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A New Multi-objective Reliability-based Robust Design Optimization Method

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Abstract: A new multi-objective reliability-based robust design optimization (M ORBRDO) model is proposed which integrats the multi-objective robustness, the reliability sensitivity robustness and the six sigma robustness design idea. The pure-quadratic polynomial functions are adopted to fit the performance objective functions (POF) and the ultimate limited state functions (ULSF) of the structure. Based on the ULSF and the checking point method, the equations of the first order reliability index are calculated. The mapping transformation method is employed when the non-normal distribution variables are included. According to the POF and the Taylor series expansion method, the equations of mean value and standard deviation of the performance objectives are deduced. In order to improve the efficiency of MORBRDO, a new reliability based design optimization strategy is proposed. The new strategy does not update the ULSF in the sub-cycle of reliability calculation process, so that the computational work is reduced remarkably. Then, the optimum design was obtained with the sequential quadratic programming method. Finally, three engineering projects, a I-beam structure, a pressure vessel and the turbine blade model lines are introduced to redesign their sizes by the new MORBRDO methodology proposed in this paper. The results prove that the new MORBRDO model and methods are correct, feasible and efficient, so it is valuable theoretically and applicative in engineering.

Keywords: robust design, response surface method, reliability, multi-objective optimization.

1 Introduction

For the past few years, the complication of multi-objective optimization, reliability based design, robust design and their combination of each other in real project had

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been paid more and more attention. The multi-objective reliability based robust design optimization (MORBRDO) takes into consideration the multi-objective. The designed product gets the expected reliability after MORBRDO and its multi-objective performances are the least sensitive to the design variables. There are several key problems must to be solved for MORBRDO: (1) calculation of structural reliability; (2)establishment of the robustness cost function; (3)the design of optimizing strategy; (4) implementation of multi-objective optimization.

When the ultimate limited state functions (ULSF) and the performance objective functions (POF) are given, the checking point method is often applied in reliability analysis, see Hasofor and Lind (1974); Rachwitz and Fiessler (1978); Santos et al (2012). The first order reliability index is numerically equal to the ratio of the mean value and standard deviation of the ULSF. However, for most of the structures in real projects, neither the ULSF nor the POF is known because of the complication of the shape, loads and boundary conditions. Theoretically, the mean value and standard deviation of the performance objectives and the structural reliability could be obtained by the Monte Carlo simulation method combined with the finite element analyses (FEA). But tremendous amount of computational work would make the Monte Carlo simulation method infeasible. Focusing on the listed problems, the researchers present a variety of methods.

When the ULSF and the POF are unknown, a dimension reduction method (DRM) is proposed to obtain the mean value and standard deviation, see Alois and Hans (2001); Rahman and Xu (2004); Lee et al (2008); Wang et al (2011) It converts the multi-dimensional integral to one dimensional integral. Similar to the Gaussian integral, it only calculates the values at some certain integral points and gets the mean value and standard deviation with the weighted sum method. The DRM provides an effective way to calculate the statistical moments, but the response surface method (RSM) is a more commonly used method. The essence of RSM is to simulate an approximate function to present the real relationship between the response and the random variables of the structure. And the approximate ULSF and POF can also be obtained by RSM, see Zhen et al. (2009); Shun et al. (2011); Adam and Tadeusz (2012). There are some commonly used forms for the response function, such as the quadratic polynomial function, see Cho JY, Oh MH.(2010); Kaminski and Szafran (2012), the Kriging model, see Andy (2009); Rajagopal and Ranjan G.(2011); Yang and Sun (2013)and the radial basis function model, see Zhou et al. (2013); Luo and Zhang (2012) Zhou (2013) compares the precision of several surrogate models and proposed a hybrid surrogate model.

The robustness cost function also often includes the mean value and standard deviation of the POF. The Taguchi quality loss function model is one of the most commonly used robustness cost functions, see Dubey and Yadava (2008). In other models, the six sigma robustness design idea is introduced, see Shimoyama (2008); Koch. et al (2004). Om et al (2010) proposes a hybrid quality loss function which included desirable as well as undesirable deviations; Lu and Zhang (2011) combines the reliability sensitivity with the performance objectives in robustness cost function.

The main challenge of MORBRDO is the low efficiency caused by large number of reliability analyses involved in the optimization procedure, especially when multi-failure modes are within the structure. To improve efficiency, a sequential method is developed, see Yu. et al (2013) which decouples reliability analysis from the optimization to construct an equivalent deterministic constraint instead of the original probabilistic one by using the locally exponential approximation. Two single-level methods are proposed respectively, see Wang (2002); Du and Chen (2004), to instead the common two-level method. Li and Azarm (2006) presented a deterministic non-gradient based approach which applied a robustness index based on sensitivity regions.

The performance objectives will not always get the optimum simultaneously, sometimes even conflict with each other, see Besharati et al (2006); Li and Zhao (2013). A usual way is to turn the multi-objective optimization problem into a single objective optimization problem, such as the main objective method and the linear weighted sum method, see Om et al (2010); Lu and Zhang (2011). This way is easy to accomplish but can get only one or parts of the Pareto solutions. The aim of multi-objective design optimization is to obtain the Pareto solutions set with multi-objective design optimization algorithm, so that the decider could choose the corresponding one according to his preference to each objective. The way of changing the weight factors based on the linear weighted sum method is one of the most commonly used methods to get the Pareto solutions set, see Dubey and Yadava (2008). In recent years, the meta-metaheuristic method, see Darian and Alexander (2009), the Multi-Objective Tabu Search method (MOTS2), see Kipouros et al (2008); Trapani et al (2012); Razzaq et al (2013) the multi-objective particle swarm optimization algorithm (MOPSO) , see Li and Zhao (2013), and the multi-objective evolutionary algorithm, see Rajagopal and Ranjan(2011), et al, are also applied to discover the full Pareto frontier for the multi-objective problems. These methods are being paid more and more attentions because they don't need gradient of the objectives, so they are particularly suited for the structure with discontinuous POF. The main challenge of these modern ways is that they still need to decrease their iterations and improve their efficiency to reduce computational burden.

In this paper, a new MORBRDO model which takes advantages of the multi-objective robustness, the reliability sensitivity robustness and the six sigma robustness design is proposed. And a new design optimization strategy is put forward to improve the optimization efficiency. The pure-quadratic polynomial functions are adopted to fit the POF and the ULSF of the structure. Then, the optimum design is obtained with the sequential quadratic programming method. Finally, three engineering projects are introduced to testify the feasibility, accuracy and efficiency of the new MORBRDO methodology.

2 Multi-objective reliability based robust design optimization (MORBRDO) model

2.1 traditional design optimization (TDO) model

$$\begin{aligned}
&\text{minmize} \quad h(\mathbf{d}) \\
&\text{s.t.} \quad g_i(\mathbf{d}) \le 0, i = 1, ..., nc \\
&\qquad \mathbf{d}^L \le \mathbf{d} \le \mathbf{d}^U, \mathbf{d} \in R^{ndv}
\end{aligned} \tag{1}$$

Where $h(\boldsymbol{d})$ is the cost function of the structure with respect to $\boldsymbol{d}$, $g_i(\boldsymbol{d})$ is the ith constraint, $\boldsymbol{d}$ is the ndv-dimention design variable vector, $\boldsymbol{d}^L$ is the lower specification limit of $\boldsymbol{d}$, and $\boldsymbol{d}^U$ is the upper. The traditional(deterministic) design optimization model does not take the random character of the design variable and the non-design variable into consideration, and its optimal result locates on the constraint boundary, so the result is sensitive to the variables. And the structural reliability after design can not reach the requirement.

2.2 A new MORBRDO model

$$\begin{aligned}
&\text{minmize } f(\mu_H, \sigma_H, \alpha|\mathbf{X}) \\
&s.t.\ P(g_i(\mathbf{X};\mathbf{d}) < 0) \le \Phi(\text{-}\beta_{ti}), \text{ and} \\
&\prod_{i=1}^{nc} P(g_i(\mathbf{X};\mathbf{d}) < 0) \le \Phi(\text{-}\beta_{t0}), i = 1, ..., nc \\
&\mathbf{d}^L + f \cdot \sigma_{\mathbf{d}} \le \mathbf{d} \le \mathbf{d}^U - f \cdot \sigma_{\mathbf{d}}, \\
&\mathbf{d} \in R^{ndv} \text{ and } \mathbf{X} \in R^{nxv}
\end{aligned} \tag{2}$$

Where $f(\boldsymbol{\mu}_H, \boldsymbol{\sigma}_H, \boldsymbol{\alpha}|\mathbf{X})$is the new MORBRDO robust cost function with respect to $\boldsymbol{\alpha}$ and $\boldsymbol{X}$, $\boldsymbol{\alpha}$ is the reliability sensitivity coefficient, $\boldsymbol{X}$ is the nxv-dimention variable vector (include design variable and non-design variable), $\boldsymbol{\mu}_H$and $\boldsymbol{\sigma}_H$are the mean value vector and the standard deviation vector of the multi-objectives, respectively. $\boldsymbol{\sigma}_d$ is the standard deviation of $\mathbf{d}$. $g_i(\boldsymbol{X};\boldsymbol{d})$ is the ith ultimate limit state function with respect to $\boldsymbol{X}$, the design is assumed to be a "failure" if $g_i(\boldsymbol{X};\boldsymbol{d}) < 0; \beta_{ti}$ is the ith target structural reliability index corresponding to the $g_i(\boldsymbol{X};\boldsymbol{d})$; $P(g_i(\boldsymbol{X};\boldsymbol{d}) < 0)$is the probability of $g_i(\boldsymbol{X};\boldsymbol{d})<0$. $\Phi(\cdot)$ is the standard normal cumulative distribution function; $\prod_{i=1}^{nc} P(G_i(\mathbf{X};\mathbf{d}) < 0) = P(G_1(\mathbf{X};\mathbf{d}) < 0) * P(G_2(\mathbf{X};\mathbf{d}) < 0) * ... * P(G_{nc}(\mathbf{X};\mathbf{d}) <$

0); β_{t0} is the target reliability index of the structure system under multi-failure modes. f is the robustness level of design variables, it guarantees the probability that the design point goes beyond the specification limits equal to or less than $\Phi(-f)$; Usually, $3 \leq f \leq 6$.

