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Mechanical Analyses of Casings in Boreholes, under Non-uniform Remote Crustal Stress Fields: Analytical & Numerical Methods

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Abstract: The methods to design the casings used in oilfields, are currently based on the assumptions that the remote crustal-stress-field is axially symmetric, in plane strain. However, most of the failures of the casings are caused by non-uniform and asymmetric far-field crustal stresses, so that it is necessary for a proper design of the casings, to investigate and understand the casing's behavior under non-uniform far-field crustal stresses. A mechanical model is first established for the system, consisting of the casing and formation, by using the plane strain theory of linear elasticity. The non-uniform crustal stress is resolved into a uniform stress field and a deviatoric stress field. The analytical formulae for the stress and deformation fields within the casing, under non-uniform remote crustal stresses, are obtained by analyzing the stress and deformation fields of the combined system separately under the two components of applied crustal remote stress fields and then combining by linear superposition. Furthermore, a finite element analysis is also used to simulate this problem numerically. The crustal stress data in some oilfield are used to compute the tractions exerted on the external surface of the casing by the crustal formation, as well as the detailed stress-field within the casing, as an example. The maximum relative differences between the analytical solutions and the numerical results are about 6.5%. Under non-uniform remote crustal stresses, the normal traction exerted on the casing has an oval distribution, and the circumferential stress at the inner wall of the casing has a peanut-like distribution. It can be predicted that the initial yield failure in the casing occurs in the direction of the minimum horizontal crustal stress. Both the analytical and numerical solutions obtained in this paper are useful for the design of casings under the non-uniform remote crustal-stresses in an oilfield.

Keywords: Casing design; Non-uniform load; Crustal stress; Analytic solution;

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Numerical simulation; Well barrier integrity

1 Introduction

Currently, the design standard of the American Petroleum Institute (API) is generally adopted in oil & gas well engineering, for the design of casings. This standard is based on assuming that a uniform load [Yin et al. (2007)] is applied on the casing. However, a casing installed in a non-uniform crustal stress field is generally subjected to non-uniform loads. Under non-uniform loads, the failure behavior of the casing is very different from that under uniform load, and the collapse strength will also be reduced [Pattillo P D et al. (1981)]. Non-uniform loads produced by creeping formations such as salt, gypsum and mudstone, are the key factors which cause the failure of the casing [Li (2010) and Fang et al. (1997)]. In the Sand well 24 at Tarim Basin in China, the damage of the casing occurred just in one week after the completion of the well, due to the severe non-uniform loads produced by the salt formation [Deng et al. (2005)]. In the Yushulin oilfield in China, the mudstone, in the section of 1700m$\sim$1750m depth in the borehole, absorbed water and crept, which caused a large non-uniform crustal stress and caused the J-55 casing to collapse [Zheng et al. (1998)]. Thus, research on the mechanical behavior and the collapse strength of casing, under non-uniform crustal stresses, is significant.

There are a few studies on the behavior of casing under non-uniform load by numerical simulation. Pattillo et al. (2004) showed that a non-uniform load caused an unacceptable oval deformation of the casing, and reduced its collapse strength. Shen (2011) found that the different Young modulus of cement sheath lead to a non-uniform load, and which had an impact on the failure of the casing. Du, et al. (2011) and Wang et al. (2003) analyzed the stress and deformation state of a casing in a non-uniform crustal stress field. Some researchers [Yin et al. (2006) and Han et al. (2001)] studied the topic using analytical methods and derived the expressions for radial and circumferential stresses in the casing. Chi et al. (2008) discussed the influence of the lateral load on the casing-strength, via a mechanical model.

The theory of the mechanics of solids is applied in this paper to derive the relations between the externally applied tractions, and the radial stress, the circumferential stress and the shear stress in the casing. These results are examined using some oilfield data. In order to test and verify the present analytical method, this problem is also simulated using the available finite element software. The maximum difference between the analytical solution and the finite element result is about 6.5%. In order to simplify the oilfield's design, the selection, strength analysis and life prediction of the casing under a non-uniform crustal stress, utility software can be developed based on the formulae and finite element analysis presented in this paper.

