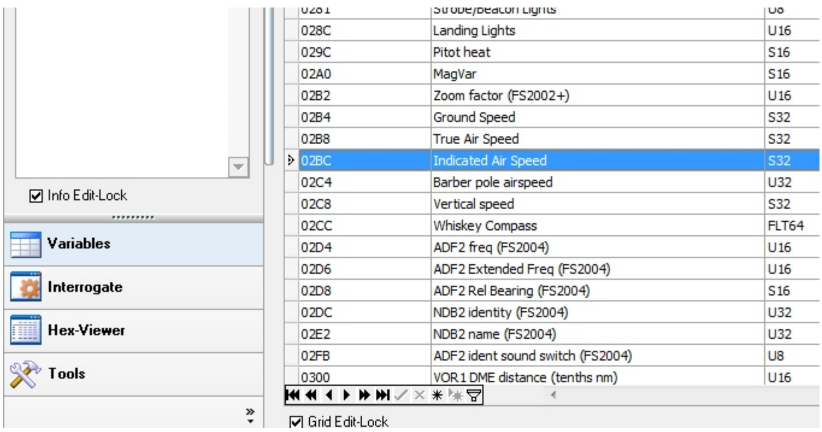

Figure 4: Part of the table FSUIPC.

The following data information is recorded in real time:

- Geographic position (latitude λ, longitude μ and altitude h), Ground speed of the aircraft from the Global Positioning System (GPS);
- Pressure-altitude, vertical/Indicated air Speeds, angle-of-attack α and angle-of-sideslip β from the air data measurement system;
- Rotations rates p, q, r, accelerations a_x, a_y, a_z , Euler angles φ, θ, ψ from Inertial Measurement Unit (IMU).

In this work, the main goal is to maintain the desired aircraft's trajectory; and to do so, we propose the following approach:

- Implementation of a real time interface between the flight simulator FS2004 and the module real time Windows target of Simulink/Matlab;
- Description and analysis of the aircraft system model;
- Development and implementation of the technique based on the combination of the robust differentiator with a dynamic adaptation of the gains and the robust controller via second order sliding mode for the design of the autopilot controller;
- Flight tests.

5 System modeling

The model describing the system state is

$$\dot{x} = f(x,t) + g(x,t).U \tag{1}$$

With x the aircraft state vector in the body frame:

$$\begin{aligned} x &= \begin{bmatrix} u & v & w & p & q & r & \varphi & \theta & \psi & \lambda & \mu & h \end{bmatrix}^T \\ &= \begin{bmatrix} x_1 & \cdots & & \cdots & & \cdots & x_{12} \end{bmatrix}^T \end{aligned} \tag{2}$$

$U = \begin{bmatrix} \delta_t & \delta_e & \delta_a & \delta_r \end{bmatrix}^T$ The control vector and δ_t, δ_e, δ_a and δ_r denoting thrust control, elevator deflection, aileron deflection and rudder deflection.
The nonlinear functions $f(x)$ and $q(x)$ are given by:

$$f(x,t) = \begin{bmatrix} f_1(x,t) & \cdots & \cdots & f_9(x,t) \end{bmatrix}^T \tag{3}$$

$$f_1(x,t) = x_2x_6 - x_3x_5 + C_{x2}x_5 + C_{x4} + C_{x5}\alpha + C_{x1}\dot{\alpha} - g\sin x_8$$

$$f_2(x,t) = x_3x_4 - x_1x_6 + C_{y2}x_4 + C_{y3}x_6 + C_{y6}\beta + C_{y1}\dot{\beta} + C_{y7} + g\sin x_9 \cos x_8$$

$$f_3(x,t) = x_1x_5 - x_2x_4 + C_{z2}x_5 + C_{z4} + C_{z5}\alpha + C_{z1}\dot{\alpha} + g\cos X_9 \cos X_8$$

$$\begin{aligned} f_4(x,t) = &-\frac{I_{zz}}{\Delta}(-I_{xz}x_4x_5 + (I_{yy} - I_{zz})x_5x_6 + C_{l2}x_4 + C_{l3}x_6) \\ &-\frac{I_{xz}}{\Delta}(-I_{xz}x_5x_6 + (I_{yy} - I_{xx})x_4x_5 - C_{n2}x_4 - C_{n3}x_6) \\ &-\frac{1}{\Delta}(I_{zz}(C_{l5}\beta + C_{l1}\dot{\beta} + C_{l7}) - I_{xz}(C_{n6}\beta + C_{n1}\dot{\beta}) + C_{n7}) \end{aligned}$$

$$f_5(x,t) = \frac{1}{I_{yy}}(I_{zz} - I_{xx})x_4x_6 + I_{xz}(x_6^2 - x_4^2) + C_{m2}x_5 + C_{m5}\alpha + C_{m1}\dot{\alpha} + C_{m4}$$

$$f_6(x,t) = -\frac{I_{xz}}{\Delta}(I_{xz}x_4x_5 - I_{xx}(I_{yy} - I_{zz})x_5x_6 - C_{l2}x_4 - C_{l3}x_6)$$
$$-\frac{I_{xx}}{\Delta}(I_{xz}x_5x_6 + (I_{yy} - I_{xx})x_4x_5 + C_{n2}x_4 + C_{n3}x_6)$$
$$-\frac{1}{\Delta}(-I_{xz}(C_{l5}\beta + C_{l1}\dot{\beta} + C_{l7}) + I_{xx}(C_{l5}\beta + C_{l1}\dot{\beta}) + C_{n7})$$

