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BEM-FEM Coupling For Acoustic Effects On Aeroelastic Stability Of Structures

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Abstract: A series of work has been carried out to develop the foundation for the computational scheme for the calculation of the influence of the acoustic disturbance to the aeroelastic stability of the structure. The generic approach consists of three parts. The first is the formulation of the acoustic wave propagation governed by the Helmholtz equation by using boundary element approach, which then allows the calculation of the acoustic pressure on the acoustic-structure boundaries. The structural dynamic problem is formulated using finite element approach. The third part involves the calculation of the unsteady aerodynamics loading on the structure using generic unsteady aerodynamics computational method. Analogous to the treatment of dynamic aeroelastic stability problem of structure, the effect of acoustic pressure disturbance to the aeroelastic structure is considered to consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, referred to as the acoustic aerodynamic analogy. Results are presented and compared to those obtained in earlier work.

Keywords: Boundary Element Method, Finite Element Method, Fluid-Structure Coupling, Computational Mechanics

1 Introduction

The foundation for the computational scheme for the calculation of the influence of the acoustic disturbance to the aeroelastic stability of the structure has been developed in earlier work [Djojodihardjo and Tendean (2004), Djojodihardjo and Safari(2005, 2006))]. Analogous to the treatment of dynamic aeroelastic stability problem of structure, in which the aerodynamic effects can be distinguished into motion independent and motion induced aerodynamic forces, the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aero-elastic problem) can

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be viewed to consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, which is known as the scattering pressure. This can be referred to as the acoustic aerodynamic analogy. Proceeding to the formulation of BE-FE coupling to treat the fluid-structure interaction, reference is made on the solution of structural-acoustic interaction problems using BEM-FEM coupling given by Holström(2001) and Meddahi, Marquez, and Selgas (2004). Applying similar approach to solve the acousto-aeroelastic problem, the present work consists of three parts. The first part involves the formulation of the acoustic wave propagation governed by the Helmholtz equation by using boundary element approach, which then allows the calculation of the acoustic pressure on the acoustic-structure boundaries. The governing Helmholtz equation will be solved using Boundary Element method, following the procedure elaborated by Wrobel (2002) and taking into considerations various techniques and development described in many recent literature, such as those elaborated by Holström (2001), Zhang, Gu and Chen (2009a, 2009b). Chen, Chen and Liang(2001), and Papacharalampopoulos, et al (2010). The second part addresses the structural dynamic problem using finite element approach. The acoustic-structure interaction is then given special attention to formulate the BEM-FEM fluid-structure coupling. The third part involves the calculation of the unsteady aerodynamic loading on the structure using a conveniently chosen unsteady aerodynamics computational method, to be utilized in the aeroelastic problem. The acoustic pressure disturbance is then superposed to the aeroelastic problem, following the acoustic aerodynamic analogy. Solution procedure can then be readily formulated. Figure 1 shows the computational strategy to treat the aeroacoustic effects on aeroelastic structure.

2 Discretization of the Helmholtz Integral Equation for the Acoustic Field

For an exterior acoustic problem, as depicted in Figure 2, the problem domain V is the free space V_{ext} outside the closed surface S. V is considered enclosed between the surface S and an imaginary surface Λ at a sufficiently large distance from the acoustic sources and the surface S such that the boundary condition on Λ satisfies Sommerfeld's acoustic radiation condition as the distance approaches infinity.

For time-harmonic acoustic problems in fluid domains, the corresponding boundary integral equation is the Helmholtz integral equation [Dowling and Ffowcs-Williams (1983)]

$$cp(R) = \int\limits_S \left(p(R)\frac{\partial g}{\partial n_0} - g(|R-R_0|)\frac{\partial p}{\partial n_0} \right) dS \tag{1}$$

where n_0 is the surface unit normal vector, and the value of c depends on the location of R in the fluid domain, and where g the free-space Green's function. R_0

denote a point located on the boundary S, as given by

$$g\left(|R-R_0|\right) = \frac{e^{-ik|R-R_0|}}{4\pi\,|R-R_0|} \tag{2}$$

To solve Eq. (1) with g given by Eq. (2), one of the two physical properties, acoustic pressure and normal velocity, must be known at every point on the boundary surface. At the infinite boundary Λ, the Sommerfeld radiation condition in three dimensions can be written as [Dowling and Ffowcs-Williams (1983)]:

$$\lim_{|R-R_0|\to\infty} r\left(\frac{\partial g}{\partial r}+ikg\right) \Rightarrow 0 \text{ as } r \Rightarrow \infty, r = |R-R_0| \tag{3}$$

which is satisfied by the fundamental solution.

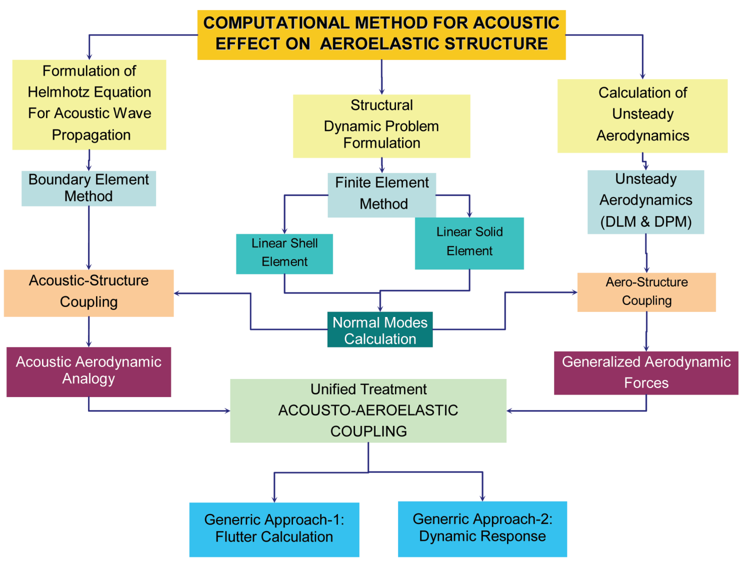


Figure 1: Computational strategy for the calculation of acoustic effects on aeroelastic structures

The total pressure, which consists of incident and scattering pressure, serves as an acoustic excitation on the structure. The integral equation for the total wave is given by

$$cp(R) - p_{inc}(R) = \int_S \left[p\left(R\right)\frac{\partial g\left(R-R_0\right)}{\partial n_0} - \frac{\partial p\left(r\right)}{\partial n_0} g\left(R-R_0\right)\right] dS \tag{4}$$

where $p = p_{inc} + p_{sc}$, and where

$$c = \begin{cases} 1 & , \mathrm{R} \in \mathrm{V_{ext}} \\ 1/2 & , \mathrm{R} \in \mathrm{S} \\ \Omega/4\pi & , \mathrm{R} \in \mathrm{S} \qquad \text{(non smooth surface)} \\ 0 & , \mathrm{R} \in \mathrm{V_{int}} \end{cases} \tag{5}$$

The Helmholtz equation is then discretized by dividing the boundary surface S into N elements. The discretized boundary integral equation becomes,

$$cp_i - p_{inc} - \sum_{j=1}^{N} \int_S p\bar{g}dS = i\rho_0\omega \sum_{j=1}^{N} \int_S gvdS \tag{6}$$

where i indicates field point, j source point and $S_j$surface element j, and for convenience, $\bar{g}$ is defined as

$$\bar{g} \equiv \frac{\partial g}{\partial n} \tag{7}$$

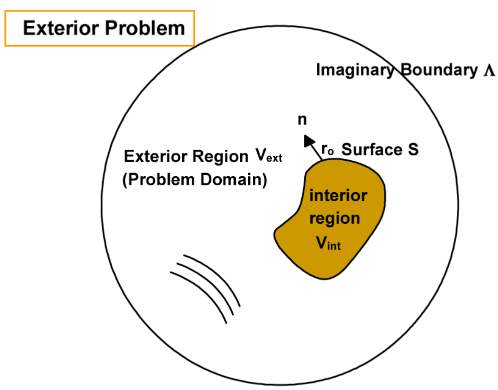


Figure 2: Exterior problem for homogeneous Helmholtz equation

Let

$$\overline{H}_{ij} = \int_{S_j} \bar{g}dS \tag{8}$$

$$G_{ij} = \int_{S_j} gdS \tag{9}$$

Substituting g in Eq.(2) to be the monopole Green's free-space fundamental solution, it follows that:

$$G_{ij} = \int_{S_j} g dS = \int_{S_j} g\left(\left|R_j - R_i\right|\right) dS = \int_{S_j} \frac{e^{ik\left|R_j - R_i\right|}}{4\pi\left|R_j - R_i\right|} dS \tag{10}$$

or, in Cartesian coordinate system,

$$G_{ij} = \int_{S_j} \frac{e^{-ik\sqrt{\left(x_j-x_i\right)^2+\left(y_j-y_i\right)^2+\left(z_j-z_i\right)^2}}}{4\pi\sqrt{(x_j-x_i)^2+(y_j-y_i)^2+(z_j-z_i)^2}} dS \tag{11}$$

where R_j is the coordinate vector of the midpoint of element j and R_i is the coordinate vector of the node i. In the development that follows, four-node iso-parametric quadrilateral elements are used throughout. To calculate $\overline{H}_{ij}$, the derivative $\bar{g}$ has to be evaluated

$$\overline{H}_{ij} = \int_{S_j} \bar{g} dS = \int_{S_j} \frac{\partial g}{\partial \hat{n}} dS = \int_{S_j} (\nabla g)^T \hat{n} dS \tag{12}$$

where

$$\hat{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \tag{13}$$

and

$$\nabla g = \begin{bmatrix} -\dfrac{xe^{-ik\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}}{4\pi\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}\left(ik+\dfrac{1}{\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}\right) \\ -\dfrac{ye^{-ik\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}}{4\pi\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}\left(ik+\dfrac{1}{\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}\right) \\ -\dfrac{ze^{-ik\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}}{4\pi\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}\left(ik+\dfrac{1}{\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2+\left(z_i-z_j\right)^2}}\right) \end{bmatrix} \tag{14}$$