2 A Mechanical model

The barrier in a well is a combined system consisting of the casing, a cement sheath and the formation. The axial displacement (along the hole) of the combined system is approximately zero, so that the problem can be simplified to be one of plane strain in the mechanics of elastic solids [Lin et al. (2006)]. It is assumed that: the casing and the cement sheath are concentric circles and the interfaces are intact with a strong bonding, and that the cement sheath may be simplified to be a part of the formation, due to its small thickness and the approximate similarity of the materials.

The inner and outer radii of the casing are a and b respectively. The remote far-field non-uniform crustal stresses can be characterized by the maximum and minimum horizontal crustal stresses denoted as σ_H and σ_h, respectively. The mechanical model of the combined system under a remote non-uniform crustal stress-field is shown in Fig. 1.

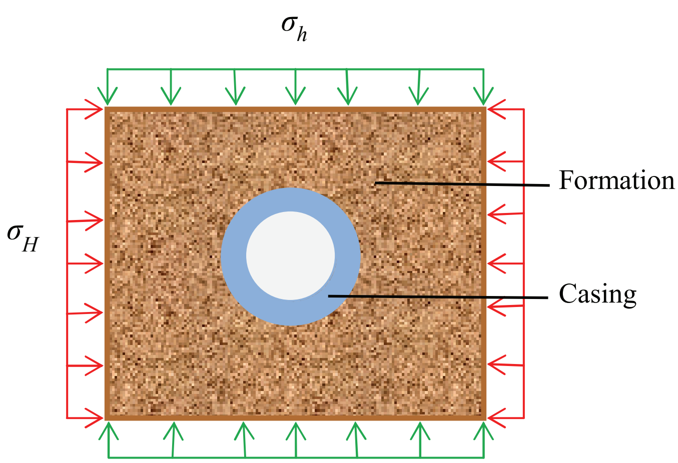

Figure 1: The mechanical model of the combined system

We use the following symbols σ (crustal direct-stress), s (crustal shear-stress) to denote:

$$\begin{cases} \sigma = \frac{1}{2}(\sigma_H + \sigma_h) \\ s = \frac{1}{2}(\sigma_H - \sigma_h) \end{cases} \tag{1}$$

Where σ_H and σ_h are the maximum and minimum planar principal crustal stresses respectively (MPa), in the far-field, in a global Cartesian system.

According to the theory of elasticity of a plane-strain medium with a hole, the normal stress and shear stress, in polar coordinates, in the formation, at a large radius c can be derived as follows:

$$\begin{cases} \sigma_r(c,\theta) = -\sigma - s\cos 2\theta \\ \tau_{r\theta}(c,\theta) = s\sin 2\theta \end{cases} \tag{2}$$

Where $\sigma_r(c,\theta)$ is the radial stress in the formation (MPa), $\tau_{r\theta}(c,\theta)$ is the shear stress in the formation (MPa), c is the radius of the formation-circle at a large distance (mm), r is the radial coordinate (mm) and θ is the angular coordinate (°).

Eq. (2) can be divided into Eq. (3) for the uniform crustal stress field (no angular variation), and Eq. (4) for the deviatoric crustal stress field (a function of θ):

$$\begin{cases} \sigma_r(c,\theta) = -\sigma \\ \tau_{r\theta}(c,\theta) = 0 \end{cases} \tag{3}$$

$$\begin{cases} \sigma_r(c,\theta) = -s\cos 2\theta \\ \tau_{r\theta}(c,\theta) = s\sin 2\theta \end{cases} \tag{4}$$

Thus, the behavior of the casing in a non-uniform crustal stress field can be divided into two simpler problems. The first problem is the mechanical behavior of the casing in a uniform far-field crustal stress field, and the second problem is the mechanical behavior in the deviatoric far-field crustal stress field. The two problems can be solved separately using the theory of linear elasticity in polar coordinates and the final solution can be obtained using the superposition method.

3 Analysis

3.1 External Tractions on the Casing

Using the above described decomposition of the far-field crustal stresses applied on the casing and the mechanics of elastic solids, we can deduce the normal and shear stress components acting on external surface of the casing, as follows:

$$\begin{cases} \sigma_n = s_1 + s_2\cos 2\theta \\ \tau_n = s_3\sin 2\theta \end{cases} \tag{5}$$

Where σ_n is the normal stress (MPa) on the external surface of the casing under far-field crustal-stress-fields, and τ_n is the shear traction (MPa) on the external surface of the casing under the far-field crustal-stress-fields.

