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Structural Topology Optimization Based on the Level Set Method Using COMSOL

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Abstract: In order to obtain smooth boundary and improve computational efficiency, a new topology optimization scheme based on the level set method is presented. Using the level set function as design variable and the volume ratio of the solid material as volume constraint, respectively, this scheme can easily implement compliance minimization structure topology optimization in associated with the reaction-diffusion equation in commercial software COMSOL. Compared with the results of solid isotropic material with penalization (SIMP) and traditional level set method, this scheme obtained a smooth geometry boundary. In the present computational scheme, the computational cost could be enormously saved without solving the complicated Hamilton-Jacobi equation restricted by Courant-Friedrichs-Lewy (CFL) condition. Two numerical examples verified the performance of the proposed structural topology optimization scheme.

Keywords: level set method, topology optimization, COMSOL, finite element.

1 Introduction

Structural optimization is a very important and popular field. Recently, as one of the most promising method in structural optimization, topology optimization has been widely investigated [Eschenauer and Olhoff (2001); Huang and Xie (2010); Rong et al. (2013); Du and Chen (2012); Olyaie et al. (2011); Matsumoto et al. (2011); Li et al. (2010)]. A variety of techniques and approaches, such as the wavelet finite element based method [Xiang et al. (2010); Xiang and Liang (2011); Liu et al. (2014)], the homogenization method [Allaire and Kohn(1993); Allaire et al. (1997); Bendsoe and Kikuchi (1988)], the Solid Isotropic Material

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with Penalization (SIMP) method[Bendsoe (1989); Bendsoe and Sigmund (2003)], and the level set method [Allaire et al. (2004); Wang et al. (2003); Osher and Santosa (2001); Luo et al.(2012)] have been developed during the past decades.

The main approach of topology optimization is the method of homogenization, in which a material model with micro-scale voids is introduced. By using the size of the microstructure parameters as topology design variables, the homogenization technique is capable of implementing topology optimization easily. However, the method may lead to numerical instabilities [Suzuki and Kikuchi (1991)].

The SIMP method is popular for its computational efficiency and conceptual simplicity [Bendsoe and Sigmund (1997, 1999); Yang and Chung (1994)]. The basic idea of the SIMP is the use of a supposed isotropic material whose elasticity tensor is assumed to be a function of penalized material parameter. However, numerical instability (checkerboard patterns, grayscales elements) and computational complexity remain to be the major difficulties.

In recent years, a new level set method has become popular in the field of topology optimization. The level set method introduced by Osher and Sethian [Osher and Sethian (1988)]is a simple and versatile method for tracking the evolution of interfaces. Sethian and Wiegmann [Sethian and Wiegmann (2000)] are first researchers to extend the level set method to structural optimization. Since then, it has gained much attention [Luo et al. (2004); Jia et al. (2011); Shojaee and Mohammadian (2012)]. Compared with the traditional topology optimization methods, the level set method is more direct and effective for structural optimization problem. However, there are also some numerical difficulties in the conventional level set methods, such as the Courant–Friedrichs–Lewy (CFL) condition [Courant et al. (1967)], the periodic re-initializations and the boundary velocity extension. The most difficult problem is how to use finite difference techniques including the traditional upwind scheme, the essentially non-oscillatory (ENO) scheme, and the central weighted essentially non-oscillatory (WENO) scheme [Sethian (1999)] to solve the Hamilton-Jacobi equation. In these techniques, the time-step size must be restricted by the CFL condition to ensure the convergence of numerical process.

For the above reasons, this paper presents a computational scheme for topology optimization based on the level set method using the commercial software COMSOL [Zimmerman (2007)]. Without the limitation of Courant-Friedrichs-Lewy (CFL) condition, this scheme can easily implement the compliance minimization structure topology optimization with a simplified reaction-diffusion equation in the partial differential equations (PDEs) module. Numerical examples illustrate that the proposed scheme has high performance.