2.3 MORBRDO cost function

MORBRDO cost function $f(\mu_H, \sigma_H, \alpha|\mathbf{X})$ is the linear weighted sum of several objectives. It can be classified to the following four types according to its characteristic:

(1) Smaller-the-better type

$$\begin{aligned} f_S(\mu_H, \sigma_H, \alpha|\mathbf{X}) = &\sum_{i=1}^{k} \left[w_{i1} \cdot sgn(\mu_{H_i}) \left(\frac{\mu_{H_i}}{\mu_{H_{i0}}} \right)^2 + w_{i2} \left(\frac{\sigma_{H_i}}{\sigma_{H_{i0}}} \right)^2 \right] \\ &+ \sum_{j=1}^{nc} \left[w_{j3} \left(\sum_{p=1}^{nxv} \alpha_{jp}^4 \bigg/ \sum_{p=1}^{nxv} \alpha_{jp0}^4 \right)^2 \right] \end{aligned} \tag{3}$$

Where μ_{H_i} and σ_{H_i} are the mean value and standard deviation of the ith performance objective, respectively. K is the number of the performance objectives; $\boldsymbol{\alpha}_j$ is the jth reliability sensitivity coefficient vector, α_{jp} is the pth element of $\boldsymbol{\alpha}_j$; $\mu_{H_{i0}}$, $\sigma_{H_{i0}}$ and α_{jp0} are the initial value of μ_{H_i}, σ_{H_i} and α_{jp}, respectively. $sgn(\cdot)$ is the sign function; w_{i1}, w_{i2} and w_{j3} are weighting factors to be determined by the designer, they are taken value in [0,1] usually.

(2) Larger-the-better type

$$\begin{aligned} f_L(\boldsymbol{\mu}_H, \boldsymbol{\sigma}_H, \boldsymbol{\alpha}|\mathbf{X}) = &\sum_{i=1}^{k} \left[w_{i1} \cdot sgn(\mu_{H_i}) \left(\frac{\mu_{H_{i0}}}{\mu_{H_i}} \right)^2 + w_{i2} \left(\frac{\sigma_{H_i}}{\sigma_{H_{i0}}} \right)^2 \right] \\ &+ \sum_{j=1}^{nc} \left[w_{j3} \left(\sum_{p=1}^{nxv} \alpha_{jp}^4 \bigg/ \sum_{p=1}^{nxv} \alpha_{jp0}^4 \right)^2 \right] \end{aligned} \tag{4}$$

(3) Nominal-the-best type

$$\begin{aligned} f_N(\boldsymbol{\mu}_H, \boldsymbol{\sigma}_H, \boldsymbol{\alpha}|\mathbf{X}) = &\sum_{i=1}^{k} \left[w_{i1} \left(\frac{\mu_{H_i} - h_{it}}{\mu_{H_{i0}} - h_{it}} \right)^2 + w_{i2} \left(\frac{\sigma_{H_i}}{\sigma_{H_{i0}}} \right)^2 \right] \\ &+ \sum_{j=1}^{nc} \left[w_{j3} \left(\sum_{p=1}^{nxv} \alpha_{jp}^4 \bigg/ \sum_{p=1}^{nxv} \alpha_{jp0}^4 \right)^2 \right] \end{aligned} \tag{5}$$

Where h_{it} and $\mu_{H_{i0}}$ are the target nominal value and the initial target nominal value of the ith performance objective function respectively

(4) Combined type

$$f_C(\boldsymbol{\mu}_H,\boldsymbol{\sigma}_H,\boldsymbol{\alpha}|\mathbf{X}) = f_S(\boldsymbol{\mu}_H,\boldsymbol{\sigma}_H,\boldsymbol{\alpha}|\mathbf{X}) + f_L(\boldsymbol{\mu}_H,\boldsymbol{\sigma}_H,\boldsymbol{\alpha}|\mathbf{X}) + f_N(\boldsymbol{\mu}_H,\boldsymbol{\sigma}_H,\boldsymbol{\alpha}|\mathbf{X}) \quad (6)$$

In equations (3)-(6), to reduce the dimensionality problem of multi-objectives, each term is normalized by the initial value $\mu_{H_{i0}} \sigma_{H_{i0}}$ and α_{jp0}, respectively. The new MORBRDO cost function not only pursuits the optimum of the mean value, but also seeks the minimum values of the standard deviation of the multi-objectives. Moreover, it explores the most robust design of the structural reliability in itself. The weighting factors reflect the preferences of the designer to each objective

3 MORBRDO based on response surface method (RSM)

In practical projects, for structural system with complicated shape and in complex stress condition, it is necessary to get the response values of volume, stress, deformation and inherent frequency by numerical computation method (such as finite element method, FEM). The real function relationships between the responses and the random variables of the structure are so complicate that they can not be obtained, so are the POF and ULSF. In terms of the listed problems, the response surface method (RSM) is an effective method had been developed in recent years. The essence of RSM is to fit an approximate specific function to substitute the real POF or ULSF. And based on the response function, it is relatively easy and feasible to carry out MORBRDO for complicated structures

3.1 The fitting of response functions

The steps to obtain the response function are showed as bellow:

(1) Choose the pure-quadratic polynomial function to fit the relationship between the response and the random variables as the following:

$$H(X) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \quad (7)$$

Where $H(X)$ is the response of the structure, $\mathbf{X} = (x_1, x_2, \cdots, x_n)$ is the random variable vector, a, b_i and c_i are the coefficients to be determined, n is the number of random variables.

(2) Take the current mean value of variables as the spread point of the response function, compute the response values at the following selected $2n$+1 sample points: the mean value point μ_X=(μ_{x_1},..., μ_{x_i},..., μ_{x_n}), $2n$ points on axis ($\mu_{x_1}, \ldots \mu_{x_i} \pm \lambda \sigma_{x_i}$,

$\ldots\mu_{x_n}$), where μ_{x_i} and σ_{x_i} are the mean value and standard deviation of x_i, respectively. λ is the location parameter of sample points, $1 \leq \lambda \leq 3$, usually. The $2n$+1 values of responses $[h_1(X), h_2(X), \ldots, h_{2n+1}(X)]$ are obtained by finite element analysis (FEA);

(3) Put the $2n$+1 sample points and $2n$+1 response value of $H(X)$ into the Eqn.(7), then a equations set include $2n$+1 variables and $2n$+1 responses can be gotten. The $2n$+1 undetermined coefficients a, b_i and c_i can be obtained conveniently by using the least square method. For convenience, we call the approximate POF and ULSF simulated with the pure-quadratic polynomial functions POF_RSM and ULSF_RSM, respectively.

3.2 Calculation of the mean value and standard deviation of POF_RSM

Suppose $H(X)$ is a certain POF_RSM (such as efficiency, stress, etc), spread $H(X)$ at the mean value point μ_X with Taylor series expansion method and get to the second order items. So:

$$H(X) = H(\mu_X) + \sum_{i=1}^{n}(b_i + 2c_i\mu_{x_i})(x_i - \mu_{x_i}) + \sum_{i=1}^{n} c_i(x_i - \mu_{x_i})^2 \tag{8}$$

The mean value of $H(X)$ is:

$$\mu_H = E(H) = H(\mu_X) + \sum_{i=1}^{n} c_i\sigma_{x_i}^2 \tag{9}$$

The standard deviation of $H(X)$ is:

$$\sigma_H = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}\left((b_i + 2c_i\mu_{x_i})(b_j + 2c_j\mu_{x_j})\rho_{ij}\sigma_{x_i}\sigma_{x_j}\right)^2} \tag{10}$$

Where ρ_{ij} is the correlation coefficient of $x_i$andx_j.

3.3 Calculation of the reliability and reliability sensitivity coefficient based on the ULSF_RSM

(1) Structural reliability index

Suppose $g_X(X)$ is the ULSF_RSM with respect to a certain failure mode, display $g_X(X)$ as below:

$$g_X(X) = d + \sum_{i=1}^{n} e_i x_i + \sum_{i=1}^{n} f_i x_i^2 \tag{11}$$

Adopt checking point method to compute the structural reliability index corresponding to Eqn. (11), suppose the checking point is x^*, hence, x^* is located on the ultimate limit state surface, so

$$g_X(x^*) = 0 \tag{12}$$

Spread $g_X(X)$ at the checking point x^* with Taylor series expansion method and get to the linear items, so

$$g_{XL} = g_X(x^*) + \sum_{i=1}^{n} \frac{\partial g_X(\mathbf{x}^*)}{\partial x_i}(x_i - x_i^*) = g_X(x^*) + \sum_{i=1}^{n}(e_i + 2f_i x_i^*)(x_i - x_i^*) \tag{13}$$

According to the definition of the structural reliability index, the first order reliability index based on checking point method is:

$$\begin{aligned}\beta = \frac{\mu_{g_{XL}}}{\sigma_{g_{XL}}} &= \frac{g_X(x^*) + \sum\limits_{i=1}^{n} \frac{\partial g_X(x^*)}{\partial x_i}(\mu_{x_i} - x_i^*)}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left(\frac{\partial g_X(x^*)}{\partial x_i}\cdot\frac{\partial g_X(x^*)}{\partial x_j}\rho_{ij}\sigma_{x_i}\sigma_{x_j}\right)^2}} \\ &= \frac{g_X(x^*) + \sum\limits_{i=1}^{n}(e_i + 2f_i x_i^*)(\mu_{x_i} - x_i^*)}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left((e_i + 2f_i x_i^*)\cdot(e_j + 2f_j x_j^*)\rho_{ij}\sigma_{x_i}\sigma_{x_j}\right)^2}}\end{aligned} \tag{14}$$

β can be calculated with iteration method.