In Eq. (5):

$$\begin{cases} s_1 = -\frac{(1-\upsilon_s)(\sigma_H+\sigma_h)}{1+\frac{1}{1-m^2}\frac{(1+\upsilon_c)}{(1+\upsilon_s)}\frac{E_s}{E_c}(1-2\upsilon_s+m^2)} \\ s_2 = -\frac{C_{22}+C_{12}}{C_{11}C_{22}-C_{12}C_{21}}\frac{2(1-\upsilon_s^2)}{(1+\upsilon_c)}\frac{E_c}{E_s}(1-m^2)^3(\sigma_H-\sigma_h) \\ s_3 = \frac{C_{21}+C_{11}}{C_{11}C_{22}-C_{12}C_{21}}\frac{2(1-\upsilon_s^2)}{(1+\upsilon_c)}\frac{E_c}{E_s}(1-m^2)^3(\sigma_H-\sigma_h) \end{cases} \tag{6}$$

Where E_s is elastic modulus of the earth-formation (MPa), E_c is elastic modulus of the casing (MPa), υ_s is Poisson's ratio of formation, υ_c is Poisson's ratio of casing. The inside and outside radii of casing are a, b (mm), and we set the symbol m=a/b.

In Eq. (6):

$$\begin{cases} C_{11} = A+\frac{(1+\upsilon_s)}{(1+\upsilon_c)}\frac{E_c}{E_s}(\frac{5}{3}-2\upsilon_s)(1-m^2)^3 \\ C_{12} = B-\frac{(1+\upsilon_s)}{(1+\upsilon_c)}\frac{E_c}{E_s}(\frac{4}{3}-2\upsilon_s)(1-m^2)^3 \\ C_{21} = C-\frac{(1+\upsilon_s)}{(1+\upsilon_c)}\frac{E_c}{E_s}(\frac{4}{3}-2\upsilon_s)(1-m^2)^3 \\ C_{22} = D+\frac{(1+\upsilon_s)}{(1+\upsilon_c)}\frac{E_c}{E_s}(\frac{5}{3}-2\upsilon_s)(1-m^2)^3 \end{cases} \tag{7}$$

In Eq. (7):

$$\begin{cases} A = (1-\frac{2}{3}\upsilon_c)+(5-6\upsilon_c)m^2+(3-2\upsilon_c)m^4+(\frac{5}{3}-2\upsilon_c)m^6 \\ B = -\frac{2}{3}\upsilon_c+2\upsilon_c m^2-2(2-\upsilon_c)m^4-(\frac{4}{3}-2\upsilon_c)m^6 \\ C = -\frac{2}{3}\upsilon_c+2\upsilon_c m^2-2(2-\upsilon_c)m^4-(\frac{4}{3}-2\upsilon_c)m^6 \\ D = (1-\frac{2}{3}\upsilon_c)-(3-2\upsilon_c)m^2+(3-2\upsilon_c)m^4+(\frac{5}{3}-2\upsilon_c)m^6 \end{cases} \tag{8}$$

We substitute Eq. (6), Eq. (7) and Eq. (8) into Eq. (5). Thus, we can obtain the normal and shear components of the external tractions on the casing.

3.2 Evaluation of Stresses inside the Casing

On the basis of elastic mechanics, the radial stress, circumferential stress and shear stress within the casing are described as follows:

$$\begin{cases} \sigma_r(r,\theta) = \frac{b^2}{b^2-a^2}(1-\frac{a^2}{r^2})s_1+[-n_1-n_2(\frac{b}{r})^4-2n_4(\frac{b}{r})^2]\cos(2\theta) \\ \sigma_\theta(r,\theta) = \frac{b^2}{b^2-a^2}(1+\frac{a^2}{r^2})s_1+[n_1+n_2(\frac{b}{r})^4+2n_3(\frac{r}{b})^2]\cos(2\theta) \\ \tau_{r\theta}(r,\theta) = [n_1-n_2(\frac{b}{r})^4+n_3(\frac{r}{b})^2-n_4(\frac{b}{r})^2]\sin(2\theta) \end{cases} \tag{9}$$

Where $\sigma_r(r,\theta)$ is the radial stress within the casing (MPa), $\sigma_\theta(r,\theta)$ is the circumferential stress within the casing (MPa), $\tau_{r\theta}(r,\theta)$ is shear stress within the casing (MPa) and r is the varying radius (mm) within the casing.

