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Multiple-damage detection using the best achievable flexibility change

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Abstract: A method based on best achievable flexibility change is presented in this paper to localize and quantify multiple damages in structures. The key process of the damage localization approach is the computation of the Euclidean distances between the measured flexibility change and the best achievable flexibility changes. The location of damage can be identified by searching for a value that is considerably smaller than others in these distances. For the multiple-damage case, a sequential damage localization approach is proposed to locate the damage sites one by one. With the suspected damaged elements determined, the flexibility sensitivity method is employed to calculate the damage extents. Three numerical examples are used to demonstrate the efficiency of the method. Results show the good efficiency and stability of the presented method on the identification of single damage or multiple damages.

Keywords: damage detection; flexibility change; best achievable; multiple damages

1 Introduction

Structural damage detection using measured dynamic data has emerged as a new research area in civil, mechanical and aerospace engineering communities in recent years. The basic idea of this technique is that modal parameters are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, changes in the physical properties will cause changes in the modal properties. Recent surveys on the technical literature, such as on Doebling S. W., Farrar C. R., Prime M. B., and Shevitz D. W. (1996) and Chang F. K. (1997), among others, show

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that extensive efforts have been developed to find reliable and efficient numerical and experimental models to identify damage in structures.

One of the approaches in detecting damage has been to use changes in the stiffness matrix of a structure. Mannan M. A. and Richardson M. H. (1990) utilized the difference in the stiffness matrices of the intact and the damaged structures to detect and locate structural cracks. Lim T. W. and Kashangaki T.A.L. (1994) referred to a best achievable eigenvector as a damage indicator. Park Y. S., Park H. S., and Lee S. S. (1988) used a stiffness error matrix method based on the difference between analytical stiffness and measured modal properties to search for damages in a structure. Gysin H. P. (1986) point out that the error matrix method is effective only when all modes of the structure are include or at least those modes that are influenced most by the damage. Based on governing equations of structural dynamics, Lin C. S. (1990) has observed that higher modal frequencies contribute to the stiffness matrix values to a greater extent. Hence, to obtain a good estimate of the stiffness matrix, one needs to measure all the modes of the structure, especially, the high frequency modes. For obvious limitations of experimental instrumentation, it is increasingly difficult to measure higher frequency response data. This presents a severe constraint on the success of techniques based on an estimation of the stiffness matrix (Pandey A. K. and Biswas M., 1995).

Another important class of damage identification methods is based on the change in the flexibility matrices of the undamaged and the damaged structures. The advantage of using flexibility instead of stiffness is that the flexibility matrix can be accurately estimated using only a few of the lower frequency modes and is very sensitive to damage. Pandey A. K. and Biswas M. (1994, 1995) developed a method for locating damage in beam type structures using changes in the flexibility matrix of the structure. Zhao J. and DeWolf J. T. (1999) demonstrated that the algorithms using modal flexibility, derived from frequencies and mode shapes, are very sensitive to local damage. Bernal D. and Gunes B. (2002a, 2002b, 2004) computed a set of load vectors from the flexibility matrix change, designated as damage location vectors to localize damage. Jaishi B. and Ren W. X. (2005, 2006) presented a damage detection method by finite element model updating using modal flexibility residual. Stutz L. T., Castello D. A., and Rochinha F. A. (2005) presents a flexibility-based continuum damage identification approach. Yan A. and Golinval J. C. (2005) studied a damage localization method by combining flexibility and stiffness methods. Necati Catbas F., Brown D., and Emin Aktan A. (2006) made use of modal flexibility for damage detection on two real-life bridges. Duan Z., Yan G., Ou J., and Spencer B. F. (2007) studied damage detection in ambient vibration using proportional flexibility matrix with incomplete measured degrees of freedom. Koo K. Y., Sung S. H., Park J. W., and Jung H. J. (2010, 2011) presents a vibration-based damage detection and quantification methods for shear buildings using the damage-induced deflections estimated by modal flexibility from ambient vibration measurements. Li Y., Zhou B., and Zhou X. (2011) presented the curvature matrix of change in flexibility as a new index of nondestructive damage detection, which is derived from change in structural flexibilities calculated from before damaging and after damaging by means of difference calculation twice, firstly to columns, and then to rows. By the matrix eigen-decomposition and flexibility sensitivity analysis, Yang Q. W. and Liu J. K. (2009) approaches the damage identification problem in a decoupled fashion: determining the number of damaged elements, localizing the damaged elements and quantifying the damage extents. Perera R., Ruiz A., and Manzano C. (2007, 2008) used the modal flexibility as one objective function in formulation of the multiple objective damage identification problems. Yang Q. W. (2009) derived the sensitivity of flexibility matrix using Neumann series expansion and presented a mixed sensitivity method to identify structural damage by combining the eigenvalue sensitivity with the flexibility sensitivity.