(2) Reliability sensitivity coefficient

The structural reliability sensitivity coefficient with respect to x_i is:

$$\begin{aligned}\alpha_{x_i} &= \frac{-\sum\limits_{j=1}^{n} \frac{\partial g_X(X^*)}{\partial x_i}\rho_{ij}\sigma_{x_j}}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left(\frac{\partial g_X(X^*)}{\partial x_i}\cdot\frac{\partial g_X(X^*)}{\partial x_j}\rho_{ij}\sigma_{x_i}\sigma_{x_j}\right)^2}} \\ &= \frac{-\sum\limits_{j=1}^{n}(e_i + 2f_i x_i^*)\rho_{ij}\sigma_{x_j}}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left((e_i + 2f_i x_i^*)\cdot(e_j + 2f_j x_j^*)\rho_{ij}\sigma_{x_i}\sigma_{x_j}\right)^2}}\end{aligned} \tag{15}$$

α_{x_i} indicates the relative contribution of the ith random variable to the standard deviation of ULSF_RSM, and it also reflects its relative influence to the first order reliability index. It is not robust when the influence of a certain variable takes the main part. Hence, it is very natural to hope that each element of $\alpha_{\mathbf{x}}$ is relatively equal.

According to Eqn.(15), $\sum\limits_{i=1}^{n} \alpha_{x_i}^2 = 1$. Let $t_i = \alpha_{x_i}^2$(i=1, 2, … , n), so that $\sum\limits_{i=1}^{n} t_i = 1$, $0 \leq t_i \leq 1$. According to the knowledge of non-equality, the smaller $\sum\limits_{i=1}^{n} t_i^2$ is, the closer the each element of t_i is, so is α_{x_i}. And the reliability index is more robust. Therefore, the following function is chosen to be the reliability sensitivity robustness objective function:

$$f(\alpha_{x_i}) = \sum_{i=1}^{n} t_i^2 = \sum_{i=1}^{n} \alpha_{x_i}^4 \tag{16}$$

(3) Dealing with non-normal distribution variables

Eqn.(14) is appropriate when all the variables of the structure are normal distribution variables. If there are non-normal distribution variables, the mapping transformation method is adopted. Its principle is to map the non-normal distribution variable into standard normal distribution variable in the constraint that the cumulative distribution function (CDF) values of both variables are equal at the checking pointx^*. Suppose y_i is a normal distribution variable and x_i is a non-normal distribution variable, $\Phi(y_i)$ and $F(x_i)$ are the CDF of y_i and x_i, respectively. To carry out the following transformation:

$$F_{x_i}(x_i) = \Phi(y_i) \tag{17}$$

So:

$$\begin{cases} x_i = F_{x_i}^{-1}[\Phi(y_i)] \\ y_i = \Phi^{-1}[F_{x_i}(x_i)] \end{cases} \tag{18}$$

Eqn.(17) for differential on both sides:

$$\begin{cases} f_{x_i}(x_i)dx_i = \varphi(y_i)dy_i \\ dx_i/dy_i = \varphi(y_i)/f_{x_i}(x_i) \end{cases} \tag{19}$$

Where $\varphi(y_i)$ and $f_{x_i}(x_i)$ are the probability density function of y_i and x_i, respectively.

Put Eqn. (18) into Eqn. (11), the ULSF_RSM with respect to $\boldsymbol{Y}$:

$$Z = g_X(X) = d + \sum_{i=1}^{n} e_i \cdot F_{x_i}^{-1}[\Phi(y_i)] + \sum_{i=1}^{n} f_i \cdot \{F_{x_i}^{-1}[\Phi(y_i)]\}^2 = g_Y(Y) \tag{20}$$

Where, $\mathbf{Y} = [y_1\ y_2\ \cdots\ y_i\ \cdots y_n\ ]$, $y_i \sim N(0,1)$. Then, Eqn.(14) and Eqn.(15) are turned into Eqn.(21) and Eqn.(22) as the following:

$$\begin{aligned}\beta &= \frac{g_Y(y^*) + \sum\limits_{i=1}^{n} \frac{\partial g_Y(y^*)}{\partial y_i}(\mu_{y_i} - y_i^*)}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left(\frac{\partial g_Y(y^*)}{\partial y_i}\cdot\frac{\partial g_Y(y^*)}{\partial y_j}\rho_{ij}\right)^2}} = \frac{g_Y(y^*) - \sum\limits_{j=1}^{n} \frac{\partial g_X(x^*)}{\partial x_j}\cdot \left.\frac{\partial x_j}{\partial y_j}\right|_{y_j^*}\cdot y_i^*}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left(\frac{\partial g_X(x^*)}{\partial x_i}\cdot\left.\frac{\partial x_i}{\partial y_i}\right|_{y_i^*}\cdot\frac{\partial g_X(x^*)}{\partial x_j}\cdot\left.\frac{\partial x_j}{\partial y_j}\right|_{y_j^*}\rho_{ij}\right)^2}} \\ &= \frac{g_X(x^*) + \sum\limits_{i=1}^{n}(e_i + 2f_i x_i^*)\cdot\left.\frac{\partial x_i}{\partial y_i}\right|_{y_i^*}\cdot\Phi^{-1}(F_{x_i}(x_i^*))}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left((e_i+2f_i x_i^*)\cdot\left.\frac{\partial x_i}{\partial y_i}\right|_{y_i^*}\cdot(e_j+2f_j x_j^*)\cdot\left.\frac{\partial x_j}{\partial y_j}\right|_{y_j^*}\rho_{ij}\right)^2}}\end{aligned} \tag{21}$$

$$\begin{aligned}\alpha_{y_i} &= \frac{-\sum\limits_{j=1}^{n}\frac{\partial g_Y(y^*)}{\partial y_i}\rho_{ij}}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left(\frac{\partial g_Y(y^*)}{\partial y_i}\cdot\frac{\partial g_Y(y^*)}{\partial y_j}\rho_{ij}\right)^2}} \\ &= \frac{-\sum\limits_{j=1}^{n}(e_j+2f_j x_j^*)\cdot\left.\frac{\partial x_j}{\partial y_j}\right|_{y_j^*}\cdot\rho_{ij}}{\sqrt{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n}\left((e_i+2f_i x_i^*)\cdot\left.\frac{\partial x_i}{\partial y_i}\right|_{y_i^*}\cdot(e_j+2f_j x_j^*)\cdot\left.\frac{\partial x_j}{\partial y_j}\right|_{y_j^*}\cdot\rho_{ij}\right)^2}}\end{aligned} \tag{22}$$

According to Eqn. (19), the below item in Eqn. (21) and Eqn. (22) is deduced:

$$\left.\frac{\partial x_i}{\partial y_i}\right|_{y_i^*} = \left.\frac{dx_i}{dy_i}\right|_{y_i^*} = \frac{\varphi(y_i^*)}{f_{x_i}(x_i^*)} = \frac{\varphi(\Phi^{-1}(F_{x_i}(x_i^*)))}{f_{x_i}(x_i^*)} \tag{23}$$

3.4 A new design optimization strategy of MORBRDO

For the structure with unknown ULSF which needs to fit the approximate ULSF with FEM, the main challenge of MORBRDO is the large number of reliability analysis within the optimization process. And the reliability is an iteration process in itself. So, it is a key step to design an efficient optimization strategy for MORBRDO which can guarantee the reliability and in the same time can reduce computational work effectively. Traditional reliability analysis based on RSM and checking point method (TDCRA_RSM) is a double cycle iteration process, as shown in Fig.1.a). In TDCRA_RSM, several iterations (k supposed) may be needed to get the convergence and reliability index. If the pure-quadratic polynomial function is adopted as the RSM model, $2n$+1 FEA are needed every time the ULSF_RSM updates, n is the number of variables of the structure. So, $k\times(2n+1)$ FEA in total are taken in a TDCRA_RSM in order to get the reliability index under current value of design variables.

In order to enhance the efficiency of reliability analysis, an improved method is proposed, as shown in Fig.1.b). The improved method does not update the ULSF_RSM. Based on the checking point method and the current ULSF_RSM, it ends the reliability analysis when the checking point convergences. We call the improved method the simplified single cycle reliability analysis method based on current ULSF_RSM (SSCRA_RSM).