2 Level set method

2.1 Basic principles

The level set method implicitly represents target structural configurations using the iso-surface of the level set scalar function. The outlines of target structures are changed by updating the level set function during the optimization process. For a design domain Ω with smooth boundary, suppose the existence of an implicit function $\varphi(x)$ satisfies

$$\left.\begin{array}{ll} \varphi(x) > 0 & x \in \Omega^{+} \text{ (material)} \\ \varphi(x) = 0 & x \in \partial\Omega \text{ (boundary)} \\ \varphi(x) < 0 & x \in \Omega^{-} \text{ (void)} \end{array}\right\} \tag{1}$$

The time derivative of the level set function namely the level set equation (Hamilton-Jacobi equation) is obtained by introducing the pseudo-time t.

$$\frac{\partial \varphi}{\partial t} + V_n |\nabla \varphi| = 0 \tag{2}$$

where V_n is the normal velocity of the implicit interface, $V_n = V \bullet \frac{\nabla\varphi}{|\nabla\varphi|}$, $n = \frac{\nabla\varphi}{|\nabla\varphi|}$, here $\nabla\varphi$ is the gradient vector of φ. By solving Eq. (2) numerically to promote the evolution of structure topology.

2.2 Level set method for compliance minimization problem

In general, the minimum mean compliance problem can be written as

$$\left.\begin{array}{ll} \min: & C(\varphi) = \int_{\Omega} \frac{1}{2} E(\varphi)\, \varepsilon(u)^{T} D \varepsilon(v)\, d\Omega \\ s.t. & \int_{\Omega} \varepsilon(u)^{T} D \varepsilon(v)\, d\Omega = \int_{\Omega} p \bullet v d\Omega + \int_{\Gamma} \tau \bullet v d\Omega \\ & \int_{\Omega} H(\varphi)\, d\Omega \le V_{\max} \end{array}\right\} \tag{3}$$

where u is the displacement vector, ε is the strain tensor, D is the elasticity matrix, p is the body force, τ is the traction applied on the boundary Γ, v is the virtual displacement, $V_{\max}$ is the volume constraint, and E is the design variable defined as

$$E(\varphi) = E_0 H(\varphi) + (1 - H(\varphi)) E_{\min} \tag{4}$$

where E_0 is the material elasticity modulus, $E_{\min}$ is the minimum elasticity modulus and $H(\varphi)$ represents the Heaviside function

$$H(\varphi) = \left\{ \begin{array}{ll} 0 & \varphi \le 0 \\ 1 & \varphi > 0 \end{array} \right. \tag{5}$$

whose derivative is expressed as the Dirac delta function $\delta(\varphi)$

$$\delta(\varphi) = \frac{d(H(\varphi))}{d\varphi} \tag{6}$$

Introducing the Lagrangian multiplier λ, Eq. (3) can be rewritten as

$$J(\varphi) = \int_{\Omega} \left[\frac{1}{2} E(\varphi) \varepsilon(u)^T D\varepsilon(v) + \lambda (H(\varphi) - V_{\max}) \right] d\Omega \tag{7}$$

where $J(\varphi)$ is the objective function.

From traditional calculus, the extremes of Eq. (7) are achieved at the point where $\frac{\partial J(\varphi)}{\partial \varphi}$. Then the shape derivatives respect to φ of $J(\varphi)$ can be derived as

$$\frac{\partial J(\varphi)}{\partial \varphi} = \int_{\Omega} \left[\frac{1}{2} (E_0 - E_{\min}) \varepsilon^T D\varepsilon + \lambda \right] \delta(\varphi) |\nabla \varphi| \delta l d\Omega \tag{8}$$

where δl is an infinitesimal variation. The relevant Euler-Lagrange equation at the extreme value point is

$$\left[\frac{1}{2} (E_0 - E_{\min}) \varepsilon^T D\varepsilon + \lambda \right] \delta(\varphi) |\nabla \varphi| = 0 \tag{9}$$

According to the steepest descent method [Wang (2004)], the advection velocity V_n can be expressed as

$$V_n = \left[\frac{1}{2} (E_0 - E_{\min}) \varepsilon^T D\varepsilon + \lambda \right] \delta(\varphi) \tag{10}$$

Then Eq. (2) can be described as

$$\frac{\partial \varphi}{\partial t} - \left[\frac{1}{2} (E_0 - E_{\min}) \varepsilon^T D\varepsilon + \lambda \right] \delta(\varphi) |\nabla \varphi| = 0 \tag{11}$$

In most cases, it is impossible to solve Eq. (11) directly. One common idea is to solve the level set equation numerically.

