 CMES, vol.94, no.1, pp.53-75, 2013

Topological Design of Structures Using a Cellular Automata Method

Yixian Du[1,2,3,4], **De Chen**[1], **Xiaobo Xiang**[1], **Qihua Tian** [1], **Yi Zhang**[1]

Abstract: Topological design of continuum structures usually involves numerical instabilities, such as checkerboards and mesh-dependency, which degenerate the manufacturability, the efficiency and the robustness of the optimal design. This paper will propose a new topology optimization method to suppress numerical instabilities occurred in the topology optimization of continua, according to the principle of error amplifier and feedback control in the control system. The design variables associated with topological design are updated based on the Cellular Automata (CA) theory. A couple of typical numerical examples are used to demonstrate the effectiveness of the proposed method in effectively suppressing numerical instabilities occurred in the numerical procedure of topology optimization.

Keywords: Topology optimization; Cellular automata; Numerical instability

1 Introduction

Topology optimization has been one of the most important but challenging techniques in the area of structural optimization [Bendsøe and Sigmund (2003)]. Topological optimization has drawn much consideration over the past two decades, and many different methods have been developed for a number of engineering applications [e.g. Kang and Tong (2008); Li and Atluri (2008); Du, Luo, Tian and Chen (2009); Luo, Tong, Luo and Wang (2009); Luo, Luo, Tong, Gao and Song (2011)]. Compared with size and shape optimization, topology optimization has a broader design space to satisfy more harsh design requirements. The aim of topology optimization is to automatically distribute a given amount of material in the

[1] College of Mechanical & Material Engineering, China Three Gorges University, Yichang 443002.
[2] Hubei Key Laboratory of Hydroelectric Machinery Design & Maintenance, Yichang 443002.
[3] Corresponding Author.
Tel: +86-717-6397560; Fax: +86-717-6397559; Email: duyixian@aliyun.com
[4] Collaborative Innovation Center for Energy Equipment of Three Gorges Region, China Three Gorges University, Yichang 443002, China.

design domain to achieve the best material layout by optimizing the design objective under specific design constraints. The typical methods for topology optimization include the homogenization method [Bendsøe and Kikuchi (1988)], the SIMP method [Zhou and Rozvany (1991); Bendsøe and Sigmund (1999)], nodal density-based methods [Matsui and Terada (2004); Kang and Wang (2011); Wang, Luo and Zhang (2012); Luo, Zhang, Wang and Gao (2013)] and the level-set based method [Wang, Wang and Guo (2003); Allaire, Jouve and Toader (2004); Luo, Tong and Wang (2008)]. With the development of modern computational techniques, topology optimization is now a more preferable industrial design tool in the stage of conceptual design.

Despite its recent development, numerical instabilities in the design are still open topics in the community of topology optimization [Sigmund and Petersson (1998)]. These numerical problems, e.g. checkerboards, local minima, mesh-dependency and gray-scale element, will result in topological designs that are unacceptable in practical engineering. The checkerboards problem refers to the formation of regions of alternating solid and void elements ordered in a checkerboard-like fashion, which should be caused by improper computational modeling with low-order finite elements [Diaz and Sigmund (1995)], with over-stiffness local regions in the structure. To a large extent, local minima appear for the nonrestricted 0-1 topology optimization problems. The schemes that employed tend to convexify the problems and produce reproducible designs. Convergence proofs of algorithms producing iterates to solve convex programs are common, while for nonconvex problems the statements usually only ensure the algorithm iterates' convergence to a nearby stationary point (which certainly need not be similar to a global solution). The mesh-dependence [Sigmund and Petersson (1998)] means a finer finite element mesh will lead to a design with more complex topological geometry. To overcome these shortcomings, there have been a lot of research endeavors with a view to solving these numerical problems [Jog and Haber (1996); Haber and Bendsøe (1996); Petersson and Sigmund (1998); Poulsen (2003); Bruggi (2008); Kang and Wang (2011); Luo, Zhang, Wang and Gao (2013); Du and Chen (2012); Zakhama, Abdalla, Smaoui, and Gürdal (2009); Li and Atluri(2008); Luo and Zhang (2012)]. And many valuable results are got through these methods.

This paper proposes a Cellular Automata (CA) algorithm to update the design variables without using the design sensitivity analysis to simplify the numerical procedure, according to the active cellular and its neighbors' state information. The concept of CA model for topology optimization was presented by Kita and Toyoda (2000). The hybrid cellular automaton (HCA) algorithm for structural optimization was presented by Tovar and co-workers (2004), which combined elements of the cellular automaton (CA) paradigm with the method of finite element analy-

ses (FEA). Faramarzir and Afshar (2012) presented a two-phase, hybrid, Cellular Automat-Linear Programming (CA-LP) method for both size and topology optimization of planar truss structures, to achieve the minimum weight of structures under stress, nodal displacement, cross-sectional area and kinematic stability constraints.

In this work, we present a new updating rule for design variables in topology optimization of continua, which is based on the CA theory and inspired by the principle of feedback control and the error amplifier in the control system. Compared with classical gradient-based optimization methods, this updating scheme does not need to calculate the sensitivity information of the objective function. And in contrast to the widely used Genetic Algorithms (GA) method [Mark, Colin, James, Adenike and Kazuhiro(2000); Wang and Tai (2005)]. The proposed algorithm computationally efficient and can effectively suppress the mesh-dependence and the gray-scale element. Basing on the SIMP material description, the design domain is discretized into a regular lattice of cellular automata. According to the strain energy density level at each iteration, the material quantity is added or removed depends on the updating rule that drives the actual strain energy density level to the desired value. The computational efficiency of the updating scheme is illustrated by three typical numerical examples. The right selection of error amplification coefficient might dramatically improve the convergence of the updating rule presented in this work.

The following sections are arranged as follows: Section 2 describes the topology optimization model using CA; Section 3 presents a new updating rule of design variables, and numerical examples to show the efficiency and accuracy of the new updating rule, followed by discussion and conclusions in Section 5.

2 Structure topology optimization model using CA

Topology optimization is often formulated as a material distribution problem, in which solid and void phases are indicated by discrete values 1 and 0, respectively. As we all know, the topology optimization of continua belongs to a family of integer programming problems with 0 and 1 discrete design variables in fact, which many gradient-based optimization algorithms cannot be directly applied to. To overcome this drawback, the discrete model is usually required to be relaxed to make material properties continuously dependent on the local amount of material. SIMP has been widely used, as a result of its conceptual simplicity, implementation easiness and computational efficiency.

