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State Estimation of Unequipped Vehicles Utilizing Microscopic Traffic Model and Principle of Particle Filter

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Abstract: The movements of vehicles equipped with various positioning systems such as global and wireless positioning ones have provided beneficial channels to acquire abundant traffic flow information for total road network. However, not all vehicles are mounted with positioning systems and not all equipped positioning facilities are always active. This paper will address how to estimate the number and the states of unequipped vehicles through a series of observations on equipped ones. The proposed estimation process initiates employing the non-analytical microscopic traffic model for particle filter to estimate the number, positions and speeds of unequipped vehicles between an equipped one and another equipped one or a specified point in front. Various kinds of possible particles are utilized for the estimation process. Each kind of particle is composed of a definite number of vehicles (quarks) with possible position and speed distributions. The movements of vehicles in each particle are described through microscopic traffic simulation model. The importance of each particle is iteratively measured by the distinction between the observed states of equipped vehicle and their estimates. The position and speed information of unequipped vehicles can be roughly abstracted through the weighted sum of simulated vehicle movements in the same kind of particles. The subtotal of weights of the same kind of particles stands for the confidence level of corresponding kind of state estimates. Numerical tests have demonstrated the favorable performance of proposed estimation approach. It provides a solution to establish traffic flow database of total road network through limited mobile positioning sensors for the application of traffic planning, route guidance and signal control.

Keywords: state estimation, traffic flow, microscopic model, particle filter.

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1 Introduction

With the wide application of global and wireless positioning systems (GPS and WPS), the mobile vehicles equipped with positioning facilities, simply called *equipped* ones, provide a convenient measure to detect the traffic flow of total road network. Out of consideration for the implementation cost, the traditional inductive loop sensors can not be deployed on all the links of road network, by which only local traffic flow data can be collected. In addition, the loop detectors fixed on roads are inconveniently and costly maintained. Although the equipped vehicles can bring abundant traffic flow data, not all vehicles are equipped with positioning systems and not all positioning facilities of equipped vehicles are always in use. In these two cases, the vehicles are simply called *unequipped* ones. This paper makes an attempt to estimate the movement states of unequipped vehicles through observing the movements of equipped ones.

Recent studies on intelligent transport systems show great interest in traffic flow estimation. A general approach was proposed to estimate the traffic states based on the extended Kalman filter [Wang and Papageorgiou (2005)]. The particle filter was utilized to estimate the traffic states of freeway network based on the cell-transmission model [Mihaylova, Boel, and Hegyi (2007)]. The adaptive maximum likelihood approach was developed to estimate the key parameters in the macroscopic traffic flow model [Ramezani, et al. (2011)]. The extended Kalman filter (EKF) was applied to the state estimation based on the discretized Lagrangian Lighthill-Whitham and Richards (LWR) model [Yuan, et al. (2012)]. The GPS-equipped vehicles were employed to monitor the urban traffic [Shi and Liu (2010)]. An alternative way was presented through the cellular phone data to capture traffic volume [Caceres, et al. (2012)]. The wireless sensor network was utilized to monitor the traffic and improve the quality and safety of mobility [Pascale, et al. (2012)].

The current traffic estimation mainly disposes of the parameters of macroscopic traffic flow models. The macroscopic models do not directly describe the interactions among vehicles and their space distributions [Daganzo (2006); Tyagi, Darbha, and Rajagopal (2008); Vikram, Mittal, and Chakroborty (2011)]. The detailed domain knowledge such as object uncertainty [Myers, et al. (2012)] and feature [Pinho and Tavares (2009)] can facilitate the effectiveness of parameter estimation. Microscopic traffic simulation model precisely reveals the intelligent and stochastic driving behaviors [Nagel and Schreckenberg (1992); Chowdhury, Santen, and Schadschneider (2000); Yoshimura (2006); Fujii, Yoshimura, and Seki (2010); Fujii and Yoshimura (2012)]. The restrictive, synergistic and autonomous movements can be represented with the microscopic traffic model [Zhou, Mi, and Yang (2012)] considering driving feedback behavior and braking reference distance [Zhou and

Mi (2012)]. Although the movements of GPS or WPS-equipped vehicles have been applied to the speed estimation and traffic monitoring of road network, little literature deals with the state estimation of unequipped vehicles through the filtering approach involving the microscopic simulation of traffic flow and the availment of data from mobile sensor-equipped vehicles.