The goal of level set based topology optimization for compliance minimization problem is to find the optimal structural boundaries $\partial\Omega$, which is implicitly expressed by the level set function φ. Therefore, we can get the optimal shape and topology of the structure by solving the Eq. (11) numerically.

3 Topology optimization scheme based on the level set method using COMSOL

3.1 Lagrangian multiplier

In Eq. (11), the Lagrangian multiplier λ can be explicitly expressed by an integration coupling variable in COMSOL. Supposing that an initial φ_0 satisfies the volume constraint condition, the constraint condition should be satisfied all the time

$$\frac{d}{dt}\left(\int_{\Omega} H(\varphi) - V_{\max}\right) = \int_{\Omega} \frac{\partial H(\varphi)}{\partial \varphi} \bullet \frac{\partial \varphi}{\partial t} d\Omega = \int_{\Omega} \delta(\varphi) \bullet \frac{\partial \varphi}{\partial t} d\Omega = 0 \tag{12}$$

Replacing the $\frac{\partial \varphi}{\partial t}$ by Eq. (11), we have

$$\lambda = -\frac{\int_{\Omega} \left[\frac{1}{2}(E_0 - E_{\min})\varepsilon^T D\varepsilon\right] \delta^2(\varphi) |\nabla\varphi| d\Omega}{\int_{\Omega} \delta^2(\varphi) |\nabla\varphi| d\Omega} \tag{13}$$

The method we used to calculate the Lagrangian multiplier λ in Eq. (13) is based on the assumption that the initial φ_0 satisfies the volume constraint condition all the time. In order to ensure accuracy, a penalty term should be added to Eq. (7)

$$\frac{1}{2}\gamma(V_{cur} - V_{\max})^2 \tag{14}$$

where γ is a penalty factor, V_{cur} is the value of volume of material in current iterative step, and $V_{\max}$ is the volume constraints.

3.2 Reaction-diffusion equation

In the conventional level set method, the CFL condition has been introduced to maintain the stability of the computational procedure. However, it leads to poor computational efficiency at the same time. Therefore, in this paper, we employ the reaction-diffusion equation to replace the level set equation Eq. (2)

$$\frac{\partial \varphi}{\partial t} + V_n |\nabla\varphi| = \alpha \Delta\varphi \tag{15}$$

where the term $V_n |\nabla\varphi|$ is regarded as the reaction term, α is the viscosity coefficient, and the term $\alpha\Delta\varphi$ is an artificial dissipation term which make this scheme be able to achieve numerical stability and improve computational efficiency easily.

Then Eq. (3) can be changed into the coupled plane stress equation and reaction-diffusion equation

$$\left.\begin{array}{l} \int_{\Omega} \varepsilon(u)^T D\varepsilon(v)\, d\Omega = \int_{\Omega} p \bullet v d\Omega + \int_{\Gamma} \tau \bullet v d\Omega \\ \frac{\partial \varphi}{\partial t} - \left[\frac{1}{2}(E_0 - E_{\min})\varepsilon^T D\varepsilon + \lambda\right] \delta(\varphi)|\nabla\varphi| = \alpha\Delta\varphi \end{array}\right\} \tag{16}$$

As a result, we can solve the fully coupled PDEs by the finite element method via commercial software COMSOL.

3.3 Optimization procedure

A description of the present optimization procedure is given as follows:

*Step*1: Choose the plane stress and PDEs in Multi-physics module of COMSOL.

*Step*2: Establish the geometry model and mesh.

*Step*3: Set initial structural parameters, integration coupling variables and expressions.

*Step*4: Set the boundary conditions.

