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Dynamic Route Guidance Based on Model Predictive Control

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Abstract: Route selections for vehicles can be equivalent to determine the optimized operation processes for vehicles which intertwine with each other. This paper attempts to utilize the whole methodology of model predictive control to engender rational routes for vehicles, which involves three important parts, i.e. simulation prediction, rolling optimization and feedback adjustment. The route decisions are implemented over the rolling prediction horizon taking the real-time feedback information and the future intertwined operation processes into account. The driving behaviors and route selection speculations of drivers and even traffic propagation models are on-line identified and adapted for the simulation prediction in next prediction horizon. The mesoscopic traffic model is utilized for the simulation prediction so as to achieve both computing efficiency and prediction accuracy, where the partial link density in front of the vehicle rather than the density of total link is utilized to calculate the vehicle propagation velocity. The path traveling time is accumulated in a way related to the departure time and the operation process of a vehicle. The system architecture is composed of two parts. One is to simulate the true traffic system with stochastic behaviors such as speed fluctuations and inclinations to obey or disobey navigation commands, and the other one is the simulation prediction, rolling optimization and feedback adjustment system. In this way, the case study of medium traffic network shows that the simulation prediction-based rolling-horizon feedback implementation can prevent possible congestion in advance. It provides an engineering solution to the real-time closed-loop prediction-based traffic navigation.

Keywords: route guidance, dynamic traffic assignment, model predictive control, network flow optimization, non-analytical iterative algorithm.

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1 Introduction

Route guidance is classified into static and dynamic types. The static route guidance assumes the link traveling time is constant and deals with the calculation of path traveling time in a simple way. The dynamic route guidance (DRG) involves reactive and predictive categories. The reactive DRG calculates the path traveling time according to the current link traveling time, while the predictive one should accumulate the predicted link traveling time along the running process of a vehicle to educe the total path traveling time. Much work still remains to be done in the area of predictive DRG.

The purpose of centralized traffic control and management is to assign vehicles to the routes with shortest path traveling time and at the same time keep road network with smooth flow. The complexity lies in that dynamic traffic assignment (DTM) for real-time route guidance should be implemented in the rolling horizon according to the feedback of traffic conditions and the prediction of future traffic demands and operations. DTM has developed along the two technological lines, i.e. the analytic and simulation approaches in terms of traffic propagation models. The analytic DTA [Ran and Boyce (1996)] depends on the macroscopic traffic propagation models [Daganzo (2006); Tyagi, Darbha, and Rajagopal (2008); Vikram, Mittal, and Chakroborty (2011)] with sound mathematical properties. The simulation-based DTA [Peeta and Mahmassani (1995a)] generally employs the microscopic or mesoscopic simulation models [Nagel and Schreckenberg (1992); Bando, Hasebe, Nakayama, Shibata, and Sugiyama (1995); Chowdhury, Santen, and Schadschneider (2000); Celikoglu and Dell'Orco (2007); Fujii, Yoshimura, and Seki (2010); Zhou, Mi, and Yang (2012)] to record the movements of vehicles such that the traffic signal control [Galán Moreno, Sánchez Medina, Álvarez Álvarez, and Rubio Royo (2009)] can be conveniently incorporated into DTA. The O-D (origin-destination) matrices should be more accurate in a short period than that in the total planning horizon, the rolling horizon implementation of DTM has been discussed [Peeta and Mahmassani (1995b); Ran, Lee, and Shin (2002)]. In order to resist disturbances and improve robustness, the splitting rate-based feedback control strategy has been proposed for the route guidance [Wang, Papageorgiou, and Messmer (2003)], and the H_∞-based control strategy has been incorporated into DTM [Kachroo, Özbay (2005)].

This paper will address the prediction-based rolling-horizon feedback implementation of DTM for the application of real-time route guidance based on the total methodology of model predictive control. Model predictive control [Camacho and Bordons (1995)] is composed of three parts, i.e. simulation prediction, rolling optimization and feedback adjustment. The application of model predictive control to *traffic signal control* has been extensively discussed [Hegyi, De Schutter, and Hellendoorn (2005); Aboudolas, Papageorgiou, Kouvelas, and Kosmatopoulos (2010)]. This paper emphasizes the integrative application of three aspects in the methodology of model predictive control to *route guidance* at the real-time network-wide level. Dynamic route guidance based on model predictive control is also called traffic navigation predictive control. We adopt the path-based approach to optimize the operation performances of total road network and vehicles.