2.1 Material interpolation scheme

In SIMP, a density-stiffness interpolation scheme is used to represent the nonlinear dependency between elemental densities and material properties. To recover the original 0 and 1 discrete material distribution, a power-law scheme [Bendsøe and Sigmund (1999)] is usually applied to penalize the intermediate densities to push the intermediate densities towards its binary bounds (0/1). The SIMP can be generally written as

$$E_e(x_e) = E_{\min} + x_e^p(E_0 - E_{\min}) \tag{1}$$

$$\rho_e(x_e) = x_e\rho_0 \ (0 \le x_e \le 1) \quad e = 1,2,\cdots,n \tag{2}$$

where E_e and E_0 denote the actual and initial Young's Modulus of the $e-$th element, respectively; p is the penalty factor (typically$p$ =3); E_{min} is the elastic modulus of the element without material; ρ_0 is the material's density of solid state, ρ_e is the material density of the eth element, x_e is its design variable, namely, the element's relative density, and n is the number of the structural elements. In addition, the design variable ρ_e need to be iteratively updated, and so the Young's Modulus is also reevaluated for the structural analysis of the next iteration; In order to insure the stability of numerical calculation, $E_{min} = E_0/1000$ is used in this work.

2.2 Mathematical model of topology optimization

Cellular Automata models have been used to simulate complex biological and physical phenomena for many years. It is an idealization of a physical system in which space and time are discrete, and physical quantities can take values on a finite set. In the previous research, the HCA method inspired in the bone remodeling process combines local evolutionary design rules with finite element analyses [Tovar, Niebur and Renaud (2004); Tovar, Neal and Glen (2004)]. It has been demonstrated to be an efficient computational non-gradient technique for the topology design of structures, since they can get the optimal topology without checkerboards. This methodology has been proven to be a globally convergent under certain assumptions with a fixed point iteration scheme and is appropriate for structural topology design problems.

In this paper, the proposed method using finite element analyses to evaluate the stress states of cellular based on the principles of fully stressed design, which makes the none-vacant cellular tend to fully stressed gradually by the cellular model iteration. Since all the elements are in the state of maximum strain energy density that the material can bear, the optimal topology can thus be obtained.

The state $\alpha_e(t)$ of the e-th cellular in the t-th iteration is defined by the design variable $x_e(t)$ and the state field variables $U_e(t)$. In this paper, we define the cellulars'

relative density x_e as a design variable and the strain energy density U_e as a state field variable. Each cellular state can be written as

$$\alpha_e(t) = \left\{ \begin{array}{l} \mathbf{x}_e(t) \\ U_e(t) \end{array} \right\} \tag{3}$$

For the design problem of a rigid structure, the mechanical energy that the structure stored can be defined as the total strain energy U. So if the structural has a high stiffness value, it means that it can stored a fewer strain energy. For a two-dimensional structure defined on a design domainΩ, Ucan be expressed as follows:

$$U = \int_{\Omega} \frac{1}{2} \sigma^T \varepsilon d\Omega \tag{4}$$

where σ and ε are the corresponding stress and the strain field vectors under the load, respectively.

The objective of the optimization problem is to obtain the minimum difference between the cellular's strain energy density and the target values that are pre-established. Thus, the topology optimization model based on the CA can be written by

$$\begin{array}{l} \min\limits_{\mathbf{x}} : \quad \sum\limits_{e=1}^{n} |\bar{U}_e - U_e^*| \\ s.t. : \mathbf{Ku} = \mathbf{F} \\ 0 \le x_e \le 1 \quad e = 1, 2, \cdots, n \end{array} \tag{5}$$

where U_e^* and $\overline{U}_e$ are the local strain energy density of the e-th cellular and the average strain energy density of the neighboring cellulars around it; $\mathbf{K}$ is the global stiffness matrix, $\mathbf{u}$ and $\mathbf{F}$ are the global displacement and force vectors, respectively; x_e is the relative density of the e-th cellular. The average value $\overline{U}_e$ of the cellular can be calculated by

$$\overline{U}_e = \frac{U_e + \sum\limits_{i=1}^{N_N} U_i}{N_N + 1} \tag{6}$$

where U_i is the cellular i's strain energy density of the neighbors. N_N is the number of neighbors defined by the CA neighbor type.

The design domain of the structure is discretized by using the finite element method. According to the theory of the finite element analysis, the strain energy of a cellular is calculated by

$$SE_e = \int_{V_e} \frac{1}{2} u_e^T \left(B_e^T D B_e \right) u_e dv = \frac{1}{2} u_e^T k_e u_e \tag{7}$$

where D is the elastic matrix; B_e is the element's strain matrix; $\boldsymbol{u}_e$ is the element's displacement matrix and k_e is the element's stiffness matrix. Therefore, the strain energy density in Eq. (6) is written as

$$U_e = \frac{SE_e}{V_e} = \frac{u_e^T k_e u_e}{2V_e} \tag{8}$$

where V_e is the element volume.

3 Updating rule of the design variables

3.1 Neighbor types of the cellular automata

CA is essentially a space-time discretization mathematical model, which consists of finite cellulars with specified state information. At certain iteration, the state information of the active cellular only associated with its neighbors. Theoretically, the cellular space in the cellular automaton can be unlimitedly extended. However, it is impossible to realize this expansion by using the computation procedure. In practice, the size of the neighbor is often limited to the adjacent cellulars, but can also be reasonably extended. The most commonly adopted neighbor types are the "Von Neumann Type", which includes four neighboring cellulars (NN=4), and the "Moore Type", which includes eight cellulars (NN=8). The neighbor cellulars can also be reduced down to an "Empty Type" (NN=0) or extended to the "Extended Moore Type" including 24 cellulars. Figure 1 shows the common neighbor types in the two-dimensional cases.