*Step*5: Select solver and set the corresponding solver parameters, such as the solution time, the *stationary linear* solver, and *time dependent* solver.

*Step*6: Solve and draw contour lines.

4 Numerical examples

In this section, two numerical examples are presented to confirm the validity of the proposed optimization scheme for two-dimensional compliance minimization problem.

Example 1. A cantilever beam

The model is shown in Fig. 1. The whole design domain is a rectangle of size $8m \times 6m$ with a fixed boundary on the left side. A vertical concentrated force F = 1 N, is loaded in the center of the right-hand side. The properties of the isotropic material are: Young's module $E = 2.06 \times 10^5$Pa, Poisson's ratio $\mu = 0.3$, and material density $\rho = 7800 kg/m^3$.The volume of material allowed for the structure is 50% of the design domain.

The evolution of cantilever beam using the present topology optimization scheme is shown in Fig. 2. Fig. 3(a)-(c) shows the process of SIMP topology optimization and Fig. 3(d)-(f) shows the process of traditional level set method topology optimization. We found that the optimal configurations in all the three methods are nearly the same, but the SIMP method and the conventional level set method have drawback of zigzag boundary, only the present scheme obtains a smooth and clear boundary.

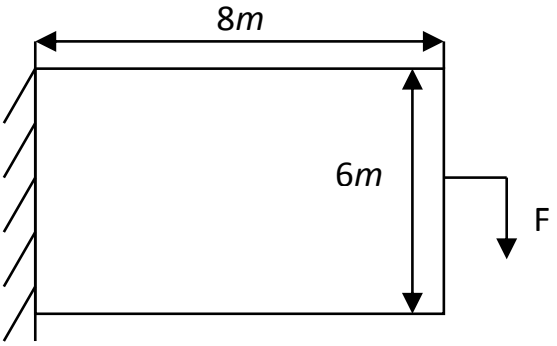

Figure 1: The geometry of a cantilever beam.

Table 1: Comparison of the performance using different methods to solve a cantilever beam.

Method	Element	Dofs	Time(s)	Iteration (step)	Linear solver (time)
SIMP	4800quadri	4941	173	78	78
Conventional level set	4800quadri	4941	103	52	52
Present method	2954tri	4671	30	115	351
Note: 'quadri' denote the linear quadrilateral element, 'tri' denote the linear triangular element.					

The computational time among the three methods are compared in Table 1 for the cantilever beam. Obviously, the proposed scheme takes the shortest time to complete the topology optimization process. Convergence is obtained within 30 s, 115 iterative steps and 351 solving times.

Example 2. A half-wheel

A half-wheel problem is considered. The design domain of a half-wheel is shown in Fig. 4. The whole design domain is a rectangle of size $8m\times\ 6m$. The left side of the grid point at the lower left-hand of corner is fixed and the vertical motion of the grid point at the lower right-hand corner is restricted. A vertical concentrated force $F = 1\ N$, is loaded in the center of the bottom side. The properties of the isotropic material are the same as the cantilever beam problem. The upper limit of the volume constraint $V_{\max}$ is set to 50% of the fixed design domain.