The complicated part of traffic navigation predictive control is to solve the fixed point problem [Liu, He, and He (2009)] $P = f(P)$ where P is the time shortest paths and f stands for calculating the time shortest paths after the propagation of traffic flow which is hard to be denoted in an analytical form. The nonlinear problem $P = f(P)$ is equivalent to F(P)=0. The iterative methods have been extensively studied for the nonlinear algebraic equations F(x)=0 utilizing the optimal descent vectors [Liu and Atluri (2011); Liu, Dai, and Atluri (2011a, 2011b); Liu and Atluri (2012)]. In this paper, we attempt to develop the non-analytical iterative algorithm to solve the problem $P = f(P)$. In each iterative step, the enumerated time shortest paths for vehicles represent the feasible directions to minimize the objective function. We utilize the mesoscopic traffic simulation model to describe the traffic propagation, which denotes the speed of a vehicle as the function of traffic density in front of that vehicle on a link. The path traveling time related to the departure time of a vehicle is calculated along the operation process of the vehicle rather than the illogical accumulation of the link traveling time as that of the reactive DRG.

The rest of the paper is organized as follows. Section 2 develops the architecture and the optimization model of traffic navigation predictive control system. In section 3, the complexity of solution process to the addressed problem is analyzed, and the solution algorithm and the simulation process are elucidated. Section 4 demonstrates the simulation results of comparative study and testifies the advantage of traffic navigation predictive control and the efficiency of the proposed solution process. Finally, the conclusions are drawn in section 5.

2 Traffic navigation predictive control

2.1 System architecture

Fig. 1 shows the architecture of traffic navigation predictive control system which generates the commands of route guidance utilizing the total mechanism of model predictive control. The routes for navigated vehicles $\mathbf{U}_a(kT)$ at time kT are given out through the optimizer which considers the effects of current route decisions on the future performances of traffic flow system in the specified prediction horizon and allows the future performance to asymptotically approach the set-point $\boldsymbol{Y}_s$.

The pseudo set-point part engenders $\mathbf{Y}_s(kT)$ according to the set-pint $\mathbf{Y}_s$ and the

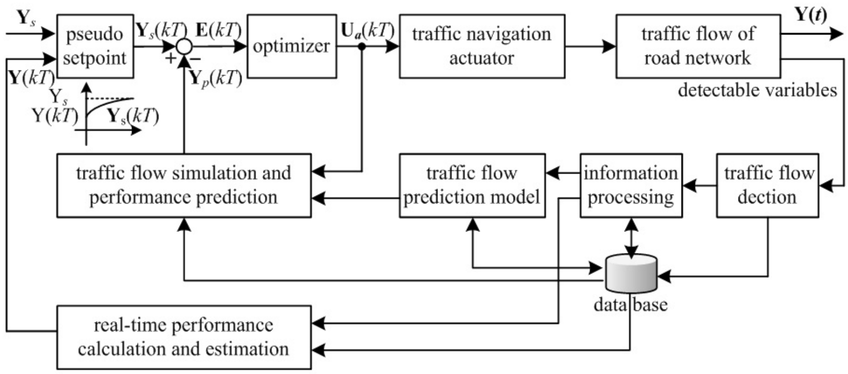

Figure 1: Architecture of traffic navigation predictive control system.

current performance $\mathbf{Y}(kT)$. The traffic navigation actuator is composed of components to transmit navigation commands through the communication network to vehicles. The traffic flow detection deals with the acquisition of traffic flow information. The information processing part is to abstract the useful information from the real-time collected raw data. The part of traffic flow prediction model is to identify the driving and path selection behaviors of drivers as well as macroscopic and mesoscopic traffic propagation models according to the real-time feedback traffic-flow data. Through the traffic flow simulation and the performance evaluation within the prediction horizon, the future performance of traffic flow system can be verified for the selected navigation strategies. The real-time performance should be calculated if it is not detectable and be estimated if the acquired information is incomplete. For example, the traffic density can not be directly detected and can be calculated through the number of vehicles on a link. If the number of vehicles on a link can not be attained, however it can be roughly estimated through the speed of link flow.