In this paper, using the concept of best achievable flexibility change, a method is developed to determine both the locations and magnitudes of structural multiple damages. The method approaches the damage location and extent problem in a decoupled fashion. The key point of the damage location algorithm lies in the formulation of the best achievable flexibility change. The damages are located by calculating the Euclidean distances between the measured flexibility change and the calculated best achievable flexibility changes. For the multiple-damage case, a sequential damage localization approach is proposed to locate the damage sites one by one. With location determined, the flexibility sensitivity method is used to determine the extents of damages. Three numerical examples is used to validate the developed technique. The results show that the locations and magnitudes of local damages can be identified by the proposed method using the noisy and incomplete modes. In the following theoretical development, it is assumed that structural damages only reduce the system stiffness matrix and structural refined FEM has been developed before damage occurrence.

2 Damage localization using the best achievable flexibility change

2.1 Single damage

For the intact and the damaged structures, the global stiffness and flexibility matrices will satisfy the following relationship:

$$F_u \cdot K_u = F_d \cdot K_d = I \tag{1}$$

where F_u and K_u are the $n \times n$ flexibility and stiffness matrices of the undamaged structure, F_d and K_d are the $n \times n$ matrices of the damaged structure, I is the $n \times n$

identity matrix. It is noted that both translational and rotational degrees of freedom are considered in the flexibility and stiffness matrices here. As is well known, damage reduces the stiffness and increases the flexibility of structures. Let ΔF and ΔK be the exact perturbation matrices that reflect the nature of the structural damage. Then the undamaged model matrices and the damaged model matrices are related as follows:

$$F_d = F_u + \Delta F \tag{2}$$

$$K_d = K_u - \Delta K \tag{3}$$

In practice, the exact ΔF cannot be obtained due to the limitation of the modal survey. But ΔF can be approximated by the first few low-frequency modes as (Pandey A. K. and Biswas M., 1994, 1995)

$$\Delta F = \sum_{j=1}^{m} \frac{1}{\lambda_{dj}} \phi_{dj}\phi_{dj}^T - \sum_{j=1}^{m} \frac{1}{\lambda_{uj}} \phi_{uj}\phi_{uj}^T \tag{4}$$

where m is the number of measured modes in modal survey, $\lambda_{uj}(\phi_{uj})$ and$\lambda_{dj}(\phi_{dj})$ are the eigenparameters of the undamaged and damaged structures, respectively. The modes of the damaged structure can be obtained by a modal survey on it, and the modal data of the undamaged structure can be obtained by solving a generalized eigenvalue problem of the undamaged FEM or through a modal test on the intact structure. A detailed discussion of modal analysis techniques is beyond the scope of this paper, and interested readers are referred to the references (Ewins D. J. (1984), Juang J. N. (1994), Maia N. M. M. and Silva J. M. M. (1997)). Substituting equations (2) and (3) into (1), one has

$$\Delta F \cdot K_d = F_u \Delta K \tag{5}$$

Rewriting equation (5) yields

$$\Delta F = F_u \Delta K F_d \tag{6}$$

When damage has occurred in the structure, the stiffness matrix perturbation ΔK can be expressed as a sum of each elemental stiffness matrix multiplied by a damage coefficient, that is

$$\Delta K = \sum_{i=1}^{N} \alpha_i K_i, \quad (0 \le \alpha_i \le 1) \tag{7}$$

where K_i is the ith elemental stiffness matrix, α_i is its damage parameter, N is the total number of elements. The value of α_i is 0 if the ith element is undamaged and α_i is 1 or less than 1 if the corresponding element is completely or partially damaged. In equation (7), it is assumed that in case of damage, all the elements in the stiffness matrix K_i will be affected by same ratio α_i. This assumption is used for mathematical simplicity since the practical damage is very complex. Substituting equation (7) into (6), one has

$$\Delta F = \sum_{i=1}^{N} \alpha_i F_u K_i F_d \tag{8}$$

According to equation (8), the changes in the flexibility could be caused by damage at a single member or at multiple members. Assume, for the time being, that the damage is caused by a single member. Without loss of generality, assume that only the ith element is damaged ($\alpha_i \neq 0$), then equation (8) reduces to

$$\Delta F = \alpha_i F_u K_i F_d \tag{9}$$

Let the jth column of ΔF and F_d be represented by Δf_j and f_{dj}, respectively. That is, $\Delta F = [\Delta f_1 \cdots \Delta f_j \cdots \Delta f_n]$ and $F_d = [f_{d1} \cdots f_{dj} \cdots f_{dn}]$. From equation (9), we have

$$\Delta f_j = \alpha_i F_u K_i f_{dj}, (j = 1 \sim n) \tag{10}$$

Let

$$E_i = F_u K_i \tag{11}$$

$$\gamma_{ij} = \alpha_i f_{dj} \tag{12}$$

Then equation (10) simplifies to

$$\Delta f_j = E_i \gamma_{ij}, (j = 1 \sim n) \tag{13}$$

The implication of equation (13) is very important. According to the theory of Linear algebra (Herstein I. N. and Winter D. J. (1988), Datta B. N. (1995)), equation (13) is valid only if the vector Δf_j is a linear combination of the columns of E_i. In other words, Δf_j must lie in the subspace spanned by the columns of E_i. That is to say, if the ith element is damaged, then the vector Δf_j will lie exactly in the subspace spanned by the columns of E_i. If not, Δf_j would not lie in the subspace spanned by the columns of E_i. According to the matrix theory, we can use the concept of the best achievable vector to evaluate whether or not Δf_j lies in the subspace spanned by the columns of E_i. The best achievable vector of Δf_j can be computed by