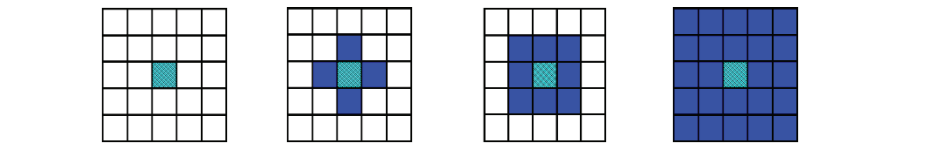
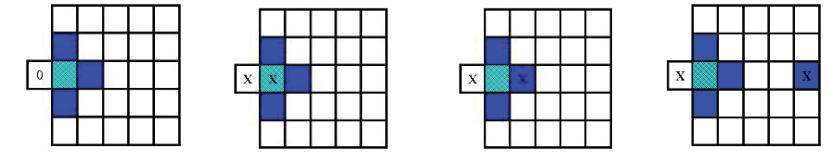


Figure 1: Neighbor types of cellular automata.

To overcome the shortcoming that cannot be unlimitedly extended in practice, it is important to extend the design domain. Four common neighbor types of the boundary cellular are indicated in Figure 2. In the Fixed type, the neighbor is completed with cellulars having a pre-assigned fixed state. An adiabatic type is obtained by duplicating the value of the cellular in an extra virtual neighbor. While in the Reflecting type, the state of the opposite neighbor is replicated by a virtual



(a) Fixed (b) Adiabatic (c)Reflecting (d) Periodic

Figure 2: Neighbor types of the boundary cellular

cellular. The Periodic type is applied for the condition that the design domain is assumed to be wrapped in a torus-like shape. In this work, the Fixed type is applied, where the extra cellulars are considered to be empty without physical or mechanical properties.

In order to make use of finite element analysis to evaluate the field states in CA, the design domain is discretized into quadrilateral elements with four nodes. Therefore, when the optimization model iteratively changes based on the proposed updating rule of design variables, there is an unique mapping relationship between the state of cellular in the topology optimization model and the finite element mesh. The change of material distribution (e.g., the Young's Modulus) in the design domain leads to the change of the structure (e.g., the strain energy), which complete the transformation from the finite element mesh to the cellular element.

3.2 Updating the design variables

The updating aim is to minimize the difference between the average strain energy density value and the pre-given value. The updating is accomplished by using the active cellular's state information and the local state information of the neighbors. The updating rule for the design variables at (t+1)-th iteration step can be written as

$$x_e(t+1) = x_e(t) + \Delta x_e(t) \tag{9}$$

where the increment of the cellular's relative density at the iteration is defined as

$$\Delta x_e(t) = f(\overline{U}_e^{(t)} - U_e^*) \tag{10}$$

where the function $f(\overline{U}_e^{(t)} - U_e^*)$ is the local control rule of the CA algorithm, which leads to the distribution of the material. The literature [Tovar, Niebur and Renaud (2004)] has proposed four different kinds of control strategies: the two-position control, the linear control, the integration control and the differentiation control.

The two-position control and the proportion control are two fundamental control methods, while the proportion-integration-differentiation (PID) control method is a complicated feedback control methd. It has been commonly used in the control field. The mathematical formula of the PID can be written as

$$\Delta x_e(t) = c_P \times (\overline{U}_e^{(t)} - U_e^*) + c_I \times \int_0^t (\overline{U}_e^{(\tau)} - U_e^*) d\tau + c_D \times \frac{d(\overline{U}_e^{(t)} - U_e^*)}{dt} \tag{11}$$

where c_p, c_I and c_D are the proportion, integration and differentiation controls' gains, respectively. The computational efficiency of the PID control has been shown in several engineering applications.

Based on the principle of feedback control and the error amplifier in the control system, this paper proposes a new updating rule of the design variables. The main idea of this method is regarding the difference between the average strain energy density value $\overline{U}_e^{(t)}$ and the pre-established value U_e^* as the error value of the feedback in each optimization step of the CA model, and then controlling the increment of the elements' relative density based on the error value.

The mathematical formula of this method can be written as

$$\Delta x_e(t) = \frac{1 - e^{-k_e \times (\overline{U}_e^{(t)} - U_e^*)}}{1 + e^{-k_e \times (\overline{U}_e^{(t)} - U_e^*)}} \tag{12}$$

where *ke* is the error amplification coefficient and its value is determined by the feedback signal, and it depends on the difference between the average strain energy density value and the pre-established value in this work. Figure 3 gives the graph of this method with different *ke*.

As Figure 3 shows, this method can make the intermediate feedback error close to -1 or 1. Using this approach, it can make the elements' relative density variations close to 0 or 1, which is therefore important to make the topological design approximate to the integer programming problem with 0 and 1 discrete design variables. With the increasing of the error amplification coefficient, the curves are approximately close to a smoothed Heaviside step function. The smoothed Heaviside function can get better results when they are included to update the element densities. In this way, the high-relative error single cellulars are enhanced. As if the feedback signal is bigger than zero, it means that the corresponding cellular's strain energy density is relatively high, and it is the cellular that with high stress. This incremental of the cellulars' relative density should be polarized to 1 because its contribution is very higher. For the low-relative error single, the effect is opposite, which means that these cellulars' corresponding incremental of relative density is weakened.

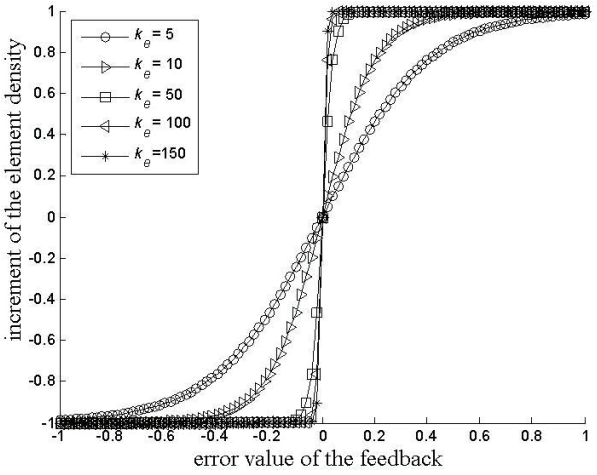


Figure 3: Graph of the design variable updating rule

To describe the optimized effectiveness intuitively, the mass fraction Q is presented in this paper. It refers to the ratio between the optimized structure's mass $M(t)$ after a certain iteration step and the initial mass of the solid design domain M_0. So it can be written as

$$Q = \frac{M(t)}{M_0} = \left(\sum_{e=1}^{N} x_e(t) v_e \right) \Bigg/ \sum_{e=1}^{N} v_e = \frac{1}{N} \sum_{e=1}^{N} x_e(t) \tag{13}$$

where *ve* is the volume of the element, N is the total number of elements used to discretize the design domain. A lower mass fraction Q represents at better optimized result.