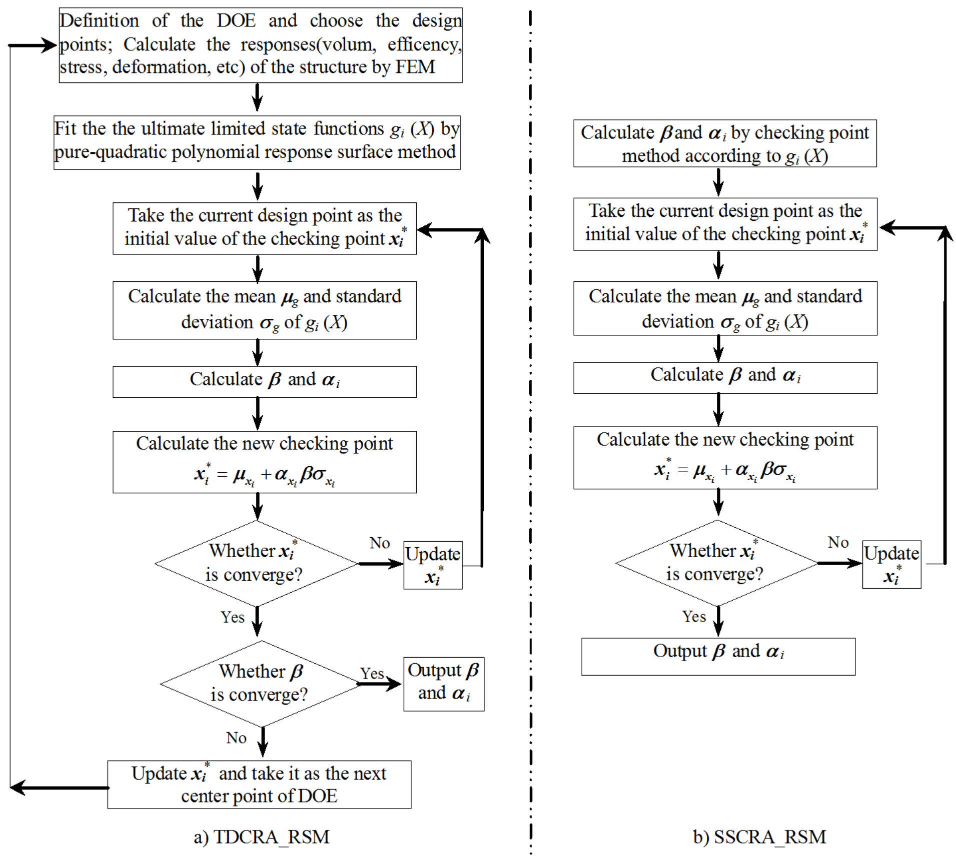


Figure 1: Flow chart of TDCRA_RSM and SSCRA_RSM.

The main distinguish between the TDCRA_RSM and SSCRA_RSM is that the TDCRA_RSM chooses the checking point as the final center point of the ULSF_RSM

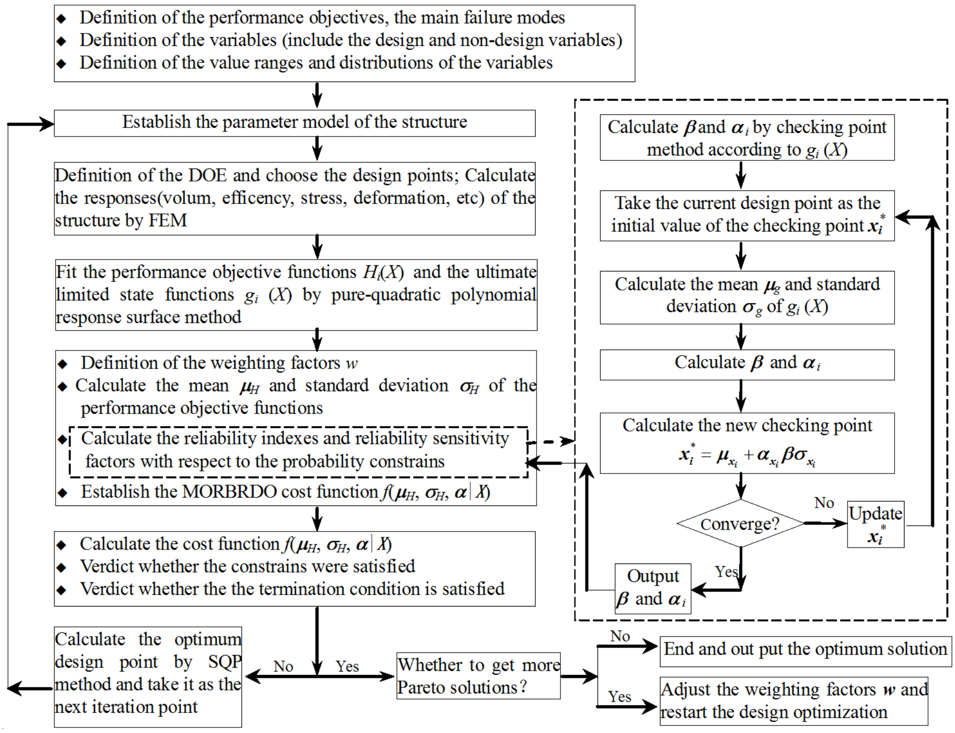


Figure 2: The new design optimization strategy of MORBRDO.

by iteration. Thus, the computational accuracy of TDCRA_RSM is higher than SSCRA_RSM. However, the computational work is also greatly larger than SSCRA_RSM. The SSCRA_RSM is also based on the checking point method and can deal with correlative and non-normal distribution variables. Although its computational accuracy is lower than TDCRA_RSM, it can satisfy the engineering requirement. The examples followed in this paper will prove this.

Based on the SSCRA_RSM, we proposed a new design optimization strategy of MORBRDO. Its workflow is showed in Fig.2. From the workflow, if the design optimization takes $m$iterations to gets its convergence, it will take $m\times(2n+1)$ FEA for the new strategy. But if the sub-cycle of reliability analysis adopts the TDCRA_RSM, it will take $m \times k\times(2n+1)$ FEA. It is obviously that the new design optimization strategy of MORBRDO improves the design optimization efficiency remarkably.

The steps of the new MORBRDO methodology are as following:

1) Determine the performance objectives, random variables (include design vari-

ables and non-design variables), rang and distributions of the variables; Determine the failure modes of the structure, such as stiffness failure and strength failure;

2) Establish the parameter model of the structure;

3) Adapt the method introduced in section 3.1 of this paper, fit the ULSF_RSM and POF_RSM;

4) Based on the POF_RSM adopt the method introduced in section 3.2 of this paper to calculate the mean values and standard deviations; Based on the ULSF_RSM, adopt the method introduced in section 3.3 of this paper to calculate the reliability index and reliability sensitivity coefficient;

5) According to Eqn.(3) $\sim$ Eqn.(6), gain the MORBRDO cost function;

6) According to the MORBRDO model in Eqn.(2) of this paper, adopt sequential quadratic programming method to obtain the new design optimization point.

7) Take the new design optimization point as the next iteration point, repeat step 3)$\sim$6) until the cost function value converge to required precision. If more Pareto solutions are needed, adjust the weighting factors and restart the design optimization.

4 Application

4.1 MORBRDO for I-beam structure

To obtain the minimum sectional area of the I-beam structure, as shown in the Fig. 3, under the concentrated force *P*. The constraints are as the following: vertical distortion less than 0.3mm; bending stress less than 160Mpa. *E* is elasticity modulus of the beam material. The variables of the I-beam structure and their statistical properties and correlation coefficients are shown in Tab.1. Please redesign the sizes of I-beam structure with MORBRDO method.

Analysis: In terms of the I-beam structure, the analytic solution of MORBRDO can be obtained. So this example can be used to testify the feasibility and validity of the method proposed in this paper.

According to the purpose of the problem, the POF is the sectional area as following:

$$H(X) = 2x_2x_4 + x_3(x_1 - 2x_4) \tag{24}$$

The vertical section area is:

$$I = \left[x_2x_1^3 - (x_2 - x_3)(x_1 - x_4)^3\right]/12 \tag{25}$$

The maximum distortion is:

$$y_{\max} = PL^3/(48EI) \tag{26}$$

Table 1: Types, ranges, statistical properties and correlation coefficients of random variables of I-beam structure.

Variables	variable type	Mean value and date range	Standard deviation	distribution	Remarks
x_1	Design	[100,800]mm	3.0 mm	Normal	Original design
x_2	Design	[100,800]mm	1.8 mm	Normal	size/mm:
x_3	Design	[9,50]mm	0.03 mm	Normal	(x_1,x_2,x_3,x_4)=
x_4	Design	[9,50]mm	0.03 mm	Normal	(750,520,15,15)
L	Non-design	2000mm	10mm	Lognormal	Correlation
P	Non-design	600kN	3 kN	Gumbel	coefficients:
E	Non-design	2×10^5Mpa	1000 Mpa	Normal	$\rho_{x_1x_4}$=0.8 $\rho_{x_2x_3}=0.8$

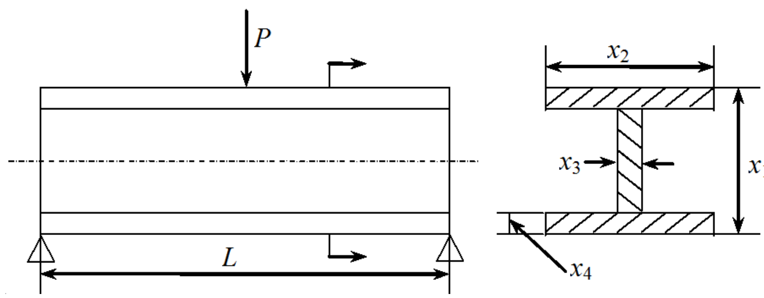


Figure 3: Sketch map of I-beam structure.

