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Numerical Analysis for the Mooring System with Nonlinear Elastic Mooring Cables

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Abstract: This paper presents numerical analysis for the mooring system with nonlinear elastic mooring cables. The equation of motion for nonlinear elastic mooring cable is established by utilizing finite element method. A marine mooring system of floating rectangular box with nonlinear elastic cables is taken as an illustrative example. The dynamic analysis, static analysis, and uniformity analysis are carried out for the polyester mooring system and the results are compared with those of the steel wire and the chain mooring system. Results from the present study can provide valuable recommendations for the design and construction of the mooring system with nonlinear elastic mooring cables.

Keywords: nonlinear elastic mooring cables, finite element method, numerical model, polyester mooring system, dynamic analysis.

1 Introduction

Compared with the steel wire or chain, the polyester cable has lighter mass and higher corrosion resistance. Currently, trends aim at station-keeping of floating structure via mooring in deeper water. Polyester cable is considered to be an attractive option for deepwater mooring systems. Polyester cable has many advantages over steel wire and chain, however one of the major challenges lies in an accurate understanding of the full-scale physical properties. Other than the steel wire or chain, the mechanical properties of polyester cable are generally nonlinear and time- dependent and exhibit viscoelasticity and viscoplasticity.

The nonlinear elastic mooring cable has significant effects on the performances of the mooring system and the calculations are more complicated than the linear ones, and they are related to dynamic responses and mooring forces of the mooring system. Therefore, it is necessary and significant to make research about for mooring system with nonlinear elastic mooring cables.

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The nonlinear elastic mooring cable has been an interest topic in marine hydrodynamics research in the past few decades. Various methods have been presented for obtaining material property of nonlinear elastic. Del Vecchio (1992) proposed a method for determining the modulus of the polyester rope at constant temperature. Francois and Davies (2008) carried out experiments on the characterization of polyester mooring cables. Liu et al. (2014) made experimental investigation on nonlinear behaviors of synthetic fiber ropes. Some researchers [Yuan et al. (2010); Huang et al. (2013); Beltrán and Williamson (2011); Ćatipović et al. (2011)] presented various computational methods for nonlinear elastic mooring cables. And, the finite element methods have been developed to solve nonlinear dynamics problems of flexible structure, nonlinear elastic-plastic material and catenary structure [Okamoto and Omura (2003); Nishioka et al. (2007); Kim et al. (2010)].

However, those methods still need to be further improved to enable easier numerical implementation utilizing classical finite element method codes, and the researches about the comparison of dynamic and static analysis between steel wire, chain and the polyester mooring system has not been seen.

In the present study, the equation of motion for a nonlinear elastic mooring cable of a mooring system is built and the finite element method is used to obtain the dynamic responses of the system. A mooring system of floating rectangular box with nonlinear elastic cable is taken as the numerical model. The dynamic analysis, static analysis, and uniformity analysis are made for the polyester mooring system.

2 Methodology

2.1 Equation of motion for the mooring system

In Fig. 1, two coordinate systems are defined to describe the motions of floating structure, i.e. the fixed reference coordinate system $O-XYZ$ with origin in the free surface and the body fixed coordinate system $o-xyz$ with origin at the centre of gravity of floating structure.

To carry out calculation of the coupled model the force and the moment of each mooring cable should be included into dynamics of a floating body. Considering the combined effects of current, wind and wave, the equation of motion for a mooring system with nonlinear elastic cable can be written as [Buchner et al. (2001)]

$$\begin{aligned}([M^m]+[M_a^\infty])\{\ddot{x}(t)\}+\int_0^t[K(t-\tau)]\{\dot{x}(t)\}\,d\tau+\\+[C]\{x(t)\}=\{F_{WA}(t)\}+\{F_M(t)\}+\{F_{WI}(t)\}+\{F_{CU}(t)\}\end{aligned} \tag{1}$$

where $[M^m]$ and $[M_a^\infty]$ are the mass and added mass matrices, respectively, $[K(t-\tau)]$ is the delay function matrix, $[C]$ is the hydrostatic resilience matrix, $\{\ddot{x}(t)\}$, $\{\dot{x}(t)\}$ and $\{x(t)\}$ are the acceleration, velocity and displacement matrices,

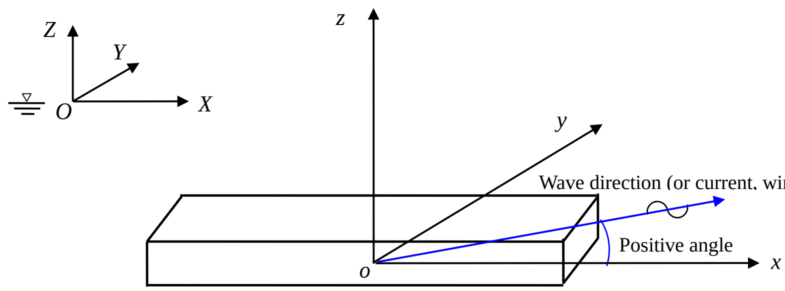

Figure 1: Definition of the coordinate systems of the mooring system.

respectively, and $\{F_{WA}(t)\}$, $\{F_M(t)\}$, $\{F_{WI}(t)\}$ and $\{F_{CU}(t)\}$ are the wave force, cable tension, wind force and current force, respectively.

The wind force provided by Ref. [OCIMF (1994)] is expressed as

$$F_{WI} = 0.5v_k^2 C_h C_s A \tag{2}$$

where v_k is the wind speed, C_h is the pressure and height coefficient, C_s is the shape factor, and A is the windward area.

The current force in Eq. (1) can be given as

$$F_{CU} = 0.5\rho C_D u^2 A_{CU} \tag{3}$$

where ρ is the density of water, C_D is the hydraulic drag coefficient, u is the flow velocity, and A_{CU} is the incident flow area.