In Eq. (9):

$$\begin{cases} n_1 = -\frac{1+m^2+2m^4}{(1-m^2)^3}s_2 + \frac{2m^4}{(1-m^2)^3}s_3 \\ n_2 = -\frac{3m^4+m^6}{(1-m^2)^3}s_2 + \frac{2m^6}{(1-m^2)^3}s_3 \\ n_3 = -\frac{1+3m^2}{(1-m^2)^3}s_2 + \frac{1-3m^2}{(1-m^2)^3}s_3 \\ n_4 = -\frac{m^2(2+m^2+m^4)}{(1-m^2)^3}s_2 - \frac{m^2(m^2+m^4)}{(1-m^2)^3}s_3 \end{cases} \tag{10}$$

Where s_1, s_2, s_3 are shown in Eq. (5).

4 A Case study

Through a comprehensive analysis and test of rocks in Liuzan block of Jidong oilfield [WANG Ximao et al. (2007)], the maximum horizontal remote crustal stress gradient (in the depth direction) is 0.021 MPa/m, and the minimum horizontal remote crustal stress gradient (in the depth-direction) is 0.017 MPa/m. Liuzan 13-16 well's tested depth is 3241m, so the maximum and the minimum horizontal remote crustal stresses are σ_H=68.061MPa, σ_h=55.097 MPa, respectively. The geometrical and material parameters of the casing and formation are shown in Table 1.

Table 1: The geometrical and material parameters

Combined system	Inside diameter (mm)	Outside diameter (mm)	Elastic modulus (MPa)	Poisson's ratio
Casing	159.4	177.8	210000	0.3
Formation	177.8	/	5000	0.45

Substituting the parameters into Eq. (6), the three parameters in the crustal stress-field exerted on the casing are calculated: s_1=61.675 MPa, s_2= 5.180 MPa, s_3=14.223 MPa. Based on the Eq. (3) $\sim$ (4), the average crustal stress produces a compressive traction on the casing, and the far-field crustal shear-stress produces a tensile traction when θ=0° and compressive traction when θ=90° on the casing. The normal traction distribution on the casing, under the far-field non-uniform crustal stresses is shown in Fig. 2. The minimum normal traction on the casing is σ_{n0}= 56.495 MPa, and the maximum normal traction on the casing is σ_{n90}= 66.855 MPa.

Therefore, in a non-uniform crustal stress field, the normal traction exerted on the casing is of an oval distribution. The maximum normal stress is in the direction of minimum horizontal crustal stress, which is the hot spot for casing failure.

The shear traction distribution on the external surface of the casing, under a non-uniform far-field crustal stress-field, is shown in Fig. 3, where it can be seen that

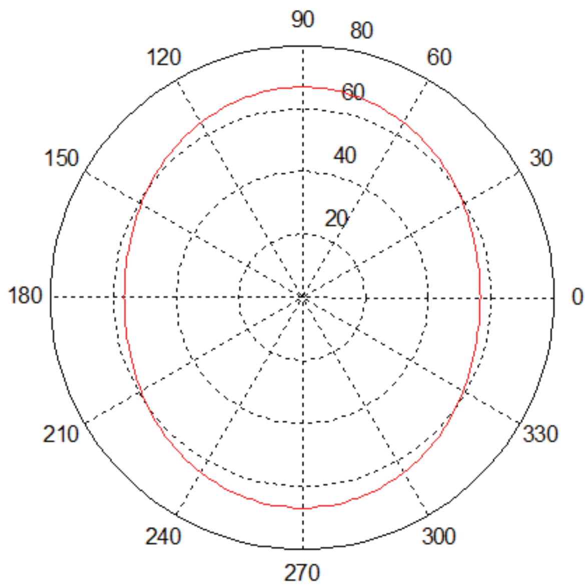

Figure 2: The normal traction exerted on the casing, under non-uniform far-field crustal stresses

the shear traction exerted on the casing is of a four-leaf clover shape. The minimum shear stress is τ_{n0}= 0 MPa, and the maximum shear stress is τ_{n45}= 14.223 MPa.