2.2 *Optimization model*

The purpose of centralized real-time traffic assignment is to strategically schedule the vehicles to less congested roads so that the smoother traffic flow in total road network and the faster traveling time for vehicles can be achieved. The route guidance based on model predictive control is to find optimal routes through feedback adjustment and simulation prediction in the rolling prediction horizon. The optimization objectives of total road network and individual vehicle are, respectively, formulated as:

$$\min\ J_1 = \frac{1}{L_\alpha}\sum_{l=1}^{L}\sum_{k=1}^{P}\alpha_l\,|y_s(l,t+kT) - y_p(l,t+kT)| \tag{1}$$

$$\min\ J_n = t'_n(o,d,t)\ n = 1,...,N \tag{2}$$

where $y_s(l,t+kT)$ and $y_p(l,t+kT)$ are the future ideal and predicted performances (such as densities) of link l at time t+kT in the prediction duration PT (k=1, ..., P), respectively. T is the simulation period, and P is the total number of prediction periods, called the prediction horizon. $y_s(l,t+kT)$ is given by $y_s(l,t+kT) = y(l,t)e^{-\lambda kT} + y_s(l,t)(1-e^{-\lambda kT})$ where $y(l,t)$ is the performance of link l at time t and λ is the asymptotic rate. α_l (l=1, ..., L) is the weighting factor where L is the number of links. L_α is a scale constant to make J_1 become an average value. $t'_n(o,d,t)$ is the predicted traveling time of vehicle n from current origin o to destination d at time t. The individual satisfaction degree of vehicle n at time t is defined as $S_n(o,d,t) = t'_{\min}(o,d,t)/t'_n(o,d,t)$ where $t'_{\min}(o,d,t)$ is the minimal traveling time of a vehicle from current origin o to destination d at time t.

For the multi-objective optimization algorithm, a large number of Pareto-optimal solutions will be enumerated, however in practice only one is selected to be implemented on the system. Thereupon, we regard the objective functions expressed in Eq. 2 as constraints, which will be discussed at next section.

Once certain routes are assigned to the corresponding vehicles, the operation performances of total network and vehicles will be predicted according to the traffic propagation models, including macroscopic, mesoscopic and microscopic simulation models. The macroscopic model does not care the detailed positions of individual vehicle on the link, and regard the vehicles entering into the link during a period as a package to calculate their link traveling time according to the relationship between flow or density and link traveling time. The microscopic model records the concrete positions of individual vehicle on the link, which reflects the interactive behaviors among vehicles with speed fluctuations incurred, such as cellular automaton and car following models. The mesoscopic model deals with the speed updates similar to the macroscopic one, however the recording approach of vehicle positions resembles that of the microscopic one. Hence, the mesocopic model can predict the traffic flow with fast speed as the macroscopic one but with moderate accuracy tantamount to the microscopic one.

Set $\mathbf{Y}(t)$ as the link densities. The prediction of link performance at time t+kT is generally denoted as

$$y_p(l,\ t+kT) = f\left(\sum_{n=1}^{N}\delta(l,n,t+kT),\ L_l\right) \tag{3}$$

where the instrumental variable $\delta(l,n,t+kT)$ denotes whether vehicle n is on link l (=1) or not (=0) at t+kT, L_l is the length of link l, and f is the function to calculate the density.

Set the mesoscopic traffic model as an example to describe the operation processes of vehicles:

$$v'(l,n,\ t+kT) = g\left(f\left(\sum_{m=1}^{N}\eta(l,m,t+kT),\ L_l - d(l,n,t+kT)\right)\right) \tag{4}$$

$$v(l,n,t+kT) = \min\left(gap/T,\ v'(l,n,t+kT)\right) \tag{5}$$

$$v'(l,t+kT) = \sum_{n=1}^{N}\delta(l,n,t+kT)v(l,n,t+kT)/\sum_{n=1}^{N}\delta(l,n,t+kT) \tag{6}$$

$$d' = d(l,n,t+kT-T) + v(l,n,\ t+kT-T)T \tag{7}$$

$$\begin{aligned} &if \quad d' \le L_l \quad d(l,n,t+kT) = d' \\ &else \qquad\qquad d(l+1,n,t+kT) = d' - L_l \end{aligned} \tag{8}$$