The linear three-dimensional potential theory is used to calculate the wave forces. The first order wave force, the second order wave force are defined by linear transfer function and quadratic transfer function, respectively, which is the most common method for calculating wave forces of wet surfaces.

The irregular wave conditions can be expressed by the Pierson-Moskowitz wave spectrum [Pillai and Prasad (2008)]

$$S(\omega) = \frac{1}{2\pi}\frac{H_s^2}{4\pi T_z^4}\left(\frac{2\pi}{\omega}\right)^5 \exp\left[-\frac{1}{\pi T_z^4}\left(\frac{2\pi}{\omega}\right)^4\right] \tag{4}$$

where H_s is the significant wave height, T_z is the zero crossing period, and ω is the wave frequency.

2.2 Equation of motion of the mooring cable

In Fig. 2, nonlinear elastic mooring cable is modeled as a space curve, which is defined by a position vector **r**. Any point at the curve is defined by an arc length of extended mooring line $\tilde{s}$. Within dynamic analysis the position vector $\mathbf{r}$ is also the function of time t.

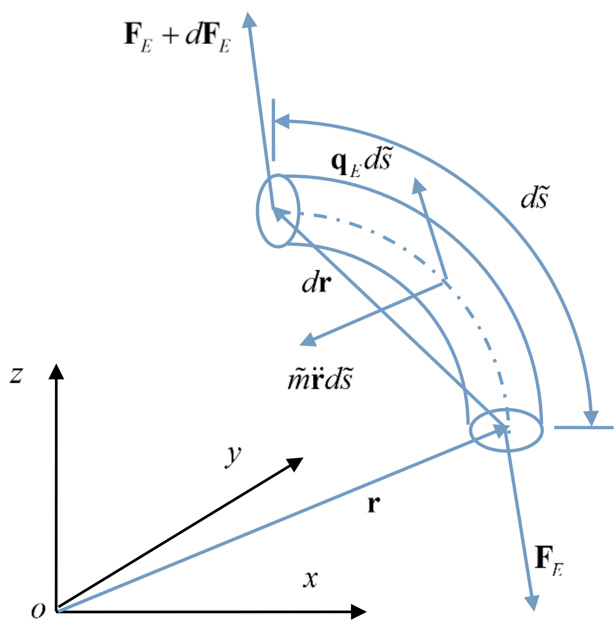

Figure 2: Definition of the mooring cable element.

The equation of motion is established by force equilibrium on an equivalent segment of the mooring line based on Newton's Second Law

$$\frac{d\mathbf{F}_E}{d\tilde{s}} + \mathbf{q}_E = \tilde{m}\ddot{\mathbf{r}} \tag{5}$$

where $\mathbf{q}_E$ is the effective distributed load vector, $\tilde{m}$ is the distributed mass of the extended mooring line, $\ddot{\mathbf{r}}$ is the acceleration vector of the segment, and $\mathbf{F}_E$ is the cross section effective force vector expressed as

$$\mathbf{F}_E = T_E \frac{d\mathbf{r}}{d\tilde{s}} \tag{6}$$

where T_E is the effective tension force.

Combining Eq. (5) and Eq. (6) leads to

$$\frac{d}{d\tilde{s}}(T_E \frac{d\mathbf{r}}{d\tilde{s}}) + \mathbf{q}_E = \tilde{m}\ddot{\mathbf{r}} \tag{7}$$

The elongation strain ε is defined as

$$\varepsilon = \frac{d\tilde{s} - ds}{ds} \tag{8}$$

where s is the arc length related to non-extended case.

From Eqs. (8) and (7), we have

$$\frac{d}{ds}\left(\frac{T_E}{1+\varepsilon}\frac{d\mathbf{r}}{ds}\right) + (1+\varepsilon)\mathbf{q}_E = (1+\varepsilon)\tilde{m}\ddot{\mathbf{r}} \tag{9}$$

The effective distributed load vector $\mathbf{q}_E$ is the summation of gravity, buoyancy, and the hydrodynamic force vector $\mathbf{q}_H$ of the cable. And $\mathbf{q}_H$ can be expressed by the Morison equation.

Thus, $\mathbf{q}_E$ can be expressed as

$$\mathbf{q}_E = -\rho\tilde{A}\mathbf{g} + \tilde{m}\mathbf{g} + \mathbf{q}_H \tag{10}$$

with

$$\tilde{A} = \frac{A}{1+\varepsilon}, \quad \tilde{m} = \frac{m}{1+\varepsilon} \tag{11}$$

where $\tilde{A}$ and $\tilde{m}$ are the cross-sectional area and distributed mass variables related to the extended case, respectively, while A and m are related to the non-extended case, ρ is the density of water, $\mathbf{g}$ is the gravitational acceleration vector.

Substituting Eq. (10) and Eq. (11) in to Eq. (9), we have

$$\frac{d}{ds}\left(\frac{T_E}{1+\varepsilon}\frac{d\mathbf{r}}{ds}\right) + m\mathbf{g} - \rho A\mathbf{g} + \mathbf{q}_H = m\ddot{\mathbf{r}} \tag{12}$$

With classical form of the elongation strain and the Lame formula, the elongation strain ε of the cable submerged in water, can be written as

$$\varepsilon = \frac{1}{E}\left[\frac{T_E}{A} - (1-2\nu)p\right] \tag{13}$$

where E is the Young's modulus, p is hydrostatic pressure of the sea water, and ν is the Poisson's ratio.

For simplicity, the conserved Poisson's ratio is adopted, i.e. the synthetic cable ν is assumed to be 0.5 [Tjavaras et al. (1998)]. Thus, Eq. (13) can be expressed as

$$\varepsilon = \frac{T_E}{AE} \tag{14}$$

An empirical formulation in Ref. [Tahar et al. (2008)] is used to represent the axial stiffness AE of the polyester cable

$$AE = RHOL(\alpha + \beta \frac{T_R}{B_s}) \cdot 10^6 \tag{15}$$

where B_s is the minimum breaking strength, T_R is the time dependant real tension, α and β are the material constants depending on the type of polyester material and their values usually come from the experimental results, and $RHOL$ is the dry weight per unit length of the cable.