For a four-node iso-parametric quadrilateral element, the pressure p and the normal velocity v at any position on the element can be defined by their nodal values and linear shape functions, i.e.

$$v(\xi,\eta) = N_1v_1 + N_2v_2 + N_3v_3 + N_4v_4 = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{15}$$

$$p(\xi,\eta) = N_1p_1 + N_2p_2 + N_3p_3 + N_4p_4 = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \tag{16}$$

where the shape functions in the element coordinate system are,

$$\begin{array}{l} N_1 = \frac{1}{4}(\xi-1)(\eta-1) \;\; N_2 = -\frac{1}{4}(\xi+1)(\eta-1) \\ N_3 = \frac{1}{4}(\xi+1)(\eta+1) \;\; N_2 = -\frac{1}{4}(\xi-1)(\eta+1) \end{array} \tag{17}$$

The four node quadrilateral element can have any arbitrary orientation in the three-dimensional space. Using the shape functions (20), the integral on the left hand side of Eq. (5), considered over one element j, can be written as:

$$\int_{S_j} p\overline{g_i}ds = \int_{S_j} \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \overline{g}_i dS \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}_j = \begin{bmatrix} \overline{h}_{ij}^1 & \overline{h}_{ij}^2 & \overline{h}_{ij}^3 & \overline{h}_{ij}^4 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}_j \tag{18}$$

while that on the right hand side

$$\int_{S_j} gvdS = \int_{S_j} \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} g_i dS \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}_j = \begin{bmatrix} g_{ij}^1 & g_{ij}^2 & g_{ij}^3 & g_{ij}{}^n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}_i \tag{19}$$

where

$$\overline{h}_{ij}^k = \int_{S_j} N_k \overline{g}_j dS \quad k = 1,2,3,4 \tag{20}$$

$$g_{ij}^k = \int_{S_j} N_k g_j dS \quad k = 1,2,3,4 \tag{21}$$

The integration in Eq. (18) and (19) can be carried out using Gauss points [Weaver and Johnston (1987), Holström (2001)]. These Gauss points in the iso-parametric system are defined as:

$$\begin{array}{l} (\xi_1,\eta_1) = \frac{1}{\sqrt{3}}(1,-1) \;\; (\xi_2,\eta_2) = \frac{1}{\sqrt{3}}(1,1) \\ (\xi_3,\eta_3) = \frac{1}{\sqrt{3}}(-1,1) \;\; (\xi_4,\eta_4) = \frac{1}{\sqrt{3}}(-1,-1) \end{array} \tag{22}$$

Substituting equations (18) and (19) into equation (6) for all elements j, there is obtained

$$c_i p_i - p_{inc} - \sum_{j=1, j\neq i}^{N} \left[\overline{h}_{ij}^{1} \ \overline{h}_{ij}^{2} \ \overline{h}_{ij}^{3} \ \overline{h}_{ij}^{4} \right] \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}_j = i\rho\omega_0 \sum_{j=1}^{N} \left[g_{ij}^{1} \ g_{ij}^{2} \ g_{ij}^{3} \ g_{ij}^{4} \right] \begin{bmatrix} v_{n1} \\ v_{n2} \\ v_{n3} \\ v_{n4} \end{bmatrix}_j \tag{23}$$

Equation (23) can be rewritten as

$$\sum_{j=1}^{N} \left[h_{ij}^{1} \ \ h_{ij}^{2} \ \ h_{ij}^{3} \ \ h_{ij}^{4} \right] \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}_j = i\rho\omega_0 \sum_{j=1}^{N} \left[g_{ij}^{1} \ \ g_{ij}^{2} \ \ g_{ij}^{3} \ \ g_{ij}^{4} \right] \begin{bmatrix} v_{n1} \\ v_{n2} \\ v_{n3} \\ v_{n4} \end{bmatrix}_j + p_{inc} \tag{24}$$

Hence the discretized equation forms a set of simultaneous linear equations, which relates the pressure p_i at field point i due to the boundary conditions p to v at source surface S_i of element i and the incident pressure p_{inc}.

In matrix form:

$$[\mathbf{H}]\{p\} = i\rho_0\omega[\mathbf{G}]\{v\} + \{p_{inc}\} \tag{25}$$

where, $\mathbf{H}$ and $\mathbf{G}$ are two $N\times N$ matrices of influence coefficients, while p and v are vectors of dimension N representing total pressure and normal velocity on the boundary elements. This matrix equation can be solved if the boundary condition $v = \partial p / \partial n$ and the incident acoustic pressure field p_{inc} are known.

At this point, a few remarks are necessary. Proper interpretation should be given to the diagonal terms of [$\mathbf{H}$] in equation (19) as implied by the original boundary integral (3), since these terms concern the evaluation of influence coefficient for which the field point is located at the source element. Accordingly, [$\mathbf{H}$] should be written as

$$[\mathbf{H}] = [\mathbf{H}]^{\mathbf{D}} + [\mathbf{H}]^{\mathbf{OD}} \tag{26}$$

i.e. the diagonal and the off-diagonal part.

The matrix $[\mathbf{H}]^D$as implied in (26) can be written as $[\mathrm{H}]^D = [\bar{H}]^D + [\mathrm{C}]$ where $\mathbf{C}$ is space angle constant as implied in Eq.(4) which is the quotient of $\Omega/4\pi$ and $\bar{H}$ is the matrix implied by the second term of eq.(23) . For a node coinciding with three or four element corners, Ω is the space angle towards the acoustic medium, and the space angle for a sphere is 4π. For a smooth surface the space angle is 2π, and $\mathbf{C} = {}^1\!/_2$.

The boundary integral equation of equation (4) fails at frequencies coincident with the interior cavity frequencies of homogeneous Dirichlet boundary conditions [Wrobel(2002)]. In the case of the formulation of the exterior problem, these frequencies correspond to the natural frequencies of acoustic resonances in the interior region. When the interior region resonates, the pressure field inside the interior region has non-trivial solution. Since the interior problem and the exterior problem shares similar integral operators, the exterior integral equation may also break down. The discretized equation of the [**H**]matrix in equation (25) becomes ill-

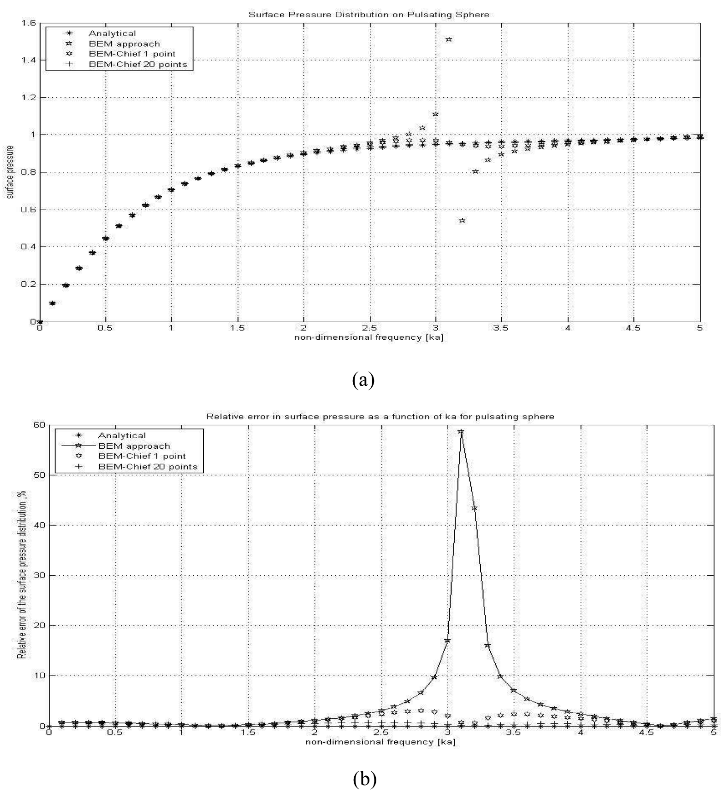


Figure 3: (a) Surface pressure distribution on pulsating sphere for analytical, BEM, and BEM-CHIEF solution for one and two CHIEF point ; (b) error of the surface pressure distribution in (a) with respect to the exact solution. 384 isometric surface elements are utilized, and the use of one (1) CHIEF point has been able to eliminate the spurious solution with reasonably good accuracy.