This paper will adopt the microscopic traffic model-based particle filter to address the above problem. Particle filter is the combination of Monte Carlo numerical integration with Bayesian inference. The position and speed of front vehicle will directly restraint the movement of the back adjacent one. If the dependency relationships are unique, assuming there exist N vehicles between two equipped ones, in theory after N time steps, the states of those N vehicles can be identified through the state observation values of the last equipped one. However, the dependency relationships are not unique. Consequently, more time steps will be required to estimate the states of unequipped vehicles. Even so, it intuitively accounts for the feasibility of proposed solution approach. It allows the variation of the dimension of state variables, i.e. multiple kinds of solutions, and moreover, the confidence levels will be assigned to them based on the matching degree of results from Monte Carlo simulations with the true case. The estimates provide sufficient data support for traffic planning, route guidance [Peeta and Mahmassani (1995); Ran and Boyce (1996)], and traffic signal control [Galán Moreno, et al. (2009)].

The rest of this paper is organized as follows. In section 2, the problem statement is represented. Section 3 develops the estimation process based on the microscopic traffic model and the principle of particle filter. The experimental results are demonstrated and elucidated in section 4. Finally, the conclusions are drawn in section 5.

2 Problem statement

2.1 General description

The vehicle movements are described as the following state equation:

$$\begin{aligned}
\dot{x}_{1,1} &= x_{1,2} \\
\dot{x}_{1,2} &= f_1(x_{0,1}, x_{0,2}, x_{1,1}, x_{1,2}) \\
&\vdots \\
\dot{x}_{n,1} &= x_{n,2} \\
\dot{x}_{n,2} &= f_n(x_{n-1,1}, x_{n-1,2}, x_{n,1}, x_{n,2})
\end{aligned} \tag{1}$$

where $x_{1,1}$, …, and $x_{n,1}$ are the positions of vehicle 1, …, and n, respectively, with $x_{1,1} > \cdots > x_{n,1}$. $x_{1,2}$, …, and $x_{n,2}$ are the speeds of vehicle 1, …, and n, respectively. f_1, ..., and f_n are the description functions of accelerations with stochastic characteristics for vehicle 1, ..., and n, respectively, which are related to the speeds and positions of the preceding adjacent vehicle and the current one.

With regard to the problem to be addressed, vehicle 1, ..., and n-1 are assumed to be unequipped and vehicle n equipped. $x_{0,1}$ and $x_{0,2}$ can be the position and speed of another equipped vehicle in front of vehicle 1. However, if $x_{0,1}$ is constant and $x_{0,2} = 0$, $x_{0,1}$ can stand for the position of a specified point in front of vehicle 1 such as intersection. The output equation is defined as

$$\begin{aligned} z_1 &= x_{n,1} + \theta_1 \\ z_2 &= x_{n,2} + \theta_2 \end{aligned} \tag{2}$$

where z_1 and z_2 are the observable outputs, and θ_1 and θ_2 are their random components.

In matrices, Eq. 1 and Eq. 2 are denoted as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bf} \\ \mathbf{z} &= \mathbf{Cx} + \boldsymbol{\theta} \end{aligned} \tag{3}$$

where $\mathbf{x} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{n,1} & x_{n,2} \end{bmatrix}^T$, $\mathbf{z} = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T$, $\mathbf{f} = \begin{bmatrix} f_1 & \cdots & f_n \end{bmatrix}^T$, $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T$, $\mathbf{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & \cdots & 0 \\ 1 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \\ 0 & \cdots & 1 \end{bmatrix}$, and $\mathbf{C} = \begin{bmatrix} 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$.

The problem is to estimate the number of vehicles within a distance of $x_{0,1} - x_{n,1}$ and the state $\mathbf{x}$ through constantly observing output $\mathbf{z}$.

2.2 Microscopic traffic simulation model

There exist stochastic safe distance maintenance and acceleration and deceleration behaviors during the driving process. The microscopic traffic simulation model has been proposed to completely describe those stochastic phenomena [Zhou, Mi and Yang (2012)]. This paper will employ the model to replicate all kinds of stochastic driving behaviors in realistic traffic. The model is outlined as follows.

(1) speed update

S_a : if $d_t \geq d_b^{\max}$, $v \to \min(v + a_{\max},\ v_{\max},\ d_t)$

S_b : elseif $d_b^{\min} < d_t < d_b^{\max}$

begin

if $d_t > d_b$, $v \to \min(v + a,\ v_{\max},\ d_t)$

elseif $d_t = d_b$, $v \to v$

else $v \to \min(\max(v - b,\ 0),\ d_t)$

end

S_c : else $v \to \min(\max(\ v - b_{\max},\ 0),\ d_t)$

(2) position update

$x \to x + v$

where

v : the current speed of the vehicle at current time.

a : the acceleration that the driver is adopting.

b : the deceleration that the driver is adopting.

