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An Analysis of the Bottomhole Assembly (BHA) in Directional Drilling, by Considering the Effects of the Axial Displacement

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Abstract: The modeling of the bottomhole assembly (BHA) is an essential problem in directional drilling. Some basic equations for predicting the performance of the BHA are presented in this paper. These equations take into account the effects of the axial displacement. The method of weighted residuals and the Newton-Raphson iterations are used to compute the nonlinear effects of the deformation of the BHA. A computer program is developed for the analysis of the BHA in order to quantitatively predict the performance of the BHA in directional drilling. In addition, a case study is presented to evaluate the effect of the axial displacement in the governing equation on the performance of the BHA. It is concluded that this effect is so small, that it can be ignored in actual calculations, and in the design and operations pertaining to directional drilling.

Keywords: directional drilling; bottomhole assembly; axial displacement; nonlinear; the method of weighted residuals

1 Introduction

Prediction of the performance of a bottomhole assembly (BHA) is an essential problem in improving the efficiency of directional drilling. So far, there have been several theories for analyzing the static behavior of the BHA under small deformation [Rafie, Ho and Chandra (1986); Williamson and Lubinski (1987); Liu, Gao, and Cui (1988)]. There are also some literatures on methodologies for the analysis of a BHA under large deformation [Ho (1986); Gao and Xu (1995); Wu and Chen (2006); Chen and Wu (2007)]. However, the formulations which take into account the effects of large deformation are always simplified, in order to improve the computational efficiency. The effect of the axial displacement in the governing

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equation, on the side force of the drill bit, is rarely discussed in the earlier literatures. In this paper, the basic equations employed in predicting the performance of the BHA are presented, and the effects of the large deformation are calculated by using the method of weighted residuals and Newton-Raphson iterations. The effect of the axial displacement is also included in the nonlinear governing equation. In addition, the effect of the axial displacement on the side force of the drill bit is evaluated through a case study.

The following hypotheses are adopted in the analysis of a BHA [Gao (1993)]:

1. Each section of the BHA remains elastic.
2. The physical & geometrical parameters are constants in each section of the BHA.
3. The drill bit is always at the center of the bottomhole plane, and there exists no bending moment at the drill bit.
4. There is a point at which the drillstring is tangential to the well wall, and above which the drillstring lies on the lower side of the wellbore.
5. The dynamical effects are ignored in the BHA analysis.

2 Coordinate Systems

It is necessary to describe the geometry of well trajectory as the directional drilling tendency of the BHA is quantified. To facilitate the discussion, three coordinate systems need to be employed, as shown in figure 1.

(1) Fixed global coordinate system *O-NEH*

The global system is fixed with respect to the compass-directions, in which the origin O is located at the center of bottomhole plane, N points to north, E points to east, and H points to vertically downward. The fixed coordinate system is necessary to describe the geometry of the well trajectory.

(2) Reference coordinate system *O-xyz*

The origin O is also located at the center of bottomhole plane as in the fixed global coordinate system, z is along the tangential direction of the wellbore axis (opposite to the drilling direction), x is perpendicular to z-axis pointing to the high side of the wellbore, and the y-axis is perpendicular to the plane in which the x-axis *and the* z-axis intersect. The reference coordinate system is needed to describe the geometry of the drillstring trajectory.

(3) Natural curvilinear coordinate system P-$\xi\eta\zeta$.