where $v(l,n,t+kT)$ refers to the velocity of vehicle n on link l at t+kT, and $v'(l,n,t+kT)$ is the initially calculated value of $v(l,n,t+kT)$. The instrumental variable $\eta(l,m,t+kT)$ denotes whether vehicle m is on link l and in front of vehicle n (=1) or not (=0) at t+kT. L_l is the length of link l. $d(l,n,t+kT)$ is the position of vehicle n on link l at t+kT. The *gap* is the distance between the head of vehicle n and the rear of its preceding adjacent one. $v'(l,\ t+kT)$ is the average velocity of flow on link $l$at t+kT. Eq. 4 indicates that the velocity of vehicle n on link l is determined by the vehicle density in front of vehicle n on that link. In most cases, the traffic propagation satisfies the first-in-first-out (FIFO) rule, and Eq. 5 ensures the realization of FIFO. Eq. 6 implies that the average velocity of link flow is determined by the speeds of vehicles on the link. Eq. 7 and Eq. 8 demonstrate the position update of vehicle n on adjacent links l and l+1 on the route of vehicle n.

Given $v'(l,t+kT)$, the link traveling time is calculated as

$$TT(l,t+kT) = \underset{M}{\arg\min} H\left(\sum_{j=0}^{M} v'(l,t+kT+jT)T - L_l\right) \tag{9}$$

where $H(x)$ is defined as

$$H(x) = \begin{cases} x & if\ x \ge 0 \\ +\infty & otherwise \end{cases}. \tag{10}$$

Similarly, the time-dependent path traveling time can be accumulated link by link.

The relationship between link speed and density has been empirically formulated as [Mihaylova, Boel, and Hegyi(2007)]:

$$v = v_f \exp\left\{-\frac{1}{a_m}\left[\frac{D}{D_{crit}}\right]^{a_m}\right\} + n \tag{11}$$

where v_f is the speed of free flow on a link, D is the link density, D_{crit} is the critical density, n is the stochastic variable, and a_m is an adjustable parameter.

3 Solution algorithm

3.1 Solution complexity

To predict the future traffic condition in the prediction horizon, the current and future traffic demands should be known. If the prediction only concerns the current traffic demands, the traffic navigation predictive control is the partially predictive type. And if the prediction involves both the current and predicted traffic demands, it is called the fully predictive one.

No matter which traffic propagation model is utilized, the consistency of flow or density with the speed should be maintained. The decision variables are the paths of navigated vehicles. There exists no direct continuous function to describe the relationship between link density or path traveling time and vehicle routes. The link traveling time is related to the newly produced traffic demands at the inlet of that link and the propagated flow from the adjacent links to that one. The path traveling time is time-dependent and rests with the propagation processes of vehicles. Since the operation processes of vehicles interact among each other, the heuristic non-analytical iterative algorithm is a feasible measure to find out the quasi-optimal solutions to the complex optimization problem.

3.2 Iterative process

The heuristic algorithm originates from two basic facts. The first is that all the vehicles should be assigned to the routes with shortest traveling time from their respective current origins to the destinations as described by Eq. 2. Another one lies in that if the flow speed of each link is fast or the link density is low, the vehicles can run with shortest times from their respective current origins to the destinations. Thus, the current time shortest paths of vehicles indicate the feasible direction to minimize the objective function J_1. Based upon these viewpoints, the iterative process is described as follows.

Step 1: Set the initial performance $J_1(t) = +\infty$ and the optimal routes $P_o(t) = NIL$ at instant t =0. And the iteration step i=1.

Step 2: Assign the routes $P(i)$ to the vehicles with shortest traveling time from their respective current origins to the destinations.

Step 3: Undertake traffic simulation utilizing propagation models to predict the future operation.

Step 4: Calculate the current performance $J_1(i)$.

Step 5: If $J_1(i) < J_1(i-1)$, record $P_o(t) = P(i)$. Continue the iterative process.

Step 6: If $|J_1(i) - J_1(i-1)| < \varepsilon$ (a small number) or $i \geq I$(the maximum iteration number), terminate the iteration process. Or, $i = i+1$, go to *Step 2*.

3.3 Simulation process

The simulation system is composed of two parts. One is to simulate the true traffic system, and the other one is the navigation part to engender the optimal routes. The true traffic system has stochastic components, such as the speed fluctuations described in Eq. 11 and the inclinations to obey or disobey the navigation commands. The navigation part is to identify the driving and path selection behaviors, adjust the prediction models, undertake the simulation prediction, and determine the quasi-optimal routes.