$v^{\max}$: the expected maximum running speed of the vehicle.

$a^{\min}$: the minimum acceleration that the driver can adopt.

$a^{\max}$: the maximum acceleration that the driver can adopt.

$b^{\min}$: the minimum deceleration that the driver can adopt.

$b^{\max}$: the maximum deceleration that the driver can adopt.

d_t : the current distance of the vehicle to the preceding adjacent one except the vehicle length l_v and the safety margin for stopping.

$d_b^{\min}$: the minimum braking reference distance of the vehicle to the preceding adjacent one when the driver adopts $b^{\max}$.

$d_b^{\max}$: the maximum braking reference distance of the vehicle to the preceding adjacent one when the driver adopts $b^{\min}$.

d_b : the braking reference distance the driver perceptually holds and intends to maintain.

Different drivers have their own sensory estimates about $d_b^{\min}$, $d_b^{\max}$ and d_b. Considering driving feedback, a and b are proposed to be $a = (1 - e^{-\alpha|d_t - d_b|})a_h$ and $b = (1 - e^{-\beta|d_t - d_b|})b_h$, respectively, where α and β are the scaled parameters and can be set as 1 for simplicity. The initial habitual acceleration a_h and deceleration b_h are temporarily chosen according to the difference of speeds between two adjacent vehicles. If acceleration is required, a_h is suggested to be stochastically selected around $a_{\min} + (a_{\max} - a_{\min}) * abs(v_f - v)/v_{\max}$ where v_f is the speed of front adjacent vehicle and *abs* denotes to get absolute value. If deceleration is demanded, b_h is recommended to be stochastically chosen around $b_{\min} + (b_{\max} - b_{\min}) * abs(v - v_f)/v_{\max}$.

3 State estimation utilizing the principle of particle filter

3.1 Particle filter

Particle filter is a technique to represent the posterior density function (*pdf*) through the randomly sampled particles with associated weights and to calculate the state estimates based on the particles and their weights [Ristic, Arulampalam, and Gordon (2004)]. Given $\mathbf{Z}_k = \{\mathbf{z}_i, i = 0, \cdots, k\}$ representing the sequence of all outputs up to instant *k*, the *pdf* $p(\mathbf{x}_k|\mathbf{Z}_k)$ can be approximated as

$$p(\mathbf{x}_k|\mathbf{Z}_k) \approx \sum_{i=1}^{N} w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) \tag{4}$$

where $\mathbf{x}_k^i$ and w_k^i stand for the state and weight of particle *i* at instant *k*, respectively. And *N* is the number of samples. Under the Bayesian inference framework, the weight is iteratively updated as

$$w_k^i \propto w_{k-1}^i p(\mathbf{z}_k \left|\mathbf{x}_k^i\right.) \, p(\mathbf{x}_k^i \left|\mathbf{x}_{k-1}^i\right.) \,/ q(\mathbf{x}_k^i \left|\mathbf{x}_{k-1}^i, \mathbf{z}_k\right.) \tag{5}$$

where the importance density $q(\mathbf{x}_k|\mathbf{x}_{k-1}, \mathbf{z}_k)$ only depends on $\mathbf{x}_{k-1}$ and $\mathbf{z}_k$. If the importance density function adopts the prior one, the weight can be simply updated as

$$\tilde{w}_k^i = w_{k-1}^i p(\mathbf{z}_k \left|\mathbf{x}_k^i\right.) \tag{6}$$

where w_k^i is the normalization of $\tilde{w}_k^i$. Subsequently, the minimum mean-square error estimate of state according to Monte Carlo numerical integration is

$$\hat{\mathbf{x}}_k = E\{\mathbf{x}_k \left| \mathbf{Z}_k \right.\} = \int \mathbf{x}_k \cdot p(\mathbf{x}_k|\mathbf{Z}_k) d\mathbf{x}_k = \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i. \tag{7}$$