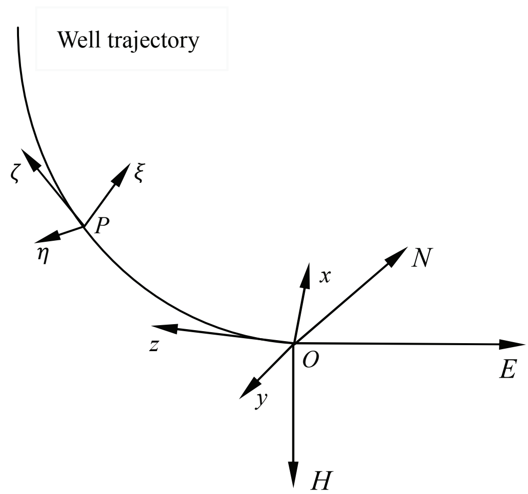

Figure 1: Coordinate systems

This coordinate system is defined by the well trajectory. With an arbitrary point P in the wellbore axis as the origin, the coordinate ξ is along the principal normal direction of the wellbore axis, η points to subnormal direction of the wellbore axis, and ζ is along the tangential direction of wellbore axis (opposite to the drilling direction).

To illustrate the relations between the above three coordinate systems clearly, we set:

$$A = \begin{bmatrix} \cos\omega_p & -\sin\omega_p & 0 \\ \sin\omega_p & \cos\omega_p & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$B = \begin{bmatrix} \cos\alpha_p\cos\phi_p & \cos\alpha_p\sin\phi_p & -\sin\alpha_p \\ \sin\phi_p & -\cos\phi_p & 0 \\ -\sin\alpha_p\cos\phi_p & -\sin\alpha_p\sin\phi_p & -\cos\alpha_p \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} \cos\alpha_0\cos\phi_0 & \cos\alpha_0\sin\phi_0 & -\sin\alpha_0 \\ \sin\phi_0 & -\cos\phi_0 & 0 \\ -\sin\alpha_0\cos\phi_0 & -\sin\alpha_0\sin\phi_0 & -\cos\alpha_0 \end{bmatrix} \tag{3}$$

where, α_0 and ϕ_0 are the inclination and the azimuth, respectively, at the drill bit ($^\circ$), α_P and ϕ_P are the inclination and the azimuth, respectively, at an arbitrary point P in the well trajectory (°), ω_P is the angle which is subtended by rotating clockwise from the high side of the wellbore to the ξ-axis at point P(°).

Then, the relations between three coordinate systems can be shown as follows:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = [C] \begin{Bmatrix} N \\ E \\ H \end{Bmatrix} \tag{4}$$

$$\begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix} = [T] \begin{Bmatrix} N \\ E \\ H \end{Bmatrix}, \ [T] = [A]\,[B] \tag{5}$$

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = [M] \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix}, \ [M] = [C]\,[B]^{-1}\,[A]^{-1} \tag{6}$$

Measured from the drill bit, L is assumed to be the arc length of an arbitrary point P in the well trajectory (m). To measure curvature of the hole, two coefficients, namely, the inclination curvature K_α and the azimuth curvature K_ϕ, are defined as the measured depth increases (°/30m). Then, we can obtain:

$$\begin{cases} \frac{d\alpha(L)}{dL} = -\frac{K_\alpha}{30} \\ \frac{d\phi(L)}{dL} = -\frac{K_\phi}{30} \end{cases} \tag{7}$$

$$\begin{cases} \alpha_p(L) = \alpha_0 + \frac{d\alpha(L)}{dL} \cdot L = \alpha_0 - \frac{K_\alpha}{30} \cdot L \\ \phi_p(L) = \phi_0 + \frac{d\phi(L)}{dL} \cdot L = \phi_0 - \frac{K_\phi}{30} \cdot L \end{cases} \tag{8}$$