From Eq. (8) and considering the symmetric set, the axial elongation condition can be obtained by an approximate form

$$\frac{1}{1+\varepsilon}(\frac{d\mathbf{r}}{ds} \cdot \frac{d\mathbf{r}}{ds}) - 1 - \varepsilon = 0 \tag{16}$$

2.3 Finite element model for the mooring cable

From Eqs. (12), (14) and (16), the final form of governing equations of the cable can be written as

$$\left[(T_E - T_E \frac{T_E}{AE} + T_E(\frac{T_E}{AE})^2) r_i'\right]' + mg_i - \rho A g_i + \mathbf{q}_i^H = m\ddot{r}_i \tag{17}$$

$$\left[1 - \frac{T_E}{AE} + (\frac{T_E}{AE})^2)\right] r_j' r_j' - 1 - \frac{T_E}{AE} = 0 \tag{18}$$

with $i, j = 1, 2, 3$. It is noted that $\mathbf{q}_i^H$ using new label to avoid confusion.

Based on Galerkin's method [Zienkiewicz et al. (2005); Tahar et al. (2008); Ćatipović et al. (2011)], the finite element discretization of the equation of motion Eq. (17) can be written as

$$(M_{ijkl} + M_{ijkl}^A)\ddot{U}_{jk} + (K_{nijkl}^0 + \lambda_m K_{nmijkl}^1 + \lambda_m \lambda_p K_{nmijkl}^2)\lambda_n U_{jk} + F_{il} + F_{il}^H = 0 \tag{19}$$

in which,

$$M_{ijkl} = -\int_0^L m A_l A_k \delta_{ij} ds \tag{20}$$

$$M_{ijkl}^A = -\int_0^L (C_A \rho A) A_l e_{ifg} A_v' e_{gjh} A_k A_z' U_{fv} U_{hz} ds \tag{21}$$

$$K_{nijkl}^0 = -\int_0^L P_n A_k' A_l' \delta_{ij} ds \tag{22}$$

$$K^1_{nijkl} = -\int_0^L \frac{1}{AE} P_n P_m A'_k A'_l \delta_{ij} ds \tag{23}$$

$$K^2_{nijkl} = -\int_0^L \frac{1}{(AE)^2} P_n P_m P_p A'_k A'_l \delta_{ij} ds \tag{24}$$

$$F_{il} = F^C_{il} + \int_0^L m g_i ds - \int_0^L (\rho A) g_i A_l ds \tag{25}$$

$$\begin{aligned} F^H_{il} = & \int_0^L (C_M \rho A) A_l e_{ifg} A'_v e_{gjh} A'_z \dot{v}_j U_{fv} U_{hz} ds + \\ & + \int_0^L (\frac{1}{2} C_D \rho A) \sqrt{[e_{abc}(v_b - A_r \dot{U}_{br} A'_s U_{cs})][e_{ade}(v_d - A_t \dot{U}_{dt} A'_u U_{eu})]} \cdot \\ & \cdot [A_l e_{ifg} A'_v e_{gjh}(v_j - A_k \dot{U}_{jk}) A'_z U_{fv} U_{hz}] ds \end{aligned} \tag{26}$$

with $a,b,c,d,e,f,g,h,m,n,p = 1,2,3$, $k,r,t,u,v,z = 1,2,3,4$, where L is the length of each element, M_{ijkl} is the mass matrix due to the own mass, M^A_{ijkl} is the added mass matrix, K^0_{nijkl}, K^1_{nmijkl}, K^2_{nmijkl} is the geometric stiffness matrix and additional stiffness matrix, respectively, δ_{ij} is the Kronecker delta, e_{ifg} is the Levi-Civita symbol, A_l and P_n are the shape functions with Hermitian polynomials, C_A, C_M and C_D are the added mass, inertial and drag coefficients, F_{il}, F^H_{il} and F^C_{il} are the total nodal force, hydrodynamic nodal force and external nodal force vector, and λ_m, λ_p, U_{jk} are unknown coefficients of time dependent.

Similarly, Eq. (18) can be written as

$$(\hat{B}^0_{mkl} + \lambda_n \hat{B}^1_{nmkl} + \lambda_n \lambda_p \hat{B}^2_{nmpkl}) U_{jl} U_{jk} + \hat{C}_{nm} \lambda_n - C_m = 0 \tag{27}$$

with the detailed coefficients expressed as

$$\hat{B}^0_{mkl} = \int_0^L P_m A'_k A'_l ds \tag{28}$$

$$\hat{B}^1_{nmkl} = -\int_0^L \frac{1}{AE} P_n P_m A'_k A'_l ds \tag{29}$$

$$\hat{B}^2_{nmpkl} = -\int_0^L \frac{1}{(AE)^2} P_n P_m P_p A'_k A'_l ds \tag{30}$$

$$\hat{C}_{nm} = -\int_0^L \frac{1}{AE} P_n P_m ds \tag{31}$$

$$C_m = -\int_0^L P_m ds \tag{32}$$

where $\hat{B}^0_{mkl}$, $\hat{B}^1_{nmkl}$, $\hat{B}^2_{nmpkl}$ is the geometric coefficient matrix and additional coefficient matrix, respectively, and C_m, $\hat{C}_{nm}$ are the vectors related to shape and stiffness.