The evolution of half-wheel using the present topology optimization scheme is

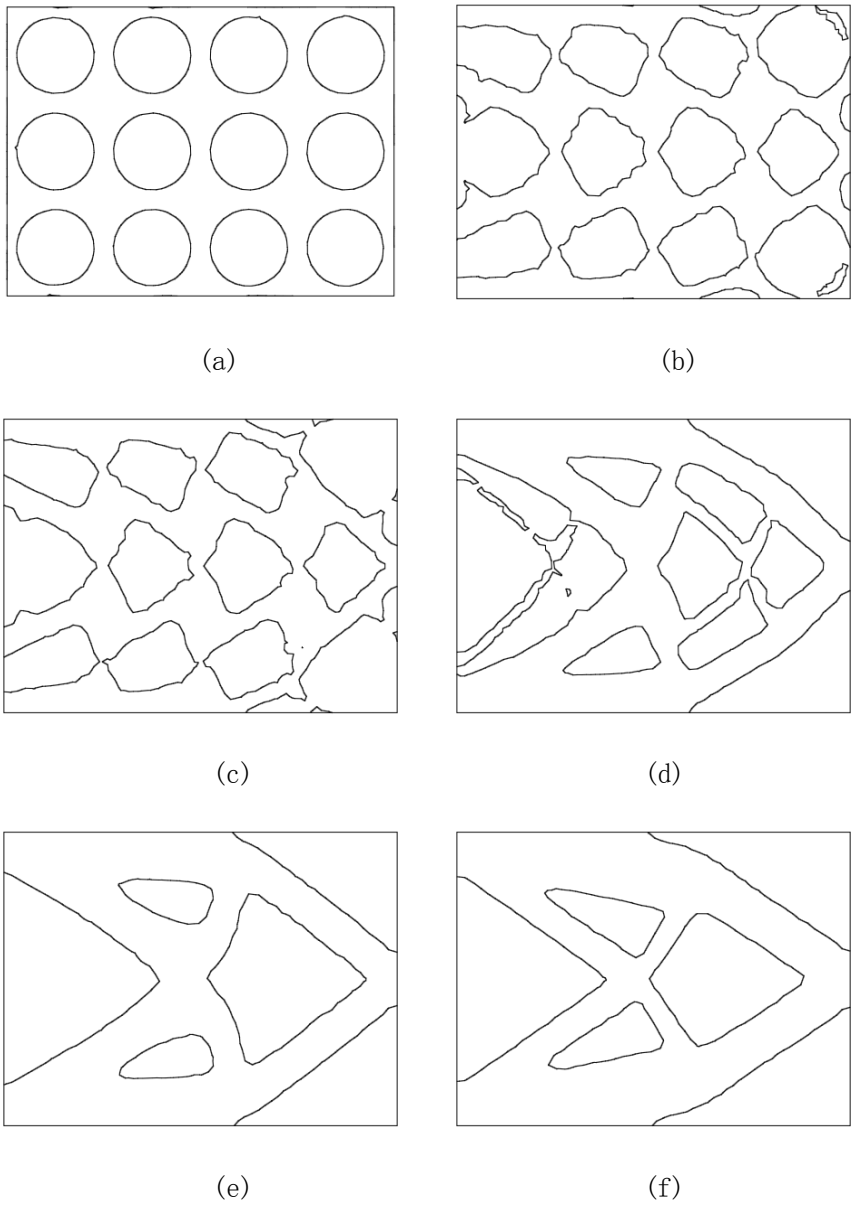

Figure 2: Process of the present topology optimization scheme for a cantilever beam: (a) initial design, (b)-(e) intermediate results, (f) optimal result.