3.3 Convergence criterion

The convergence criterion depends on the type of local control rule used to update the design variables. In this paper, the convergence criterion is introduced as follows

$$\frac{|\Delta M(t)| + |\Delta M(t-1)|}{2} \leq \varepsilon \tag{14}$$

where $\Delta M(t) = M(t) - M(t-1)$, and it refers to the mass' relative variation between the t-th and the $(t-1)$-th iteration, namely, $M(t)$and $M(t-1)$, respectively. ε is a small fraction of the total mass of the solid structure M_0, and $\varepsilon = 0.001 M_0$ is used in this paper.

3.4 Flowchart of the proposed method

The flowchart of the proposed method is displayed in figure4. It contains four steps, and each step is explained as follows

Step 1. Set the initial value. Define the design domain, material's property and load parameters, discretize the design domain with finite cellulars, and then set the initial value of the cellular.

Step 2. Structural analysis. The structural numerical analysis is completed. Set boundary conditions, define loads, construct elemental stiffness matrix, assemble the global stiffness matrix, and then solve the state equation.

Step 3. Optimization. In this step, the optimization is performed using the updating rule of the design variables. During the optimization, the variables are updated, and the iteration is being processed until the corresponding convergence criterion is satisfied.

Step 4. Convergence. If the convergence criterion is satisfied, the optimization will stop. Otherwise, it will return to the second step to continue the optimization.

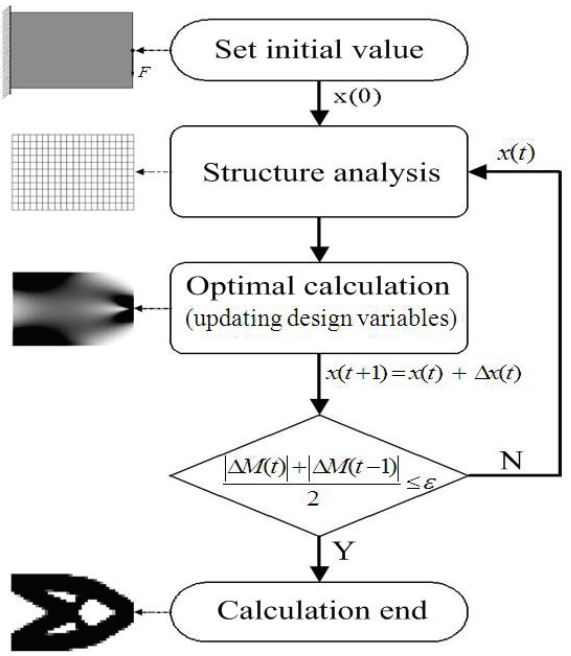


Figure 4: Flowchart of the Hybrid Cellular Automata algorithm

4 Numerical examples

Three topology optimization numerical examples are presented in this section to demonstrate the effectiveness of the proposed method, and the capability to suppress the numerical instabilities of checkerboards and mesh-dependence. In the former two examples, three methods are used to solve the given problem, namely, the proposed method in this paper, the proportion control method and the PID control method. The former two examples are employed to analysis the effect of different error amplification coefficients, the comparison between optimal topologies obtained by these three methods and the effect of the checkerboards' suppression. In the last example, the proposed approach will be used to illustrate its suppression effect for the numerical instability of mesh-dependence.

4.1 Numerical example 1: The optimization of the cantilever

Figure 5: Design domain of a cantilever beam structure

As indicated in Figure 5, the design domain of the problem is a 0.8m×0.5m rectangle, discretized with 80×50 low-order quadrilateral finite elements . The material's Young's Modulus E is 210 GPa and Poisson's ratio v is 0.3. The load F is 1000 N, located at the middle point of the right side of the structure. The left side of the structure is fixed. The initial design of the structure is a homogeneous distribution of solid material with the value of 1.0. The total strain energy is U_0 =78.019Nmm, while each cellular's objective value of local strain energy is $U_e^* = U_0/(80\times50)$=1.950×10^{-2}N/mm^3.

Table 1: Effect of different $k_e$on the optimal topologies

The coefficientk_e	5	10	50	100	150
Optimal topologies					
Strain energy	93.401	93.254	93.544	93.115	94.278
Iteration Number	64	38	15	53	44
The mass ratio	0.493	0.486	0.475	0.476	0.481

(1) Results and analysis

Table 1 shows the optimal topologies and the corresponding data with different error amplification coefficientske. Figures 6 and 7 indicate the structural strain energy$U$ and the mass ratio F of the optimal topology, respectively. From table 1, it can be seen that different ke can cause different results. When the value of ke is relatively small (e.g. ke = 5 or 10) or is relatively large (e.g. ke = 100 or 150), the corresponding iteration numbers are bigger and the mass ratios are larger than that of ke = 50. But the optimal topological strain energy for each ke is nearly 93.5. In addition, from Figures 6 and 7, it can also be seen that if the value of the error amplification coefficient is too large, the structural strain energy and mass ratio curves show strong oscillation phenomenon.

So we can draw the conclusion that when the error amplification coefficient ke is equal to 50, it can produce better optimal topology, and needs a lesser iteration number. More importantly, the proposed method can effectively suppress the numerical instability phenomenon of checkerboards.

(2) Comparison with different methods

Table 2 indicates the comparison result between three different methods: this paper's proposed method, the proportion control and the PID control for the given example. Figures 8 and 9 are the corresponding iterative curves of strain energy and mass ratio, respectively.