$$\Delta f_{ij}^{a} = \overline{E_i}(\overline{E_i})^{+} \Delta f_j, (j = 1 \sim n) \tag{14}$$

where $\overline{E_i}$ is the matrix E_i where the zero columns have been removed to enhance computational efficiency, and the superscript + denotes the generalized inverse. For $j = 1 \sim n$, equation (14) can be assembled as

$$\Delta F_i^a = \overline{E_i}(\overline{E_i})^+ \Delta F \tag{15}$$

where $\Delta F_i^a = [\Delta f_{i1}^a \cdots \Delta f_{ij}^a \cdots \Delta f_{in}^a]$. The matrix ΔF_i^a is defined as the best achievable flexibility change. If the damage is caused by the ith element, then the matrices ΔF_i^a and ΔF will be identical. If not, the two matrices will be different. We can use the Euclidean distance between the two matrices to evaluate whether or not ΔF_i^a equals ΔF. The distance between the two matrices can be computed using the Frobenious norm

$$d_i = \|\Delta F - \Delta F_i^a\|_F \tag{16}$$

where $\|\cdot\|_F$ represents the Frobenious norm. If the perfect data are presented, the damaged element will has zero distance ($d_i = 0$) and all others will have nonzero values. For a structure that has N structural members, a damage localization vector, of length N, can be defined as

$$d = \left[\frac{d_1}{d_{\max}}, \cdots, \frac{d_i}{d_{\max}}, \cdots \frac{d_N}{d_{\max}}\right]^T \tag{17}$$

where $d_{\max}$ is the largest value in all distances, i.e., $d_{\max} = \max(d_1, \cdots, d_i, \cdots d_N)$. For the measured data with truncation and noise, $\frac{d_i}{d_{\max}}$ will be equal or close to zero if damage is located in element i and all other coefficients will be populated with nonzero entries. As a result, the location of damage can be determined by searching for a value that is considerably smaller than others in the vector d.

2.2 Multiple damages

When there is more than one damaged element in the structure, the examination of the vector d in equation (12) may not provide a clear indication of the damaged members. If the vector d does not indicate the damaged member clearly, it may imply that there are multiple damaged members. To examine multiple damages, a sequential damage localization approach is proposed. In this approach, damaged members are detected one at a time in a screening process. For convenience of the following discussion, without loss of generality, two-damage case is used to illustrate the screening process (other multiple damage cases are also valid).

The first step is to select the single most probable damaged member by searching for a value that is considerably smaller than others in the vector d of equation (12). Without loss of generality, the ith element is assumed to be the damaged member obtained in the first step and the rest are candidate members. If the rth element is another damaged member, equation (8) reduces to

$$\Delta F = \alpha_i F_u K_i F_d + \alpha_r F_u K_r F_d \tag{18}$$

where K_r is the rth elemental stiffness matrix and α_r is its damage parameter. The following derivation is similar to the process between equation (9) and equation (17). From equation (18), the jth column of ΔF can be expressed as

$$\Delta f_j = \alpha_i F_u K_i f_{dj} + \alpha_r F_u K_r f_{dj}, (j = 1 \sim n) \tag{19}$$

Let

$$E_r = F_u K_r \tag{20}$$

$$\gamma_{rj} = \alpha_r f_{dj} \tag{21}$$

Substituting equations (11), (12), (20) and (21) into (19), one has

$$\Delta f_j = E_i \gamma_{ij} + E_r \gamma_{rj}, (j = 1 \sim n) \tag{22}$$

Removing the zero columns in the matrices E_i and E_r and the corresponding entries in the vector γ_{ij} and γ_{rj}, equation (22) reduces to

$$\Delta f_j = \overline{E_i}\gamma_{ij}^{'} + \overline{E_r}\gamma_{rj}^{'} \tag{23}$$

Equation (23) can be rearranged as

$$\Delta f_j = E_{ir} \gamma_{irj} \tag{24}$$

where the matrix E_{ir} and the vector γ_{irj} are given as

$$E_{ir} = [\ \overline{E_i} \quad \overline{E_r}\ ] \tag{25}$$

$$\gamma_{irj} = \left\{ \begin{array}{c} \gamma_{ij}^{'} \\ \gamma_{rj}^{'} \end{array} \right\} \tag{26}$$