3.2 Estimation process

Step 1: Initialize various kinds of particles

Set the domain of possible speeds of unequipped vehicles and $b^{\max}$. Determine $d_b^{\min}$ and the maximum number $N^{\max}$ of vehicles within the distance of $x_{0,1} - x_{n,1}$. The kind number of particles is $N^{\max}$. For each kind number $n \in [1,\ N^{\max}]$, generate N_n^p particles with corresponding speeds and braking reference distances for *n* vehicles within the distance of $x_{0,1} - x_{n,1}$. Each braking reference distance is engendered by $d_b = d_b^{\min} + (d_b^{\max} - d_b^{\min}) * rand$ where *rand* is a random variable yielding to certain probability distribution. The total number of particles is $N^p = \sum_{n=1}^{N^{\max}} N_n^p$. Assign weights $q_0^{n,m} = 1/N^p (n = 1,\ \ldots,\ N^{\max}; m = 1,\ ..,\ N_n^p)$ to N^p particles.

Step 2: Set simulation step *k*=1

Step 3: Predict

FOR n=1, ..., $N^{\max}$

FOR m=1, ..., N_n^p

FOR j=1, ..., n

Undertake microscopic traffic simulation as described in Section 2.2.

END FOR

END FOR

END FOR

If $x_{0,1}$ is located by the equipped vehicle 0, it should be added to n vehicles to lead their movements. The position and speed of vehicle 0 are known. Through microscopic traffic simulation, the prediction $\mathbf{x}_k^{n,m}$ and $\mathbf{z}_k^{n,m}$, i.e. the states and the outputs of the mth one of the nth kind of particles, can be attained at instant k.

Step 4: Update weights

Step 4.1: Initially update weights

FOR n=1, ..., $N^{\max}$

FOR m=1, ..., N_n^p

$$\tilde{q}_k^{n,m} = q_{k-1}^{n,m} p(\mathbf{z}_k^{n,m} \left| \mathbf{x}_k^{n,m} \right.)$$

END FOR

END FOR

Step 4.2: Normalize the weights

FOR n=1, ..., $N^{\max}$

FOR m=1, ..., N_n^p

$$q_k^{n,m} = \tilde{q}_k^{n,m} / \sum_{n=1}^{N^{\max}} \sum_{m=1}^{N_n^P} \tilde{q}_k^{n,m}$$

END FOR

END FOR

Step 5: Output state estimates and their likelihoods

FOR n=1, ..., $N^{\max}$

FOR m=1, ..., N_n^p

$$\hat{q}_k^{n,m} = q_k^{n,m} / \sum_{m=1}^{N_n^P} q_k^{n,m}$$

END FOR

$$\hat{\mathbf{x}}_k^n = \sum_{m=1}^{N_n^p} \mathbf{x}_k^{n,m} \hat{q}_k^{n,m}$$

$$\mathrm{P}_k^n = \sum_{m=1}^{N_n^P} q_k^{n,m}$$

END FOR

Step 6: Resample

If $\hat{N}_{eff} = 1/\sum_{n=1}^{N^{\max}} \sum_{m=1}^{N_n^p} (q_k^{n,m})^2 < N_{eff}^{\max}$

FOR n=1, ..., $N^{\max}$

(1) Construct the cumulative sum of weights (CSW) $c^{n,m}$

$$c^{n,0} = 0$$

FOR m=1, ..., N_n^p

$$c^{n,m} = c^{n,m-1} + \hat{q}_k^{n,m}$$

END FOR

(2) Resample the particles and reinitialize the weights

Start at the bottom of CSW: j=1

Draw a starting point u^1 from the uniform distribution $U(0,\ 1/N_n^p)$

FOR m=1, ..., N_n^p

Move along the CSW: $u^m = u^1 + (m-1)/N_n^p$

WHILE $u^m > c^{n,j}$

$$j = j+1$$

END WHILE

Assign sample: $\mathbf{x}_k^{n,m} = \mathbf{x}_k^{n,j}$

Assign weight: $q_k^{n,m} = 1/N^p$

END FOR

END FOR

END IF

Step 7: $k = k$+1. If $k < K^{\max}$, go to *Step 3*, else stop.

4 Numerical results

4.1 Parameter configuration

The numerical experiments attempt to estimate the number, speed and position of unequipped vehicles between two equipped ones. The variation domain of initial speeds of vehicles is between 40*km/h* and 60*km/h*. $l_v = 5m$, $b^{\min} = 1m/s^2$, $b^{\max} = 5m/s^2$, $a^{\min} = 0$, $a^{\max} = 6m/s^2$, and $v^{\max} = 60km/h$. θ_1 and θ_2 are both assumed to yield to the normal distribution $N(0,\ 5^2)$, whose joint probability density function is $N(0,\ 0, 5^2, 5^2, 0)$. $N^p = 100$ and $K^{\max} = 100$. The simulation period T is $1s$.