conditioned when the exciting frequency is close to the interior frequencies, thus providing an erroneous acoustic loading matrix. This problem could be overcome by using the CHIEF [Schenck(1976), Chen, Chen and Liang(2001)] or Burton-Miller method [Burton and Miller(1971), Chen, Chen, Kuo and Liang(2001)], or a recent technique utilizing SVD and Fredholm alternative theorem [Chen, Chen, Kuo and Liang(2001), Chen, Chen & Chen (2006), Chen, Chen, LEE and LEE (2012)]. To avoid non-uniqueness problem, reference [13] describes special treatment to be carried out to inspect whether the **H** matrix is ill-behaved or not by utilizing SVD updating technique. The present method, however, resorts to the utilization of CHIEF method, if eq. (25) ill-behaved. Such approach applied to the present method has proved to be successful, as indicated in Figure 3.

3 BEM-FEM Acoustic-Aeroelastic Coupling (AAC)

Following [Weaver and Johnston (1987)], the BE region is treated as a super finite element and its stiffness matrix is computed and assembled into the global stiffness matrix, and identified as the coupling to finite elements. The state of affairs is schematically depicted in Figure.4(a).

The utilization of FEM on the structural domain leads to a system of simultaneous equations which relate the displacements at all the nodes to the *nodal forces*. In the BEM, on the other hand, a relationship between nodal displacements and *nodal tractions* is established. Representing the elastic structure by FE model, the structural dynamic equation of motion is given by [Bisplinghoff, Ashley and Halfman(1955)]

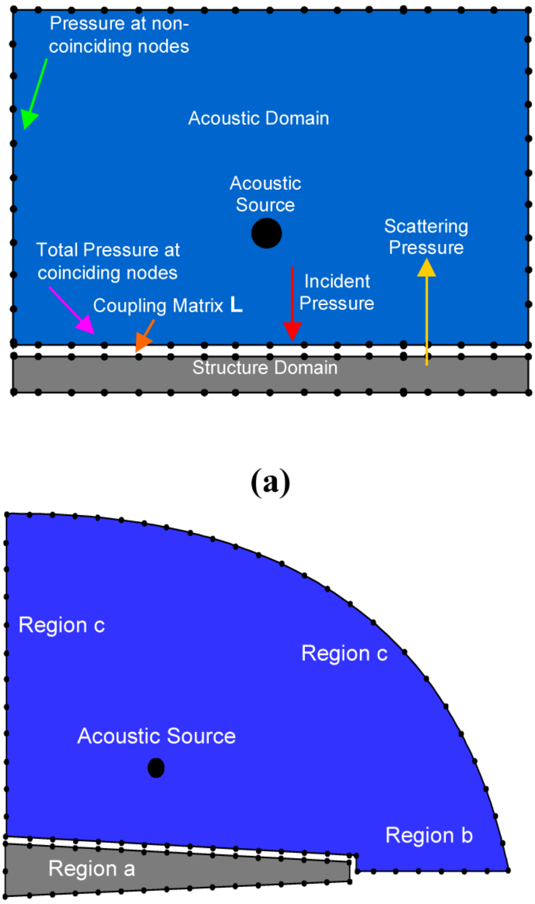
$$[\mathbf{M}]\{\ddot{x}\}+[\mathbf{C}]\{\dot{x}\}+[\mathbf{K}]\{x\}=\{\mathbf{F}\} \tag{27}$$

where **M**, **C** and **K** are structural mass, damping and stiffness, respectively, which are expressed as matrices in a FE model, while **F** is the given external forcing function vector, and $\{x\}$ is the structural displacement vector. The incorporation of the self excited aerodynamic effects to the structural dynamics equation can be written as [Bisplinghoff, Ashley and Halfman (1955), Rodden and Johnson(1994) and Zona-Tech(1992)]:

$$[\mathbf{M}]\{\ddot{x}\}+[\mathbf{C}]\{\dot{x}\}+[\mathbf{K}]\{x\}-q_{\infty}[\mathbf{A}(ik)]\{x\}=\{0\} \tag{28}$$

where $\mathbf{A}(ik)$ is an aerodynamic influence coefficient after applying aero-structure coupling from the control points of aerodynamic boxes to the structural finite element grid points as elaborated in [Djojodihardjo and Safari (2006)].

Taking into account the acoustic pressure p on the structure at the fluid-structure interface as a separate excitation force, the acoustic-structure problem can be obtained from Eq.(28) by introducing a fluid-structure coupling term given by $[\mathbf{L}]\{p\}$.



(a)

(b)

Figure 4: (a).Schematic of Fluid-Structure Interaction Domain (b).Schematic of FE-BE problem representing quarter space problem domains for half wing

It follows *that*

$$[\mathbf{M}]\{\ddot{x}\} + [\mathbf{C}]\{\dot{x}\} + [\mathbf{K}]\{x\} - q_\infty [\mathbf{A}(ik)]\{x\} + [\mathbf{L}]\{p\} = \{\mathbf{F}\} \tag{29}$$

where $\mathbf{L}$ is a coupling matrix of size $M \times N$ in the BEM/FEM coupling thus formulated. M is the number of FE degrees of freedom and N is the number of BE nodes on the coupled boundary. For the BE part of the surface at the fluid-structure interface a, Eq.(19) can be utilized.

The global coupling matrix **L** transforms the acoustic fluid pressure acting on the nodes of boundary elements on the entire fluid-structure interface surface a to nodal forces on the finite elements of the structure. Hence **L** consists of n assembled local transformation matrices L_e, given by

$$L_e = \int\limits_{S_e} N_F^T n N_B dS \tag{30}$$

in which N_F is the shape function matrix for the finite element and N_B is the shape function matrix for the boundary element. It can be shown that:

$$N_F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} [N_i] \tag{31}$$

The rotational parts in N_F are neglected since these are considered to be small in comparison with the translational ones in the BE-FE coupling, consistent with the assumptions in structural dynamics as, for example, stipulated in [Bisplinghoff, Ashley and Halfman (1955)].

For the normal fluid velocities and the normal translational displacements on the shell elements at the fluid-structure coupling interface, a relationship, which takes into account the velocity continuity over the coinciding nodes, should be established. This relationship is given by

$$\mathrm{v} = i\omega\left(\mathbf{T}.x\right) \tag{32}$$

Similar to **L**, **T** *(n × m)* is also a global coupling matrix that connects the normal velocity of a BE node with the translational displacements of FE nodes obtained by taking the transpose of the boundary surface normal vector n[Beer and Watson (1992), Holström (2001)]. The local transformation vector T_e can then be written as:

$$T_e = n^T \tag{33}$$

The presence of an acoustic source can further be depicted by Figure4(b). Three regions are considered, i.e. a, b and c; region a is the fluid-structure interface region, where FEM mesh and BEM mesh coincide and region b and c is the region where all of the boundary conditions (pressure or velocity) are known.

For the coupled FEM-BEM regions, BEM equation can now be written as:

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \mathbf{H}_{13} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \mathbf{H}_{23} \\ \mathbf{H}_{31} & \mathbf{H}_{32} & \mathbf{H}_{33} \end{bmatrix} \begin{Bmatrix} p_a \\ p_b \\ p_c \end{Bmatrix} = i\rho_0\omega \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \mathbf{G}_{13} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \mathbf{G}_{23} \\ \mathbf{G}_{31} & \mathbf{G}_{32} & \mathbf{G}_{33} \end{bmatrix} \begin{Bmatrix} v_a \\ v_b \\ v_c \end{Bmatrix} + \begin{Bmatrix} p_{inc_a} \\ p_{inc_b} \\ p_{inc_c} \end{Bmatrix} \tag{34}$$

Considering $\mathbf{v}_a = i\omega(\mathbf{T}.x)$, BEM equation can be modified as:

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \mathbf{H}_{13} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \mathbf{H}_{23} \\ \mathbf{H}_{31} & \mathbf{H}_{32} & \mathbf{H}_{33} \end{bmatrix} \begin{Bmatrix} p_a \\ p_b \\ p_c \end{Bmatrix} = i\rho_0\omega \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \mathbf{G}_{13} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \mathbf{G}_{23} \\ \mathbf{G}_{31} & \mathbf{G}_{32} & \mathbf{G}_{33} \end{bmatrix} \begin{Bmatrix} i\omega[\mathbf{T}]x \\ v_b \\ v_c \end{Bmatrix} + \begin{Bmatrix} p_{inc_a} \\ p_{inc_b} \\ p_{inc_c} \end{Bmatrix} \tag{35}$$

or:

$$\begin{aligned} \mathbf{H}_{11}p_a + \mathbf{H}_{12}p_b + \mathbf{H}_{13}p_c &= -\rho_0\omega^2\mathbf{G}_{11}\mathbf{T}x + i\rho_0\omega\mathbf{G}_{12}v_b + i\rho_0\omega\mathbf{G}_{13}v_c + \{p_{inc_a}\} \\ \mathbf{H}_{21}p_a + \mathbf{H}_{22}p_b + \mathbf{H}_{23}p_c &= -\rho_0\omega^2\mathbf{G}_{21}\mathbf{T}x + i\rho_0\omega\mathbf{G}_{22}v_b + i\rho_0\omega\mathbf{G}_{23}v_c + \{p_{inc_b}\} \\ \mathbf{H}_{31}p_a + \mathbf{H}_{32}p_b + \mathbf{H}_{33}p_c &= -\rho_0\omega^2\mathbf{G}_{31}\mathbf{T}x + i\rho_0\omega\mathbf{G}_{32}v_b + i\rho_0\omega\mathbf{G}_{33}v_c + \{p_{inc_c}\} \end{aligned} \tag{36}$$

If the pressure boundary condition on b (p_b), velocity boundary condition on c (v_c), and the incident pressure on a,b and c are known, by taking to the left side all the unknown the above equation can be written as:

$$\begin{aligned} \rho_0\omega^2\mathbf{G}_{11}\mathbf{T}x + \mathbf{H}_{11}p_a - i\rho_0\omega\mathbf{G}_{12}v_b + \mathbf{H}_{13}p_c &= -\mathbf{H}_{12}p_b + i\rho_0\omega\mathbf{G}_{13}v_c + \{p_{inc_a}\} \\ \rho_0\omega^2\mathbf{G}_{21}\mathbf{T}x + \mathbf{H}_{21}p_a - i\rho_0\omega\mathbf{G}_{22}v_b + \mathbf{H}_{23}p_c &= -\mathbf{H}_{22}p_b + i\rho_0\omega\mathbf{G}_{23}v_c + \{p_{inc_b}\} \\ \rho_0\omega^2\mathbf{G}_{31}\mathbf{T}x + \mathbf{H}_{31}p_a - i\rho_0\omega\mathbf{G}_{32}v_b + \mathbf{H}_{33}p_c &= -\mathbf{H}_{32}p_b + i\rho_0\omega\mathbf{G}_{33}v_c + \{p_{inc_c}\} \end{aligned} \tag{37}$$

Since the pressure p on FEM equation lies in region a, Eq. (22) can be written as

$$[\mathbf{M}]\{\ddot{x}\} + [\mathbf{C}]\{\dot{x}\} + [\mathbf{K}]\{x\} - q_\infty[\mathbf{A}(ik)]\{x\} + [\mathbf{L}]\{p_a\} = \{\mathbf{F}\} \tag{38}$$

where p_a is the total acoustic pressure resulting from the application of acoustic disturbance force to the structure, which consists of the incident acoustic pressure p_{inc} and scattering acoustic pressure p_{sc}. The scattering acoustic pressure will be dependent on the dynamic response of the structure due to the incident acoustic pressure. Following the general practice in structural dynamics, solutions of Eq. (31) are sought by considering synchronous motion with harmonic frequency ω. Correspondingly, Eq. (31) reduces to:

$$\left[\mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M}\right]\{\bar{x}\} - q_\infty[\mathbf{A}(ik)]\{x\} + [\mathbf{L}]\{\bar{p}_a\} = \{\overline{\mathbf{F}}\} \tag{39}$$

where

$$x = \bar{x}e^{i\omega t}; p_a = \bar{p}_a e^{i\omega t} \tag{40}$$

or, dropping the bar sign for convenience, but keeping the meaning in mind, Eq. (39) can be written as

$$\left[\mathbf{K}+i\omega\mathbf{C}-\omega^2\mathbf{M}\right]\{x\}-q_\infty\left[\mathbf{A}\left(ik\right)\right]\{x\}+\left[\mathbf{L}\right]\{p_a\}=\{\mathbf{F}\} \tag{41}$$

Combining Eq. (37) and (41), the coupled BEM-FEM equation can then be written as:

$$\begin{bmatrix} \left[\mathbf{K}+i\omega\mathbf{C}-\omega^2\mathbf{M}\right]-q_\infty\left[\mathbf{A}\left(ik\right)\right]\{x\} & \mathbf{L} & 0 & 0 \\ \rho_0\omega^2\mathbf{G}_{11}T & \mathbf{H}_{11} & -i\rho_0\omega\mathbf{G}_{12} & \mathbf{H}_{13} \\ \rho_0\omega^2\mathbf{G}_{21}T & \mathbf{H}_{21} & -i\rho_0\omega\mathbf{G}_{22} & \mathbf{H}_{23} \\ \rho_0\omega^2\mathbf{G}_{31}T & \mathbf{H}_{31} & -i\rho_0\omega\mathbf{G}_{32} & \mathbf{H}_{33} \end{bmatrix}\begin{Bmatrix} x \\ p_a \\ v_b \\ p_c \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ -\mathbf{H}_{12}p_b+i\rho_0\omega\mathbf{G}_{13}v_c+p_{inc_a} \\ -\mathbf{H}_{22}p_b+i\rho_0\omega\mathbf{G}_{23}v_c+p_{inc_b} \\ -\mathbf{H}_{32}p_b+i\rho_0\omega\mathbf{G}_{33}v_c+p_{inc_c} \end{Bmatrix} \tag{42}$$

This equation forms the basis for the treatment of the fluid-structure interaction in a unified fashion. The solution vector consisting of the displacement vector of the structure and total acoustic pressure on the boundaries of the acoustic domain, including the acoustic-structure interface, can be obtained by solving Eq. (42) as a dynamic response problems.

4 Further Treatment for AAC; Acoustic-Aerodynamic Analogy

At this point, the solution approach philosophy is in order. Analogous to the treatment of dynamic aeroelastic stability problem of structure, in which the aerodynamic effects can be distinguished into motion independent (self-excited) and motion induced aerodynamic forces, the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aero-elastic problem) can be viewed to consist of structural motion independent incident acoustic pressure (excitation acoustic pressure) and structural motion dependent acoustic pressure, which is known as the scattering pressure. However the scattering acoustic pressure is also dependent on the incident acoustic pressure. The consequence of such treatment has been adopted in the above section and will further be implemented in the subsequent development.

For aeroelastic calculation purposes, further treatment to simplify Eq. (42) will be carried out. Since the pressure boundary condition on b (p_b= 0) and velocity boundary condition on c (v_c = 0), Eq. (37) can be written as:

$$\rho_0 \omega^2 \mathbf{G}_{11} \mathbf{T} x + \mathbf{H}_{11} p_a - i\rho_0 \omega \mathbf{G}_{12} v_b + \mathbf{H}_{13} p_c = p_{inc_a} \tag{43a}$$

$$\rho_0 \omega^2 \mathbf{G}_{21} \mathbf{T} x + \mathbf{H}_{21} p_a - i\rho_0 \omega \mathbf{G}_{22} v_b + \mathbf{H}_{23} p_c = p_{inc_b} \tag{43b}$$

$$\rho_0 \omega^2 \mathbf{G}_{31} \mathbf{T} x + \mathbf{H}_{31} p_a - i\rho_0 \omega \mathbf{G}_{32} v_b + \mathbf{H}_{33} p_c = p_{inc_c} \tag{43c}$$

Since $\mathbf{G}_{22}$ and $\mathbf{H}_{33}$ are square matrices, equation (43b) and (43c) can be written as

$$v_b = -\frac{i}{\rho_0 \omega} \left[\mathbf{G}_{22}\right]^{-1} \left(\rho_0 \omega^2 \mathbf{G}_{21} \mathbf{T} x + \mathbf{H}_{21} p_a + \mathbf{H}_{23} p_c - p_{inc_b}\right) \tag{44a}$$

$$p_c = -\left[\mathbf{H}_{33}\right]^{-1} \left(\rho_0 \omega^2 \mathbf{G}_{31} \mathbf{T} x + \mathbf{H}_{31} p_a - i\rho_0 \omega \mathbf{G}_{32} v_b - p_{inc_c}\right) \tag{44b}$$