As before, equation (15) is valid only if the vector Δf_j is a linear combination of the columns of E_{ir}. In other words, Δf_j must lie in the subspace spanned by the columns of E_{ir}. That is to say, if the ith and rth elements are damaged, then the vector Δf_j will lie exactly in the subspace spanned by the columns of E_{ir}. If not, Δf_j would not lie in the subspace spanned by the columns of E_{ir}. Again, we can use the concept of the best achievable vector to evaluate whether or not Δf_j lies in the subspace spanned by the columns of E_{ir}. The new best achievable vector of Δf_j can be computed by

$$\Delta f_{irj}^{a} = E_{ir}(E_{ir})^{+}\Delta f_j, (j = 1 \sim n) \tag{27}$$

For $j = 1 \sim n$, equation (27) can be assembled as

$$\Delta F_{ir}^{a} = E_{ir}(E_{ir})^{+}\Delta F \tag{28}$$

If the damages are caused by the ith and rth elements, then the matrices ΔF_{ir}^{a} and ΔF will be identical. If not, the two matrices will be different. Using the Frobenious norm, the distance between the two matrices can be computed

$$d_{ir} = \|\Delta F - \Delta F_{ir}^{a}\|_F \tag{29}$$

If the exact data are used, the damaged elements will has zero distance ($d_{ir} = 0$) and all others will have nonzero values. For the other $N-1$ structural members except the ith damaged element identified in the first step, a new damage localization vector, of length $N-1$, can be defined as

$$d' = \left[\frac{d_{i1}}{d_{i\max}}, \cdots, \frac{d_{ir}}{d_{i\max}}, \cdots, \frac{d_{i(N-1)}}{d_{i\max}}\right]^T \tag{30}$$

where $d_{i\max}$ is the largest value in all distances, i.e., $d_{i\max} = \max(d_{i1}, \cdots, d_{ir}, \cdots, d_{i(N-1)})$. For the measured data, $\frac{d_{ir}}{d_{i\max}}$ will be equal or close to zero if damage are located in elements i and r, and all other coefficients will be populated with nonzero entries. As a result, another single most probable damaged element can be selected using the new d' vector. For other multiple damage cases, the above screening process is continued until all the damaged members are located. The numerical examples in section 4 will illustrate how this approach is used to locate multiple damages.

3 Damage quantification using the best achievable flexibility change

By using the above described localization approach, the possible damaged elements have been determined. It is sometimes necessary to determine the extent of structural damage. In this part, the flexibility sensitivity is used to quantify damage. Without loss of generality, supposing that the number of the possible damaged elements is q and the corresponding damage factors are $\alpha_1, \alpha_2, \cdots, \alpha_q$, respectively. Then equation (7) reduces to

$$\Delta K = \sum_{i=1}^{q} \alpha_i K_i, \quad (0 \le \alpha_i \le 1) \tag{31}$$

Substituting equation (2) into (6), one has

$$\Delta F = F_u \Delta K F_u + F_u \Delta K \Delta F \tag{32}$$

Neglecting the high-order term in equation (32), one obtains

$$\Delta F = F_u \Delta K F_u \tag{33}$$

Substituting equation (31) into (33) yields

$$\Delta F = \sum_{i=1}^{q} \alpha_i (F_u K_i F_u) \tag{34}$$

From equation (34), the unknown damage parameters α_i can be readily computed by manipulating the matrix equation (34) into a set of linear equations. As will be shown is Section 4, the suspected damaged elements can be assessed again using the damage quantification scheme to determine the true sites and the extents of damage.

4 Numerical Examples

4.1 Example 1

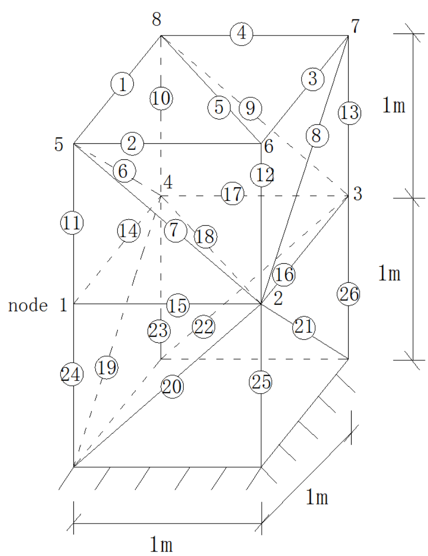


Figure 1: A space truss structure for example 1

The space truss structure used in this example is shown in Figure 1. The structure was modeled using 26 truss rod elements and 12 nodes (4 restrained), for a total of 24 DOF (every node has three DOFs). The basic parameters of the structure are as follows: Young's modulus $E = 200GPa$, density $\rho = 7.8 \times 10^3 Kg/m^3$, and cross-sectional area $A = 0.004m^2$. Two damage cases are considered in the example. The first one is a single-damage case that element 24 is damaged with a stiffness loss of 15%. The second case has two damages in which both elements 11 and 24 have 20% reduction in stiffness.

Table 1 presents the resulting comparative Euclidean distances (i.e., the vector$d$) of the single-damage case obtained by two sets of modal data: the complete and exact modes, and the first four modes with 5% noise. Measurement noise was simulated by adding proportional noise to each of the simulated measured mode shapes in the following way (Perera R., Ruiz A. , and Manzano C. (2007)):

$$\phi_{ij}^k = \phi_{ij}(1 + \eta \zeta_{ij}^k) \tag{35}$$

where ϕ_{ij}^k is the jth component of the ith mode contaminated with noise for the kth measurement, η is the degree of noise and ζ_{ij}^k is a random number in the range [-1, 1]. Such simulations were carried out 6 times and the average of these simulations is used for damage detection.