$$f_7(x,t) = x_4 + x_5.\sin x_7.\tan x_8 + x_6.\cos x_7.\tan x_8$$

$$f_8(x,t) = x_5.\cos x_7 - x_6.\sin x_7$$

$$f_9(x,t) = \frac{1}{\cos x_8}.[x_5.\sin x_7 + x_6.\cos x_7]$$

$$g(X,t) = \begin{bmatrix} \frac{F_{prop}.cos(\alpha_m)}{m} & C_{x3} & 0 & 0 \\ 0 & 0 & C_{y4} & C_{y5} \\ \frac{F_{prop}.sin(\alpha_m)}{m} & C_{z3} & 0 & 0 \\ 0 & 0 & a_1 & a_2 \\ 0 & C_{m3} & 0 & 0 \\ 0 & 0 & a_3 & a_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\Delta = I_{xz}^2 - I_{xx}.I_{zz}$$

$$a_1 = -\frac{(I_{zz}C_{l4} - I_{xz}C_{n4})}{\Delta}$$

$$a_2 = -\frac{(I_{zz}C_{l6} - I_{xz}C_{n5})}{\Delta}$$

$$a_3 = -\frac{(I_{xx}C_{n4} - I_{xz}C_{l4})}{\Delta}$$

$$a_4 = -\frac{(I_{xx}C_{n5} - I_{xz}C_{l6})}{\Delta}$$

The changing mass $m(t)$ is

$$m(t) = m_0 - c.t \tag{4}$$

$m_0 = m_{aircraft} + m_{fuel}$ is the total weight equal $1090Kg$, $c(t)$ is the cumulated fuel consumption.

The following condition must always hold:

$m_{carburant} - c.t \geq 0.$

The aircraft motor position has a pitch and a yaw offset orientation angles. In the case of our aircraft, the pitch setting is $\alpha_m = 0.394 = 200deg$, and the yaw setting is $\beta_m = 0.00$. The engine propulsion force is written in the body frame reference [Boiffier, (1998)]:

$$F = F_p.\begin{pmatrix} cos\beta_m cos\alpha_m \\ sin\beta_m \\ cos\beta_m sin\alpha_m \end{pmatrix}\delta_t$$
$$F_p = \frac{K_m.\rho}{V_a} \tag{5}$$

V_a is the aerodynamic velocity, K_m is a constant and σ_t is the throttle position (between 0.0 and 1.0).
The coefficients C_x, C_y,......,C_m are defined in Table 2.

Table 2: Expression of the modified aerodynamic coefficients.

Aer. coef. C_x	Aer. coef. C_y	Aer. coef. C_z
$C_{x1} = \frac{QSC_{x\dot{\alpha}}}{m}$ $C_{x2} = \frac{QScC_{xq}}{mV}$ $C_{x3} = \frac{QSC_{x\delta e}}{m}$ $C_{x4} = \frac{QSC_{x0}}{m}$ $C_{x5} = \frac{QSC_{x\alpha}}{m}$	$C_{y1} = \frac{QSbC_{y\dot{\beta}}}{2mV}$ $C_{y2} = \frac{QSbC_{yp}}{2mV}$ $C_{y3} = \frac{QSbC_{yr}}{2mV}$ $C_{y4} = \frac{QSC_{y\delta a}}{m}$ $C_{y5} = \frac{QSC_{y\delta r}}{m}$ $C_{y6} = \frac{QSC_{y\beta}}{m}$ $C_{y7} = \frac{QSC_{y0}}{m}$	$C_{z1} = \frac{Q.S.c.C_{z\dot{\alpha}}}{m.V}$ $C_{z2} = \frac{QScC_{zq}}{mV}$ $C_{z3} = \frac{QSC_{z\delta e}}{m}$ $C_{z4} = \frac{QSC_{z0}}{m}$ $C_{z5} = \frac{QSC_{z\alpha}}{m}$
Aer. coef. C_m	Aer. coef. C_n	Aer. coef. C_l
$C_{m1} = \frac{QSc^2C_{m\dot{\alpha}}}{V}$ $C_{m2} = \frac{QSc^2C_{mq}}{V}$ $C_{m3} = \frac{QScC_{m\delta e}}{I_{yy}}$ $C_{m4} = QScC_{m0}$ $C_{m5} = QScC_{m\alpha}$	$C_{n1} = \frac{QSb^2C_{n\dot{\beta}}}{2V}$ $C_{n2} = \frac{QSb^2C_{np}}{2V}$ $C_{n3} = \frac{Q.S.b^2.C_{nr}}{2.V}$ $C_{n4} = Q.S.b.C_{n\delta a}$ $C_{n5} = Q.S.b.C_{n\delta r}$ $C_{n6} = Q.S.b.C_{n\beta}$ $C_{n7} = QSbC_{n0}$	$C_{l1} = \frac{QSb^2C_{l\dot{\beta}}}{2V}$ $C_{l2} = \frac{QSb^2C_{lp}}{2V}$ $C_{l3} = \frac{QSb^2C_{lr}}{2V}$ $C_{l4} = QSbC_{l\delta a}$ $C_{l5} = QSbC_{l\beta}$ $C_{l6} = QSbC_{l\delta r}$ $C_{l7} = QSbCl0$

6 Analysis of the piloting

The aircraft dynamic analysis confirms that Roll and Yaw moments equations $f_4(x)$ and $f_6(x)$ have the same shape and are similar. This observation enforces us to find a control method which allows avoiding the singularity problem. In order to do so, we propose to control the longitudinal speed u by the thrust control δ_t, the bank angle φ by the aileron deflection δ_a, the pitch angle θ by the elevation deflection δ_e and the azimuth angle ψ by the aileron and elevation deflections δ_a, δ_e . The rudder deflection δ_r is used in the landing and the taking off. To make a turn, we use bank to turn procedure which needs aileron and elevator deflections. It is based on human piloting techniques.
We propose the following output vector:

$$y = \begin{bmatrix} u & \varphi & \theta & \psi \end{bmatrix}^T \tag{6}$$

The kinematic model is represented by the equations expressing $f_7(x)$, $f_8(x)$ and $f_9(x)$. Notice that the expression of $f_9(x)$ contains a singularity when $x_8 = \pm\frac{\pi}{2}$ where the terms tgx_8 and $secx_8 = \frac{1}{cosx_8}$ are infinite. Such conditions occur in aerobatic manoeuvres where the aircraft loops or climbs at a near vertical angle. Two techniques are used to overcome these problems. The pitch angle can be constrained so that the computation results in a valid floating point number. For example, $tg89.5 = 114.6$ and this value can be used in computations when the pitch attitude is between 89.50 and 90.50.