4.2 One test sample

There exist 4 vehicles between two equipped vehicles within the initial distance of 227*m*, whose initial speeds are between 40*km*/*h* and 60*km*/*h*. We design 6 kinds of particles, i.e. kind 1, 2, ..., and 6, corresponding to which there exist 0, 1, ..., and 5 vehicles between two equipped ones, respectively. Fig. 1 shows the speeds and positions of the true vehicles in solid lines and the estimated ones in broken lines from the particles of kind 5 in each of whom there exists the same number of vehicles as that of true vehicles. Fig. 2, as an example, demonstrates the estimated speeds and positions in broken lines from the particles of kind 3 in each of whom there exist 2 vehicles between two equipped vehicles. Fig. 3 represents the variation processes of likelihoods of the estimates from 6 kinds of particles. The vehicle numbers shown in the legend of Fig. 3 include two equipped vehicles. From Fig. 1 to Fig. 3, we can learn that the identified number, speeds and positions of vehicles between two equipped ones can approach respective true values with great confidence level. The likelihoods of the estimates from the particles where the number of vehicles between two equipped ones is not consistent with the true one gradually degenerate towards 0. Fig. 4 and Fig. 5 display the total and partial posterior density functions, respectively. The posterior density function describes the likelihood that there exists a vehicle within the interval $[(k-1)l_v, kl_v]$ (k=1, 2, ...), which is obtained from all the particles and only normalized by N^p. It should be noted that one particle may occupy multiple positions on the road at certain instant. At the phase of speed adjustment, the positions of vehicles in particles are fairly uncertain. Therefore, there exist great fluctuations for the posterior density functions from 0*s* to 20*s*. When the vehicles have entered into the steady following state, the positions of vehicles in particles are relatively stable. As a result, the posterior density functions from 80*s* to 100*s* are evenly distributed at the corresponding positions of road.

4.3 Stochastic test

Fig. 6 illustrates the results of 200 random tests where the number of true vehicles is stochastically engendered. Fig. 6 (a) and (b) show the estimation errors of speeds and positions of the last equipped vehicles, which to some degree reflect the estimation errors of the front other vehicles because the speeds and positions are updated from the front vehicle to the back one. Fig. 6 (c) demonstrates the numbers of true vehicles and the identified ones including two equipped vehicles. Fig. 6 (d) depicts the maximum likelihoods corresponding to the results in Fig. 6 (a)-(c). Fig. 6 further testifies that the proposed estimation process can attain the estimates of the number, positions and speeds of unequipped vehicles between two equipped vehicles with remarkable accuracy and confidence levels.

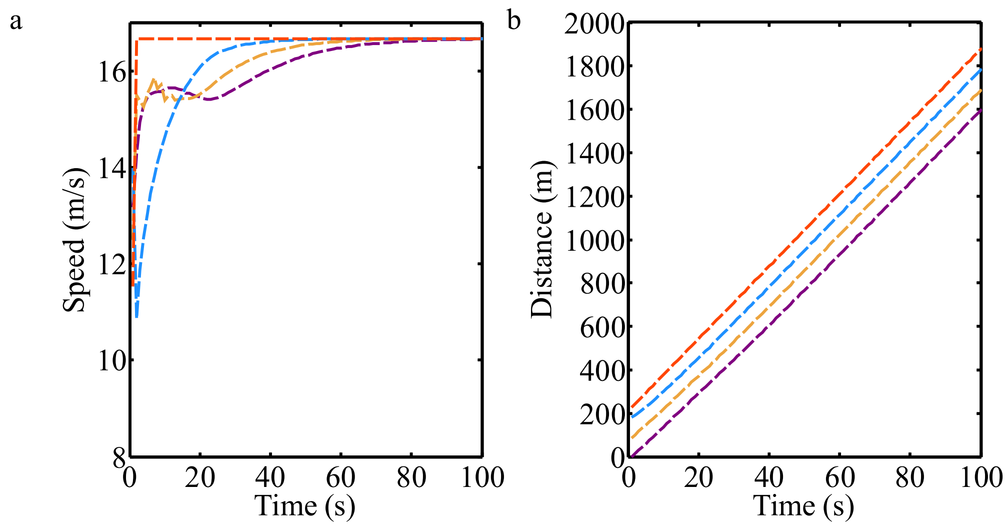


Figure 1: Speeds and positions of true vehicles and their estimates from the particles of kind 5. (a) speed, and (b) position.