Table 2 shows that the strain energies U of the optimal structures obtained by the three methods are all approximate to 93. But compared with the other two conventional methods, the total iteration number is 15 while the other two methods need 17 and 16 iteration steps, respectively. So the proposed method converges relatively fast. From the final mass ratio Q, we can get the similar conclusion that the new

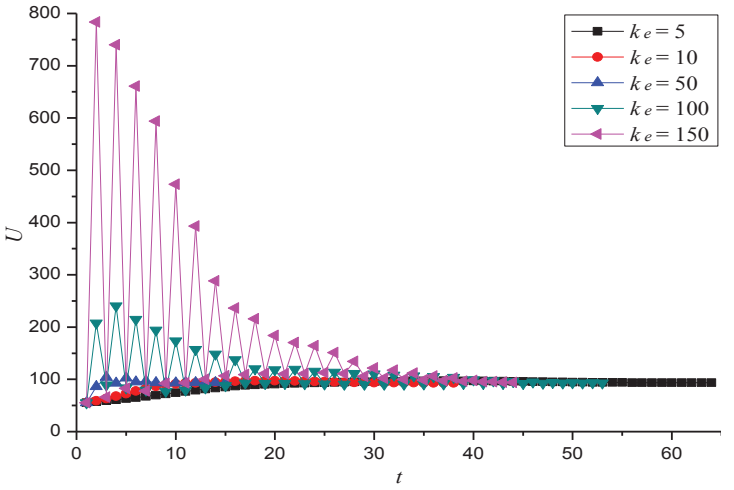


Figure 6: Iterative curves of structure strain energy with different*ke*

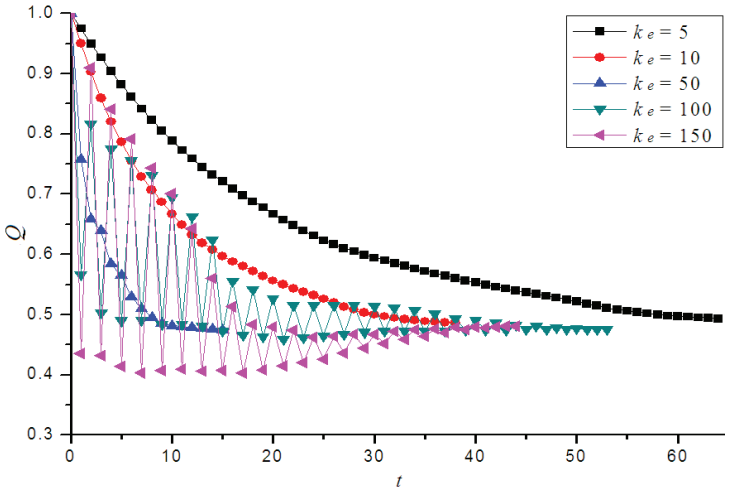


Figure 7: Iterative curves of mass ratio with different*ke*

Table 2: Comparison with different methods

Methods	Strain energy U	Iterations t	Mass ratio$Q$
The proposed method	93.544	15	0.475
The proportion control	93.215	17	0.479
The PID control	93.996	16	0.476

approach can lead to a lighter topology design if the other conditions are kept the same. From Figure 8 and Figure 9, we can see that the iterative history curves of strain energy and mass ratio are similar, respectively. It means that the proposed method is effective in overcoming the numerical difficulties.

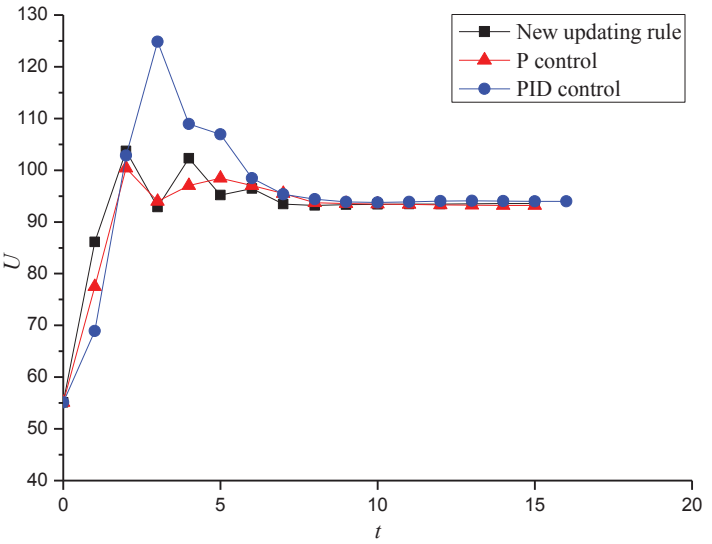


Figure 8: Iterative curves of strain energy with different methods

4.2 Numerical example 2: The optimization of the Michell structural

Figure 10 shows the design domain of a Michell type structure with a 0.6m×0.3m rectangle, discretized with 60×30 quadrilateral finite elements. The material's Young's Modulus E is 210 GPa and Poisson's ratio v is 0.3. The vertical load F is 1000N is applied on the bottom middle node. And the initial design of the structure is a homogeneous distribution of solid material with the value of 1.0. The total strain energy is U_0 =28.571Nmm, while each cellular's objective value of local strain energy is $U_e^* = U_0/(60\times30)$=1.587×10^{-2}N/mm^3.

Table 3 gives the optimal topologies by the proposed method with different error amplification coefficients. Figures 11 and 12 indicate the structure's strain energy$U$ and the mass ratio F of the optimal design, respectively. Similarly, if other conditions are kept the same, a better design with 18 iterations and mass ratio (0.454) can be obtained when *ke* is equal to 50. In addition, the convergence of the structural strain energy and mass ratio are conservative and remain stable in the optimization. This example further shows that the proposed method can conduct topology optimization without experiencing the numerical instability of checkerboards.

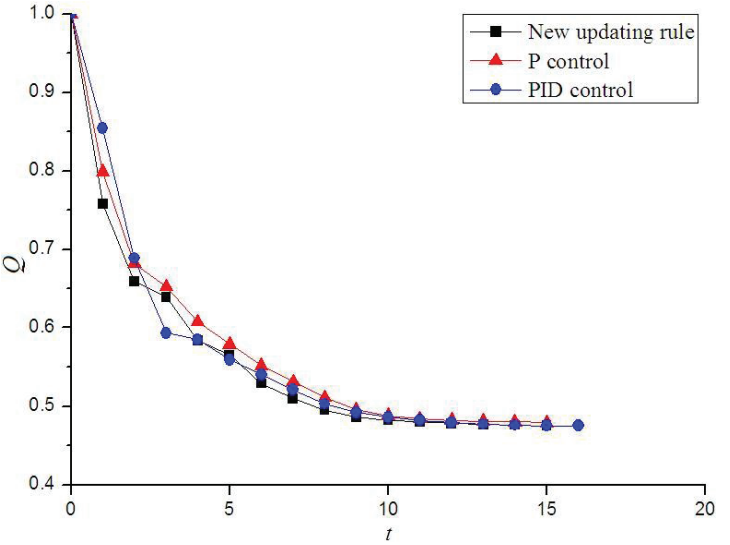


Figure 9: Iterative curves of mass ratio with different methods

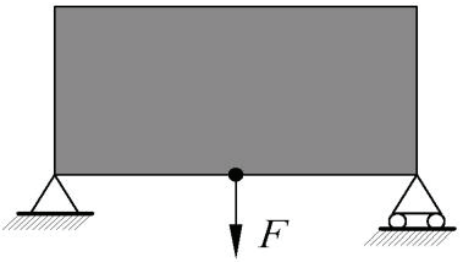


Figure 10: Design domain of the Michell type structure

Table 3: The effect of different*ke* on the optimal topologies

The coefficient *ke*	5	10	50	100	150
Optimal topologies					
The strain energy	35.031	35.178	35.337	34.751	37.966
Iteration number	83	52	18	22	71
The mass ratio	0.475	0.464	0.454	0.456	0.460