The Newton-Raphson iterative method [Kreyszig (1993)] is used to solve Eq. (19) and Eq. (27) as follows

$$\begin{pmatrix} \hat{J}_{ijkl}^{11(n+1,k)} & \hat{J}_{nil}^{12(n+1,k)} \\ \hat{J}_{mjk}^{21(n+1,k)} & \hat{J}_{mn}^{22(n+1,k)} \end{pmatrix} \begin{pmatrix} \Delta U_{jk} \\ \Delta \lambda_n \end{pmatrix} = - \begin{pmatrix} \hat{R}_{il}^{1(n+1,k)} \\ \hat{R}_{m}^{2(n+1,k)} \end{pmatrix} \tag{33}$$

with

$$\hat{J}_{ijkl}^{11(n+1,k)} = \frac{4}{\Delta t^2} \hat{M}_{ijkl}^{(n+1,k)} + \hat{K}_{nijkl}^{(n+1,k)} \lambda_n \tag{34}$$

$$\hat{J}_{nil}^{12(n+1,k)} = \hat{K}_{nijkl}^{(n+1,k)} U_{jk}^{(n+1,k)} \tag{35}$$

$$\hat{J}_{mjk}^{21(n+1,k)} = -(\hat{B}_{mkl}^0 + \lambda_n \hat{B}_{nmkl}^1 + \lambda_n \lambda_p \hat{B}_{nmpkl}^2)^{(n+1,k)} U_{jl}^{(n+1,k)} \tag{36}$$

$$\hat{J}_{mn}^{22(n+1,k)} = -\frac{1}{2} \hat{C}_{mn} - \frac{1}{2} (\hat{B}_{nmkl}^1 + 2\lambda_p \hat{B}_{nmpkl}^2)^{(n+1,k)} U_{jl}^{(n+1,k)} U_{jk}^{(n+1,k)} \tag{37}$$

$$\begin{aligned} \hat{R}_{il}^{1(n+1,k)} =& (\frac{4}{\Delta t^2} \hat{M}_{ijkl}^{(n+1,k)} + \hat{K}_{nijkl}^{(n+1,k)} \lambda_n) U_{jk}^{(n+1,k)} + \hat{F}_{il}^{(n+1,k)} \\ & - \hat{M}_{ijkl}^{(n+1,k)} (\frac{4}{\Delta t^2} U_{jk}^n + \frac{4}{\Delta t} V_{jk}^n + V_{jk}^n) \end{aligned} \tag{38}$$

$$\hat{R}_{m}^{2(n+1,k)} = -\frac{1}{2} \begin{pmatrix} (\hat{B}_{mkl}^0 + \lambda_n \hat{B}_{nmkl}^1 + \lambda_n \lambda_p \hat{B}_{nmpkl}^2)^{(n+1,k)} \\ U_{jl}^{(n+1,k)} U_{jk}^{(n+1,k)} + \hat{C}_{mn} \lambda_n^{(n+1,k)} - C_m \end{pmatrix} \tag{39}$$

where $\hat{J}_{ijkl}^{11}$, $\hat{J}_{nil}^{12}$, $\hat{J}_{mjk}^{21}$ and $\hat{J}_{mn}^{22}$ compose a Jacobian matrix while $\hat{R}_{il}^1$ and $\hat{R}_m^2$ are parts of a residual vector, and k in superscripts denotes the iteration number within a time step.

For a singe finite element, Eq. (33) is written as

$$\left(\hat{J}\right)^{(n+1,k)} (\Delta y) = -\left(\hat{R}\right)^{(n+1,k)} \tag{40}$$

where $\hat{J}$, Δy, $\hat{R}$ are the corresponding terms in the Eq. (33).

Iterative calculation is written as

$$(y)^{(n+1,k+1)} = \begin{cases} (y)^{(n)} + (\Delta y) & for\ first\ iteration \\ (y)^{(n+1,k)} + (\Delta y) & for\ other\ iteration \end{cases} \tag{41}$$

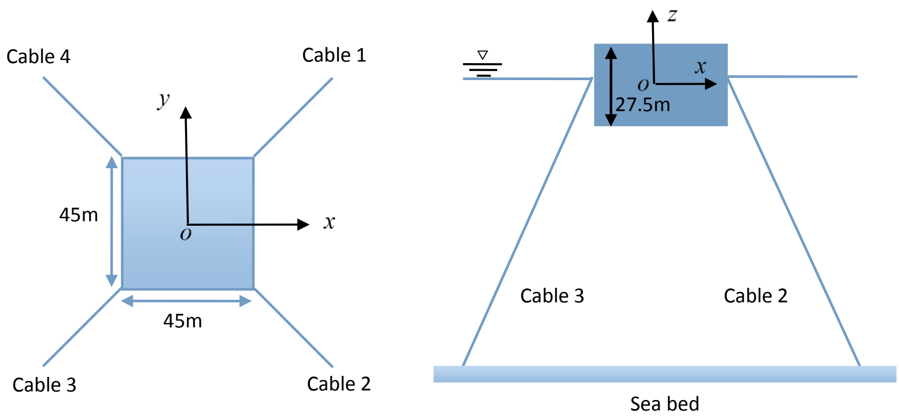

Figure 3: Layout of the mooring systems.

3 Numerical results and discussion

A marine mooring system of floating rectangular box is taken as an illustrative example. Fig. 3 shows the layout of the mooring system. The physical parameters are listed in Table 1. Some physical parameters are taken from the existed studies [Arcandra (2001); Tahar et al. (2008); Huang et al. (2013)].

As structural scale is large we should consider diffraction condition to define the corresponding position of lines in the water and diffraction units. The main contributions of wave forces are the slowly varying wave drift forces and mean wave drift forces. Since the structure has blunt shaped rectangular section, shape resistance of drag force is prominent. For simplicity, it can be assumed that the force due to wind and sea current has only constant components. The value of the wind force and current force can be calculated by simplified engineering formulas, i.e. Eq. (2) and Eq. (3), and coefficients of the formulas are obtained by the physical experiments, professional standards [OCIMF (1994); MOT of PRC (2001)].

3.1 Dynamic analysis

The mooring system located in the ocean for long time and is subjected to various complicated environmental forces like ocean wave, current, wind, etc, which is random and time-varying and may induce the failure of the whole structure. The anchoring cables have significant effects on the mooring performances of the floating structure, and are related to system's motion response and force. Thus, it is necessary and significant to make dynamic analysis about the mooring system with different cable material in the time domain.