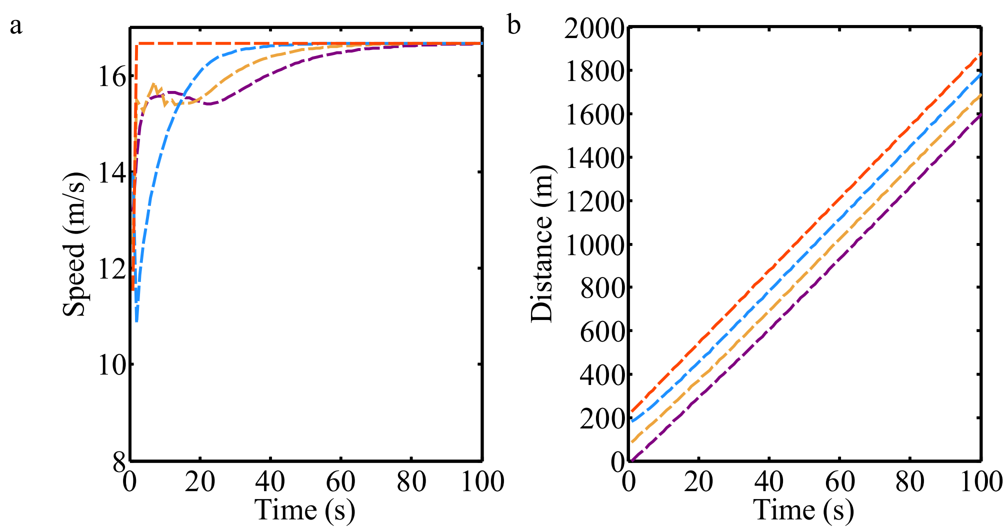


Figure 2: Estimates of speeds and positions from the particles of kind 3. (a) speed, and (b) position.

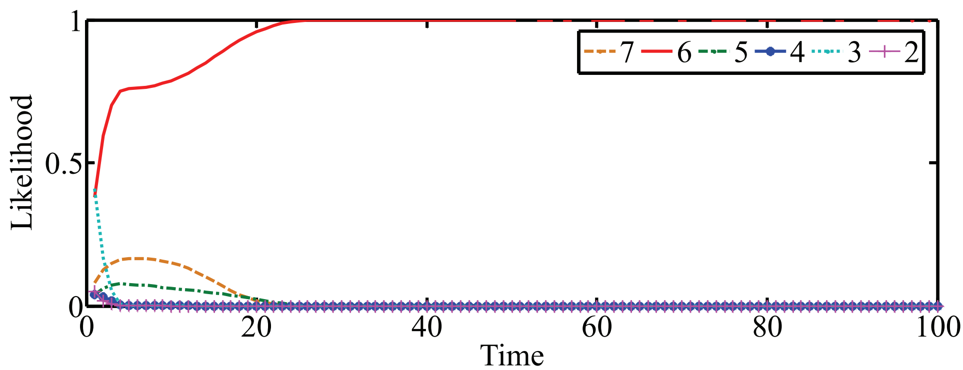


Figure 3: Likelihoods of estimates from 6 kinds of particles.