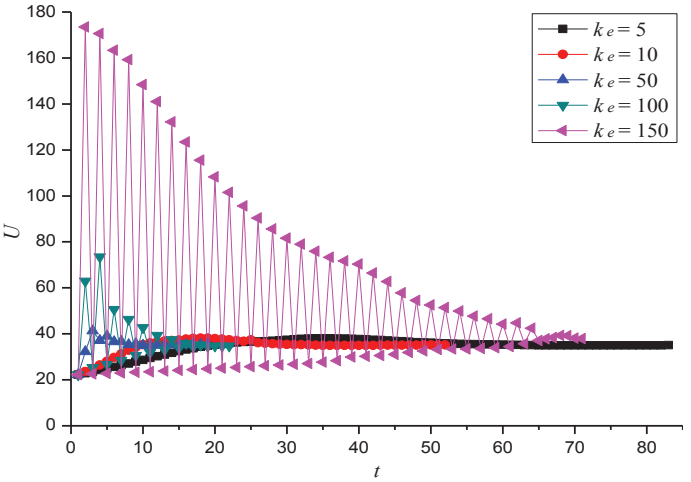


Figure 11: Iterative curves of structure strain energy with different*ke*

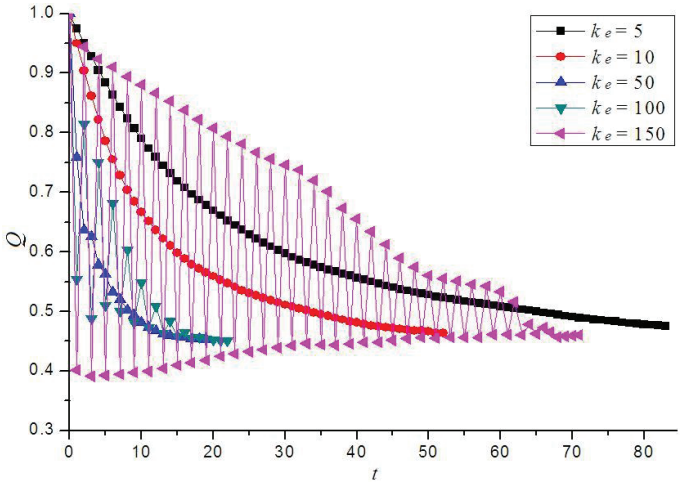


Figure 12: Iterative curves of mass ratio with different*ke*

Table 4 shows the comparison result of the three different methods, namely, the proposed method, the proportion control and the PID control of this example. Figures 13 and 14 are the corresponding iterative curves of strain energy and mass ratio, respectively. From Table 4, as well as Figure 13 and Figure 14, it can be found that these three methods can get similar optimal topologies. But the proposed method converges relatively fast (i.e. 18 steps). From this example, a conclusion can also be drawn that the proposed updating rule for design variables is efficient in the numerical procedure.

Table 4: Comparison with different methods

Methods	Strain energy U	Iteration number t	Mass ratio$Q$
The proposed method	35.337	18	0.454
The proportion control	35.374	21	0.454
The PID control	35.574	21	0.455

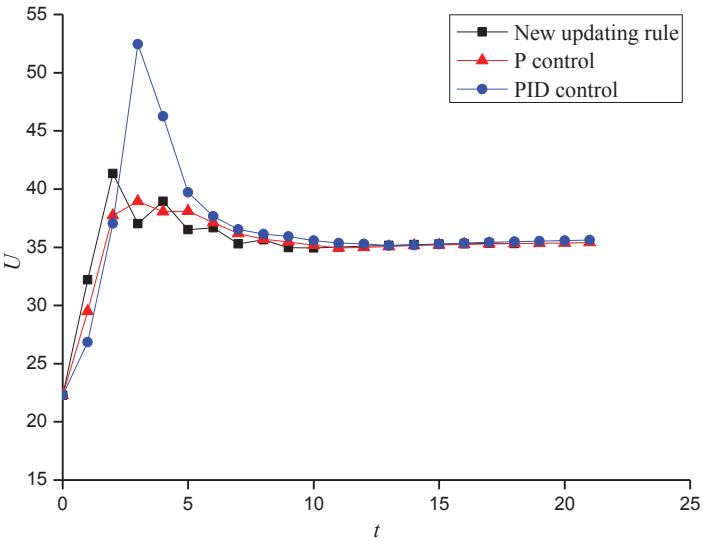


Figure 13: Iterative curves of strain energy with different methods

From the above discussions, it can be seen that when the proposed methodology is applied in the topology optimization of continuum structures, it can lead to better topological designs with less iterations, lower mass ratio and the final strain energy is more approximate to the objective value, and can effectively suppress the numerical instability of checkerboards.