Substituting equation (44b) into equation (43b)

$$\rho_0 \omega^2 \mathbf{A}_{21} \mathbf{T} x + \mathbf{B}_{21} p_a - i\rho_0 \omega \mathbf{A}_{22} v_b + \mathbf{B}_{23} p_{inc_c} - p_{inc_b} = \{0\} \tag{45}$$

where

$$\begin{aligned} \mathbf{A}_{21} &= \left(\mathbf{G}_{21} - \mathbf{H}_{23}\left[\mathbf{H}_{33}\right]^{-1} \mathbf{G}_{31}\right) \\ \mathbf{B}_{21} &= \left(\mathbf{H}_{21} - \mathbf{H}_{23}\left[\mathbf{H}_{33}\right]^{-1} \mathbf{H}_{31}\right) \\ \mathbf{B}_{23} &= \left(\mathbf{H}_{23}\left[\mathbf{H}_{33}\right]^{-1}\right) \\ \mathbf{A}_{22} &= \left(\mathbf{G}_{22} - \mathbf{H}_{23}\left[\mathbf{H}_{33}\right]^{-1} \mathbf{G}_{32}\right) \end{aligned} \tag{46}$$

Since $\mathbf{A}_{22}$ is square matrix, Eq. (45) can be written as:

$$v_b = -\frac{i}{\rho_0 \omega} \left[\mathbf{A}_{22}\right]^{-1} \left(\rho_0 \omega^2 \mathbf{A}_{21} \mathbf{T} x + \mathbf{B}_{21} p_a + \mathbf{B}_{23} p_{inc_c} - p_{inc_b}\right) \tag{47}$$

Substituting equation (44a) into equation (43c)

$$\rho_0 \omega^2 \mathbf{C}_{\mathbf{31}} \mathbf{T} x + \mathbf{A}_{\mathbf{31}} p_a + \mathbf{A}_{33} p_c + \mathbf{B}_{32} p_{inc_b} - p_{inc_c} = \{0\} \tag{48}$$

where

$$\begin{aligned} \mathbf{C}_{\mathbf{31}} &= \left(\mathbf{G}_{31} - \mathbf{G}_{32}\left[\mathbf{G}_{22}\right]^{-1} \mathbf{G}_{21}\right) \\ \mathbf{A}_{\mathbf{31}} &= \left(\mathbf{H}_{31} - \mathbf{G}_{32}\left[\mathbf{G}_{22}\right]^{-1} \mathbf{H}_{21}\right) \\ \mathbf{A}_{33} &= \left(\mathbf{H}_{33} - \mathbf{G}_{32}\left[\mathbf{G}_{22}\right]^{-1} \mathbf{H}_{23}\right) \\ \mathbf{B}_{32} &= \left(\mathbf{G}_{32}\left[\mathbf{G}_{22}\right]^{-1}\right) \end{aligned} \tag{49}$$

Since $\mathbf{A}_{33}$ is square matrix, Eq. (48) can be written as:

$$p_c = -\left[\mathbf{A}_{33}\right]^{-1}\left(\rho_0\omega^2\mathbf{C_{31}}\mathbf{T}x + \mathbf{A_{31}}p_a + \mathbf{B}_{32}p_{inc_b} - p_{inc_c}\right) \tag{50}$$

Substituting Eq. (47) and (50) into equation (43a)

$$\rho_0\omega^2\mathbf{D}_{11}\mathbf{T}x + \mathbf{E}_{11}p_a + \mathbf{F}_{12}p_{inc_b} + \mathbf{F}_{13}p_{inc_c} = \{p_{inc_a}\} \tag{51}$$

where

$$\begin{aligned}
\mathbf{D}_{11} &= \left(\mathbf{G}_{11} - \mathbf{G}_{12}\left[\mathbf{A}_{22}\right]^{-1}\mathbf{A}_{21} - \mathbf{H}_{13}\left[\mathbf{A}_{33}\right]^{-1}\mathbf{C}_{31}\right)\\
\mathbf{E}_{11} &= \left(\mathbf{H}_{11} - \mathbf{G}_{12}\left[\mathbf{A}_{22}\right]^{-1}\mathbf{B}_{21} - \mathbf{H}_{13}\left[\mathbf{A}_{33}\right]^{-1}\mathbf{A}_{31}\right)\\
\mathbf{F}_{12} &= \mathbf{G}_{12}\left[\mathbf{A}_{22}\right]^{-1} - \mathbf{H}_{13}\left[\mathbf{A}_{33}\right]^{-1}\mathbf{B}_{32}\\
\mathbf{F}_{13} &= \mathbf{H}_{13}\left[\mathbf{A}_{33}\right]^{-1} - \mathbf{G}_{12}\left[\mathbf{A}_{22}\right]^{-1}\mathbf{B}_{23}
\end{aligned} \tag{52}$$

Since $\mathbf{E}_{11}$ is square matrix

$$p_a = -\left[\mathbf{E}_{11}\right]^{-1}\left(\rho_0\omega^2\mathbf{D}_{11}\mathbf{T}\{x\} - p_{inc_a} + \mathbf{F}_{12}p_{inc_b} + \mathbf{F}_{13}p_{inc_c}\right) \tag{53}$$

Matrix $\mathbf{E}_{11}$ and $\mathbf{D}_{11}$ are also a square matrix, finally by substituting equation (53) into equation (41) BEM-FEM aero-acoustic-structure coupling can be obtained as:

$$\begin{aligned}
&\left[\mathbf{K} + i\omega\mathbf{C} - \omega^2\mathbf{M}\right]\{x\} - q_\infty\left[\mathbf{A}(ik)\right]\{x\}\\
&\quad + \left[\mathbf{L}\right]\left(-\left[\mathbf{E}_{11}\right]^{-1}\left(\rho_0\omega^2\mathbf{D}_{11}\mathbf{T}\{x\} - p_{inc_a} + \mathbf{F}_{12}p_{inc_b} + \mathbf{F}_{13}p_{inc_c}\right)\right) = \{\mathbf{F}\}
\end{aligned} \tag{54}$$

Incident pressure on region b and c will not influence the stability problem associated with the structures, and may at this point be disregarded. Hence, without considering damping matrix $\mathbf{C}$ Eq. (54) simplifies to:

$$\begin{aligned}
&\left[\mathbf{K} - \omega^2\mathbf{M}\right]\{x\} - q_\infty\left[\mathbf{A}(ik)\right]\{x\}\\
&\quad -\rho_0\omega^2\left[\mathbf{L}\right]\left[\mathbf{E}_{11}\right]^{-1}\left[\mathbf{D}_{11}\right]\left[\mathbf{T}\right]\{x\} = -\left[\mathbf{L}\right]\left[\mathbf{E}_{11}\right]^{-1}\{p_{inc_a}\} + \{\mathbf{F}\}
\end{aligned} \tag{55}$$

or

$$\left[\mathbf{K} - \omega^2\mathbf{M}\right]\{x\} - q_\infty\left[\mathbf{A}(ik)\right]\{x\} - \rho_0\omega^2\left[\mathbf{F}_{ac_{sc}}(k_w)\right]\{x\} = \{\mathbf{F}_{ac_{inc}}(k_w)\} + \{\mathbf{F}\} \tag{56}$$

where

$$\begin{aligned}
\left[\mathbf{F}_{ac_{sc}}(k_w)\right] &= \left[\mathbf{L}\right]\left[\mathbf{E}_{11}\right]^{-1}\left[\mathbf{D}_{11}\right]\left[\mathbf{T}\right]\\
\{\mathbf{F}_{ac_{inc}}(k_w)\} &= -\left[\mathbf{L}\right]\left[\mathbf{E}_{11}\right]^{-1}\{p_{inc_a}\}
\end{aligned} \tag{57}$$

Eq. (56) will not be solved directly since the size of the mass and stiffness matrices of the aircraft model are very large. Instead one uses the modal approach where the structural deformation $\{x\}$ is transformed to the generalized coordinate $\{q\}$ given by the following relation:

$$x = \Phi q \tag{58}$$

where Φ is the modal matrix whose columns contain the lower order natural modes. Pre multiplying by Φ^T and converting dynamic pressure q_∞ into reduced frequency (k) as elaborated in [Djojodihardjo and Safari (2006)], Eq. (56) can then be written as:

$$\begin{aligned}\Phi^T \left[\mathbf{K} - \omega^2 \left(\mathbf{M} + \frac{\rho}{2}\left(\frac{L}{k}\right)^2 [\mathbf{A}(ik)] + \rho_0 [\mathbf{F}_{ac_{sc}}(k_w)]\right)\right] \Phi \{q\} = \\ \Phi^T \{\mathbf{F}_{ac_{inc}}(k_w)\} + \Phi^T \{\mathbf{F}\}\end{aligned} \tag{59}$$

since all of the acoustic terms are functions of wave number (k_w) Eq. (59) can be solved by utilizing iterative procedure.

Incorporation of the scattering acoustic term along with the aerodynamic term in the second term of Eq.(59) can be regarded as one manifestation of the acoustic-aerodynamic analogy followed in this approach.