Table 1: Basic parameters of the numerical model.

Designation	Parameter	Value
Structural parameters	Length (m)	45
	Width (m)	45
	Height (m)	27.5
	Draft (m)	20
	Mass (t)	41512
	Mass moment of inertia (kgm^2) I_{xx}	4.5316E10
	Mass moment of inertia (kgm^2) I_{yy}	4.2748E10
	Mass moment of inertia (kgm^2) I_{zz}	4.8877E10
Environmental parameters	Density of water (kg/m^3)	1025
	Depth of water (m)	80
	Maximum wave zero crossing period (s)	60
	Maximum significant wave height (m)	2
	Maximum current velocity (m/s)	4
	Maximum wind velocity (m/s)	13.8
	Direction ($^\circ$)	60
Mooring parameters	Pretension (kN)	20
	Numbers of cable	4
	Length of mooring cable (m)	110
	Diameter of mooring cable (m)	0.12
	Minimum breaking load (N)	Steel wire 7.250E6, Chain 2.155E6, Polyester 2.70E6
	Dry weight (kg/m)	Steel wire 70.37, Chain 88.22, Polyester 10.8
	Stiffness (N)	Steel wire 1.36E8, Chain 3.06E8, Polyester (dynamic, linearized stiffness 1.50E8, $\alpha = 2.5, \beta = 2.0$)

Fig. 4 shows the effect of cable length on mean dynamic mooring force of mooring system, the variational laws of curves of cable 1 and cable 3 are similar, with the increase of the cable in the range of 106∼116 m, the mean dynamic mooring force is decreased. Due to the effect of dynamic stiffness of the polyester, the variational laws of curves are not linear. With the increase of the cable length, there is a fraction of cable lying on the seabed, and it does not have any influence on the cable tension because its weight is counteracted by the seabed reaction. Thus, it can be seen that, the dynamic mooring force of the two cables do change abruptly from 110 m to 112 m, and the decrease slows down thereafter.

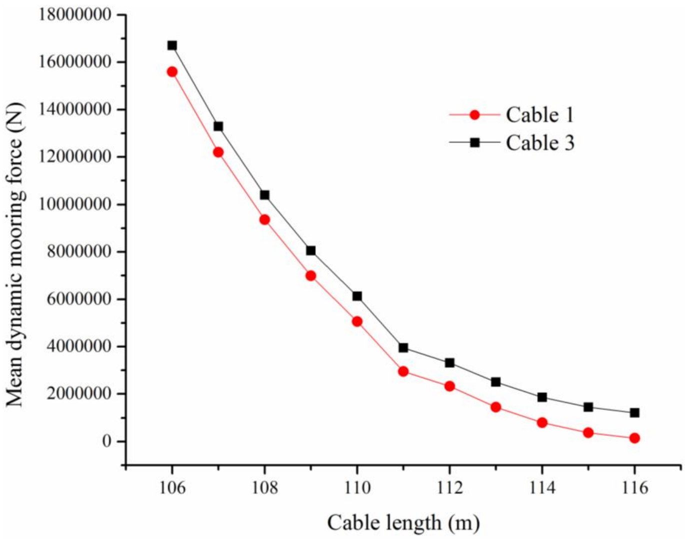

Figure 4: Mean dynamic mooring force of the polyester mooring system.

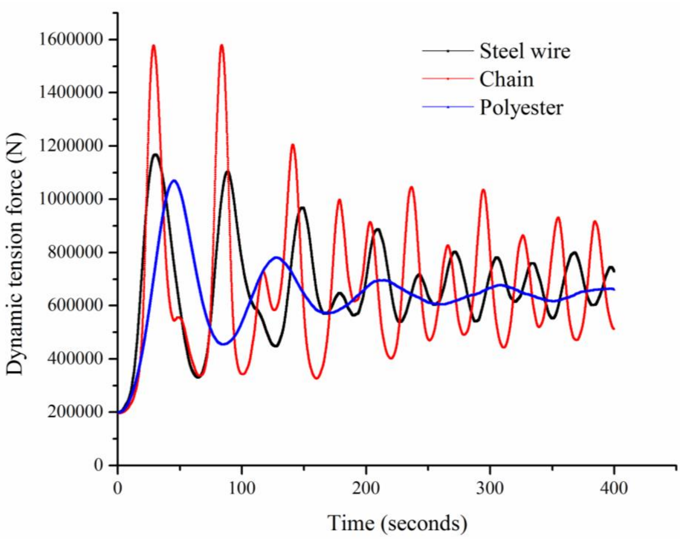

Figure 5: Dynamic mooring tension force of the cable 3.

In Fig. 5, comparing with chain and steel wire, when with same pretension, the polyester has smallest value of dynamic tension force and fastest rate of decay. The maximum value of the dynamic tension force of polyester is only about 67.72% and 91.58% of those in chain and steel wire, respectively. Due to the capacity in absorbing dynamic tension force energy and little weight in water, the polyester can effectively suppress the dynamic tension force of the mooring system, while chain and steel wire need to enhance their minimum breaking load to avoiding be broken in extreme environmental conditions.

Figs. 6-9 shows examples of the dynamic motion response time history signal of mooring system with three kinds of mooring cables. In the present case, the surge motion dominates over sway motion due to the upstream direction of the environmental force more close to surge direction. The value of surge and sway motion response of polyester mooring system is largest, while the value of roll and pitch motion response is smallest in the three kind of mooring cables. It appears that the polyester has significant suppression effect on the rotational degree of freedom of the roll and pitch motion, while the suppression effect on the translational degrees of freedom of the surge and sway motion is limited. The reason is that the dynamic tension force is mainly provided by the horizontal dynamic motion response, generally speaking, smaller dynamic tension force lead to larger horizontal dynamic motion response.