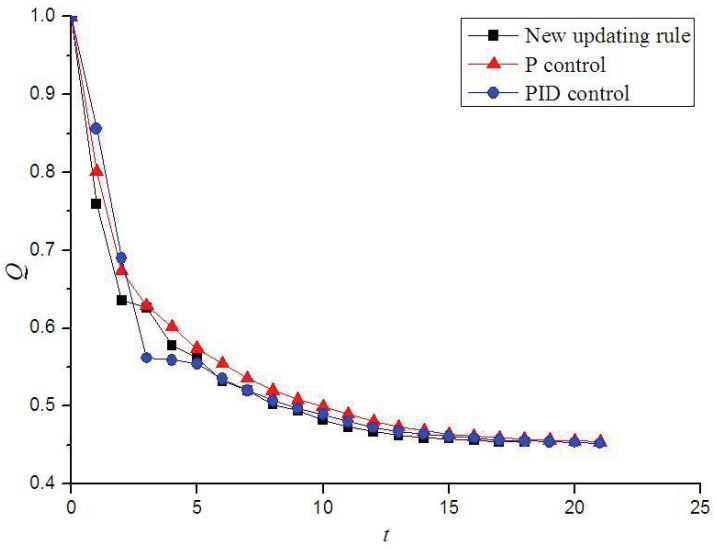


Figure 14: Iterative curves of mass ratio with different methods

4.3 Numerical example 3: The optimization of the MBB-beam

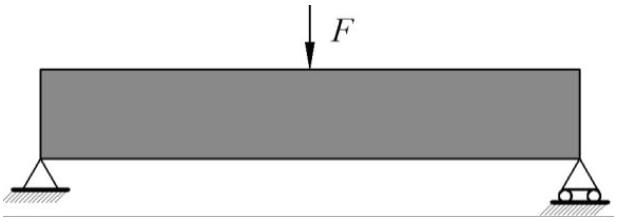


Figure 15: Design domain of the MBB-beam

As shown in Figure 15, the design domain of a MBB type beam is with a 1.20m×0.20m rectangle area. The material's Young's Modulus E is 210 GPa and Poisson's ratio v is 0.3. The load F is 1000 N, located at the middle point of the upper side. The initial design of the structure is a homogeneous distribution of solid material with the value of 1.0. The total strain energy is U_0 =257.1431Nmm.

This numerical example is given to further demonstrate the ability of the proposed method in overcoming the mesh-dependence in the topology optimization. Table 5 indicates the optimal topologies when the design domain is discretized into four different numbers of elements, namely, 180 ×30, 240 ×40, 300 ×50 and 360 ×60.

Figures 17 and 18 are the corresponding iterative curves of strain energy U and mass ratio Q with different elements, respectively.

Table 5: Optimal topologies with different elements

Finite elements	Optimal topologies	Strain energy U	Iteration Number t	Mass ratio Q
180×30		269.729	20	0.42
240×40		275.000	22	0.405
300×50		276.157	18	0.404
360×60		278.736	20	0.395

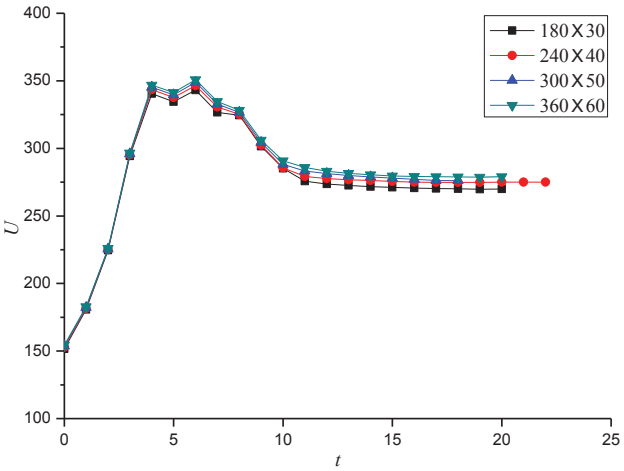


Figure 16: Iterative curves of strain energy with different elements

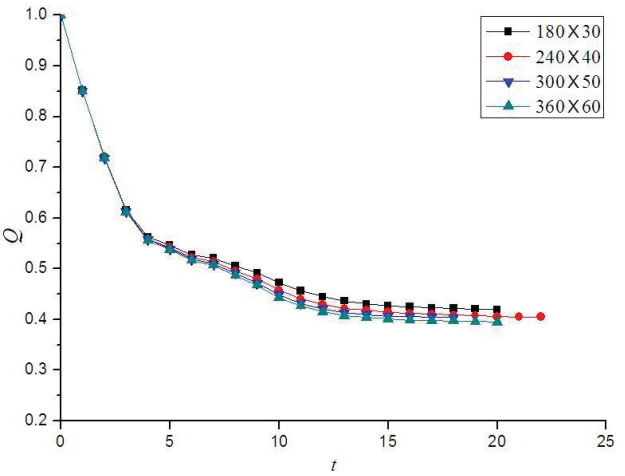


Figure 17: Iterative curves of mass ratio with different elements

As can be seen from Table 5, the optimal topologies' final strain energy, the total iterations and the mass ratio are 275, 25 and 0.4, respectively. The most important point is that we can obtain the identical topology designs, although the design domain is discretized with four different numbers of finite elements. All the optimal topologies are distinct and there is not any numerical instability of mesh-dependence in optimization procedures. On the other hand, the optimal topologies remain unchanged subject to the refinement of the finite elements. It indicates that the proposed method is robust in the optimization. Figures 16 and 17 also show that when this approach is applied to the topology optimization of continuum structures, the strain energy and mass ratio's changing process are similar to the conventional methods, respectively. Therefore, this example denotes that the proposed method can effectively suppress the mesh-dependence.

5 Conclusions

Based on the CA theory and inspired by the principle of feedback control and the error amplifier in the control system, this paper proposes a new topology optimization approach. The core aspect of this algorithm is to obtain the optimal design by the full stress design criteria, while avoid the calculation of the sensitivity information in the design process. The numerical examples show that the optimal topologies obtained by using the proposed method are with higher rigidity and less iterations, when the parameter of error amplification coefficient is appropriate. More importantly, the new updating rule can overcome numerical instability problems such as

checkerboards and mesh-dependence. The proposed method can be extended to handle more complex topology optimization problems from the point of practical engineering applications.

Acknowledgements

This research was partially funded by National Natural-Science-Foundation-of-China (51105229; 51305232; 51275274), Natural Science Foundation of Hubei Province of China (2010CDB10805), National Science Foundation for Distinguished Young Scholars of Hubei Province of China (2013CFA022) and Open Fund of Hubei Key Laboratory of Hydroelectric Machinery Design & Maintenance (2012KJ X07).

References

Allaire, G.; Jouve, F.; Toader, A. M. (2004): Structural optimization using sensitivity analysis and a level-set method. *Journal of Computational Physics,* vol. 194, pp. 363-393.

Bendsøe, M. P.; Kikuchi, N. (1988): Generating optimal topology in structural design using a homogenization method. *Computer Methods in Applied Mechanics and Engineering,* vol. 71, pp. 197-224.

Bendsøe, M. P.; Sigmund, O. (1999): Material interpolation schemes in topology optimization. *Archive of Applied Mechanics,* vol. 69, pp. 635-654.