Based on Eq. (9), the radial stress, the circumferential stress and the shear stress within the casing are calculated. The radial stress and shear stress are relatively small, whose maximum values on the inner wall of the casing are 3.411×10^{-13} MPa and 1.421×10^{-12} MPa respectively. The circumferential stress within the casing is the main stress, because the minimum value itself is 411.623 MPa and the maximum value is 845.347 MPa. The circumferential stress distribution at the inner surface of the casing is shown in Fig. 4. The circumferential stress at different phase angle is of a "peanut" distribution, and the maximum value is in the direction of the minimum horizontal remote crustal stress. Thus, it can be predicted that the initial yield failure in the casing generally occurs in the minimum horizontal crustal stress direction.

According to the Mises yield criterion, the casing yield occurs when the stress at a point on the inner wall of the casing reaches a yield value. Li et al. (2006) defined the equivalent external pressure on the casing as follows:

$$\bar{p} = -s_1 + \frac{2}{1-m^2}\left|m^2 s_3 - (1+m^2)s_2\right| \tag{11}$$

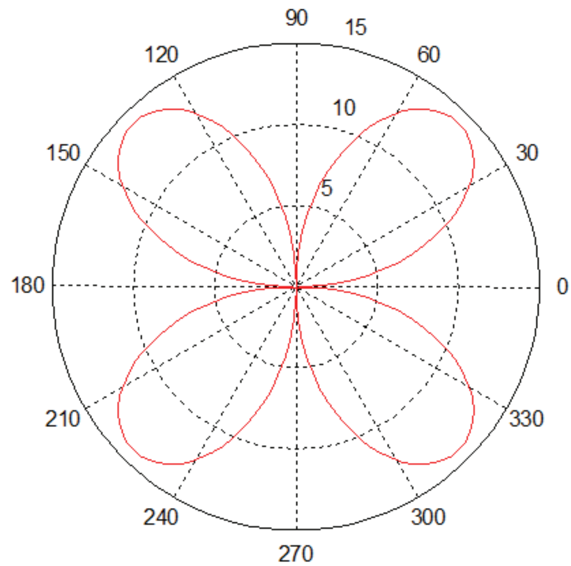

Figure 3: The shear traction exerted on the casing, under non-uniform far-field crustal stresses

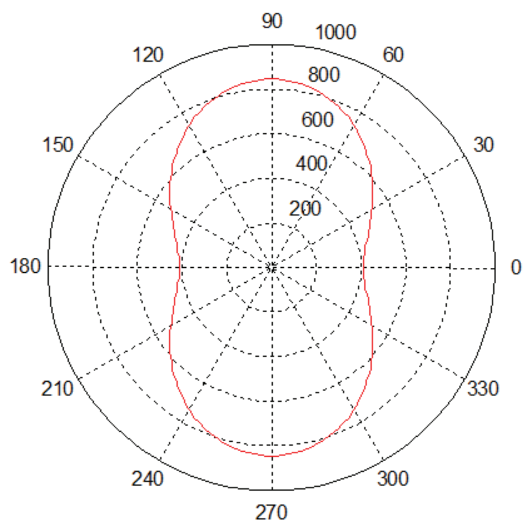

Figure 4: The circumferential stress distribution at the inner wall of the casing

Where $\bar{p}$ is the equivalent external pressure (MPa) on the casing. Based on the Eq. (11), we can calculate $\bar{p}$=82.956 MPa. The external pressure exerted on the casing is $p = -s_1$=61.675 MPa, when only a uniform far-field crustal-stress-field is considered. The existence of no-uniform factors s_2 and s_3 increase the casing pressure as follows:

$$k=(\bar{p} - p)/p = 34.5\% \tag{12}$$

Where k is the percentage increases of casing pressure due to no-uniform crustal-stresses. The conclusion is that the non-uniform far-field crustal stresses greatly increase the risk of collapse of the casing.