Step 1: t=0.

Step 2: Load vehicles produced at instant t.

Step 3: Calculate the movements with randomness for the vehicles who have not arrived at their destinations according to the current optimal routes $P_o(t)$.

Step 4: If $MOD(t,\ T_i) = 0$ where T_i is the time interval to update routes, recalculate the optimal routes from instant t according to the feedback information from the simulated true traffic system.

Step 5: If $t \geq T_t$ (the termination instant), stop. Else, $t = t+T$, go to *Step 2*.

4 Simulation results

4.1 Road network

We utilize the arterial roads of Beijing city as shown in Fig. 2 to undertake simulation. It includes 50 nodes and 168 links. The same road segment between two nodes but with different running directions for vehicles is denoted as different links. A certain scale of feasible routes between nodes are recorded beforehand to save the calculation burden of time shortest paths. At the inlet of each link, the vehicle is stochastically produced with its own running destination every 18s. In total, 10106 vehicles are generated for the single-lane road network. The vehicles will be loaded at their born instants. The simulation period T=18s, and the prediction horizon P=200. The routes will be recalculated for the running vehicles every $3T$. For the true movements in road network, we assume the stochastic variable in Eq. 11 yields to the normal distribution $N(0, 5)$.

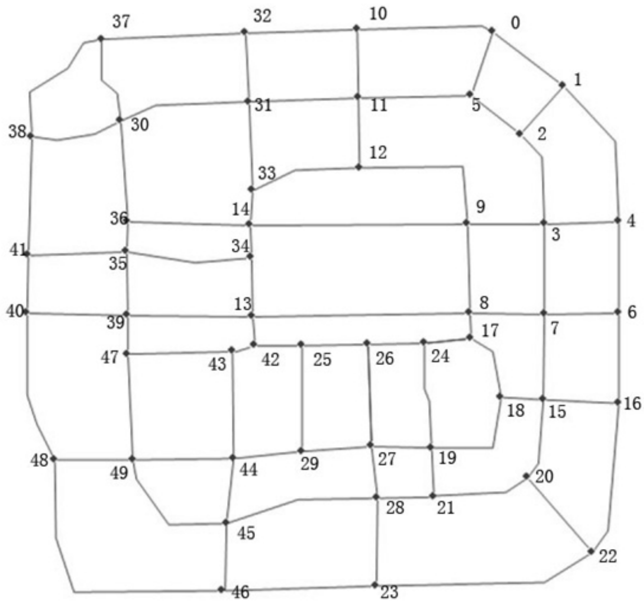

Figure 2: Road network.

4.2 *The performance of road network*

Fig. 3 (a) and (b) demonstrate the link densities under the cases of static route guidance based on the distance shortest paths and of traffic navigation predictive control based on the time shortest paths, respectively. The density is measured by $D = L_l/L_v$ where L_l is the link length and L_v is the average vehicle length including the safety margin distance between two adjacent vehicles. Set $L_v = 5m$. From Fig. 3 (a), we can learn that if all the vehicles run along the distant shortest paths, some links will be incredibly congested because their densities are greater than 1. In the traffic navigation predictive control, we let $\mathbf{Y}(t)$ be the link densities with $y_s(l,t) = 0.3$ and $\lambda \rightarrow +\infty$. If $y_s(l,t+kT) < y_p(l,t+kT)$, $\alpha_l = 1$; otherwise $\alpha_l = 0$. Fig. 3 (b) indicates that although the partial traffic navigation predictive control is utilized, it still can prevent the possible link congestion and assign the vehicles to the routes with shortest traveling times. The system- and user-optimal objectives can be both achieved utilizing the proposed approach to engender the navigation commands.