The numerical error introduced by this approximation only occurs at this extreme flight attitude where its effects on the aircraft behavior may not be apparent. The commonly used method is to use quaternion [Allerton (2009)]. In this work, we propose the adaptive differentiators via sliding mode because they are very robust and simpler to implement than the quaternion technique. They need only the sliding mode surface.

7 Application of the adaptive differentiators for second order sliding mode

7.1 Review of high order sliding mode control

The state equations of the nonlinear system are given by:

$$\begin{aligned} \dot{x} &= f(x,t) + g(x,t).U \\ S &= S(x,t) \end{aligned} \tag{7}$$

and $S(x,t)$ is the sliding mode surface. For our case $S = y - y_d$, where y_d is the desired output signal.

The task is to vanish the output S in finite time and to keep $S \equiv 0$.
According to the conception of system relative degree, there are two conditions [Bandyopadhyay (2006)].

Relative degree = 1, if and only if $\frac{\partial \dot{S}}{\partial U} \neq 0$;

Relative degree ≥ 2, if $\frac{\partial S^{(i)}}{\partial U} = 0$

$$(i = 1, 2, ..., r-1), \frac{\partial S^{(r)}}{\partial U} \neq 0 \tag{8}$$

The aim of the first order sliding mode control is to force the state to move on the switching surface $S(t,x)$. In high order sliding mode control, the purpose is to force the state to move on the switching surface $S(t,x) = 0$ and to keep its $(m-1)^{th}$ first successive derivatives null. In the case of second order sliding mode control, the following relation must be verified:

$$S(t,x) = \dot{S}(t,x) = 0 \tag{9}$$

In arbitrary order sliding mode control, the core idea is that the discrete function acts on a higher order sliding mode surface, making

$$S(t,x) = \dot{S}(t,x) = \cdots = S^{(r-1)} = 0 \tag{10}$$

Suppose the relative degree of system (7) is r, generally speaking, when the control input U first time appears in r-order derivative of S while $\frac{dS^{(r)}}{dU} \neq 0$, we take r-order derivative of S for the output of system (6), $S, \dot{S}, \ddot{S}, \cdots, S^{(r-1)}$ can be obtained. They are continuous function for all the x and t. However, corresponding discrete control law U acts on $S^{(r)}$.
So, the following expression can be obtained

$$S^{(r)} = a(t,x) + b(t,x).U \tag{11}$$

Therefore, high order sliding mode control is transformed to stability of r order dynamic system (7), (8). Through the Lie derivative calculation, one can directly check that [Salgado (2004); Huangfu, Yigeng (2011)].

$$\begin{aligned} b &= L_g L_f^{r-1} S = \frac{dS^{(r)}}{dU} \\ a &= L_f^r S \end{aligned} \tag{12}$$

The sliding mode equivalent control is $U_{eq} = -\dfrac{a(t,x)}{b(t,x)}$. At present, the aim of the control is to design a discrete feedback control, so that the new system converges into origin on the r order sliding mode surface within limited time. However, in equation (7), both $a(t,x)$ and $b(t,x)$ are bounded function. There are positive constants K_m, K_M and C so that

$$\begin{aligned} &0 \prec K_m \leq b(t,x) \leq K_M \\ &|\ a(t,x)\ | \leq C \end{aligned} \tag{13}$$

7.2 Controller construction

Let p be a positive number. Denote

$$\begin{aligned}
&\Sigma_{0,r} = S \\
&\Sigma_{1,r} = \dot{S} + \beta_1.N_{1,r}.sign(S); \\
&\Sigma_{i,r} = S^{(r)} + \beta_i.N_{i,1}.sign(\Sigma_{i-1,r}) \quad i = 1,....,r-1 \\
&N_{1,r} = |\ S\ |^{\frac{(r-1)}{r}} \\
&N_{i,r} = \left(|\ S\ |^{\frac{p}{r}} + |\ \dot{S}\ |^{\frac{p}{r-1}} .....+ |\ S^{(i-1)}\ |^{\frac{p}{(r-i+1)}}\right)^{\frac{(r-i)}{p}} \\
&N_{r-1,r} = \left(|\ S\ |^{\frac{p}{r}} + |\ \dot{S}\ |^{\frac{p}{(r-1)}} .....+ |\ S^{(r-2)|^{\frac{p}{2}}}\right)^{\frac{1}{p}}
\end{aligned} \tag{14}$$

where β_1, β_2, ..., β_{r-1} are positive numbers.
In the above formulae, $sign(.)$ denotes the usual sign function and when the argument is a vector, then $sign(.)$ denotes a vector which elements are the signs of the vector elements.

7.2.1 Theorem 1 [Levant (1998); Levant (2003)]

Let system (6) have relative degree r with respect to the output S and (11) be fulfilled. Then with propely chosen positive parameters β_1, β_2, ..., β_{r-1} controller

$$U = -\gamma.sign(\Sigma_{r-1,r}(S,\dot{S},\ddot{S},....,S^{(r-1)})) \tag{15}$$

provides for the appearance of r-sliding mode $S \equiv 0$ attracting trajectories in finite time.
Certainly, the number of choices of β_i is infinite. Here are a few examples with β_i tested for $r \preceq 3$, p being the least common multiple of $1,2,...,r$.