The maximum bending moment is:

$$M_{\max} = PL/4 \tag{27}$$

The maximum normal stress is:

$$\sigma_{\max} = (M_{\max}/I)\cdot x_1/2 \tag{28}$$

The ULSF with respect to stiffness failure mode is:

$$g_1(X) = 0.3 - y_{\max} \tag{29}$$

The ULSF with respect to strength failure mode is:

$$g_2(X) = 160 - \sigma_{\max} \tag{30}$$

The smaller-the-better type cost function in Eqn. (3) is chosen for the MORBRDO of the I-beam structure.

TDO and MORBRDO had been implemented to the I-beam structure. The optimization results are shown in Tab. 2. Where β_t is the optimum result of structural system index and $\beta_{\mathrm{M\_}C}$ is the optimum result with Monte-Carlo method, which had taken the advantage of the analytic equations of Eqn. (24)∼Eqn. (30).

According to Tab.2, the mean value and standard deviation of the performance objective (sectional area of the beam) decreased remarkably after MORBRDO compared with the initial design. The structural reliability meets the requirement. When compared the MORBRDO with the Monte-Carlo method, which are regarded as the most accurate one, the differences of the results are small. So, the MORBRDO are accurate and reliable. Different weighting factors caused different design results and revealed the preferences of the weighting factors to the objectives. In this paper, the MORBRDO cost 165 FEA in the whole 11 iterations. If the TDCRA_RSM which had been introduced in section 3.4 in this paper is adopted, it will cost 825 FEA. But the optimization results will be almost the same as the results in Tab.2.

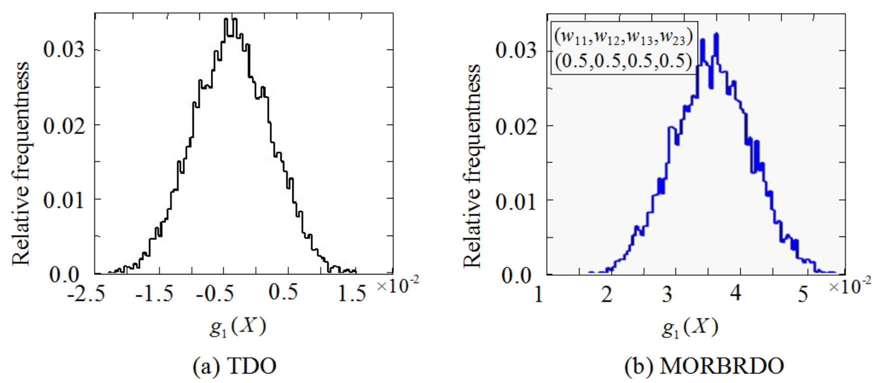


Figure 4: Statistics histogram of $g_1(X)$.

Statistics histograms of $g_1(X)$ after designed by TDO and MORBRDO respectively are showed in Fig.4. After TDO, the optimum cross-sectional area of the beam is the minimum. So, it could utmost to reduce the cost of raw materials for TDO. But, because of the randomness of the variables, the mean value of $g_1(X)$ is about zero, the optimum point is near the border of the failure surface and the structural reliability is just about 50%; After MORBRDO, however, the optimum point is far away from the border of the failure surface and the structural reliability is about $\Phi(6)$.

Table 2: Results of MORBRDO of I-beam structure.

Items	Weight $(w_{11},w_{12},w_{13},w_{23})$	$(\mu_H,\sigma_H,\sum_{p=1}^{n}\alpha_{1p}^4,\sum_{p=1}^{n}\alpha_{2p}^4)$	Design variables optimum Values / (x_1,x_2,x_3,x_4)	β_{t1}	β_{t2}	β_{M_C}	Iter.	No. of FEA
Initial		(26400,79.53,0.611,0.539)	(750,520,15,15)	23.0	310.8	24.7		
TDO		(15399,55.4,0.612,0.0.576)	(800,464.5,9.0,9.0)	0.0	132.8	0.014	11	55
MORBRDO f=6 $\beta_{ti}=\beta_{t0}=6$	(0.5,0.5,0.5,0.5)	(17263,58.99,0.483, 0.583)	(782,522.1,9.18,9.83)	6.00	172.3	6.11	11	165
	(1,0,0,0)	(17263,59.02,0.483, 0.583)	(782,520.4,9.18,9.86)	6.00	172.5	6.12	11	165
	(0,1,0,0)	(17260,58.94,0.484, 0.589)	(782,528.2,9.18,9.71)	6.00	172.4	6.09	11	165
	(0,0,1,0)	(22993,73.41,0.459, 0.509)	(750,519,9.18,15.78)	21.6	300.0	305.4	11	165
	(0,0,0,1)	(24094,76.09,0.458, 0.493)	(772,519,9.18,16.68)	27.5	333.4	340.0	11	165

4.2 MORBRDO for the pressure vessel

Pressure vessel is a kind of important equipment being widely used in engineering, to design a cylindrical pressure vessel with standard elliptical head (as shown in Fig.5). The total volume is $80m^3$ and the pressure that the vessel bears is P_c. The vessel is made of 16MnR steel whose allowable stress is S_σ and welded joint coefficient ϕ is 0.85. The length-diameter ratio constraint is $3 \leq L/D \leq 20$. Types, ranges, statistical properties and correlation coefficients of random variables and other parameters of the pressure vessel are shown in Tab.3. Please design the sizes of pressure vessel with MORBRDO method.

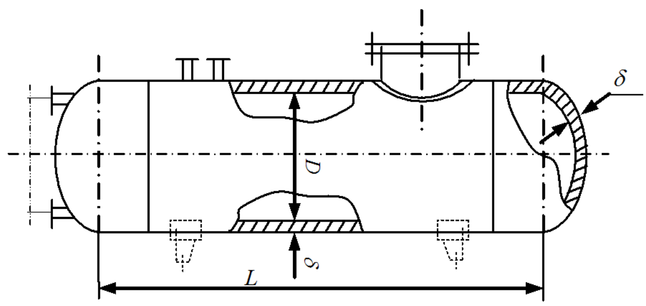


Figure 5: Sketch map of pressure vessel.

Table 3: Types, ranges, statistical properties of the random variables of pressure vessel.

Variables	variable type	Mean value and date range	Standard deviation	distribution	Remarks
D	Design	[0,+∞] mm	$0.005D$	Normal	Correlation coefficient: $\rho_{D\delta} = 0.75$, $\rho_{DL} = 1.0$
δ	Design	[0,+∞] mm	0.005δ	Normal	
L	Design	[0,17000] mm	$0.005L$	Normal	
P_c	Non-design	0.8MPa	0.192MPa	Lognormal	
S_σ	Non-design	340MPa	27.2Mpa	Normal	

Analysis: Under internal pressure, the maximum stress is at the circumferential direction and the apex of the standard elliptical head. And the value is:

$$\sigma_{\max} = P_c(D+\delta)\delta/2 \tag{31}$$

The ULSF with respect to strength is:

$$g(x) = \phi \cdot S_\sigma - \sigma_{\max} \tag{32}$$

The POF (the total mass) is:

$$H(D,\delta) = \rho\pi(D+\delta)\delta L + \frac{1}{12}\rho\pi\left[(D+2\delta)^2(D+4\delta) - D^3\right] \tag{33}$$

Where ρ is the material density of the vessel, ρ =7850Kg/m^3.
The total volume $V = \frac{1}{4}\pi D^2 L + \frac{1}{12}\pi D^3$, so:

$$L = 4V/(\pi D^2) - D/3 \tag{34}$$

The stiffness constraint is:

$$\delta \geq 3mm \tag{35}$$

The stability constraint is:

$$\delta - 15\% \cdot D \geq 0 \tag{36}$$

The smaller-the-better type cost function in Eqn. (3) is chosen for the MORBRDO of the pressure vessel. The optimization results are shown in Tab.4. After MORBRDO, the structural reliability reached the target value, the inner diameter D and the thickness δ of the pressure vessel were decreased, but the length was increased.

4.3 MORBRDO for the turbine blade model lines

The turbine blade is the key parts of the gas turbine, and the aerodynamic efficiency, structural strength and intrinsic frequency, etc, are closely connected with blade model lines, see Nikita and Keane (2011); PENG and YANG. (2012). In terms of the guide blade, the maximum aerodynamic efficiency is expected. For the rotating blade, because it is rotating under the high-speed and high temperature currents of gas, it bears very complicated stress. Therefore, it is necessary to design its model lines with MORBRDO method.

4.3.1 Parameter modeling of turbine blade model line

Taking the advantage of the basic geometry characteristic parameters of the turbine blade model line such as the blade width B, setting angle γ_p, radius of leading edge r_1 and trailing edge r_2, flow inlet angle β_1 and flow outlet angle β_2 wedge angle of leading edge φ_1 and trailing edge φ_2, we describe the blade model lines with three cubed Bezier curve, as shown in Fig.6.

According to the coordinates of G, U_1, U_2, D, the equations of the suction side line curve with respect to three cubed Bezier curve are:

$$\begin{cases} x_{SUC}(t) = x_G(1-t)^3 + x_{U_1}3(1-t)^2t + x_{U_2}3(1-t)t^2 + x_Dt^3 \\ y_{SUC}(t) = y_G(1-t)^3 + y_{U_1}3(1-t)^2t + y_{U_2}3(1-t)t^2 + y_Dt^3 \end{cases} \tag{37}$$

Table 4: Size design optimization results of pressure vessel.