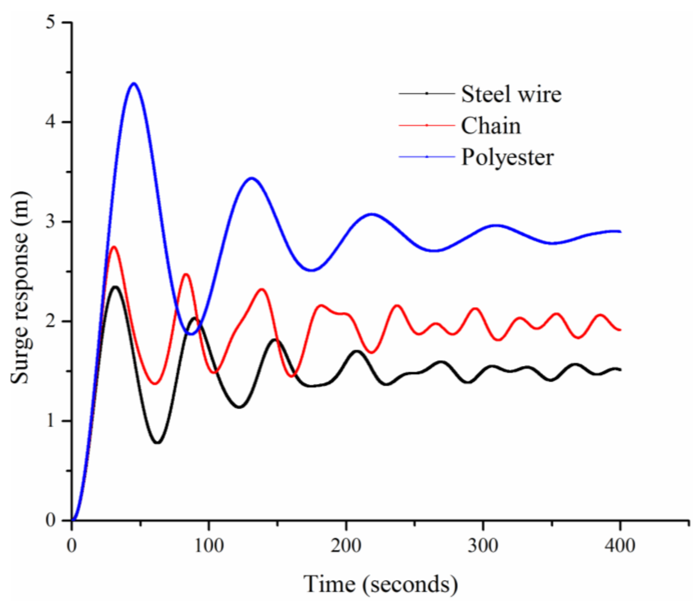

Figure 6: Dynamic surge motions of the mooing system.

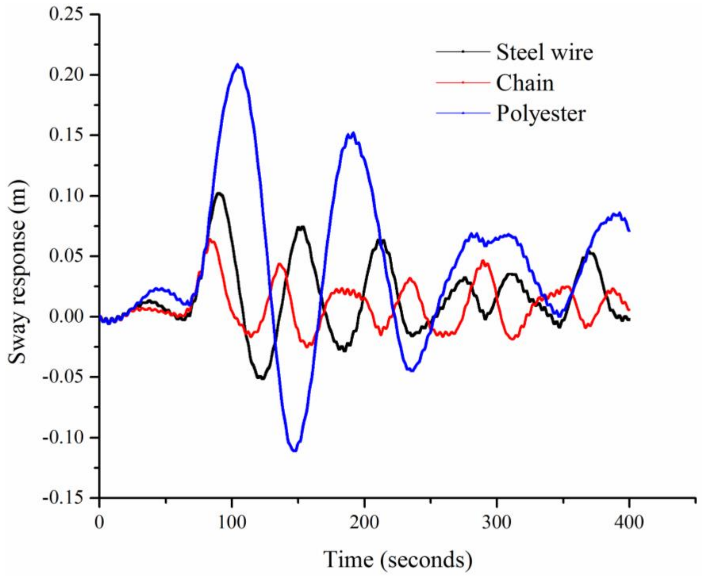

Figure 7: Dynamic sway motions of the mooing system.

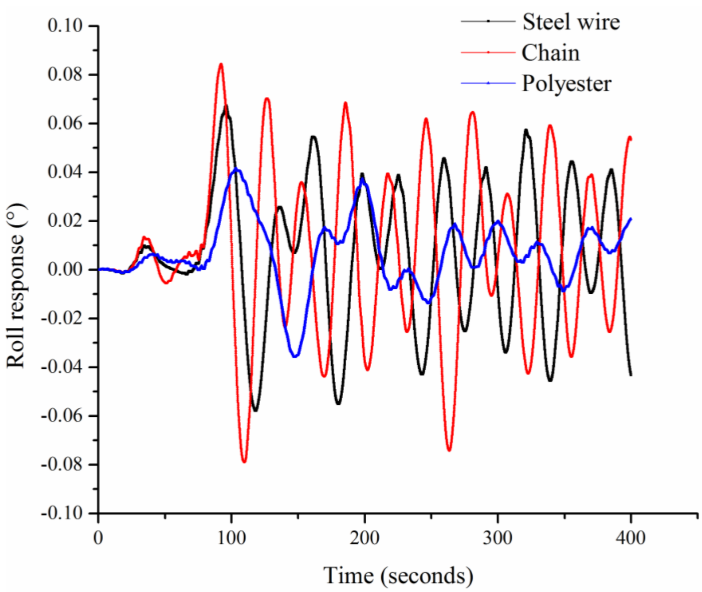

Figure 8: Dynamic roll motions of the mooing system.

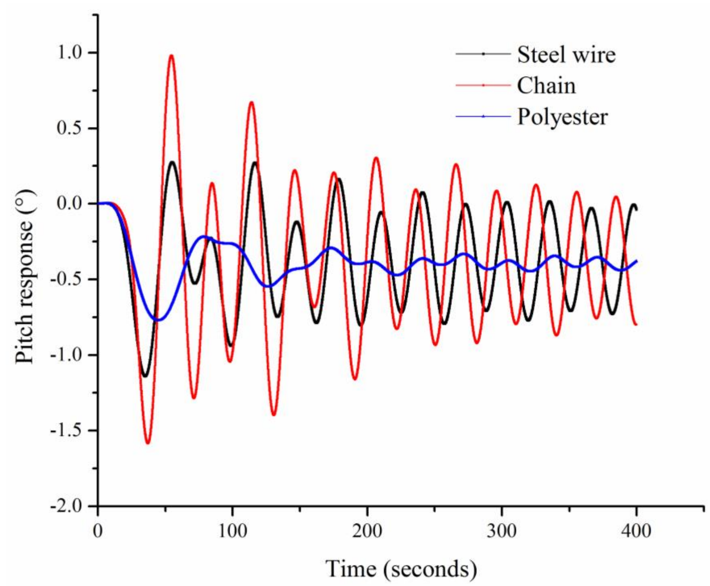

Figure 9: Dynamic pitch motions of the mooing system.