The sliding mode controller is given:

$$
\begin{aligned}
&1. \quad U = -\gamma.sign(S) \\
&2. \quad U = -\gamma.sign\left(\dot{S} + \mid S \mid^{\frac{1}{2}} .sign(S)\right) \\
&3. \quad U = -\gamma.sign\left(\ddot{S} + 2.(\mid S \mid^{3} + \mid S \mid^{2})^{\frac{1}{6}}\right) .sign\left(\dot{S} + \mid S \mid^{\frac{2}{3}} .sign(S)\right)
\end{aligned}
\tag{16}
$$

From the above equation (15) we can also see that, when $r = 1$, the controller is traditional relay sliding mode control; when $r = 2$, in fact, the controller is a super twisting algorithm of second order sliding mode.

Getting the differentiation of a given signal is always essential in automatic control systems. We often need to differentiate a variable or a function. So there are a lot of numerical algorithms for this issue. The same situation appears in the design of high order sliding mode controller (15) that needs to calculate the derivative values of sliding mode variable.

7.3 Differentiators for higher order sliding mode

For sliding mode algorithm, higher gains values can improve accuracy, but this leads to an amplification of noise in the estimated signals. The compromise between these two criteria (accuracy, robustness to noise ratio) is difficult to achieve. On the one hand, these values must increase the gains values in order to derive a signal sweeping of certain frequency ranges. On the other hand low gains values must be imposed to reduce noise amplification. Our goal is to develop a differentiation algorithm in order to have a good compromise between error and robustness to noise ratio especially to guarantee a good accuracy for certain frequency ranges, regardless of the gains setting of the algorithm. To satisfy at best these criteria, we propose a new version of the differentiators of higher order sliding modes with a dynamic adaptation of the gains:

- Second-order differentiator for the control of the Euler angles φ, θ and ψ;
- First order differentiator for the control of longitudinal speed u.

7.4 For the Euler angles φ, θ and ψ

The relative degrees are:

$r_\varphi = r_\theta = r_\psi = 2$

The control input can be chosen as following:

$$U = -\gamma.sign\left(\dot{S} + \mid S \mid^{\frac{1}{2}} .sign(S)\right) \tag{17}$$

Where

$$\begin{aligned} U &= \begin{bmatrix} \delta_e & \delta_a & \delta_r \end{bmatrix}^T \\ S &= \begin{bmatrix} S_\varphi & S_\theta & S_\psi \end{bmatrix}^T \\ \gamma &= \begin{bmatrix} \gamma_\varphi & 0 & 0 \\ 0 & \gamma_\theta & 0 \\ 0 & 0 & \gamma_\psi \end{bmatrix} \end{aligned} \tag{18}$$

We propose the sliding mode surfaces from the differentiator:

$$\begin{cases} S_0 = z_0 - y_d \\ S_1 = z_1 - v_0 \\ S_2 = z_2 - v_1 \end{cases} \tag{19}$$

Where the desired vector state variables and the outputs of the differentiator are defined by:

$$\begin{cases} y_d = \begin{bmatrix} \varphi_d & \theta_d & \psi_d \end{bmatrix}^T \\ z_0 = \begin{bmatrix} z_{0\varphi} & z_{0\theta} & z_{0\psi} \end{bmatrix}^T \\ z_1 = \begin{bmatrix} z_{1\varphi} & z_{1\theta} & z_{1\psi} \end{bmatrix}^T \\ z_2 = \begin{bmatrix} z_{2\varphi} & z_{2\theta} & z_{2\psi} \end{bmatrix}^T \end{cases} \tag{20}$$

v_0 and v_1 are given by the adaptive second order differentiator.

$$\begin{cases} \dot{z}_0 = v_0 \\ v_0 = -\hat{\lambda}_0 \mid S_0 \mid^{\frac{3}{4}} sign\left(S_0\right) - K_0.S_0 + z_1 \\ \dot{z}_1 = v_1 \\ v_1 = -\hat{\lambda}_1 \mid S_1 \mid^{\frac{2}{3}} sign\left(S_1\right) - K_1.S_1 + z_2 \\ \dot{z}_2 = v_2 \\ v_2 = -\hat{\lambda}_2. \mid S_2 \mid^{\frac{1}{2}} .sign(S_2) - \hat{\lambda}_3. \int_0^t .sign(S_2)dt - K_2.S_2 \end{cases} \tag{21}$$

where $K_1, K_2, K_3 \succ 0$.
The dynamic adaptation of the gains $\dot{\hat{\lambda}}_i, i \in \{1,2,3\}$ are given by:

$$\begin{cases} \dot{\hat{\lambda}}_0 =\mid S_0 \mid^{\frac{3}{4}} .sign(S_0)S_0 \\ \dot{\hat{\lambda}}_1 =\mid S_1 \mid^{\frac{2}{3}} .sign(S_1)S_1 \\ \dot{\hat{\lambda}}_2 =\mid S_2 \mid^{\frac{1}{2}} .sign(S_2)S_2 \\ \dot{\hat{\lambda}}_3 = S_2 \int_0^t sign(S_2)dt \end{cases} \tag{22}$$

In case of using the differentiator, variable S is considered as given input of the differentiator. Then, the output of differentiator z can be used to estimate corresponding order derivative of S (Figure 6).
The reduction of the noise is assumed by the presence of the linear term K_iS_i in the equation of each output i of the adaptive algorithm. This linear term can be expressed as the law of the equivalent control which allows the reduction of the chattering effect. The addition of this continuous term smooths the output noise due to a low gain values. If the chosen values of these gains become very low, the convergence time of the algorithm becomes slow. Therefore, the choice of the convergence gains remains difficult and is based on a compromise between reduction of the noise and the convergence time of the adaptive differentiator. It should also be noted that in the presence of noise, it is necessary to impose the small initial values of the dynamic gains to reduce the effect of the discontinuous control. Moreover, the presence of integral term in the expressions of the dynamic gains provides also the smoothing of the estimated derivatives. The application of the differentiators with dynamic adaptation of the gains via sliding mode controller in FS2004 is shown in the following figure:

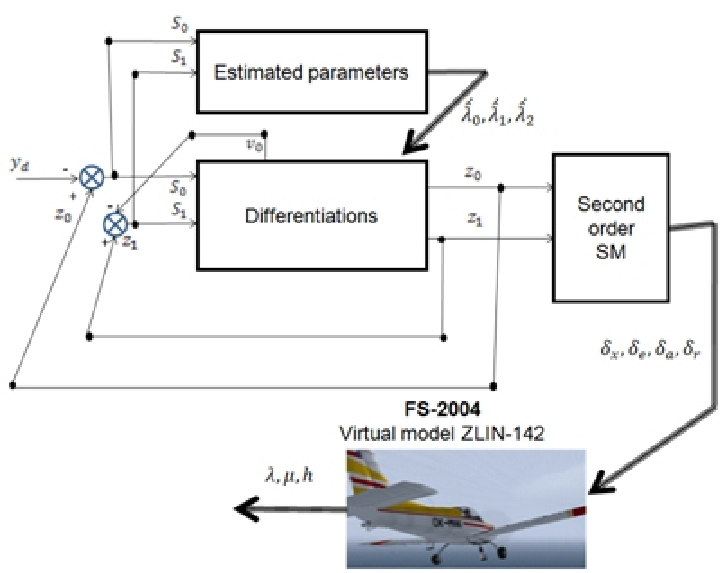

Figure 5: Application of the adaptive differentiators for sliding mode controller in FS2004.

7.5 Simulation results

We run the Flight Simulator FS2004 and the interface with the module Real Time Windows Target of Simulink/Matlab. The taking off of the aircraft Zlin-142 was done by the keyboard. Then, we run our software to transmit the control inputs based on the adaptive differentiators via second order sliding mode to the autopilot controller in order to maintain the desired trajectory.

The input signals to the upper and lower saturation values of the control laws are used to respect the actuators bounds. Scaled functions are added to take into account the actuators resolutions.

The robust differentiator via sliding mode technique is used to recover the desired signal. Several flight tests were realized to demonstrate the effectiveness of the combined controller/differentiator. We chose the parameters $K_{0,i} = 50$ and $K_{1,i} = 50$, where $i = \varphi, \theta, \psi$.

The desired signal injected and the output differentiators are shown in figure 6.

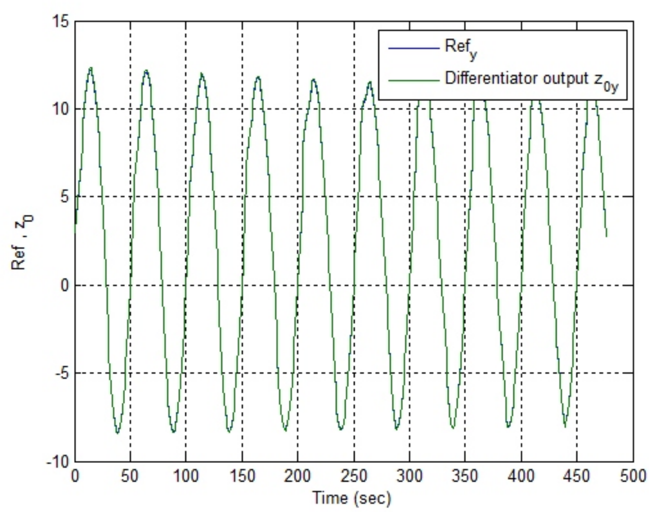

Figure 6: Reference and output differentiator.

We notice that the outputs of the differentiators $z_{0,j}$ where $i = \varphi, \theta, \psi$ follows the references φ_d , θ_d and ψ_d perfectly.

The surfaces sliding mode $S_{0,\varphi,\theta,\psi}$ are small (see Figure 8).

The Figure 9 show the error between the output differentiator z_1 and v_0. The signal z_1 follows v_0.

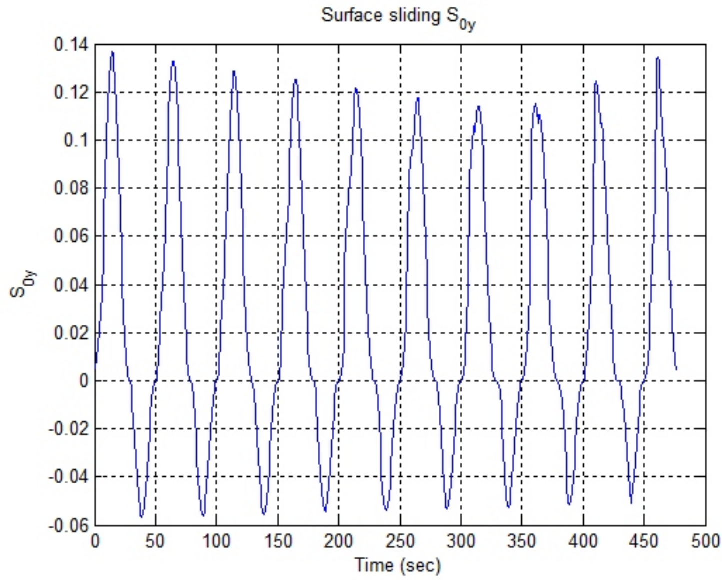

Figure 7: Surface sliding S_0.