Items		Weight (w_{11},w_{12},w_{13})	$(\mu_H,\sigma_H,\sum_{p=1}^{n}\alpha_{1p}^4)$	Design variables optimization Value / (D,δ,L)	β_t	$\beta_{\mathrm{M\_}C}$	Iter.	No. of FEA
Original			(18539,121.7,0.742)	(2600,15,14200)	6.45	6.38		
TDO			(3943,26.2,0.812)	(2392,3.59,17000)	0.43	0.42	6	18
MORBRDO	f=6,β_{ti}=β_{t0}=6	(0.5,0.5,0.5)	(16306,108.5,0.750)	(2426,14.94,16498)	6.00	6.02	5	45
	f=3,β_{ti}=β_{t0}=3	(0.5,0.5,0.5)	(7552,50.17,0.789)	(2409, 6.90,16748)	3.00	2.99	6	54

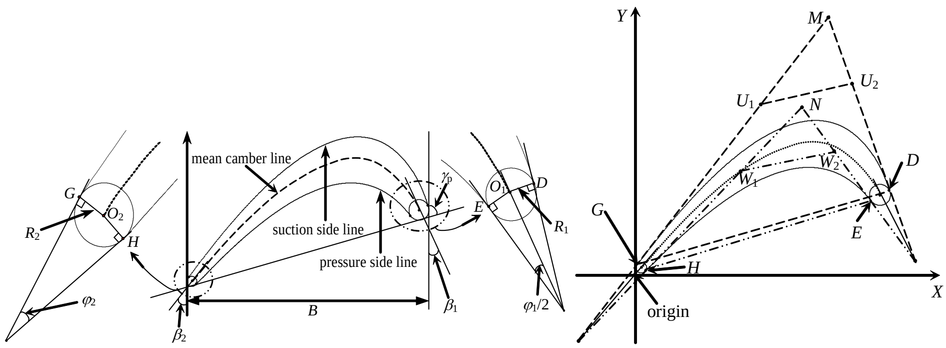


Figure 6: Description of blade model lines with three cubed Bezier curve.

Where $0 \leq t \leq 1$, and the pressure side line equations could be obtained in the similar way. The coordinates of G, U_1, U_2, D and the detailed equations of the model lines expressed by the basic geometry characteristic parameters of the turbine blade had been deduced by the authors, see PENG and YANG (2012).

4.3.2 *Parameter modeling of blade flow field and heat-fluid-solid coupling analysis*

Based on the parameter equation of the blade model line, the parameter models of the blade and flow field are established with Pro/Engineering software. The parameter models under original design are shown in Fig.6.

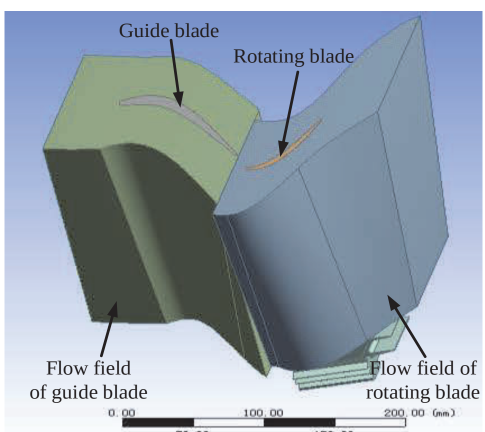


Figure 7: Blade and flow field models.

In order to obtain the aerodynamic efficiency, stress and deformation, etc of the blade accurately, the heat-fluid-solid coupling analysis is adopted. The coupling analysis is accomplished with Ansys Workbench 12.1. The flow field analysis chooses the shear stress transport (SST) as the turbulence model. According to the dates of the stand tests and the actual working records of the turbine under design working condition, the loads inflicted to the flow field analysis are as the following: to inflict the flow field inlet with mass flow rate and average temperature; to inflict the flow field outlet with average static pressure; stationary inertial frame is chosen for the guide blade flow field; the non-inertial frame which is rotating around the main shaft of the turbine at a certain angular velocity is chosen for the rotating flow field; in each reference frame, the flow field boundary is heat insulation, solid wall, zero speed condition without slip; choose the CFX frozen rotor model to transfer the calculation dates between the guide blade flow field and the rotating blade flow field; adopt general grid interface(GGI) method to map the dates. The flow analysis obtains the distribution of the temperature, pressure and gas velocity of the flow field and blade surface. And then the aerodynamic efficiency of the blade is calculated. After, the temperature and pressure of the blade surface of the flow analysis together with the angular velocity are conducted as loads of the stress analysis. Then the stress, deformation and intrinsic frequency, etc are obtained by stress analysis. Partial results of the heat-fluid-solid coupling analysis of the blade are shown in Fig. 8- Fig.10.

4.3.3 MORBRDO of blade model lines

(1) Choice of POF

For both of the guide blade and rotating blade, we choose the aerodynamic efficiency and the blade volume as the main POF.

MORBRDO cost function: The aerodynamic efficiency is always expected to increase. But the volume is expected to decrease, so the combined type in Eqn. (6) is chosen for the turbine blade model lines MORBRDO.

Because the guide blade is stationary, so the stress and deformation are much smaller than the rotating blade. In this paper, we mainly take the rotating blade into consideration and carry out the stress and reliability analyses to it.

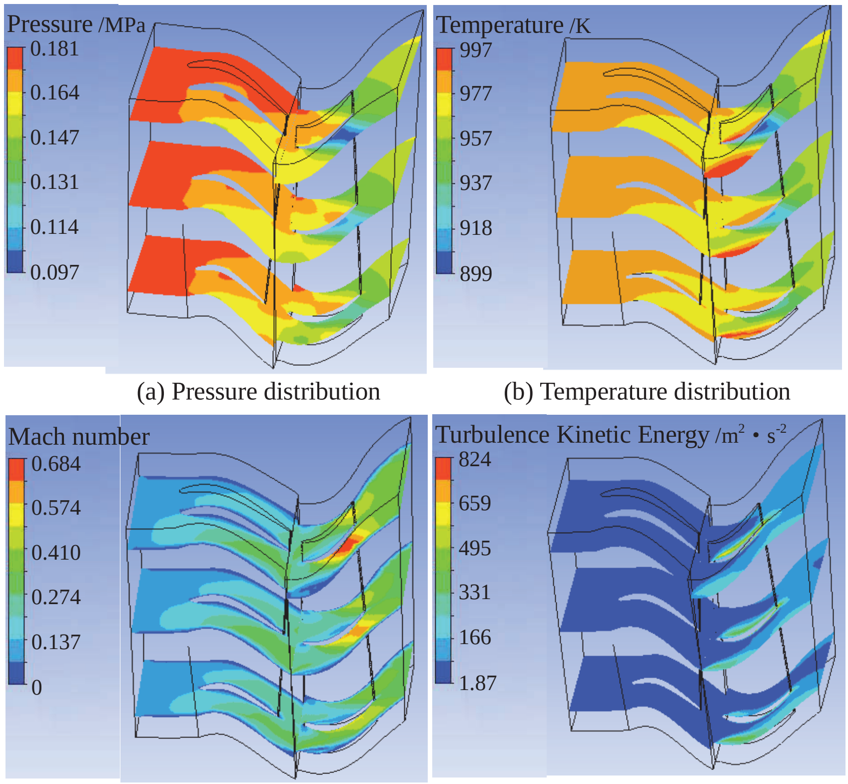
(2) Probability constraints of rotating blade

Strength probability constraint is:

$$P(S-\sigma_s>0) \geq \Phi(\beta_{ts}) \tag{38}$$

Stiffness probability constraint is:

$$P(\xi-\xi_s>0) \geq \Phi(\beta_{t\xi}) \tag{39}$$



(a) Pressure distribution (b) Temperature distribution

(c) Mach number distribution (d) Turbulence Kinetic Energy distribution

Figure 8: The pressure, temperature, Mach number and turbulence kinetic energy distributions of the flow field at 10%, 50% and 90% blade height.

Resonance frequency probability constraint is :

$$P(|F - k \cdot F_I| \geq 7.5Hz) \geq \Phi(\beta_{tF}) \tag{40}$$

Where S, ξ and F are the maximum stress, maximum deformation and natural frequency of the rotating blade under design working condition, respectively. ξ_s is the allowed deformation; σ_s is the yield strength of blade at the working temperature. F_I is the rotational frequency of the turbine, k is the number of even-distributed excitation sources, usually only k=1 and k=2 are being taking into consideration. β_{ts}, $\beta_{t\xi g}$ and β_{tF} are the target reliability indexes with respect to strength, stiffness and vibration failure modes respectively.

(3) MORBRDO cost function for turbine blade

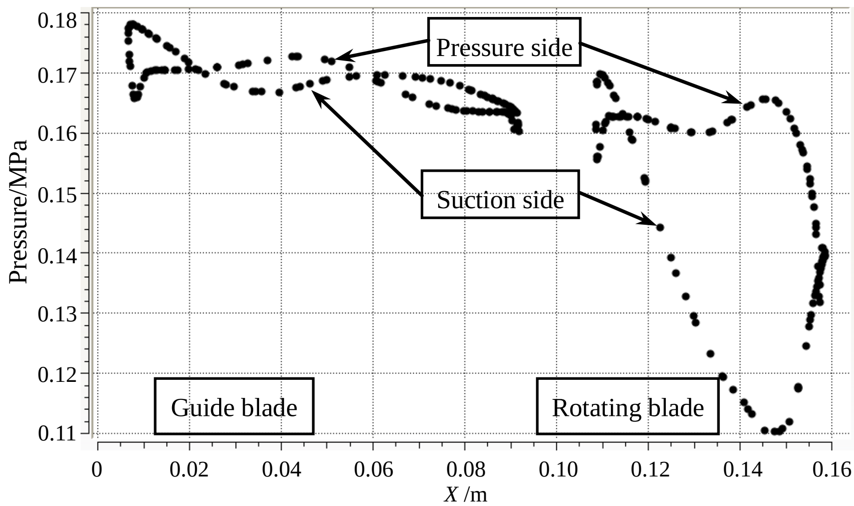


Figure 9: Pressure changing with the blade model line at the 50% blade height.