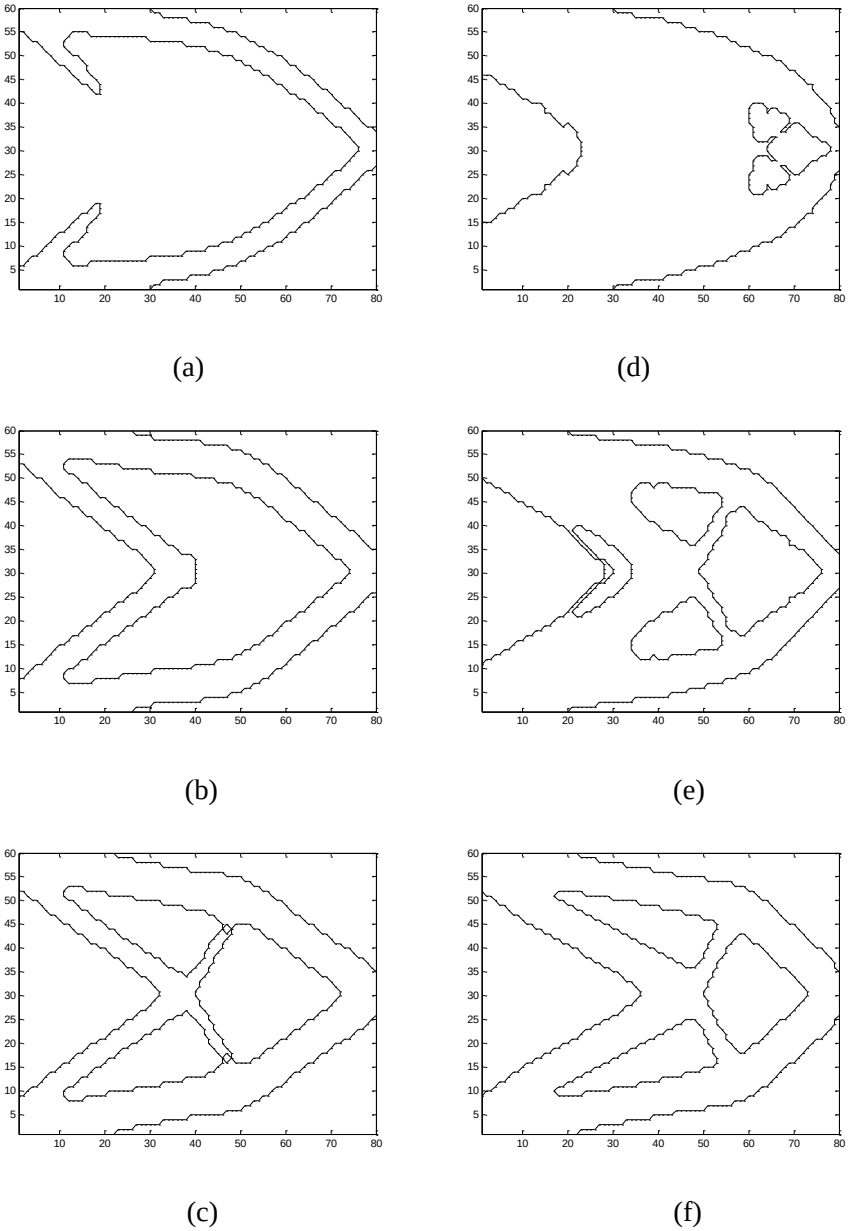

Figure 3: Process of SIMP and traditional level set method topology optimization for a cantilever beam: (a)-(b) intermediate results of SIMP, (c) optimal result of SIMP, (d)-(e) intermediate results of traditional level set method, (f) optimal result of traditional level set method.

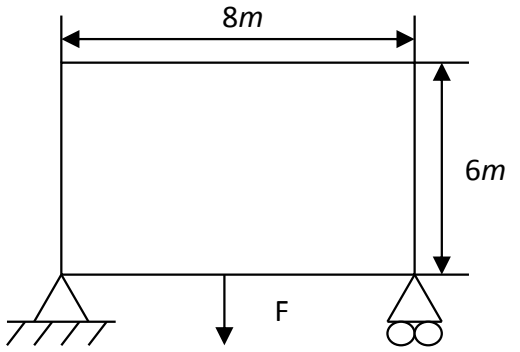

Figure 4: The geometry of a half-wheel.

Table 2: Comparison of the performance of different methods for half-wheel.

Method	Element	Dofs	Time(s)	Iteration (step)	Linear solver (time)
SIMP	4800quadri	4941	146	74	74
Conventional level set	4800quadri	4941	82	46	46
COMSOL	2927tri	4632	25	104	326
Note: quadri denote the linear quadrilateral element, tri denote the linear triangular element.					

shown in Fig. 5. Fig. 6(a)-(c) shows the process of SIMP topology optimization and Fig. 6(d)-(f) shows the process of traditional level set method topology optimization for the same half-wheel. As shown in Figs.5 and 6, the optimal results of all the three methods are basically the same except that a little different because of numerical error, but the results of the SIMP method and the conventional level set method have shortcoming of zigzag boundary. Compared with the traditional two methods, the proposed scheme achieves a smooth and clear boundary.