Bendsøe, M. P.; Sigmund, O. (2003): Topology optimization: Theory, Methods, and Applications, Springer, Berlin Heidelberg.

Bruggi, M. (2008): On the solution of the checkerboard problem in mixed-FEM topology optimization. *Computers and Structures,* vol. 86, pp. 1819-1829.

Diaz, A.; Sigmund, O. (1995): Checkerboard patterns in layout optimization. *Structural and Multidisciplinary Optimization,* vol. 10, pp. 40-45.

Du, Y. X.; Luo, Z.; Tian, Q.; Chen, L. (2009): Topology optimization for thermo-mechanical compliant actuators using meshfree methods. *Engineering Optimization,* vol. 41, pp. 753-772.

Du, Y. X.; Chen, D. (2012): Suppressing Gray-Scale Elements in Topology Optimization of Continua Using Modified Optimality Criterion Methods. *CMES: Computer Modeling in Engineering & Sciences,* vol. 86, no. 1, pp. 53-70.

Faramarzi, A.; Afshar, M. H. (2012): Application of cellular automata to size and topology optimization of truss structures. *Scientia Iranica.* vol. 19, no. 3, pp. 373–380.

Haber, R. B.; Jog, C. S.; Bendsϕe, M. P. (1996): A new approach to variable-topology shape design using a constraint on perimeter. *Structural and Multidisciplinary Optimization,* vol. 11, pp. 1-12.

Jog, C. S.; Haber, R. B. (1996): Stability of finite elements models for distributed-parameter optimization and topology design. *Computer Methods in Applied Mechanics and Engineering,* vol. 13, pp. 203-226.

Kang, Z.; Tong, L. Y. (2008): Integrated optimization of material layout and control voltage for piezoelectric laminated plates. *Journal of Intelligent Material Systems and Structures,* vol. 19, pp. 889-903.

Kang, Z.; Wang, Y. Q. (2011): Structural topology optimization based on non-local Shepard interpolation of density field. *Computer Methods in Applied Mechanics and Engineering,* vol. 200, pp. 3515-3525.

Kita, E.; Toyoda, T. (2000): Structural design using cellular autómata. *Structural and Multidisciplinary Optimization,* vol.19, no.1, pp. 64-73.

Li, S.; Atluri, S. N. (2008): The MLPG mixed collocation method for material orientation and topology optimization of anisotropic solids and structures. *CMES: Computer Modeling in Engineering & Sciences,* vol. 30, pp. 37-56.

Li, S.; Atluri, S. N. (2008): Topology-optimization of structures based on the MLPG mixed collocation method. *CMES: Computer Modeling in Engineering & Sciences,* vol.26, no.1, pp.61-74.

Luo, Z.; Tong, L.; Wang, M. Y. (2008): Design of distributed compliant micromechanisms with an implicit free boundary representation. *Structural and Multidisciplinary Optimization,* vol. 36, pp. 607-621.

Luo, Z.; Tong, L.; Luo, J.; Wang, M. Y. (2009): Design of piezoelectric actuators using a multiphase level set method of piecewise constants. *Journal of Computational Physics,* vol. 228, pp. 2643-2659.

Luo, Z.; Luo, Q.; Tong, L.; Gao, W.; Song, C. (2011): Shape morphing of laminated composite structures with photostrictive actuators via topology optimization. *Composite Structures,* vol. 93, no. 2, pp. 406-418.

Luo, Z.; Zhang, N. (2012): A Multi-Criteria Topology Optimization for Systematic Design of Compliant Mechanisms. *CMC: Computers, Materials & Continua,* vol. 28, No. 1, pp. 27-56.

Luo, Z.; Zhang, N.; Wang, Y.; Gao, W. (2013): Topology optimization of structures using meshless density variable approximants. *International Journal for Numerical Methods in Engineering,* vol. 93, pp. 443-464.

Matsui, K.; Terada, K. (2004): Continuous approximation of material distribution for topology optimization. *International Journal for Numerical Methods in*

Engineering, vol. 59, pp. 1925-1944.

Mark, J. J.; Colin, C.; James, D.; Adenike, A.; Kazuhiro, S. (2000): Continuum structural topology design with genetic algorithms. *Comput. Methods Appl. Mech. Engrg.,* vol.186 pp.339-356.

Petersson, J.; Sigmund, O. (1998): Slope constrained topology optimization. *International Journal for Numerical Methods in Engineering,* vol. 41, pp. 1417-1434.

Poulsen, T. A. (2003): A new scheme for imposing a minimum length scale in topology optimization. *International Journal for Numerical Methods in Engineering,* vol. 57, pp. 741-760.

Sigmund, O.; Petersson, J. (1998): Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards. *mesh-dependency and local minima, Structural and Multidisciplinary Optimization,* vol. 16, pp. 68-75.

Tovar, A.; Niebur, G. L.; Renaud, J. E. (2004): Bone structure adaptation as a cellular automaton optimization process , the 45^{th} AIAA/ASME/ASCE/AHS/ASC structures. Structural Dynamics and Materials Conference, AIAA 2004-17862, Palm Springs, California.

Tovar, A.; Neal, M. P.; Glen, L. N. (2006): Topology optimization using a hybrid cellular automaton method with local control rules. *Journal of Mechanical Design,* vol. 128, no. 6, pp. 1025-1216.

Wang, M. Y.; Wang, X. M.; Guo, D. M. (2003): A level set method for structural topology optimization. *Computer Methods in Applied Mechanics and Engineering,* vol. 192, pp. 227-246.

Wang, S. Y.; Tai, K. (2005): Structural topology design optimization using Genetic Algorithms with a bit-array representation. Comput. *Methods Appl. Mech. Engrg.,* vol.194, pp.3749–3770.

Wang, Y.; Luo, Z.; Zhang, N. (2012): Topological optimization of structures using a multilevel nodal density-based approximant. *CMES: Computer Modeling in Engineering and Sciences,* vol.84, pp.229-252.

Zhou, M.; Rozvany, G. I. N. (1991): The COC algorithm, Part II: topological, geometry and generalized shape optimization. *Computer Methods in Applied Mechanics and Engineering,* vol. 89, pp. 197-224.

Zakhama, R.; Abdalla, M. M.; Smaoui, H.; Gürdal, Z. (2009): Multigrid implementation of cellular automata for topology optimization of continuum structures. *CMES: Computer Modeling in Engineering and Sciences,* vol.51, no.1, pp.1-24.