The axial stiffness AE of the polyester is no longer constant but varies with the magnitude of dynamics tension force, and the dynamics tension force is time varying in irregular wave conditions. Thus, complicated nonlinear coupling effect can be occurred between the mooring cable and its mooring system. The Ref. [Tahar et al. (2008)] noted that the heave motion and roll/pitch angles are little affected by both the polyester mooring stiffness and tension, but the horizontal offset of surge and sway motions are affected signally.

Figs. 10-12 shows examples of the phase diagram in surge, sway, and heave direction, respectively. Due to little effects by the polyester cable, there is no nonlinear damping added in the heave direction, Fig. 12 shows the low frequency motion in heave direction. The motional characteristic is of periodic cycle, and the heave motion goes around at centre-of-gravity position of floating structure, and the magnitude value is roughly constant. The surge motion dominates over sway motion leads to prominent mooring-induced damping due to the significant effects of the polyester. Fig. 10 shows characteristic of surge motions in damping decay, and the stationary focal point is around at the point of 3 m. In Fig. 11, the phase diagram in sway direction shows motional characteristic of chaos, the periodic motion is broken by the nonlinear coupling effect of the polyester cable with the mooring system.

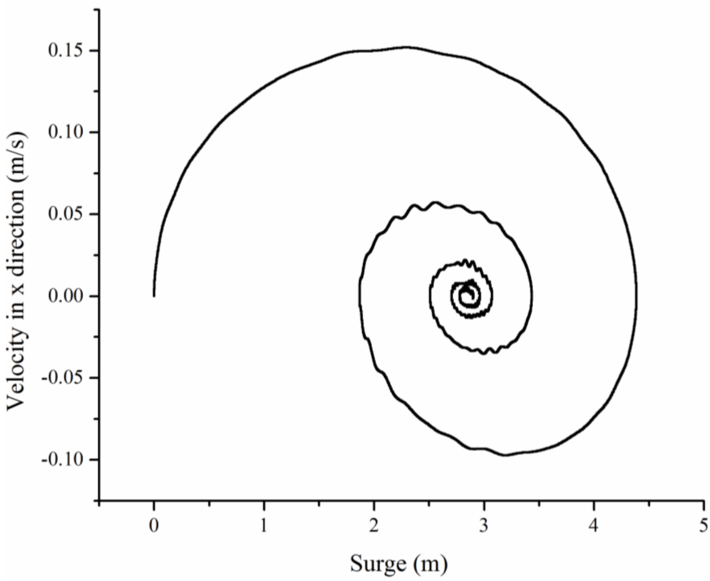

Figure 10: Phase diagram in surge direction.

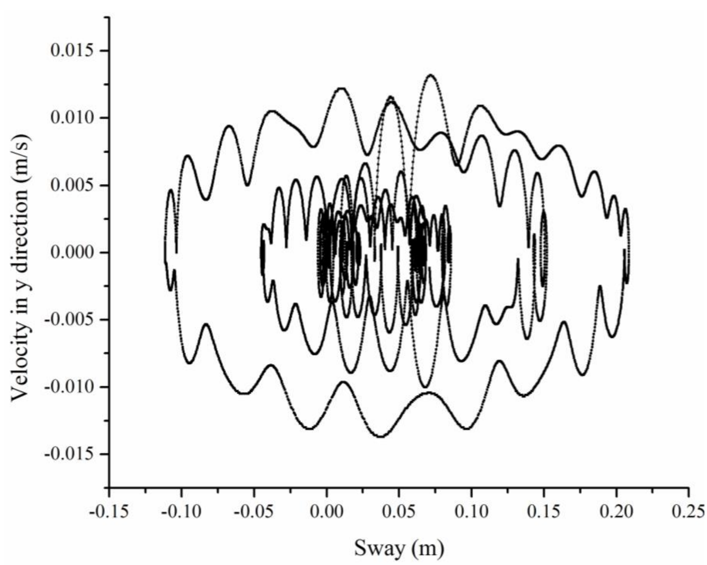

Figure 11: Phase diagram in sway direction.

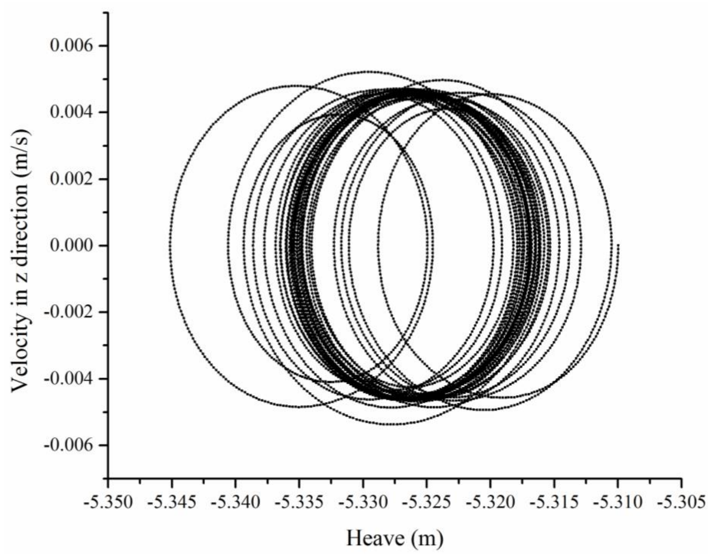

Figure 12: Phase diagram in heave direction.

3.2 Static analysis

Horizontal restoring stiffness plays very important role in global performance of the mooring system, and its values depends on the stiffness, self-weight, and mean minimum breaking load of the mooring cable, which necessitates careful consideration during the early stage of design.