The calculation time among the three methods are compared in Table 2 for the half -wheel. Obviously, the present scheme completed the topology optimization process with the shortest time. It only takes 25 s to find the optimal structure with 104 iterative steps and 326 solving times.

From the above two numerical simulations and comparisons, it can be concluded that the proposed topology optimization scheme is able to achieve smooth boundary and improve computational efficiency.

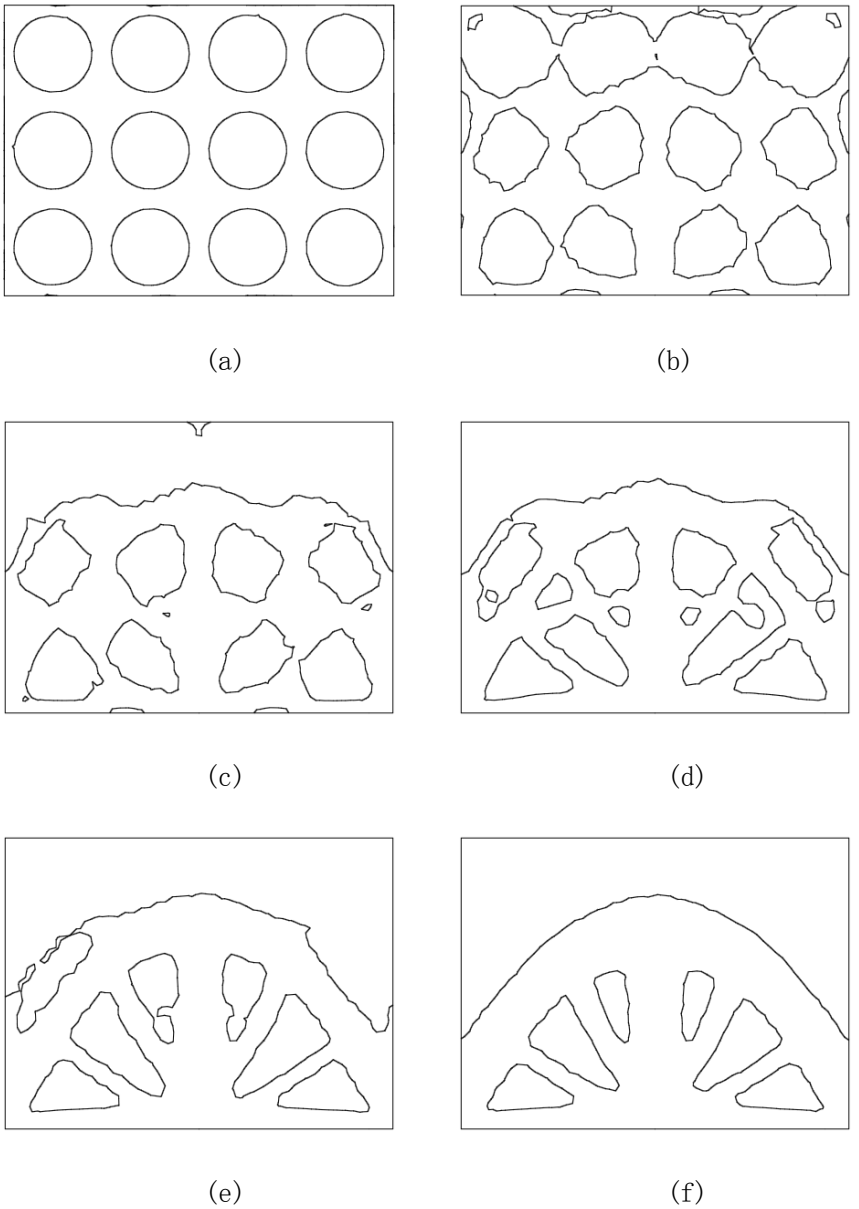

Figure 5: Process of the present topology optimization scheme for a half-wheel: (a) initial design, (b)-(e) intermediate results, (f) optimal result.