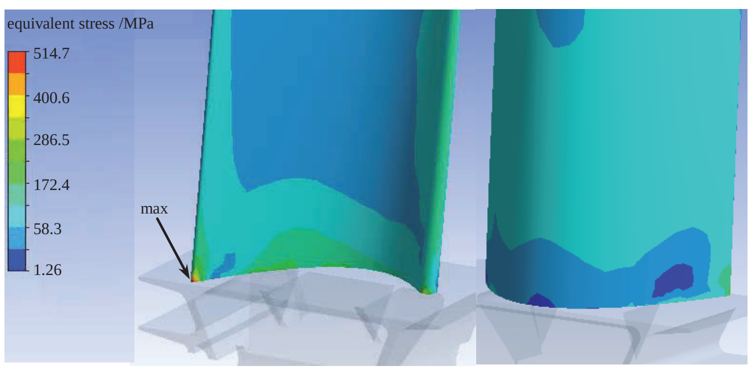


Figure 10: Equivalent stress distribution of rotating blade.

1. for turbine guide blade, the MORBRDO cost function is

$$
\begin{aligned}
f_{\mathrm{CJ}}(\mu_H, \sigma_H, \alpha|\mathbf{X}) =& w_{11} \cdot \left(\frac{\mu_{\mathrm{EJ}_0}}{\mu_{\mathrm{EJ}}}\right)^2 + w_{12}\left(\frac{\sigma_{\mathrm{EJ}}}{\sigma_{\mathrm{EJ}_0}}\right)^2 \\
&+ w_{21} \cdot \left(\frac{\mu_{\mathrm{VJ}}}{\mu_{\mathrm{VJ}_0}}\right)^2 + w_{22}\left(\frac{\sigma_{\mathrm{VJ}}}{\sigma_{\mathrm{VJ}_0}}\right)^2
\end{aligned}
\tag{41}
$$

2. for turbine rotating blade, the MORBRDO cost function is

$$f_{\mathrm{CD}}(\mu_H, \sigma_H, \alpha|\mathbf{X}) = w_{11} \cdot \left(\frac{\mu_{\mathrm{ED}_0}}{\mu_{\mathrm{ED}}}\right)^2 + w_{12}\left(\frac{\sigma_{\mathrm{ED}}}{\sigma_{\mathrm{ED}_0}}\right)^2 + w_{21} \cdot \left(\frac{\mu_{\mathrm{VD}}}{\mu_{\mathrm{VD}_0}}\right)^2 + w_{22}\left(\frac{\sigma_{\mathrm{VD}}}{\sigma_{\mathrm{VD}_0}}\right)^2 + \sum_{j=1}^{3}\left[w_{j3}\left(\sum_{p=1}^{nxv}\alpha_{jp}^4 \Big/ \sum_{p=1}^{nxv}\alpha_{jp0}^4\right)^2\right] \tag{42}$$

In Eqn.(41) and Eqn.(42), μ_{EJ}, μ_{MJ}, μ_{ED} and μ_{MD} are the mean value of the aerodynamic efficiency of guide blade, volume of guide blade, efficiency of rotating blade and volume of rotating blade, respectively. σ_{EJ}, σ_{MJ}, σ_{ED} and $\sigma_{\mathrm{MD}}$are the standard deviation of the aerodynamic efficiency of guide blade, volume of guide blade, efficiency of rotating blade and volume of rotating blade, respectively. $\mu_{\mathrm{EJ}_0}$is the value of μ_{EJ} when the blade is in initial design, so as to other items. The last item in Eqn. (42) is the objective of the reliability sensitivity coefficient with respect to Eqn.(38)- Eqn.(40).

(4) Variables definition

In MORBRDO, the design parameters are regarded as random variables. Moreover, the non-design random variables also must be taken into consideration. In terms of the turbine blade, the non-design variables and other parameters are shown in Tab.5. The design variables and their ranges are shown in Tab.6. Where, M_{inlet}, T_{inlet}, and P_{outlet} are the mass flow rate, inlet average temperature and outlet average static pressure of the flow field, respectively.

Table 5: Statistic parameters and distributions of the non-design variables of turbine blade.

Non-design Variables	Mean value	Standard deviation	distribution	Remarks
σ_s	550MPa	$0.02\sigma_s$	Normal	ξ_s=0.006mm; $\beta_{ts}=\beta_{t\xi}=\beta_{tF}$=4; β_{t0}=4;f=4; Correlation coefficient: ρ_{MP}=0.4,ρ_{TP}=0.75
F_I	54.5Hz	$0.03F_I$	Normal	
M_{inlet}	0.975/kg·s^{-1}	$0.01M_{inlet}$	lognormal	
T_{inlet}	710/°F	$0.01T_{inlet}$	Normal	
P_{outlet}	0.1495 MPa	$0.01P_{outlet}$	Gumbel	

The weighting factors w_{ij} reflects the preference of the designer to each objective. For turbine blade, w_{ij} is chosen in four cases, as shown in Tab.6.

According to the sensitivity analysis based on DOE and FEA, six parameters including β_1, β_2, φ_1, φ_2, R_1 and R_2 , as shown in Tab.7, which have the major influences on the aerodynamic efficiency and stress of the blade are chosen to be optimized.

Table 6: Weighting factors of the MORBRDO cost function of the turbine blade.

	weighting factors	w_{11}	w_{12}	w_{21}	w_{22}	w_{j3}	Volume mean value	Efficiency mean value	Remarks
original	Guide blade						173289	0.9281	Without optimization
	Rotating blade						108749	0.7565	
Case 1	Guide blade	0.8	0.8	0.2	0.2		185557	0.9421	Aerodynamic efficiency preference
	Rotating blade	0.8	0.8	0.2	0.2	0.5	115836	0.7978	
Case 2	Guide blade	0.2	0.2	0.8	0.8		165452	0.9024	Volume preference
	Rotating blade	0.2	0.2	0.8	0.8	0.5	101638	0.7413	
Case 3	Guide blade	1	1	0	0		193649	0.9465	Maximum aerodynamic efficiency
	Rotating blade	1	1	0	0	0	125368	0.8036	
Case 4	Guide blade	0	0	1	1		160973	0.8767	Minimum volume
	Rotating blade	0	0	1	1	0	98867	0.7258	

(5) MORBRDO results

According to the new design optimization strategy of MORBRDO in section 3.4, we have achieved the design optimization with respect to the above four cases in Tab.6. Take the case 1 for instance, the whole MORBRDO takes 9 iterations (350 hours in total) to achieve optimization. The optimum value of the design variables and performance objectives are shown in Tab.7∼Tab.9. After MORBRDO, the aerodynamic efficiencies of the guide blade and the rotating blade are increased by 1.5% and 5.4% respectively; the maximum stress and the maximum deformation of the rotating blade are decreased by 16.8% and 8.5% respectively; and the standard deviations of the above performance objectives are decreased too, so the robustness of the blade is increased. What's more, the structural system reliability reached 99.99%, but before MORBRDO it is only 89.4%. Fig.11 shows the comparing of turbine blade model line figures before and after MORBRDO. Fig.11∼ Fig.13 show the comparing of the turbulence kinetic energy changing along with the blade model line at the 10%, 50% and 90% blade height before and after MORBRDO, respectively. The turbulence kinetic energy reflects the loss of kinetic energy of the blade. So Fig.12 ∼ Fig.14 show that, after MORBRDO, the maximum loss of kinetic energy of the blade surface at the 10%, 50% and 90% blade height decrease by 14.9%, 11.1% and 17.6% respectively. However, the means and standard deviations of the blade volumes increased a little, so the cost of the blade material would be increase finally.

For turbine blade, the aerodynamic efficiency and the volume would not get the optimum at the same time, as shown in Fig.15. The mean value of the aerodynamic efficiency and volume of the guide blade and rotating blade in Case 1 to Case 4 are shown in Tab.6 and Fig.15. The results show that the original design is a feasible design but not a Pareto solution. By changing the weighting factors and redesign, we can get the Pareto frontier of the turbine blade model line design optimization solutions, as shown in Fig.15.

Therefore, MORBRDO can ensure the structural reliability and enhance the robustness of the structure on the one hand; but on the other hand, it may sacrifice some other desirable performances, such as the costs, time, quality, etc. So the MORBRDO could be regarded as a trade-off compromise design method.

5 Conclusions

In this paper, a new MORBRDO model and a new design optimization strategy were proposed. Based on the pure-quadratic polynomial function model, the reliability calculation equations with non-normal distribution variables, the mean value and standard deviation equations of performance objectives were deduced. The I-

Table 7: Comparing of blade model line parameters before and after design optimization (Case 1).