Further method of approach for the solution of the acousto-aeroelastic problem is then dealt with. Following the same procedure as developed in earlier work [Djojodihardjo and Tendean (2004), Djojodihardjo and Safari (2005)], the acoustic excitation is incorporated by coupling it to the unsteady aerodynamic load in the flutter stability formulation. Linearity and principle of superposition has been assumed. Hence the acoustic loading can be superposed to the aerodynamic loading on the structure, and form the modified aeroelastic equation (acousto-aeroelastic equation) of the structural dynamic problem associated with acoustic and aerodynamic excitation. In the earlier work, tacit consideration is only given to the incident acoustic pressure as the acoustic excitation, without considering the acoustic scattering effects, and without considering $\mathbf{L}$. The incident pressure p_{inc} was also assumed to belong to a certain class that allows its incorporation in the aerodynamics term. Thus eq. (42) was treated in a decoupled fashion. Such approach has given instructive results.

In the present development, rigorous consideration has been devoted to the acoustic scattering problem. Two generic approaches to solve Eq. (42) can be followed. The first is to solve Eq.(42) as a stability equation in a "unified treatment", and the disturbance acoustic pressure already incorporates the total pressure, i.e. the incident plus the scattering acoustic pressure. The treatment of the incident acoustic pressure p_{inc} in Eq. (59) follows similar approach adopted for the scattering acoustic pressure by tuning p_{inc} so that it behaves like the aerodynamic terms in the modal

Eq.(59). This is considered logical for the problem considered since the intention is to look at its enhancement effect to the aerodynamic one, thus only a class of p_{inc} will meet the eigenvalue requirements of Eq.(59). This approach will be elaborated further in the following section. The second generic approach is to solve Eq.(42) as a dynamic response problem due to acoustic excitation. The left hand side of Eq.(42) incorporates the scattering acoustic pressure term. Appropriate algebraic manipulation is carried out to allow modal approach of p_{inc}[Djojodihardjo (2007)].

5 Acoustically Modified Flutter Formulation (Stability Problem) using (K-Method)

For the calculation of the influence of acoustic effect on aeroelastic stability problem, Eq. (59) can be further formulated by making special treatment to the acoustic incident force and without other external forces (**F**=0) in the right hand side. This treatment can be made by "tuning" that term to behave like the aerodynamic terms in generalized variables, in addition to the treatment of the scattering acoustic pressure. This assumption has been made by assuming linearity and principle of superposition. This assumption allows the superposition of the acoustic loading to the aerodynamic loading on the structure, and form the modified aeroelastic equation (acousto-aeroelastic equation) of the structural dynamic problem associated with acoustic and aerodynamic excitation. Define

$$\left\{\mathbf{F}^*_{ac_{inc}}(k_w)\right\} = \Phi^T\left\{\mathbf{F}_{ac_{inc}}(k_w)\right\} \tag{60}$$

Then Eq.(59) can be written as:

$$\left[\mathbf{K}^* - \omega^2\left(\mathbf{M}^* + \frac{\rho}{2}\left(\frac{L}{k}\right)^2\left[\mathbf{A}^*(ik)\right] + \rho_0\left[\mathbf{F}^*_{ac_{sc}}(k_w)\right]\right)\right]\{q\} = \omega^2\left[\mathbf{F}^{**}_{ac_{inc}}(k_w)\right]\{q\} \tag{61}$$

where

$$\mathbf{M}^* = \Phi^{\mathrm{T}}\mathbf{M}\Phi = \text{generalized mass matrix} \tag{62a}$$

$$\mathbf{K}^* = \Phi^{\mathrm{T}}\mathbf{K}\Phi = \text{generalized stiffness matrix} \tag{62b}$$

$$\mathbf{A}^*(\mathrm{ik}) = \Phi^{\mathrm{T}}\mathbf{A}(\mathrm{ik})\Phi = \text{generalized aero matrix} \tag{62c}$$

$$\mathbf{F}^*_{ac_{sc}}(k_w) = \Phi^T\left[\mathbf{F}_{ac_{sc}}(k_w)\right]\Phi \tag{62d}$$

$$\left(\mathbf{F}^{**}_{ac_{inc}}\right)_i = \frac{\left(\mathbf{F}^*_{ac_{inc}}\right)_i}{\omega_i^2 q_i} \tag{62e}$$

As modified aeroelastic stability problem, Eq.(61) can be written as

$$\left[\mathbf{K}^{*}-\omega^{2}\left(\mathbf{M}^{*}+\frac{\rho}{2}\left(\frac{L}{k}\right)^{2}\left[\mathbf{A}^{*}\left(ik\right)\right]+\rho_{0}\left[\mathbf{F}^{*}_{ac_{sc}}\left(k_{w}\right)\right]+\left[\mathbf{F}^{**}_{ac_{inc}}\left(k_{w}\right)\right]\right)\right]\{q\}=\{0\} \tag{63}$$

which can be simplified as:

$$\left[\mathbf{M}^{**}-\lambda\mathbf{K}\right]q=0 \tag{64}$$

where

$$\left[\mathbf{M}^{**}\right]=\left[\mathbf{K}^{*}-\omega^{2}\left(\mathbf{M}^{*}+\frac{\rho}{2}\left(\frac{L}{k}\right)^{2}\left[\mathbf{A}^{*}\left(ik\right)\right]+\rho_{0}\left[\mathbf{F}^{*}_{ac_{sc}}\left(k_{w}\right)\right]+\left[\mathbf{F}^{**}_{ac_{inc}}\left(k_{w}\right)\right]\right)\right] \tag{65}$$

and

$$\lambda=\frac{1+i\mathrm{g}}{\omega^{2}} \tag{66}$$

Eq. (65) is solved as an eigenvalue problem for a series of values for parameters $k$and ρ. Since $\mathbf{M}^{**}$ is in general a complex matrix, the eigenvalues λ are also complex numbers. For n structural modes, there are n eigenvalues corresponding to n modes at each k. The air speed, frequency and structural damping are related to the eigenvalue λ as follows:

$$\omega_{f}=\frac{1}{\sqrt{Re(\lambda)}} \tag{67a}$$

$$U_{f}=\frac{\omega_{f}b}{k} \tag{67b}$$

$$g=\frac{Im\left(\lambda\right)}{Re\left(\lambda\right)} \tag{67c}$$

To evaluate the flutter speed, $V-g$ and $V-f$ diagrams are constructed [Bisplinghof, Ashley and Halfman (1955), Dowell (ed) (1980)]. The V-g diagram plots the structural damping as a function of velocity, and the V-f diagram plots the frequency as a function of velocity. The flutter critical speeds is indicated in the $V-g$ diagram as the lowest velocity Vat which the $g$curve crosses the Vaxis from its negative (stable region) to its positive value (unstable region), i.e. when g= 0.

6 Numerical Results

6.1 Acoustic Boundary Element Simulation

In order to verify the validity the boundary elements acoustic models, a numerical test case is conducted to test the validity of the method. To avoid complexity, the acoustic source is assumed to be a monopole source which creates the acoustic pressure. This acoustic pressure is interacting with the unsteady aerodynamic forces.

For a pulsating sphere an exact solution for acoustic pressure a at a distance r from the center of a sphere with radius a pulsating with uniform radial velocity U_a is

$$p\left(r\right)=\frac{a}{r}U_a\frac{iz_0ka}{1+ika}e^{-ik(r-a)} \tag{68}$$

where z_0 is the acoustic characteristic impedance of the medium and k is the wave number.

Figure 5 shows the discretization of the surface elements of an acoustics pulsating sphere representing a monopole source. BEM calculation for scattering pressure from acoustic monopole source will be compared with exact results. Figure 6 which shows the excellent agreement between the computational procedure developed and the exact one.