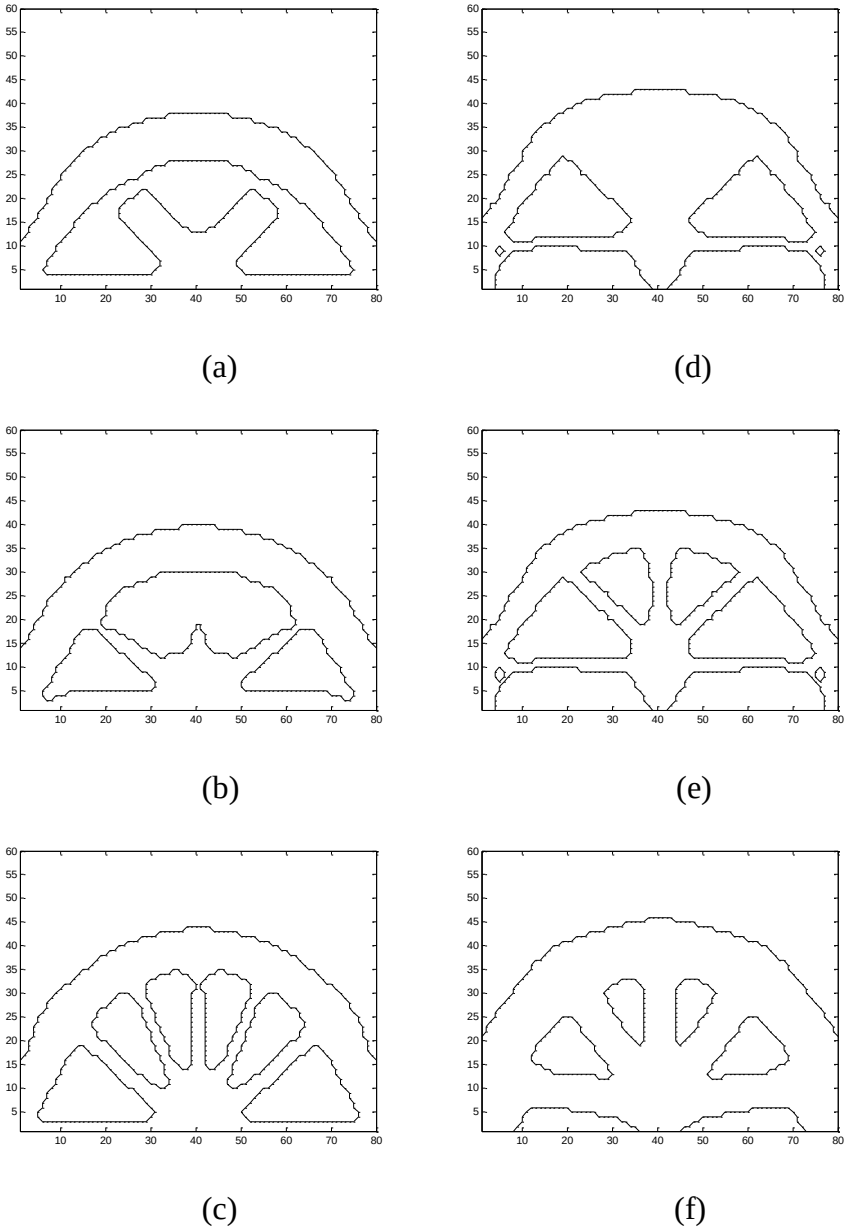

Figure 6: Process of SIMP and traditional level set method topology optimization for a half-wheel: (a)-(b) intermediate results of SIMP, (c) optimal result of SIMP, (d)-(e) intermediate results of traditional level set method, (f) optimal result of traditional level set method.

5 Conclusion

This paper proposed a topology optimization scheme based on the level set method using COMSOL. Unlike the traditional level set method whose computational efficiency restricted by the Courant-Friedrichs-Lewy (CFL) condition, the present structural optimization scheme can easily implement compliance minimization structure topology optimization with the simplified reaction-diffusion equation in commercials software COMSOL. Numerical examples illustrate that the proposed scheme is effective and is able to obtain smooth geometry boundary and further improve the computational efficiency.

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