In the present study, a static cable analysis is conducted in order to obtain the tension-displacement characteristics of the mooring system. In Fig. 13, in both cable cases, the restoring force increases with the static surge offset Comparing with the steel wire and the chain mooring system, the static-offset curve of polyester shows hardening effect, it is observed that the restoring force provided by the polyester mooring system is medium when the surge offset is below 0.7m and significantly greater than the other two one when the offset above 0.7m. The difference value of the restoring force between provided by the polyester and steel wire is -13000 N in the point of 0.2 m, while is larger than 54000 N in the points of 1.0 m, and it can expect that the difference value huge when the surge offset increases. This is expected because the modulus of a polyester cable increases with the increase in tension, while the modulus of steel wire and the chain remain unchanged.

The result shows that the polyester mooring line system provides larger nonlinear horizontal stiffness to the mooring system, which can enhance the global perfor-

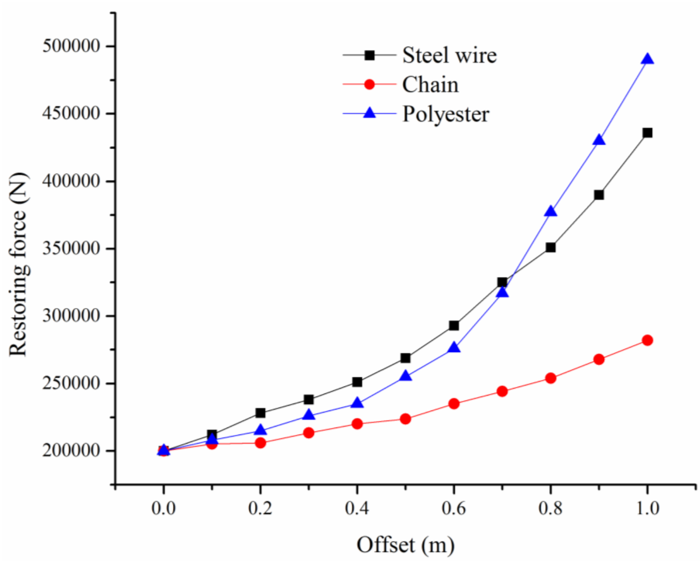

Figure 13: Static surge offset curve with same pretension of the mooring system.

mance of the mooring system located in complex ocean environmental conditions, especially in deeper water.

3.3 Uniformity analysis

In mooring system, if there are huge discrepancies between mooring cables, slack in one or more cables could be encountered, while taut in the other cables which undertake most of the force of the mooring system. The slack of cable might induce the temporary dynamic instability of the floating structure for loss of restraint. Besides, when the cable transits alternatively between the states of slack and taut, the instant tension force of a large value i.e. impulse force might occur, which might cause a sudden breakage of the cable, beside, the fatigue failure can be produced easier.

Uniformity analysis is made for reduce the risk of the slack phenomenon. The standard deviations of peak mooring force and mean mooring force of each four cables are taken to make analysis about the uniformity of the mooring forces. 100 computational cases with variational environmental parameters are calculated. And the mean values of those cases are taken to make comparisons about the three kind of cable.

In Fig. 14, the standard deviation of peak value and mean value of polyester is only about 48.89%, 23.12% of those in chain, respectively. The polyester has minimum

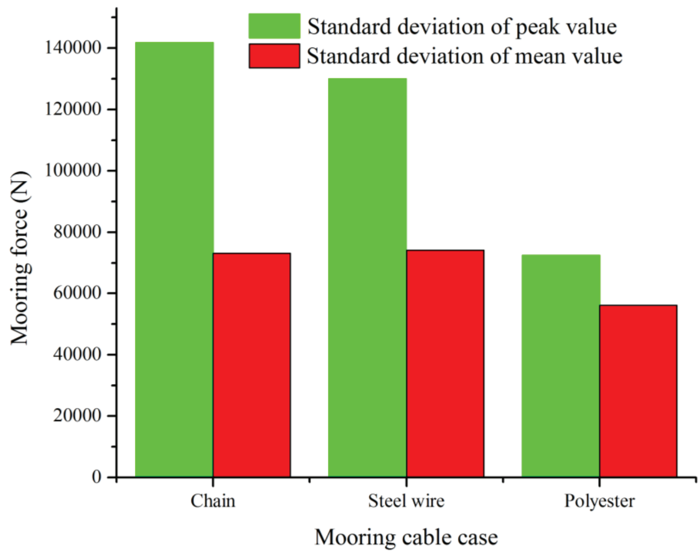

Figure 14: Uniformity of the mooring forces.

standard deviation which implies polyester has higher uniformity of mooring force than steel wire and chain, indicating that polyester has the lowest risk of causing a sudden breakage of the cable. Zhang (2008) noted that, for different cable material, when with some pretension, smaller density and larger modulus can suppress slack phenomenon better, it can be used to explain the aforementioned results.

3.4 Discussion

To keep the safety of the mooring system, the numerical simulation in the design stage for the mooring system located in complicated marine environment is necessary and significant, while physical model experiments are generally expensive and time consuming. The polyester mooring cable has a highly non-linear load-extension curve complicating the mooring system design, thus the numerical method has been seen less as set, and this disadvantage restricted its wide practical application.

The numerical model and methodology based on the finite element method has been successfully developed in this present study. Thus, the engineers can easily use the present method to make the numerical simulation to get more knowledge of the static and dynamic behavior of the mooring system with nonlinear elastic mooring cables. Steel wire or chain produce a nearly linear load-extension curve over the range of loading involved in most mooring designs, and their mechanical properties are simple and have mature use and experience, thus, make the comparison with them, the polyester mooring system can get the useful design consults. Overall, the present study can promote widely practical application of the nonlinear elastic mooring cables.

4 Conclusions

A mathematical method for dynamics motion of nonlinear elastic mooring cable is built utilizing finite element method. The marine mooring system of floating rectangular box with nonlinear elastic cables is taken as an illustrative numerical example. The dynamic analysis, static analysis, and uniformity analysis are made for the polyester mooring system while comparing with the steel wire and the chain mooring system. The presented study can be used to obtained valuable recommendations for the design and construction of the mooring system with nonlinear elastic mooring cables.

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