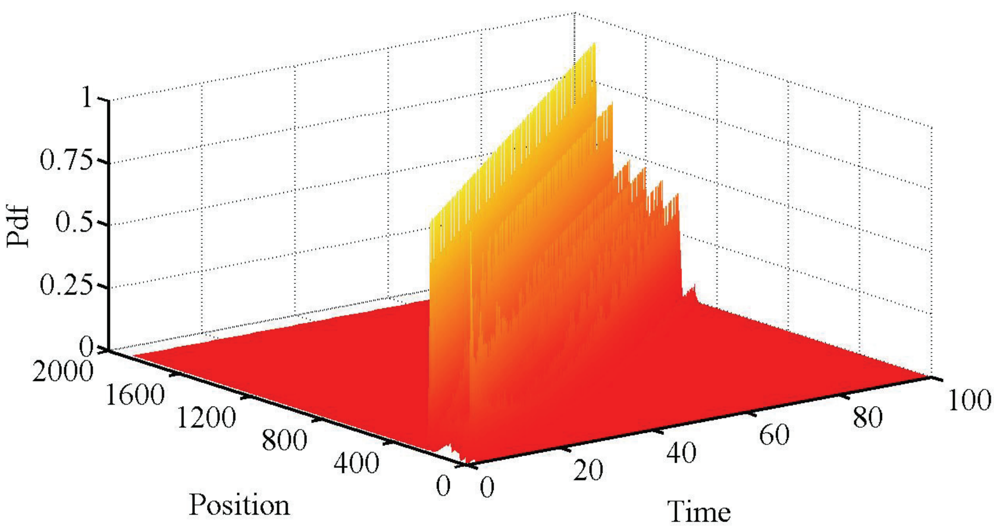
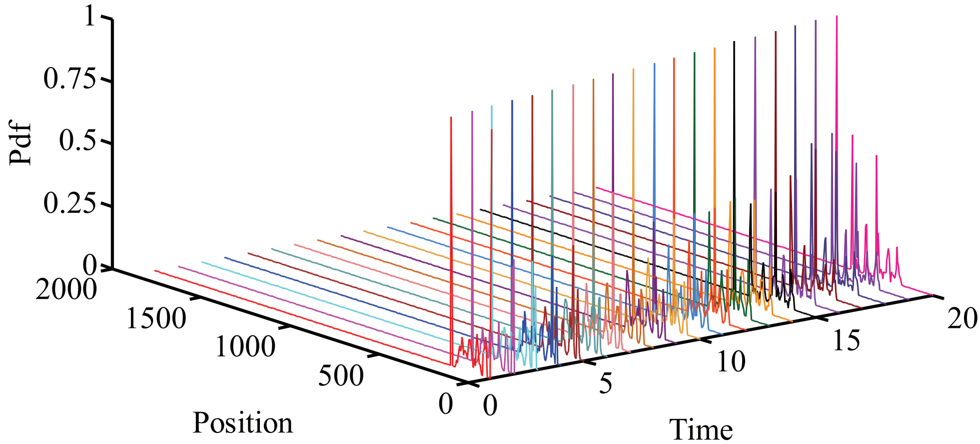


Figure 4: Total posterior density function.

a



b

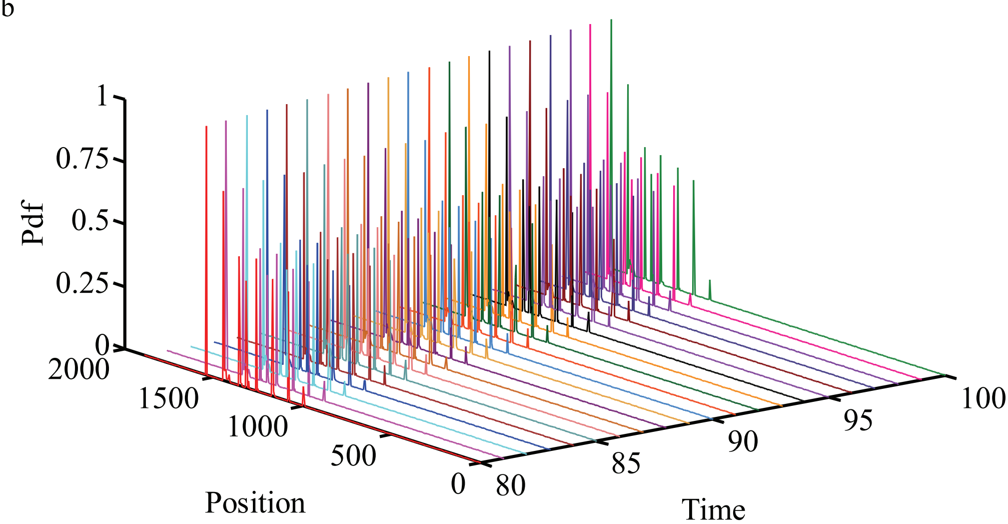


Figure 5: Partial posterior density function. (a) from 0s to 20s, and (b) from 80s to 100s.