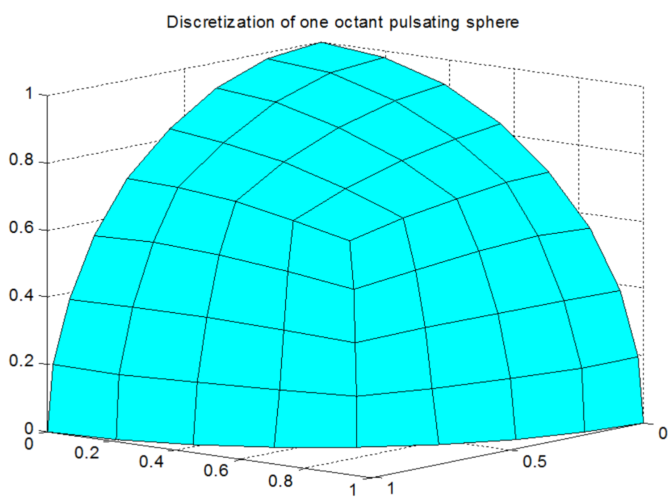


Figure 5: Discretization of one octant pulsating sphere

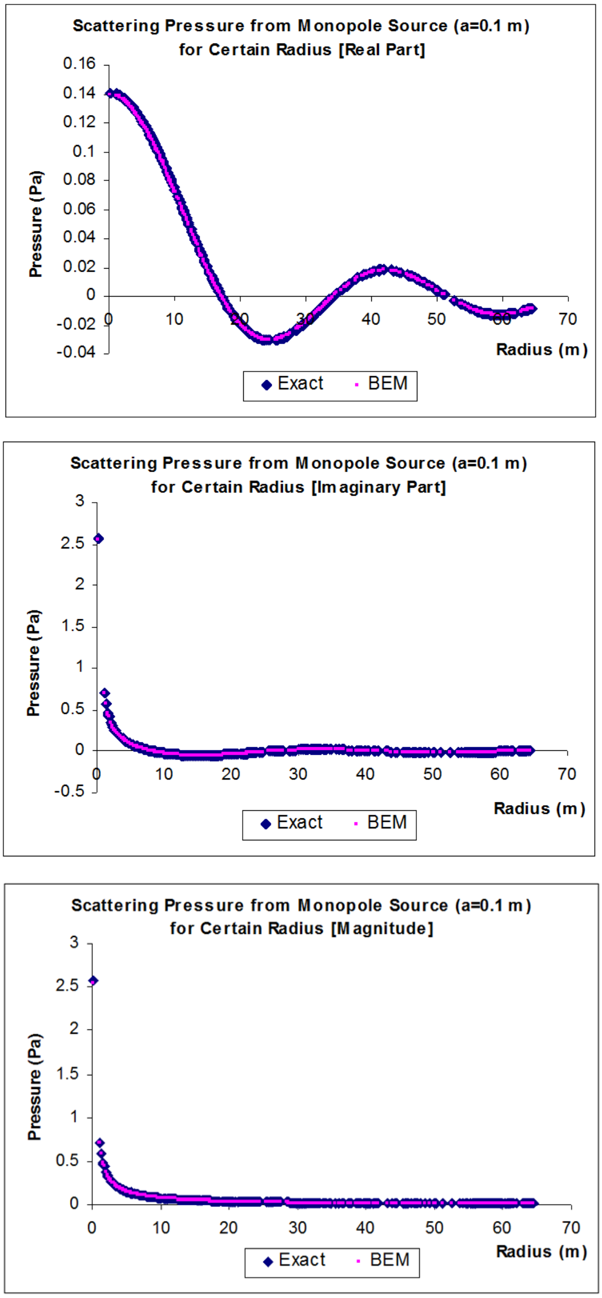


Figure 6: Comparison of monopole source exact and BEM scattering pressure results

The calculation depicted in **Figure 6** was based on the assumption of f=10 Hz, ρ=1.225 Kg/m^3, and c=340 m/s. The excellent agreement of these results with exact calculation serves to validate the developed MATLAB® program for further utilization.

6.2 Coupled BEM-FEM Numerical Simulation

The BAH wing structure [Bisplinghoff, Ashley and Halfman (1955)] and the surrounding boundary representing quarter space of the problem are discretized as shown in Figure 7. The problem domain is divided into two parts. First the near field region is a quarter space with radius of two times the BAH wing span and is relatively more densely discretized; second, the intermediate to far field region is a quarter space with radius ten times the BAH wing span and is less densely discretized compared to the near field region, and is modeled with BEM only.

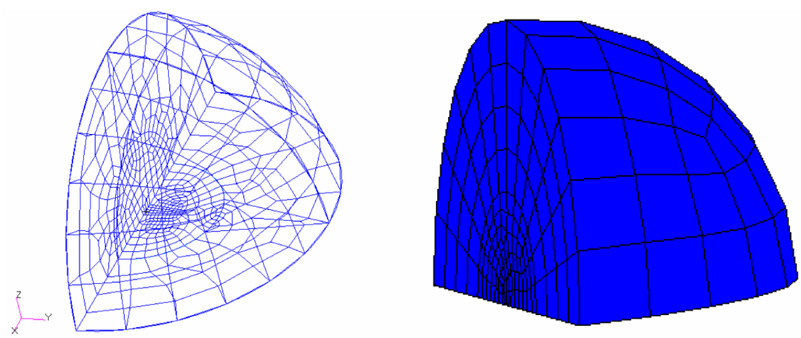


Figure 7: 3-D domain representing of BAH wing structure and its surrounding boundary

The BAH wing which is modeled as FEM and BEM is subjected to an excitation due to an acoustic monopole source; the acoustic medium is air with density ρ =1.225 kg/m^3 and the sound velocity is c = 340 m/s. The monopole acoustic source is placed at the intersecting line of the half span and half chord planes of the BAH wing structure, and at about 0.1 m above the wing surface. The monopole source has the frequency of 10 Hz, radius of a = 0.1 m. The result of applying Eq.(62) for F = 0 is presented as the incident, total and scattering pressures drawn as color-coded diagrams in Figure 8, which qualitatively exhibit the expected behavior. It could be added that in the example considered, CHIEF method has also been utilized in the BEM part to take care of the fictitious frequency problem, so that such phenomena can be eliminated in the BEM-FEM acoustic-structure coupling.

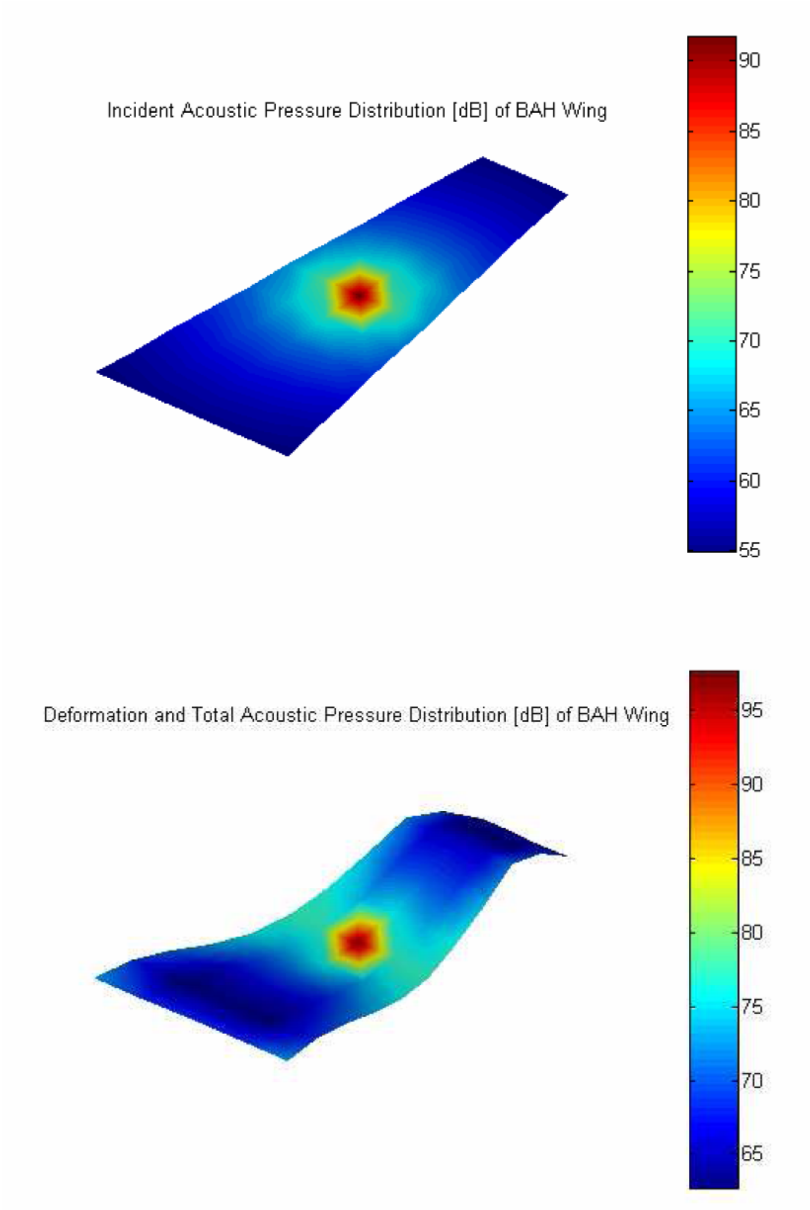
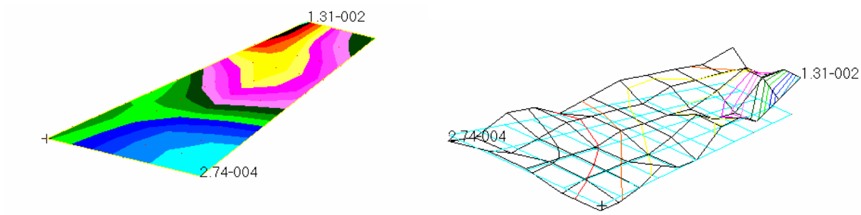


Figure 8: Pressure distribution on symmetric equivalent BAH wing; a) Incident pressure [dB] from monopole acoustic source as an acoustic excitation and b) deformation and total acoustic pressure response [dB]

6.3 Flutter Calculation for Coupled Unsteady Aerodynamic and Acoustic Excitations



(a) (b)

Figure 9: (a) Unsteady aerodynamics pressure distribution (C_p), and (b) Mode shape of the wing structure when flutter occurs

In earlier work [Djojodihardjo and Tendean (2004), Djojodihardjo and Safari (2005)], the acoustic excitation is incorporated by coupling it to the unsteady aerodynamic load in the flutter stability calculation. Linearity and principle of superposition has been assumed.