When the complete and exact modes are used in damage identification, inspection of column 2 in Table 1 indicates that a single damage occurred in the element 24 because it has zero distance. When only the first four modes with 5% noise are used in the calculation, examination of column 3 in Table 1 shows that there are possibilities of damage at element 24, element 11, or both element 24 and 11 because the distances at these locations are consistently smaller than the others. Determination of the damaged element(s) will not be made until the damage magnitudes are computed for the suspicious damaged members. As will be seen more for other damage cases, a situation like this occurs frequently when the incomplete and noisy modes are used for damage detection. To determine the damaged element(s) for the single-damage case when the incomplete and noisy modes are used, the following damage quantification procedure is employed. Assuming that elements 24 and 11 are both damaged, their corresponding damage extents are computed using equation (34). The resulting magnitudes of damage are as follows: $\alpha_{24} = 0.1801$ and $\alpha_{11} = 0.0107$. This means that a single damage occurred at element 24 and the calculated damage extent has slight deviation from the true value (0.15) because of the modal truncation, noise, and approximate process in equation (32). Thus, the damage detection process is completed.

In order to study the impact of the order of the model on the efficiency, Table 2 gives the comparative Euclidean distances for the single-damage case using the

Table 1: The comparative Euclidean distances (d) when element 24 is damaged (Example 1)

Element number	The complete and exact modes	The first four modes with 5% noise
1	0.9922	0.994
2	0.99	0.9959
3	0.9253	0.9283
4	0.9731	0.9834
5	0.9783	0.9715
6	0.94	0.9442
7	0.9317	0.9442
8	0.8852	0.8957
9	0.8823	0.9142
10	0.9009	0.9194
11	0.558	**0.5759**
12	0.9991	0.999
13	0.9029	0.8998
14	0.9994	0.9969
15	0.9997	0.9968
16	0.9997	0.9987
17	0.9675	0.9548
18	0.9263	0.9349
19	0.9534	0.9737
20	0.9293	0.9624
21	0.9675	0.9871
22	0.9755	0.9908
23	0.9967	0.9993
24	**0**	**0.3638**
25	1	1
26	0.8185	0.8627

first two modes and the first six modes. When only the first two modes are used, it is obvious from column 2 in Table 2 that elements 11 and 24 are both probable damaged locations because of their relatively smaller index. Using equation (34), their damage extents can be computed as $\alpha_{24} = 0.1228$ and $\alpha_{11} = 0.0629$. The above results show that element 11 was wrongly detected to be damaged member when only the first two modes are used. For the case of using the first six modes, inspection of column 3 in Table 2 clearly indicated that element 24 was the most probable damaged member for its index is far less than the others. Again, the damage extents of elements 24 and 11 can be computed as $\alpha_{24} = 0.1781$ and $\alpha_{11} = 0.0105$. Compared with the results obtained by the first four modes, it is obvious that the damage detection results become more accurate as the number of modes used in the calculation increases.

Table 3 shows the damage localization results for the multiple damage case using the first six modes with 5% noise. Inspection of column 2 in Table 3 indicated that element 24 was the most probable damaged member for its index is less than the others. So element 24 is selected to be the damaged member in this step. Then the other elements are combined with element 24 in turn to calculate the new Euclidean distances and the resulting new damage localization vector $d^{'}$ is given in column 3 of Table 3. Now element 11 is clearly identified as another damaged member for its index is far less than the others. The magnitudes of damage are computed for elements 11 and 24 as $\alpha_{11} = 0.2296$ and $\alpha_{24} = 0.2386$, which have 14.8% and 19.3% relative errors, respectively. Elements 11 and 24 are thus confirmed to be the damaged members.

To compare the localization performance of the proposed method and the traditional flexibility difference method, Figure 2(a) and (b) present the results obtained by the flexibility difference method using the complete and exact modes for locating damage. The flexibility difference method was proposed by Pandey and Biswas (Pandey A. K. and Biswas M. (1994)), which consists in calculating the flexibility change matrix and then observing the maximum value of each column:

$$\Delta = \left\{\delta_{ij}\right\} = F_d - F_u, \quad \overline{\delta_j} = \max_i \left|\delta_{ij}\right| \tag{36}$$