The location of an arbitrary point P in the well trajectory, in the fixed global coordinate system *O-NEH* can be defined as (N^*, E^*, H^*), and its displacement components in the reference coordinate system *O-xyz* can be assumed as (x^*, y^*, z^*). Deducing from equations (4) and (5) we obtain the following relations:

$$\begin{cases} N^* = \int_0^L T_{31} dL = -\int_0^L \sin\alpha_p(L)\cos\phi_p(L)\,dL \\ E^* = \int_0^L T_{32} dL = -\int_0^L \sin\alpha_p(L)\sin\phi_p(L)\,dL \\ H^* = \int_0^L T_{33} dL = -\int_0^L \cos\alpha_p(L)\,dL \end{cases} \tag{9}$$

$$\begin{cases} x^* = \cos\alpha_0\cos\phi_0 \cdot N^* + \cos\alpha_0\sin\phi_0 \cdot E^* - \sin\alpha_0 H^* \\ y^* = \sin\phi_0 \cdot N^* - \cos\phi_0 \cdot E^* \\ z^* = -\sin\alpha_0\cos\phi_0 \cdot N^* - \sin\alpha_0\sin\phi_0 \cdot E^* - \cos\alpha_0 \cdot H^* \end{cases} \tag{10}$$

3 A differential equation and its solution

3.1 Nonlinear governing equation

Measured from the reference coordinate system *O-xyz*, the displacement $\overrightarrow{r}(s)$ and the load $\overrightarrow{h}(s)$ per unit length of the drill-string can be expressed as follows:

$$\overrightarrow{r}(s) = u(s)\overrightarrow{i} + v(s)\overrightarrow{j} + w(s)\overrightarrow{k} \tag{11}$$

$$\overrightarrow{h}(s) = h_x(s)\overrightarrow{i} + h_y(s)\overrightarrow{j} + h_z(s)\overrightarrow{k} \tag{12}$$

where s is the arc length measured from the drill bit, $(\overrightarrow{i}, \overrightarrow{j}, \overrightarrow{k})$ are base vectors in the reference coordinate system *O-xyz*, (u, v, w) are the displacement components in the reference coordinate system *O-xyz*, (h_x, h_y, h_z) are the load components in the reference coordinate system *O-xyz*.

During the drilling process, BHA will be subjected to gravity, buoyancy, weight on bit (WOB) and torque. With the consideration of the axial displacement, the nonlinear governing equation for BHA analysis is [Gao (1996)]:

$$\begin{cases} EI\cdot u^{(4)} - EI\cdot\left[(u'')^2+(v'')^2+(w'')^2\right]\cdot u'' + M_t\cdot(v'''w' - v'w''') - T\cdot u'' - h_x = 0 \\ EI\cdot v^{(4)} - EI\cdot\left[(u'')^2+(v'')^2+(w'')^2\right]\cdot v'' + M_t\cdot(u'w''' - u'''w') - T\cdot v'' - h_y = 0 \end{cases} \tag{13}$$

$$\begin{cases} ()' = \frac{d()}{ds}, \quad ()'' = \frac{d^2()}{ds^2}, \quad ()''' = \frac{d^3()}{ds^3}, \quad ()^{(4)} = \frac{d^4()}{ds^4} \\ h_x = -q\cdot\sin\alpha_0 \\ h_y = 0 \\ T = -P + q\cdot\cos\alpha_0\cdot s \end{cases} \tag{14}$$

where, EI is flexural rigidity of the BHA per unit length (kNm^2), M_t is the torque exerted on the BHA per unit length (kNm), T is the axial force in the BHA per unit length (kN), q is the resultant of gravity and buoyancy on the BHA per unit length (kN/m), and P is the z-axis force at the end of each section of the BHA (kN).

Assuming that the drillstring is inextensible [Gao (1994)], we obtain:

$$(u')^2 + (v')^2 + (w')^2 = 1 \tag{15}$$

From equation (15), the terms of axial displacement are isolated, as follows:

$$w' = \left(1 - (u')^2 - (v')^2\right)^{0.5} \tag{16}$$

$$w'' = \frac{1}{2}\left[1-\left(u'\right)^2-\left(v'\right)^2\right]^{-0.5}\left(-2u'u''-2v'v''\right) \tag{17}$$

$$w''' = -(u'u''+v'v'')^2/\left[1-(u')^2-(v')^2\right]^{(1.5)} - \left[(u'')^2+(v'')^2+u'u'''+v'v'''\right]/\left[1-(u')^2-(v')^2\right]^{(0.5)} \tag{18}$$

3.2 Boundary conditions

3.2.1 Condition at the drill bit

From hypothesis (4), the displacement of the BHA at the drill bit is assumed to be zero. In other words:

$$u_{01} = v_{01} = 0 \tag{19}$$

where u_{01} and v_{01} are the displacement components of BHA at the drill bit in the reference coordinate system *O-xyz* (m).