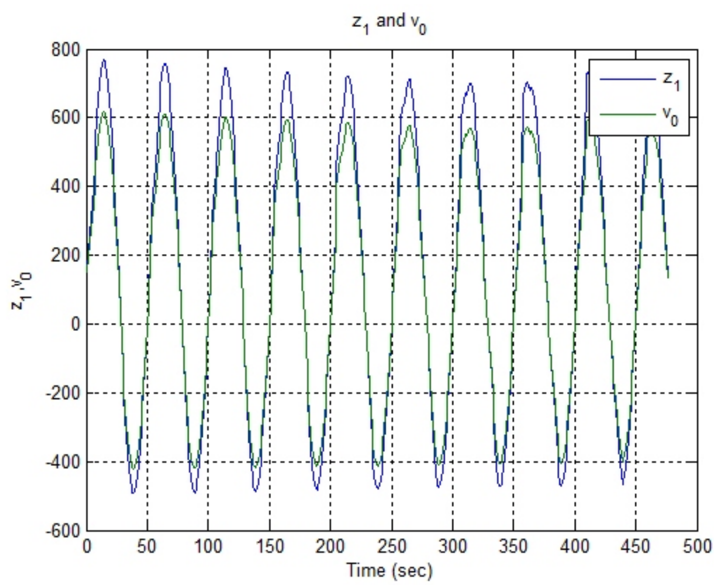

Figure 8: Output differentiator z_1 and signal v_0.

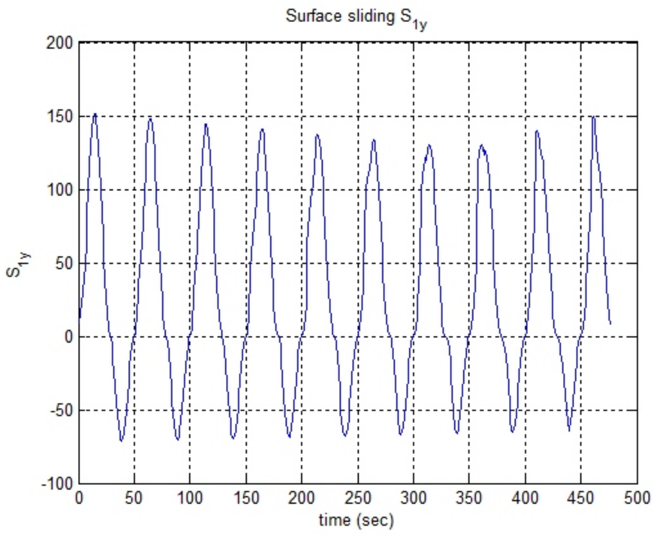

Figure 9: Surface sliding mode S_1.

The surface sliding mode S_1 is shown in Figure 10.

The input signals to the upper and the lower saturation values of the aileron, rudder and elevator deflections are used to respect the virtual Joystick (PPjoy) bounds. Upper limit: 62767, lower limit: 1.

Airwrench gives the following data:

- Aileron parameters: Aileron area $1.30m^2$, aileron up angle limit $28deg$, aileron down angle limit $20deg$;
- Elevator parameters: Elevator area $2.23m^2$, Elevator up angle limit $32deg$, Elevator down angle limit $30deg$.
- Rudder parameters: Rudder area $0.72m^2$, Rudder angle limit $22deg$.

The aileron, elevator and rudder deflections are shown in figures 10, 11, and 12. We notice the absence of the chattering phenomenon.

The evolution parameters $\hat{\lambda}_0$, $\hat{\lambda}_1$ and $\hat{\lambda}_2$ are shown in Figure 13. It's noticed that they increase gradually with the variation of the surfaces $S_{0,\varphi,\theta,\psi}$ and $S_{1,\varphi,\theta,\psi}$.

The flight tests demonstrate the robustness of the differentiator via second order sliding mode. It makes it possible to ensure a better derivation of the desired input signal in real time and this to ensure a good accuracy of tracking the desired trajectory.

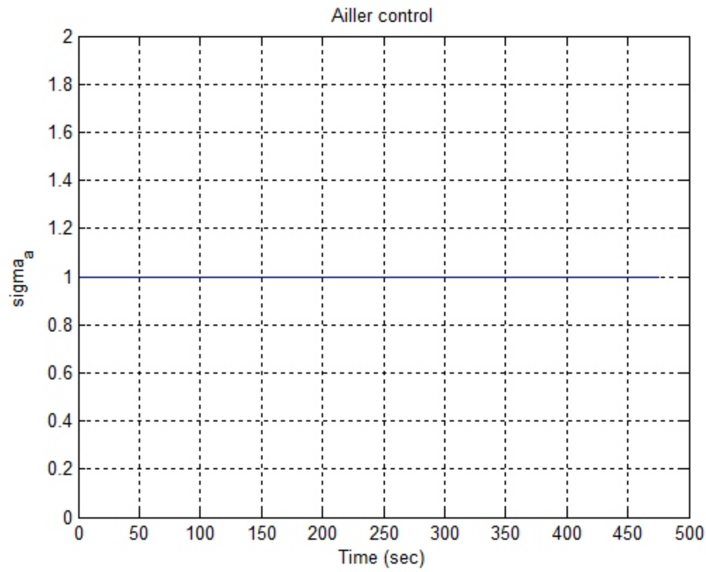

Figure 10: Ailler control σ_a.