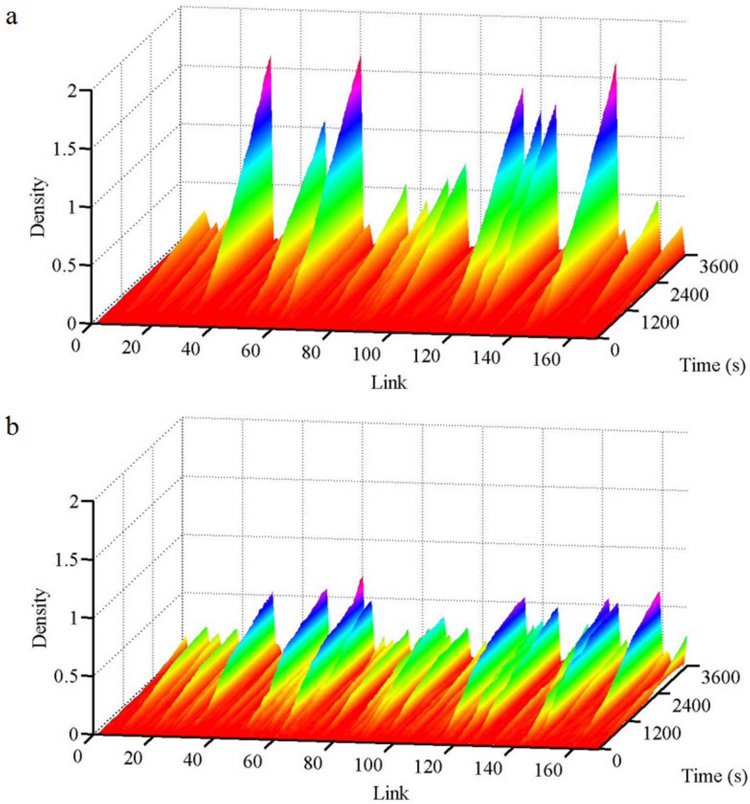

Figure 3: Link density. (a) is for static route guidance. (b) is for traffic navigation predictive control.

4.3 Consistency of density and speed

In the simulation of vehicle operation processes, the mesoscopic traffic simulation model is utilized. For the individual vehicle on a link, its speed is modeled to be related to the partial link density in front of the vehicle. The speed of link flow is the average of vehicle speeds on the link. Such an approach apparently prevents the impractical phenomenon that the operation of the front vehicle is affected by that of the rear one. Fig. 4 depicts two representative link densities and speeds, which reflects the statistical law in most cases that the link speed decreases with the increase of link density.

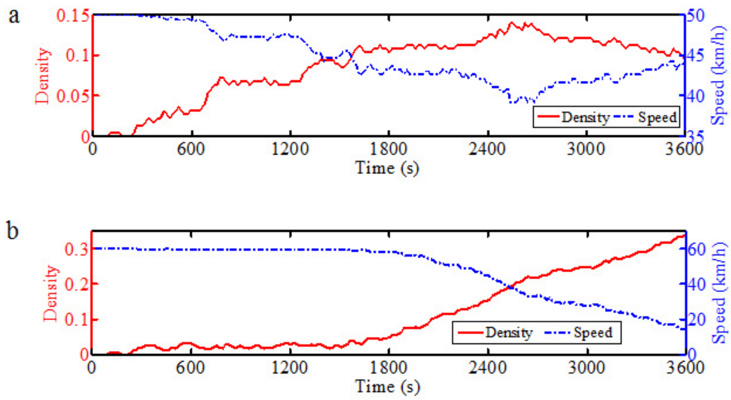

Figure 4: Samples of the consistency between density and speed on a link. (a) is for link 50. (b) is for link 135 in the case of Fig. 3 (b).

4.4 Iterative process

The routes are updated every $3T$ according to the current and predicted traffic conditions. Fig. 5 represents the iteration processes at $9T$, $39T$, $60T$, $78T$, $87T$, $96T$, $105T$, $123T$ and $189T$. From those figures, it can be learned that the feasible-direction non-analytical iterative algorithm possesses the convergent property although the local oscillations may happen. Therefore, it can facilitate finding out the quasi-optimal solutions within a few iteration steps for the engineering implementation.

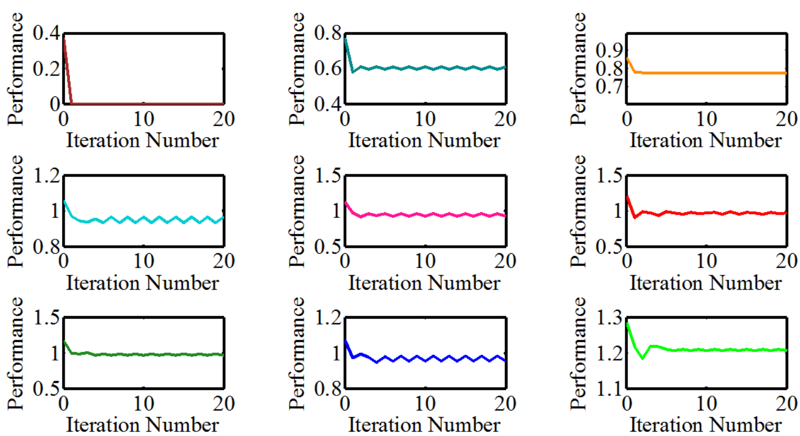

Figure 5: Samples of the iteration processes.