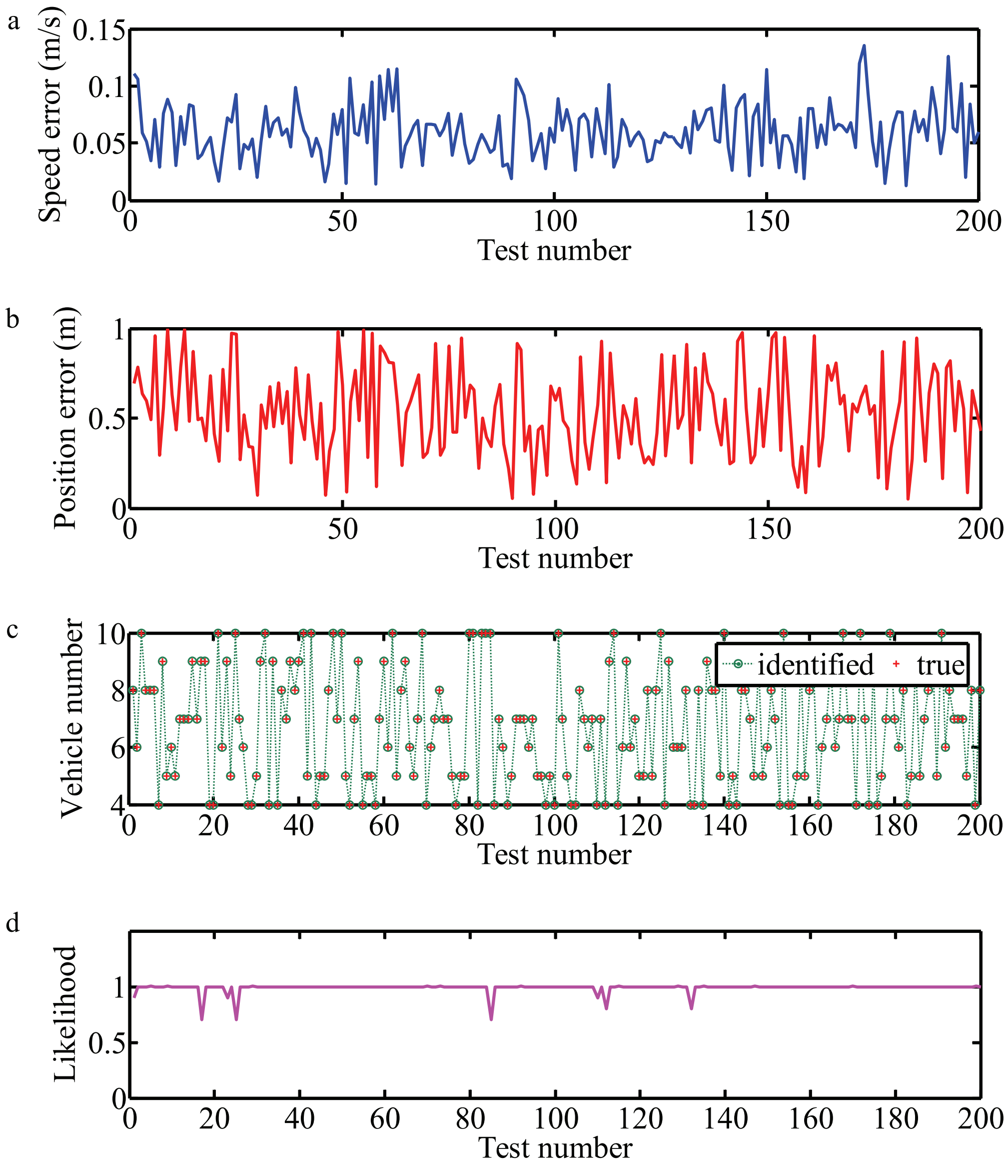


Figure 6: Results of 200 tests. (a) speed estimation error. (b) position estimation error. (c) number of true vehicles and its estimate. (d) maximum likelihood.

5 Conclusions

We have addressed how to estimate the number, speeds and positions of vehicles between an equipped vehicle and another equipped one or a specified point ahead through successively acquiring the movement information of last equipped vehicle. The principle of particle filter based on the microscopic traffic model is utilized to the estimation process. The particles are decomposed into several groups according to the dimension of state variables or the number of vehicles each particle involves. The weights of particles are measured by the differences of states between the equipped vehicle and the corresponding simulated one. The second normalized weights of particles in each group are utilized for the estimation of vehicle movements. Each group of particles thereupon possesses the respective estimate whose likelihood is measured by the accumulation of the first normalized weights of particles in that group. We have demonstrated the encouraging results with maximum likelihood utilizing the proposed estimation approach. In realistic traffic, especially at the steady following state, the number and the states of unequipped vehicles within a specified distance can be roughly speculated through observing the movements of equipped vehicle located in the end. Although the speculation is sometimes not unique, the various solutions are gauged by the associated likelihoods in the proposed approach. The further work is worthy to be undertaken for the state estimation when there exist complex driving behaviors such as lane changing and local car following.

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