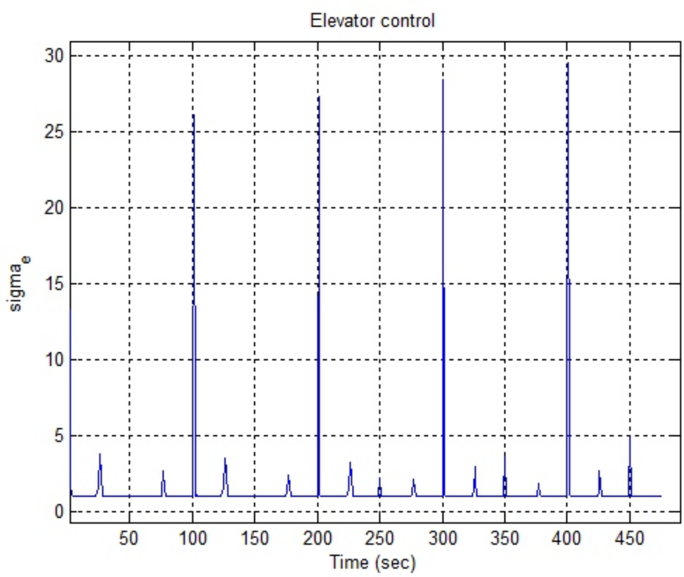

Figure 11: Elevator control σ_e.

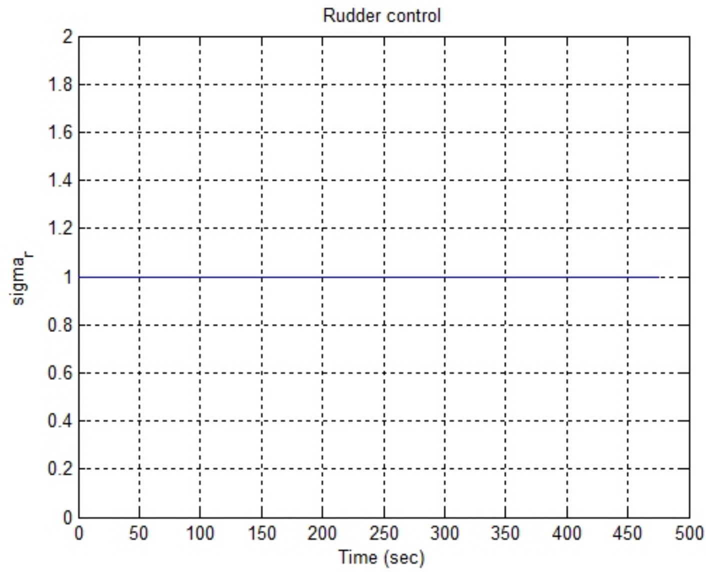

Figure 12: Rudder control σ_r.

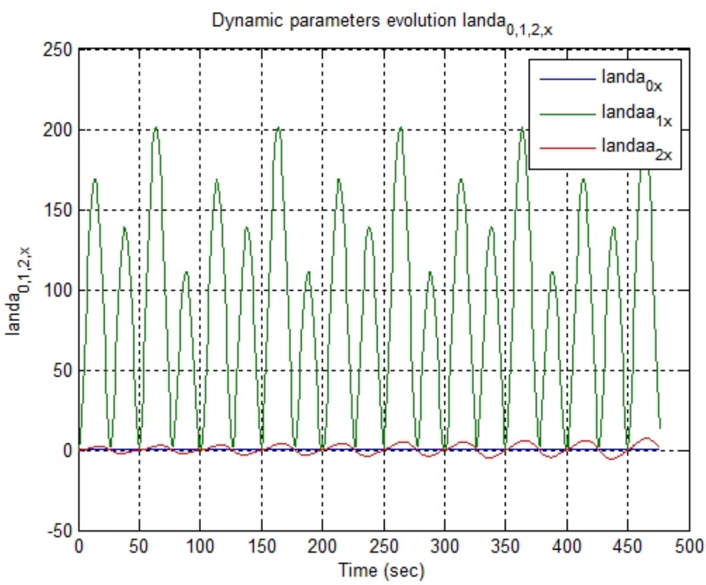

Figure 13: Dynamic parameters evolution.

7.6 For the control of the longitudinal speed u

The relative degree is:

$r_u = 1.$

The control input can be chosen as following:

$$\delta_t = -\gamma.sign(S_{0u}) \tag{23}$$

Where $S_{Ou} = z_{Ou} - u_d$ and $\gamma \succ 0$.
We propose the adaptive super twisting:

$$\begin{cases} \dot{z}_{0u} = v_{0u} \\ v_{0u} = -\hat{\lambda}_{0u} \mid S_{0u} \mid^{\frac{1}{2}} .sign(S_{0u}) - K_u.S_{0u} + z_{1u} \\ \dot{z}_{1u} = v_{1u} \\ v_{1u} = -\hat{\lambda}_{1u}. \int_0^t sign(S_{0u})dt \end{cases} \tag{24}$$

where $K_u \succ 0$.
The dynamic adaptation of the gains are given by:

$$\begin{cases} \dot{\hat{\lambda}}_{0u} = \mid S_{0u} \mid^{\frac{1}{2}} .sign(S_{0u})S_{0u} \\ \dot{\hat{\lambda}}_{1u} = S_{0u}. \int_0^t sign(S_{0u})dt \end{cases} \tag{25}$$

8 Simulation results

We chose the parameter $\gamma = 62767$.
The reference is the longitudinal speed u expressed in m/s. We notice the presence of the error between the reference and the output differentiator (figure 14). This error varies between 1.8 and $6m/s$ (see figure 15).
We notice that they increase gradually with the variation of the surface S_{0u}.
The simulations results are:

- The output differentiator follows the reference;
- The tracking error is acceptable;
- Absence of the chattering phenomenon.