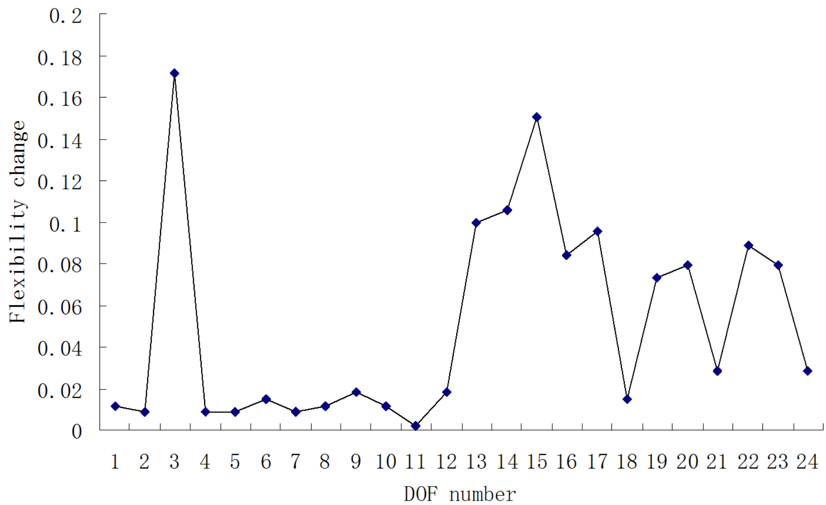
However, as shown in the reference of Pandey A. K. and Biswas M. (1994), damage localization using $\overline{\delta_j}$ depends on geometrical condition of the structure. For instance, damage in a cantilever beam is located at the point where $\overline{\delta_j}$ just appear different from zero, while damage in a simply supported beam is located at the point corresponding to the maximum of $\overline{\delta_j}$. So human experience and judgment may be necessary for a general application. For this example, Figure 2 only shows that the variations of flexibility have the maximums at the DOF corresponding to the damage element, but it is difficult to indicate exact damage location and the number of damaged elements even if the complete and exact modes are used. It appears that damage localization directly based on flexibility change is heuristic and applicable only for particular types of structures (such as beam-like structures), with some post-calculations sometimes being necessary for a general application.

Table 2: The comparative Euclidean distances (d) when element 24 is damaged (Example 1)

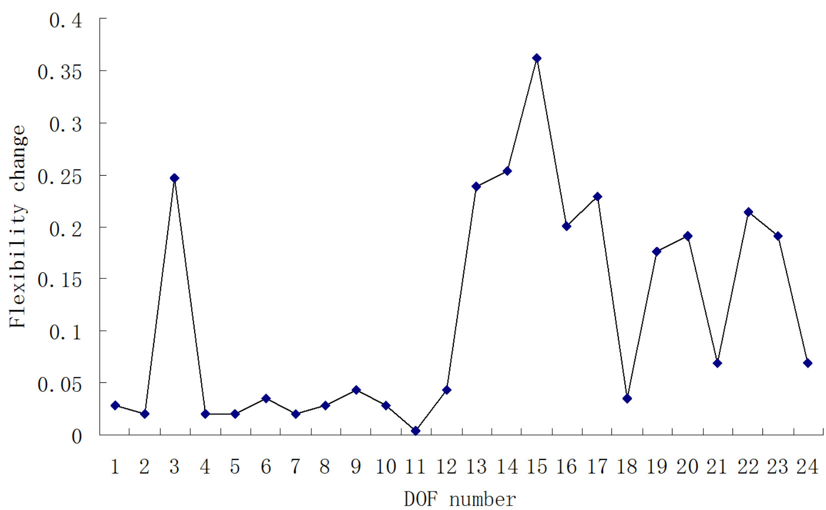
Element number	The first two modes with 5% noise	The first six modes with 5% noise
1	0.9856	0.9861
2	0.9831	0.9857
3	0.9013	0.922
4	0.9722	0.9594
5	0.9805	0.98
6	0.9259	0.9372
7	0.8986	0.9121
8	0.8403	0.8778
9	0.9001	0.8617
10	0.913	0.8878
11	**0.5835**	**0.5315**
12	0.9997	0.9989
13	0.8423	0.8968
14	0.9996	1
15	0.9991	0.9993
16	0.9987	0.9993
17	0.9801	0.9748
18	0.9381	0.9281
19	0.9382	0.9454
20	0.8732	0.919
21	0.9326	0.9623
22	0.9698	0.9692
23	0.9876	0.9962
24	**0.5861**	**0.1631**
25	1	0.9999
26	0.8141	0.7949

Table 3: Damage localization results when elements 11 and 24 are damaged (Example 1)

Element number	The vector d	The vector d' (obtained after the damage at element 24 has been detected)
1	0.9899	0.9963
2	0.9895	0.9995
3	0.9172	0.9902
4	0.9712	0.9982
5	0.9744	0.9946
6	0.9245	0.9755
7	0.917	0.9794
8	0.8703	0.9811
9	0.8717	0.9881
10	0.8902	0.9874
11	**0.3422**	**0.3167**
12	0.9985	0.999
13	0.8853	0.9754
14	0.9994	1
15	0.9996	0.9999
16	0.9996	0.9999
17	0.9642	0.9963
18	0.9209	0.9953
19	0.9466	0.9922
20	0.9192	0.9879
21	0.9623	0.9936
22	0.972	0.9964
23	0.9953	0.9973
24	**0.2512**	/
25	1	1
26	0.7898	0.9612



(a) The flexibility change using the complete and exact modes when element 24 is damaged (Example 1)



(b) The flexibility change using the complete and exact modes when elements 11 and 24 are damaged (Example 1)

Figure 2:

4.2 Example 2

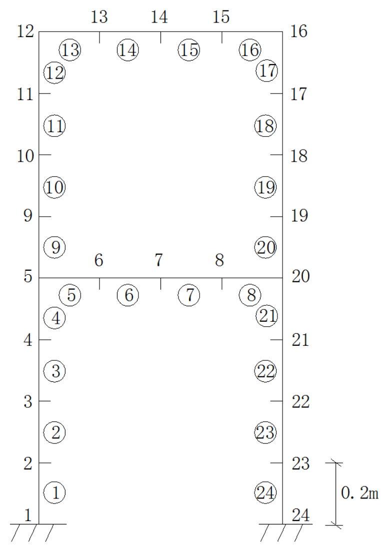


Figure 3: A two-storey frame structure for Example 2

The second example is a two-storey frame structure as shown in Fig.3 with a rectangular cross-section of 0.14m $\times$ 0.24m. This frame was modeled with 48 equal elements of 0.2m in length. Every node has three DOFs, an axial displacement, a transverse displacement and a rotation. The properties of this structure are: cross-sectional area $A = 0.0336m^2$; moment of inertia $I = 1.6128 \times 10^{-4}m^4$; Young's modulus $E = 200GPa$; shearing modulus of elasticity $G = 1.3461 \times 10^{10}N/m^2$; Poisson's ratio $v = 0.3$; density $\rho = 2500kg/m^3$. Only horizontal modal displacements are assumed to be 'measured' in the columns while for the beams only the vertical components of mode shapes are 'measured'. In other words, only the first six modes and 22 out of 66 DOFs are used in the present study to simulate incomplete modal data in the real situation.

Multiple damages are simulated in elements 2 and 5 with stiffness losses both of 20%. Table 4 shows the damage localization results for the multiple damage case using the first six modes with 5% noise. Measurement noise was simulated in the same way of Example 1. Inspection of column 2 in Table 4 indicated that element 5 was the most probable damaged member for its index is far less than the others. Consequently element 5 is selected to be the damaged member in this step. And then the other elements are combined with element 5 in turn to calculate the new Euclidean distances and the resulting new damage localization vector $d^{'}$ is given in column 3 of Table 4. Now element 2 can be detected to be another damaged member for its index is less than the others. To avoid missing the other potential damaged members, element 1 is also selected to be suspected damaged member for it has the second smallest index in column 3 of Table 4. Using equation (22), the magnitudes of damage are computed as $\alpha_1 = -0.0016$, $\alpha_2 = 0.2371$and $\alpha_5 = 0.2384$. From the results, elements 2 and 5 are thus confirmed to be the damaged members.

Table 4: Damage localization results when elements 2 and 5 are damaged (Example 2)

Element number	The vector d	The vector $d^{'}$(obtained after the damage at element 5 has been detected)
1	0.2737	0.402
2	0.1968	**0.178**
3	0.3818	0.5393
4	0.3951	0.6902
5	**0.0782**	/
6	0.3158	0.9937
7	0.3657	0.9894
8	0.2826	1
9	0.7636	0.6976
10	0.9036	0.8461
11	0.7157	0.9211
12	0.6059	0.8653
13	0.7633	0.8796
14	0.9744	0.8955
15	0.9527	0.9379
16	0.765	0.9687
17	0.6438	0.9567
18	0.8156	0.9644
19	1	0.976
20	0.8799	0.9733
21	0.5804	0.9214
22	0.5697	0.935
23	0.3677	0.933
24	0.4032	0.9173

4.3 Example 3

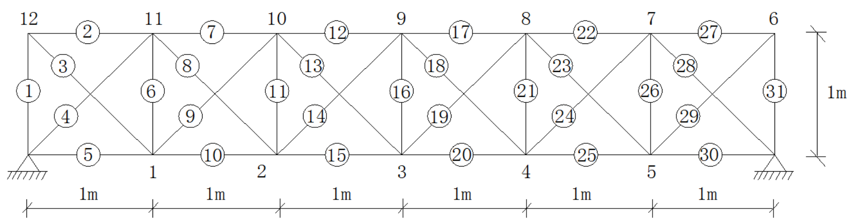


Figure 4: Thirty-one-bar truss structure for example 3

The 31-bar truss structure used in this example is shown in Figure 4, which is used to verify the proposed method for the symmetric structure. The basic parameters of the symmetric structure are as follows: Young's modulus $E = 200GPa$, density $\rho = 7.8 \times 10^3 Kg/m^3$, and cross-sectional area $A = 0.004m^2$. Multiple damages are simulated in elements 12 and 23 with stiffness losses both of 15%. Measurement noise of mode shapes was simulated in the same way of Example 1. And the measurement noise of natural frequencies was also simulated in the similar way:

$$\lambda_j^k = \lambda_j(1 + \eta \zeta_j^k) \tag{37}$$

where λ_j^k is the jth eigenvalue (the square of frequency) contaminated with noise for the kth measurement, η is the degree of noise and ζ_j^k is a random number in the range [-1, 1]. It is widely recognized that the natural frequencies are least contaminated by measurement noise and can generally be measured with good accuracy. Messina A., Williams J. E., and Contursi T. (1998) suggest a standard error of $\pm 0.15\%$ as a benchmark figure for natural frequencies measured in the laboratory with the impulse hammer technique. In contrast, mode shape estimates have error levels as much as 20 times worse than those in the corresponding natural frequency estimates. So the degree of frequency noise in equation (25) is assumed to be 0.15 ($\eta = 0.15$). As before, such simulations were carried out 6 times and the average of these simulations is used for damage detection. Figure 5 presents the result obtained by the flexibility difference method using the first six modes with noise. From Figure 5, similar conclusion can be drawn that it is difficult to indicate exact damaged elements. Using the proposed best achievable flexibility change method, Table 5 shows the damage localization results for the multiple damage case using the first six modes. Inspection of column 2 in Table 5 indicated that element 12 was the most probable damaged member for its index is far less than the others. Consequently element 12 is selected to be the damaged member in this step. And then the other elements are combined with element 12 in turn to calculate the new Euclidean distances and the resulting new damage localization vector d' is given in column 3 of Table 5. Now element 23 can be detected to be another damaged member for its index is less than the others. To avoid missing the other potential damaged members, element 24 is also selected to be suspected damaged member for it has the second smallest index in column 3 of Table 5. Using equation (22), the magnitudes of damage are computed as $\alpha_{12} = 0.1747$, $\alpha_{23} = 0.1712$and $\alpha_{24} = -0.0013$. From the results, elements 12 and 23 are thus confirmed to be the damaged members. It has been shown that the proposed method performs well for the symmetric structure even if the errors in frequencies and mode shapes inevitably make the damage assessment more difficult.

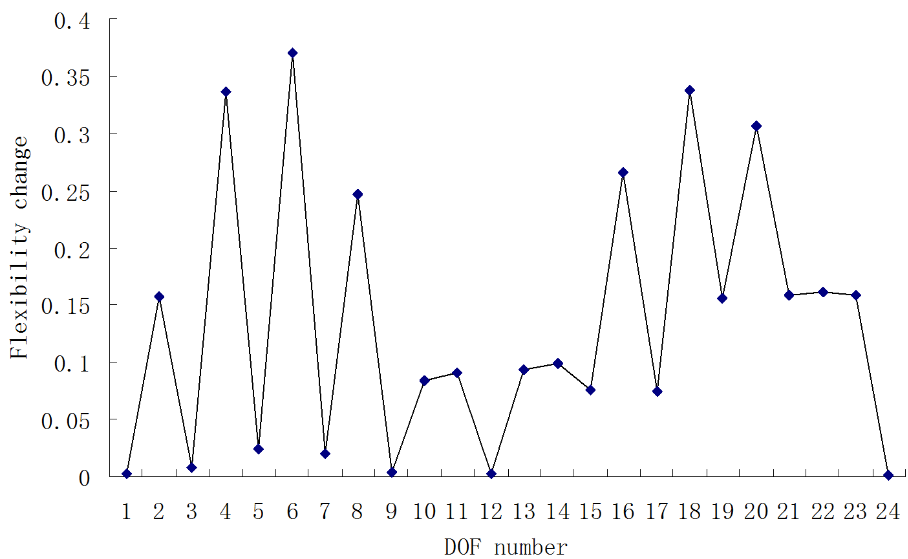


Figure 5: The flexibility change using the first six modes with noise when elements 12 and 23 are damaged (Example 3)

Table 5: Damage localization results when elements 12 and 23 are damaged (Example 3)

Element number	The vector d	The vector d' (obtained after the damage at element 12 has been detected)
1	0.9366	0.9989
2	0.8498	1.0000
3	0.7997	0.9915
4	0.6117	0.9742
5	0.8791	0.9879
6	0.7900	0.9887
7	0.4355	0.9893
8	0.8333	0.9968
9	0.5786	0.9988
10	0.9846	0.9951
11	0.7761	0.9985
12	**0.1052**	/
13	0.9902	0.9458
14	0.8203	0.9893
15	0.6378	0.9757
16	0.7475	0.9968
17	0.3509	0.9206
18	0.7209	0.9976
19	0.9996	1.0000
20	0.7402	0.9402
21	0.8096	1.0000
22	0.5895	0.9732
23	0.5271	**0.3988**
24	0.8209	0.7850
25	1.0000	0.9926
26	0.7825	0.9954
27	0.9123	0.9886
28	0.6312	0.8940
29	0.8461	0.9837
30	0.8037	0.8853
31	0.9512	0.9980

5 Conclusions

A new method for structural damage identification was developed in this study, which is based on the best achievable concept. The best achievable flexibility change is a projection of a measured flexibility change onto the subspace that is defined by the undamaged analytical model. Damage location can be determined by the Euclidean distance between the measured flexibility change and the best achievable flexibility change. With location identified, the magnitude of damage can be computed by the flexibility sensitivity method. The first advantage of the proposed method is that it can directly determine the damage locations at the element level rather than at the structural DOF level. The second advantage of the method is that the number of variables is very reduced in the stage of damage quantification, since only the elements belonging to the zones identified previously as damaged are now assumed to be damaged. Three numerical examples are used to exercise this process and measurement noise is also simulated in damage detection. The results show that the proposed method can determine accurately both the location and magnitude of structural damage with the incomplete and noisy modes. It has been shown that the proposed procedure may be a promising method in structural damage detection. Future research on the technique should be carried out to demonstrate the procedure using experimentally measured data.

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