3.2.2 Conditions at the tangency point

The tangency point is defined as the location at which the pipe is tangential to the wall of the hole. Assuming that the displacement components of well trajectory, at the tangency point in the reference coordinate system *O-xyz* are (x_T^*, y_T^*, z_T^*), and the pseudo-radius of the wellbore at the tangency point is r_T (mm), we have the boundary conditions at the tangency point:

$$\begin{cases} u_T = x_T^* - 10^{-3} r_T \\ \qquad \cdot(\cos\alpha_0\cos\phi_0\cos\alpha_T\cos\phi_T + \cos\alpha_0\sin\phi_0\cos\alpha_T\sin\phi_T + \sin\alpha_0\sin\alpha_T) \\ v_T = y_T^* - 10^{-3} r_T\left(\sin\phi_0\cos\alpha_T\cos\phi_T - \cos\phi_0\cos\alpha_T\sin\phi_T\right) \end{cases} \tag{20}$$

$$\begin{cases} u'_T = -\cos\alpha_0\cos\phi_0\sin\alpha_T\cos\phi_T - \cos\alpha_0\sin\phi_0\sin\alpha_T\sin\phi_T + \sin\alpha_0\cos\alpha_T \\ \qquad -10^{-3} r_T\times(\cos\alpha_0\cos\phi_0 D + \cos\alpha_0\sin\phi_0 C - \frac{K_\alpha}{30}\sin\alpha_0\cos\alpha_T) \\ v'_T = -\sin\phi_0\sin\alpha_T\cos\phi_T + \cos\phi_0\sin\alpha_T\sin\phi_T - 10^{-3} r_T\times(\sin\phi_0 D - \cos\phi_0 C) \end{cases} \tag{21}$$

$$\begin{cases} u''_T = -\cos\alpha_0\cos\phi_0 A - \cos\alpha_0\sin\phi_0 B + \frac{K_\alpha}{30}\sin\alpha_0\sin\alpha_T - 10^{-3} r_T \\ \cdot\left[\cos\alpha_0\cos\phi_0(\frac{K_\alpha}{30}A + \frac{K_\phi}{30}C) + \cos\alpha_0\sin\phi_0(\frac{K_\alpha}{30}B - \frac{K_\phi}{30}D) - (\frac{K_\alpha}{30})^2\sin\alpha_0\sin\alpha_T\right] \\ v''_T = -\sin\phi_0 A + \cos\phi_0 B - 10^{-3} r_T\left[\sin\phi_0(\frac{K_\alpha}{30}A + \frac{K_\phi}{30}C) - \cos\phi_0(\frac{K_\alpha}{30}B - \frac{K_\phi}{30}D)\right] \end{cases}$$

(22)

$$\begin{cases} A = -\frac{K_\alpha}{30}\cos\alpha_T\cos\phi_T + \frac{K_\phi}{30}\sin\alpha_T\sin\phi_T \\ B = -\frac{K_\alpha}{30}\cos\alpha_T\sin\phi_T - \frac{K_\phi}{30}\sin\alpha_T\cos\phi_T \\ C = \frac{K_\alpha}{30}\sin\alpha_T\sin\phi_T - \frac{K_\phi}{30}\cos\alpha_T\cos\phi_T \\ D = \frac{K_\alpha}{30}\sin\alpha_T\cos\phi_T + \frac{K_\phi}{30}\cos\alpha_T\sin\phi_T \end{cases} \tag{23}$$

where, u_T and v_T are the displacement components of the drillstring trajectory at the tangency point in the reference coordinate system O-xyz (m), α_T and ϕ_T are the inclination and the azimuth, respectively, at the tangency point (°).