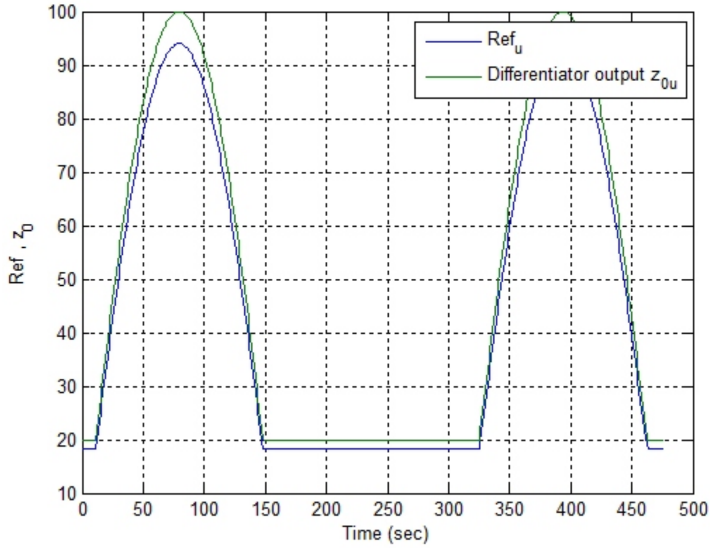

Figure 14: Reference and output differentiator.

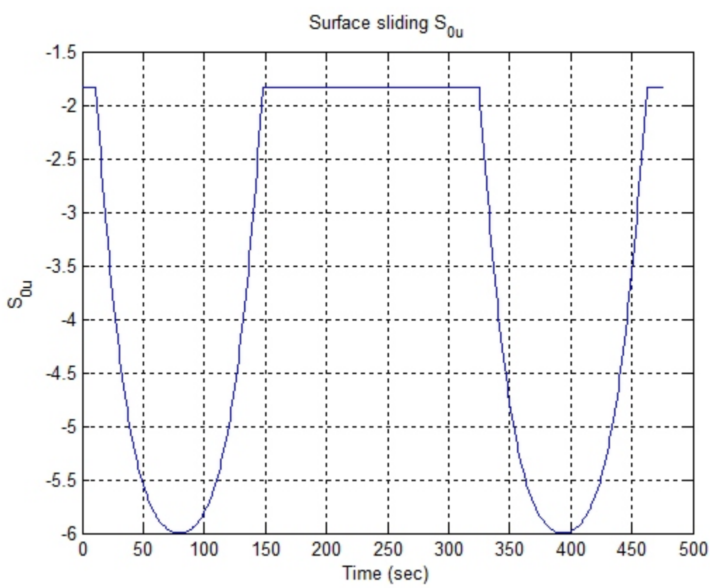

Figure 15: Surface sliding mode S_{0u}.

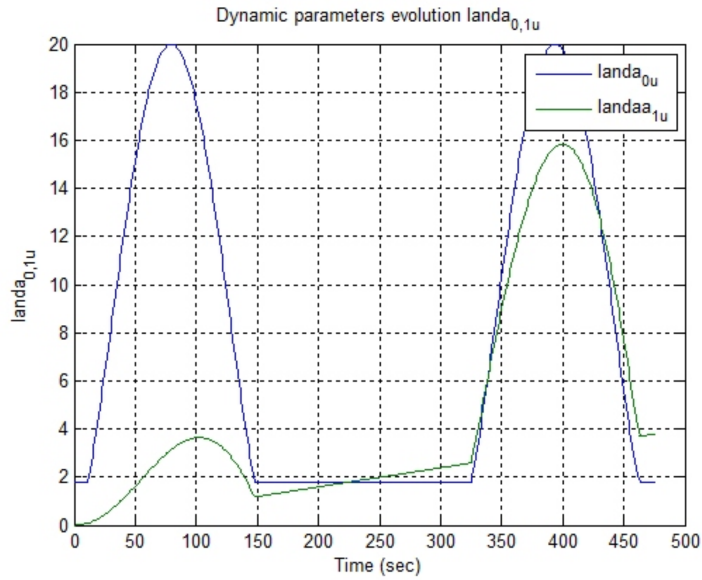

Figure 16: Dynamic parameters evolution.

9 Conclusion

In this paper, a combination of the robust differentiator with a dynamic adaptation of the gains and the robust controller via second order sliding mode for an aircraft autopilot has been presented. Our approach uses the environment simulator (FS2004) to reduce the design process complexity.

The aircraft dynamic analysis confirms that Roll and Yaw moments equations are similar and have the same shape. This observation enforced us to find a method of control which permits avoiding the singularity problem. To solve this problem, we proposed a new version of the differentiators for higher order sliding modes with a dynamic adaptation of the gains approach. This technique is more robust and simpler to implement than the quaternion one and only needs the information about the sliding mode surface.

The first order Sliding mode autopilot controller is characterized by its robustness and takes account of model uncertainties and external disturbances. Unfortunately, the application of this control law is confronted to the serious problem of the chattering phenomenon. To prevent this drawback, adaptive differentiators for the second order sliding mode controller were designed and applied.

For sliding mode algorithm, choosing higher gains values can improve accuracy but this leads to an amplification of noise in the estimated signals. The compromise between these two criteria (accuracy, robustness to noise ratio) is generally difficult to achieve. On the one hand, these values must increase the gains values in order to derive a signal sweeping certain frequency ranges. On the other hand, low gains values must be imposed to reduce noise amplification. Hence, we developed a differentiation algorithm in order to get a good compromise between error and robustness to noise ratio and at the same time guarantee a sufficient accuracy for a specific frequency range, regardless the gains setting of the algorithm. To satisfy at best these criteria, we have proposed a new version of the adaptive differentiators of:

- First order differentiator for the control of longitudinal speed u;
- Second-order differentiator for the control of the Euler angles φ, θ and ψ.

Consequently, using this approach we obtained the following results:

i) Absence of the chattering phenomenon in the control signals inputs;

ii) Higher accuracy of the convergence of the system towards surface, owing to the fact that the system is governed by the expression: $S = \dot{S} = 0$.

The flight tests demonstrate the robustness of the new version adaptive differentiators for the second order sliding mode. The former ensures a better derivation of the desired input signal in real time and this ensures a good accuracy in term of tracking for a desired reference.

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