Items	guide blade root			guide blade tip			rotating blade root			rotating blade tip		
Par.	range	Orig.	Opt.	range	Orig.	Opt.	range	Orig.	Opt.	range	Orig.	Opt.
R_1/mm	[3.0,4.5]	3.7	3.5	[1.2,2.0]	2.5	2.5	[2.0,4.0]	3.0	3.5	[1.2,2.0]	1.51	1.55
R_2 /mm	[0.4,1.5]	0.7	0.9	[0.4,1.5]	0.7	0.7	[0.4,1.5]	0.8	1.0	[0.4,1.5]	0.8	0.8
β_1	[20^o, 40^o]	26.96°	30°	[20°, 40°]	26.5°	29.5°	[20°, 40°]	27.0°	30°	[60°, 85°]	75°	76°
β_2	[60°, 80°]	73.30°	73°	[60°, 80°]	74°	72.5°	[30°, 50°]	40.2°	41.5°	[20°, 40°]	33.5°	34.5°
φ_1	[15°, 30°]	28.15°	22°	[15°, 30°]	23°	20°	[15°, 30°]	20°	23°	[8°, 20°]	10°	12°
φ_2	[4°,10°]	5.97°	7.5°	[4°,10°]	5.47°	6.5°	[4°,10°]	7°	7.0°	[4°,10°]	5°	6.5°

Table 8: Optimization results of target parameters of rotating blade (Case 1)

Items		aerodynamic efficiency	Volume /mm^3	maximum deformation /$\times 10^{-3}$m	maximum Stress /MPa	first order resonant frequency/ Hz	second order resonant frequency/ Hz
mean	Orig.	0.7565	108749	4.47	514.25	291.32	622.12
	Opt.	0.7978	115836	4.22	440.76	286.86	623.5
standard deviation	Orig.	0.035	3589	0.265	26.52	3.28	4.31
	Opt.	0.034	3642	0.233	23.92	2.79	4.24
structural reliability index			items	stiffness	strength	first order resonant	second order resonant
			Orig.	5.77	1.25	5.79	5.25
			Opt.	7.64	4.15	5.57	5.66

Table 9: Optimization results of target parameters of guiding blade (Case 1).

Items		aerodynamic efficiency	volume / mm^3
mean	Orig.	0.9281	173289
	Opt.	0.9421	185557
standard deviation	Orig.	0.039	4935
	Opt.	0.041	5052

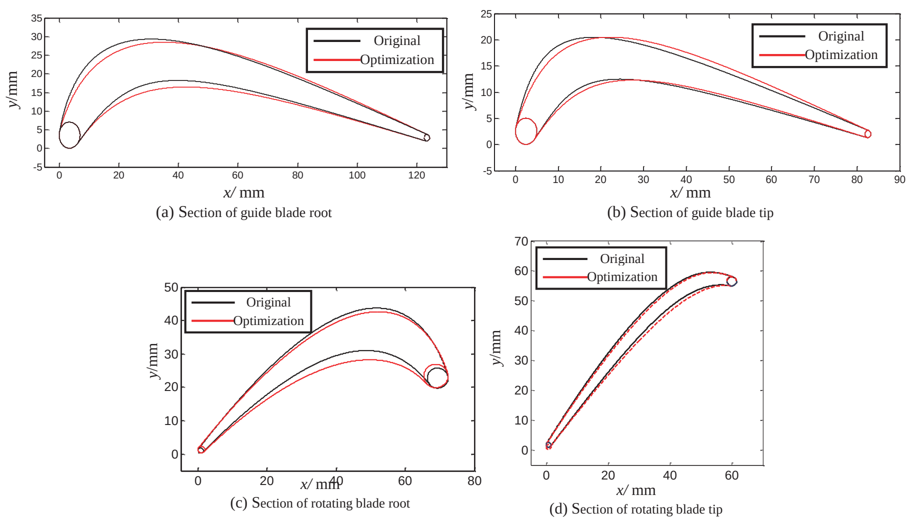


Figure 11: Comparing of turbine blade model line figures before and after MORBRDO (Case 1).

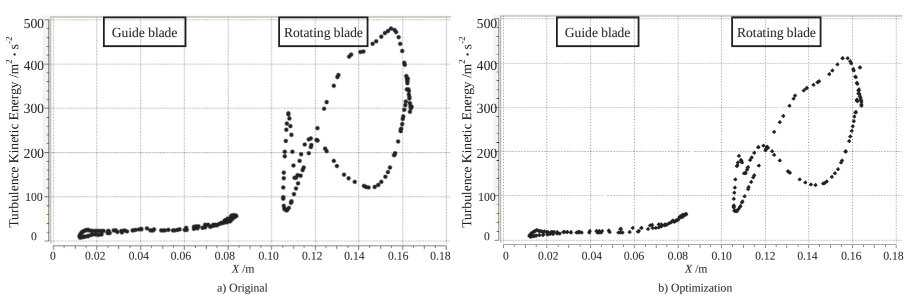


Figure 12: Compare of the turbulence kinetic energy changing along with the blade model line at the 10% blade height before and after MORBRDO (Case 1)

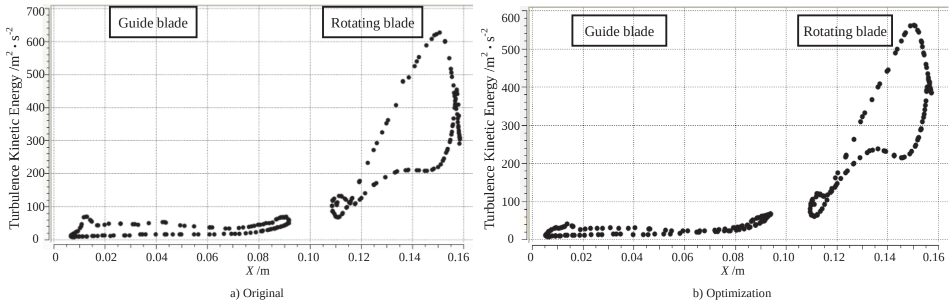


Figure 13: Compare of the turbulence kinetic energy changing along with the blade model line at the 50% blade height before and after MORBRDO (Case 1).

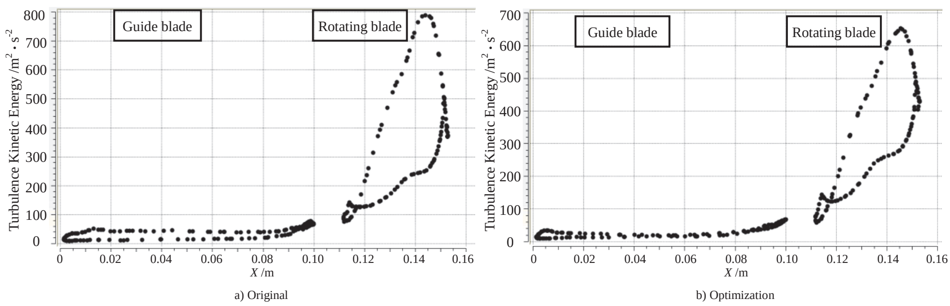


Figure 14: Compare of the turbulence kinetic energy changing along with the blade model line at the 90% blade height before and after MORBRDO (Case 1).

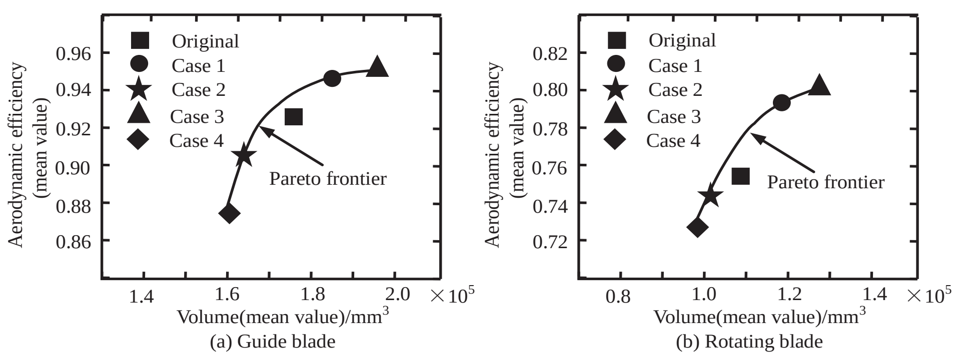


Figure 15: Sketch map of the relationship between original design and Pareto solutions.

beam structure, a pressure vessel and the turbine blade model lines were introduced to testify the MORBRDO methodology. The main concluding remarks could be summarized as below:

(1) The new MORBRDO model takes advantages of the multi-objective robustness, the reliability sensitivity robustness and the six sigma robustness design idea. It considers the multi-objectives and multi-failure modes simultaneously, ensures the system reliability and enhances the robustness of the structure.

(2) In terms of the structure with unknown ULSF and POF, the new MORBRDO model adopts the pure-quadratic polynomial function to obtain approximate ULSF and POF. It makes the design optimization feasible and relatively easy to implement.

(3) The new design optimization strategy does not update the ULSF_RSM, so it reduces computational work of FEA and improves optimization efficiency remarkably.

(4) Two real projects of I-beam structure and pressure vessel with analytic solutions had been introduced and had verified the accuracy and feasibility of the new MORBRDO methodology. Then the new methodology was also adopted to redesign the turbine blade model lines. And the Pareto frontier of the turbine model line MORBRDO solutions is obtained. So it could be inferred that the new MORBRDO methodology could be applied to the complicated structures.

(5) For the MORBRDO, in order to improve the robustness and guarantee the reliability of the structure, it may sacrifice some other desirable performances, such as manufacturing costs, time, etc. In actual engineering, the designer can adjust the target reliability index, the robustness level or the weighting factors to trade off between the quality enhancement and the costs decrease.

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