3.3 Conditions of continuity

The conditions of continuity can be summarized as follows. Firstly, the characteristic parameters of physics and mechanics governing the problem, within each section of BHA, are continuous. Secondly, the displacement, the inclination and the bending moment, are continuous between two adjacent sections of the BHA. Thirdly, if the BHA contacts with the well wall, the shear force within BHA will be discontinuous at the contact point.

3.4 Constraint condition of the wellbore

The deformation of the drillstring will definitely be constrained by the wellbore geometry. Assume that the displacement components along the drillstring trajectory in the natural curvilinear coordinate system P-$\xi\eta\zeta$ are (f_ξ, f_η, f_ζ), the following relations are deduced:

$$\sqrt{f_\xi^2 + f_\eta^2} \le r_c \times 10^{-3} \tag{24}$$

$$\begin{Bmatrix} f_\xi \\ f_\eta \\ f_\zeta \end{Bmatrix} = [M]^{-1} \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix} \tag{25}$$

$$\begin{cases} \Delta x = u - x^* \\ \Delta y = v - y^* \\ \Delta z = w - z^* \approx 0 \\ r_c = (DH - DS)/2 \end{cases} \tag{26}$$

where, r_c is pseudo-radius of wellbore (mm), DH is diameter of wellbore (mm), DS is outer diameter of drill-string (mm).

3.5 Solution

The complicated BHA configuration is divided into independent sections. As is well known, the general solution for the governing differential equation of beam deflection is polynomial. Therefore, it is possible to consider polynomials of various orders as the trial functions, while employing the method of weighted residuals. Since the orders of the trial functions determine the accuracy of the solution, here we arbitrarily set:

$$\begin{cases} u = C_7 s^6 + C_6 s^5 + C_5 s^4 + C_4 s^3 + C3 s^2 + C_2 s + C_1 \\ v = C_{14} s^6 + C_{13} s^5 + C_{12} s^4 + C_{11} s^3 + C_{10} s^2 + C_9 s + C_8 \end{cases} \tag{27}$$

where C_1, C_2, C_3, …, C_{14} are the parameters in trial functions for each section of BHA.

Substituting equations (16), (17), (18) and (27) into equation (13), the residuals within each section of BHA are obtained. To eliminate the residuals, six equations are deduced by using the method of moment:

$$\begin{cases} \int_0^L R ds = 0 \\ \int_0^L s \cdot R ds = 0 \\ \int_0^L s^2 \cdot R ds = 0 \end{cases} \tag{28}$$

where R is the residual within each section of BHA, L is the length of each section of BHA (m).

As there are fourteen unknown parameters in the trial functions, there are six equations obtained from equation (28), together with eight boundary conditions aforementioned, the deflection of each section of BHA can be determined.

In detail, the fourteen equations established as above are nonlinear. To solve these equations, the iterative method of New-Raphson can be employed, in which it is feasible to set the initial values of all fourteen unknowns as zero. Besides, the method of numerical integration is required for deducing six equations from equation (28).

After the deflection of each section of BHA is determined, we need to change the location of the tangency point until the bending moment at the drill bit is zero:

$$\left(u''_{01}\right)^2 + \left(v''_{01}\right)^2 = 0 \tag{29}$$

Finally, the inclination force and azimuth force at the drill bit are obtained:

$$\begin{cases} F_x = -EI_1 \cdot u_{01}''' - EI_1 \cdot \left[(u_{01}'')^2 + (v_{01}'')^2 + (w_{01}'')^2 \right] \cdot u_{01}' \\ \qquad -M_{t1} \cdot (v_{01}'' w_{01}' - v_{01}' w_{01}'') + T_1 \cdot u_{01}' \\ F_y = -EI_1 \cdot v_{01}''' - EI_1 \cdot \left[(u_{01}'')^2 + (v_{01}'')^2 + (w_{01}'')^2 \right] \cdot v_{01}' \\ \qquad -M_{t1} \cdot (w_{01}'' u_{01}' - w_{01}' u_{01}'') + T_1 \cdot v_{01}' \end{cases} \tag{30}$$

where (u_{01}, v_{01}, w_{01}) are the displacement components of BHA at the drill bit in the reference coordinate system *O-xyz* (m), F_x is the inclination force at the drill bit, which is the component in the vertical plane that contains the drill bit axis (kN), F_y is the azimuth force at the drill bit which is the component in the horizontal plane and perpendicular to the wellbore axis (kN), EI_1 is the flexural rigidity of drill-string at the drill bit (kNm^2), M_{t1} is the torque that suffered by drill-string at the drill bit (kNm), T_1 is the axial force in drill-string at the drill bit (kN).

4 A Case study

Employing the aforementioned method, a program for predicting BHA performance is developed. Because the nonlinear effect of the BHA deformation is usually sensitive to hole curvature, hole curvature is chosen to illustrate the effect of axial displacement on the side force of the drill bit. Two assemblies are used to illustrate this effect.

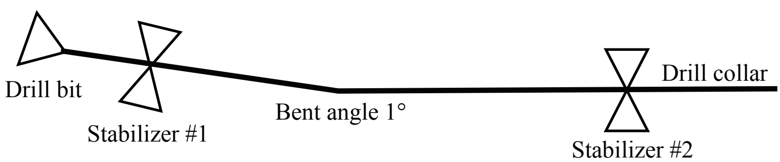

Figure 2: Steerable BHA with two stabilizers and one bent angle

BHA #1 is a conventional steerable system with a 1 degree bend, called a bent housing in a positive displacement motor (PDM), as shown in figure 2. It relies on the bend and stabilizers for a usually precise directional control. This BHA type has also been widely used in the South China Sea. Figure 3 demonstrates the effect of hole curvature on drill bit side force for BHA #1. In this analysis, the BHA consisted mainly of 215.9mm drill bit, 171.45mm PDM, 165.1mm drill collar, 212.7mm stabilizer #1 and 206.3mm stabilizer #2. The distance between the drill bit and the stabilizer #1 is 0.76m. The distance between the drill bit and the bent angle is 1.55m. The distance between two stabilizers is 7.64m.

BHA #2 is an imaginary assembly for microhole drilling. It can be thought of as a small-sized BHA #1. Figure 4 illustrates the effect of hole curvature on drill bit side force for BHA #2. In this analysis, the BHA consisted mainly of 88.9mm drill bit, 73.0mm PDM, 73.0mm drill collar, 85.73mm stabilizer #1 and 82.55mm stabilizer #2. The distance between the drill bit and the stabilizer #1 is 0.32m. The distance between the drill bit and the bent angle is 0.66m. The distance between two stabilizers is 3.25m.