4.5 Feedback adjustment

We design two cases to demonstrate how the feedback adjustment takes effect. In case I, 25% vehicles are assumed to disobey the navigation commands, while in case II, 50% ones are assumed. The vehicles disobeying the commands are identified to run along the distance shortest paths. The feedback information will be reflected in the route optimization. Fig. 6 (a) and (b) display the link densities for case I and II, respectively. Apparently, the performance will deteriorate with the number increase of disobedient vehicles, but it is still better than that in case all vehicles run along the distance shortest paths.

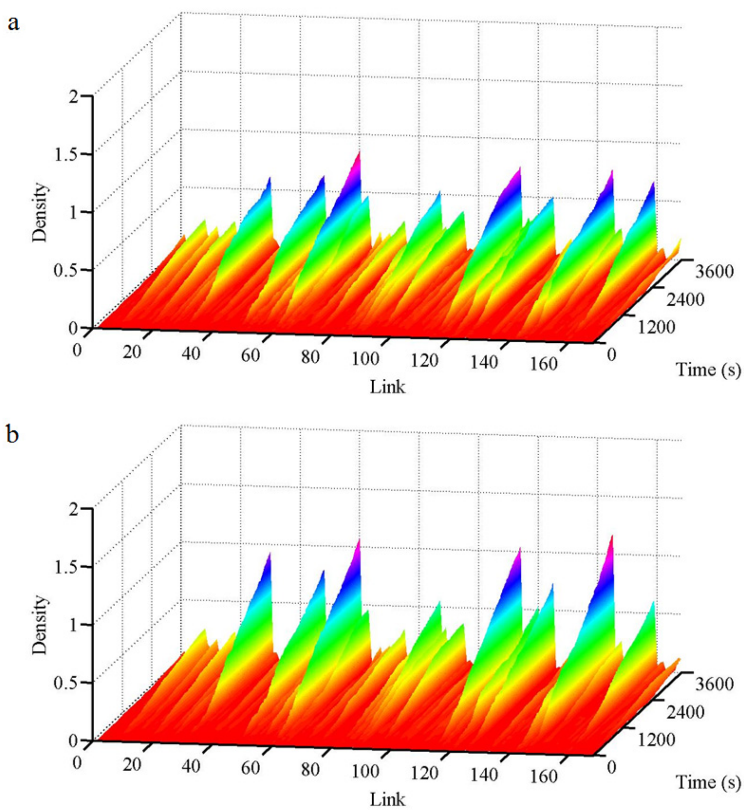

Figure 6: Link density. (a) is for case I: 25% vehicles disobey the navigation commands. (b) is for case II: 50% vehicles disobey the navigation commands.

5 Conclusions

We have proposed one kind of engineering-oriented solution to DTM for the real-time route guidance based on model predictive control, i.e. simulation prediction-based rolling-horizon feedback implementation. For the stochastic and time-variant traffic system, there exists the randomness of traffic demands and propagations. The real-time feedback will be beneficial for the DTM to engender the valid commands for route guidance. The current route decision is established on the basis that the simulated traffic operations in the prediction horizon are the compromise between the system- and user-optimal performances. Through the feasible-direction non-analytical iterative algorithm, the time shortest paths are gradually enumerated and the instantaneous optimal solutions are reserved through the comparison between system performances obtained at iteration steps. The time shortest path of a vehicle is overviewed in its operation process where the accumulation of link traveling times has the instant-related logicality. The mesoscopic simulation model describes the speed of a vehicle related to the density in front of that vehicle on a link. The crowded links are predicted and furthermore averted because of long traveling times during the enumeration processes of time shortest paths. In the end, the quasi-optimal solutions are attained to be implemented during the future certain periods. This process is repeated over the rolling horizon to periodically update the routes in view of stochastic traffic demands and propagations.

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