Hence the acoustic loading can be superposed to the aerodynamic loading on the structure, and form the modified aeroelastic equation (acousto-aeroelastic equation) of the structural dynamic problem associated with acoustic and aerodynamic excitation. In the earlier work, tacit consideration was only given to the incident acoustic pressure as the acoustic excitation, without considering the acoustic scattering effects, and without considering **L**.

The incident pressure p_{inc} was also assumed to belong to a certain class that allows its incorporation in the aerodynamics term. Thus Eq.(62) was treated in a decoupled fashion, Such approach has resulted in the delay of the inception of flutter. Figure 9 shows the unsteady aerodynamics pressure (C_p) and mode shape of the structure when flutter occurs.

In the present development, rigorous consideration has been devoted to the acoustic scattering problem. by solving Eq.(62) as a stability equation in a ”unified treatment”, and the disturbance acoustic pressure already incorporates the total pressure, which has been “tuned” to behave like the aerodynamic terms in the modal Eq.(62). The solution is exhibited as Figure10, which shows that the flutter inception is delayed at a higher speed also.

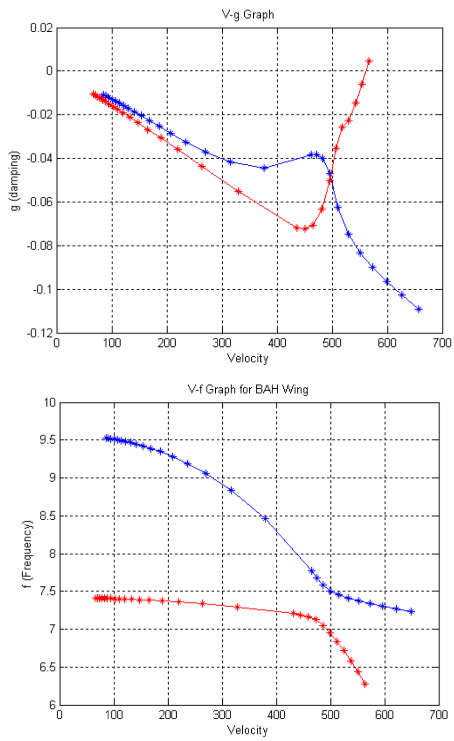
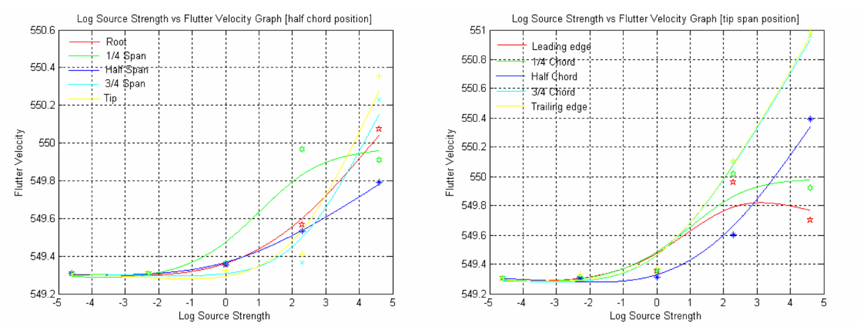


Figure 10: Damping and frequency diagram for BAH wing calculated using V-g method written in MATLAB® for the acousto-aeroelastic problem (the total acoustic pressure already incorporates the scattering pressure).

7 AAC Parametric Study

The computational scheme for the distribution of acoustic pressure on the surface of the pulsating sphere, the total acoustic pressure for coupled BEM-FEM problem, and the influence of placing an acoustic monopole above a three dimensional wing (a BAH wing) to the flutter velocity, by using coupled BEM-FEM formulation for the acoustic incident pressure induced by the monopole source have been validated using NASTRAN® [Rodden and Johnston(1994)] and ZAERO® [Zona Tech (2004)]. The calculation of the unsteady aerodynamic terms in Eq.(57) is carried out using Doublet Point Method as elaborated in [Houbolt (1969), Ueda and Dowell (1982)], and developed into a routine written in MATLAB® , as elaborated in [Djojodihardjo and Safari (2006)].

It is of interest to look into some simple applications to obtain the usefulness of the method. Along this line, several parametric studies are carried out. The first study looks into the influence of the intensity of the acoustic source on the flutter stability by varying its location above the wing. Figure11(a) indicates that the most effective way in placing the acoustic monopole source is on the tip of the wing and Figure11(b) indicates that the most effective way in placing the acoustic monopole source is on the trailing edge of the wing.



(a) (b)

Figure 11: The Influence of Acoustic Monopole Source Intensity on flutter velocity as a function of Monopole position; (a) at midchord along wing span, (b) at wing-tip section along the chord

Next the influence of the distance between the acoustic source and the wing on the flutter stability is investigated. Figure12 exhibits the results of such study and indicates that the most effective way in placing the acoustic monopole source is on the nearest distance from the wing. These results serve to indicate the logical trend of such problem, which will be useful for further practical applications. However, the favorable effect of the introduction of a monopole source closer to the wing should be accompanied by the increase of its strength.

8 Concluding Remarks

The computational scheme for the calculation of the influence of the acoustic disturbance to the aeroelastic stability of a structure has been developed using a unified treatment by applying acoustic aerodynamic analogy.

By considering the effect of acoustic pressure disturbance to the aeroelastic structure (acousto-aero-elastic problem) to consist of structural motion independent incident acoustic pressure and structural motion dependent acoustic pressure, the scattering acoustic pressure can be grouped to-gether in the aerodynamic term of the

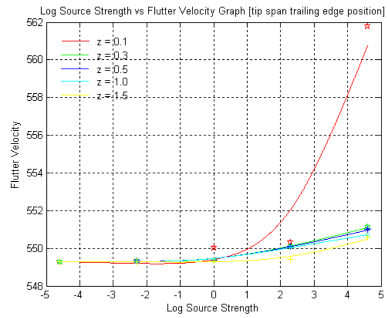


Figure 12: The Influence of Acoustic Monopole Source Strength on Flutter Velocity as a function of the distance of the Monopole Source above the wing from the tip-chord point of the wing-tip section

aeroelastic equation. By tuning the incident acoustic pressure, it can also be incorporated along with the scattering acoustic term, forming the acousto-aeroelastic stability equation. For this purpose the topology of the problem domain has been defined to consist of those subjected to acoustic pressure only and that subject to acoustic structural coupling, which is treated as acousto-aeroelastic equation. Using BE and FE as appropriate, an integrated formulation is then obtained as given by the governing Eq. (42), which relates all the combined forces acting on the structure to the displacement vector of the structure. The solution of Eq. (42) – and after using modal approach in structural dynamics, Eq.(59) - can be obtained by solving Eq.(63) as a stability equation in a "unified treatment", and the disturbance acoustic pressure already incorporates the total pressure (incident plus scattering pressure), which has been "tuned" to behave like the aerodynamic terms in the modal Eq.(61). Such approach allows the application of the solution of the acousto-aeroelastic stability equation in the frequency domain using V-g method. Such technique forms the first generic approach to solve Eq.(42). Alternatively, the acousto-aeroelastic equation part can also be treated as a dynamic response problem, which forms the second generic approach and which has been dealt with in [Djojodihardjo and Safari (2006) and [Djojodihardjo (2007)].

The method developed has been demonstrated to be capable of solving the acoustic-aero-elastomechanic coupling problem. Specifically, the results of both generic approaches to the example worked out show that the presence of acoustic excitation at a frequency near the original flutter frequency can delay the flutter inception, thus confirming our expectation. Further improvements in the computational technique based on efficient algorithm and specific numerical behavior may take advantage of the work of Schanz (2010) and Wu, Liu and Jiang (2012).

List of Symbols

$[AIC]$: Aerodynamics Influence Coefficient
$[A(ik)]$: Unsteady Aerodynamics Matrix
b : wing chord/span chosen for convenience
$[\mathbf{C}]$: Viscous Damping
c : constant for BEM equation, or speed of sound
$[\mathbf{F}]$: External Forces
G_{ij} : influence coefficient matrices
g : free space green function
H_{ij} : influence coefficient matrices
$[\mathbf{K}]$: Stiffness Matrix
k : reduced frequency
k_w : wave-number, in the Helmholtz equation
$[\mathbf{L}]$: fluid-structure coupling matrix
$[\mathbf{M}]$: Mass Matrix
N : shape function, as implied by the context
n_0 : surface normal vector
p : acoustic pressure
p_{inc} : incident acoustic pressure
p_{sc} : scattering acoustic pressure
q : generalized coordinates
q_∞ : dynamic pressure of the fluid surrounding the structure
R_0 : a point in boundary surface
S : bounding surface
V_f : flutter speed
v : normal velocity vector
δ : Kronecker's delta function
λ : wave length
ρ : air density

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