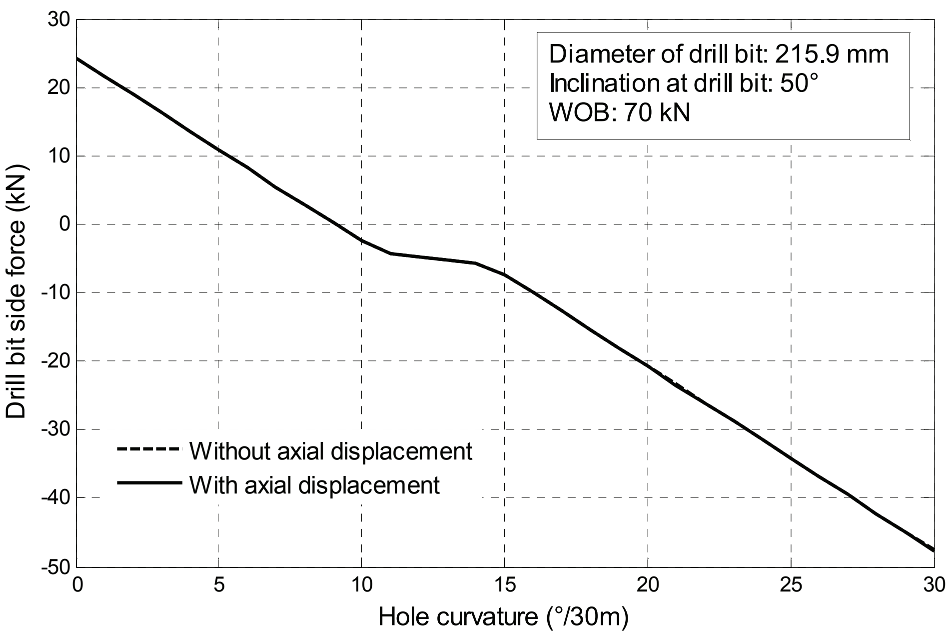

Figure 3: The effect of hole curvature on drill bit side force for BHA #1

In figures 3 and 4, the negative drill bit side force means the BHA is predicted to possess a dropping tendency. As shown in these two figures, the effect of the axial displacement on the drill bit side force is very small, for both the conventional BHA and the small-sized BHA. This effect can be ignored to improve the computational efficiency.

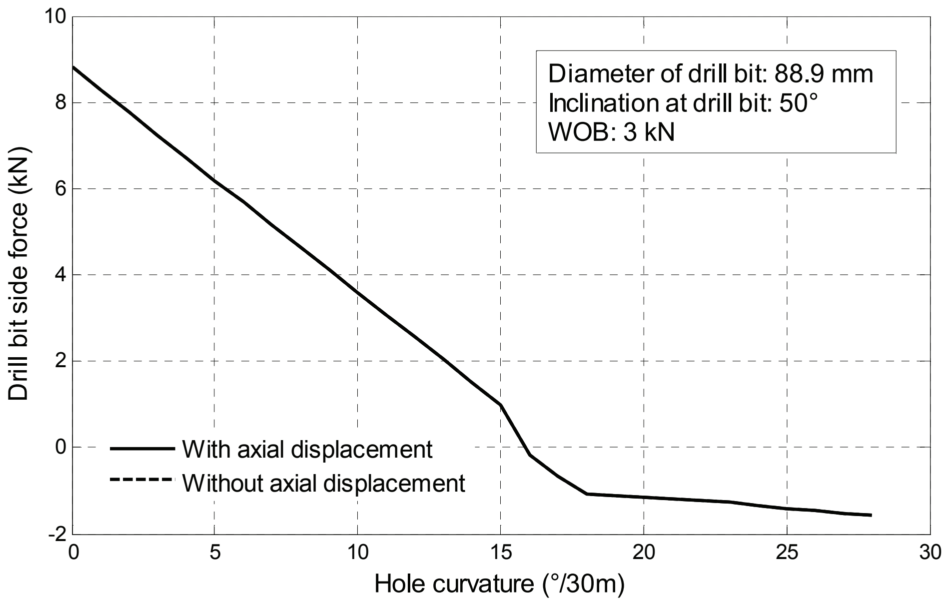

Figure 4: The effect of hole curvature on drill bit side force for BHA #2

5 Conclusions

1. The method of weighted residuals and the Newton-Raphson iterations are used to solve the nonlinear governing equation of the BHA deflection. It is feasible to set the initial values of all unknowns as zero during the iterative computation for BHA analysis.

2. The effect of the axial displacement in the governing equation on the drill bit side force is so small, that it can be ignored, in order to improve the computational efficiency in BHA analysis.

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