5 A Comparison of the analytical solution with numerical results

Numerical simulation is widely used to analyze complex structures. Gao et al. (2012) studied the stress distribution in a defective casing using a FEA software and achieved good results. Here, to verify the presently derived analytical solutions, the FEA software ANSYS12.0 is used to simulate the casing behavior in a non-uniform crustal stress field. The FEA model of the combined system consisting of casing, cement sheath and formation is built, which is shown in the Fig. 5. The two horizontal crustal stresses are imposed at the far-field boundary of the crustal formation.

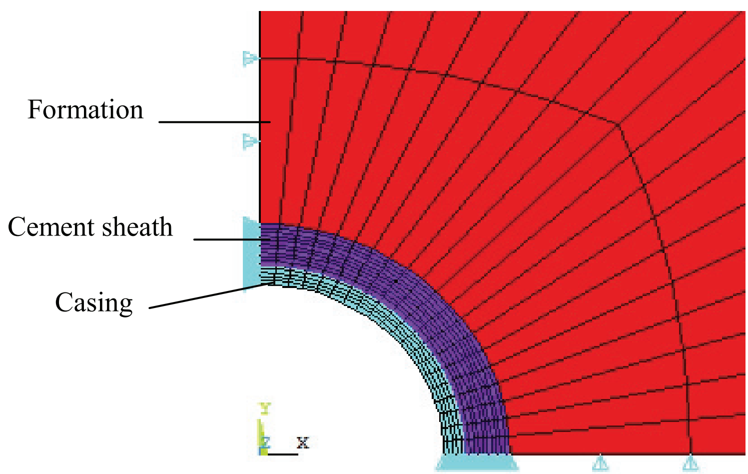

Figure 5: FEA model of the combined system

Based on the above parameters and data in section 4, Numerical simulations have been performed. As a result, the radial stress, the circumferential stress and the shear stress within the casing are obtained in the form of nephogram. The circumferential stress nephogram of the casing is shown in Fig. 6.

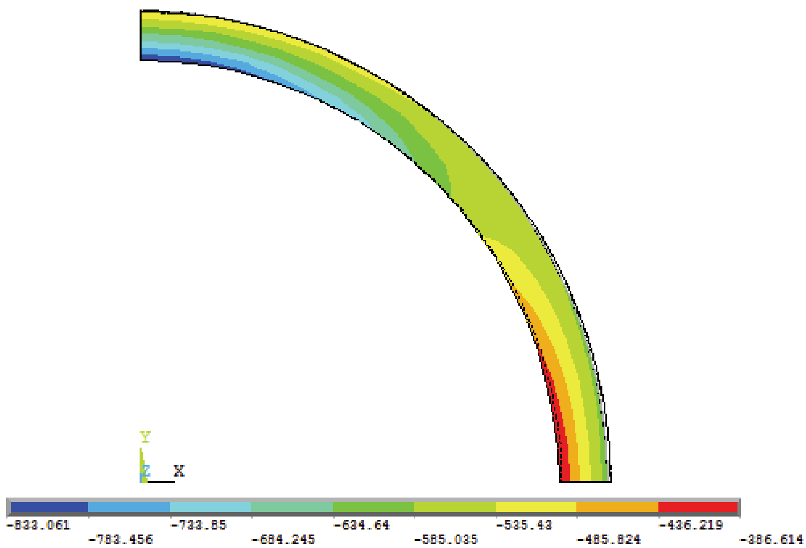

Figure 6: The circumferential stress nephogram of casing

The circumferential stress at the inner wall of the casing can be extracted from the numerical results. These numerical results are compared with the circumferential stress obtained by analytical solution, and the comparison is shown in Fig. 7.

The results of the analytical solution and the numerical simulation are very close, and the maximum deviation is 6.5%. Both the analytical solution and the numerical results in this paper are accurate and effective. Based on these methods new application software will be developed to guide the design of casings under the non-uniform far-field crustal stresses, in oilfields.

6 Conclusions

(1) The non-uniform remote crustal stress field can be resolved into a uniform crustal stress field and a deviatoric crustal stress field. The analytical formulae to determine the detailed stress-state within the casing can be obtained by superposing the solutions obtained under these two divided stress fields.

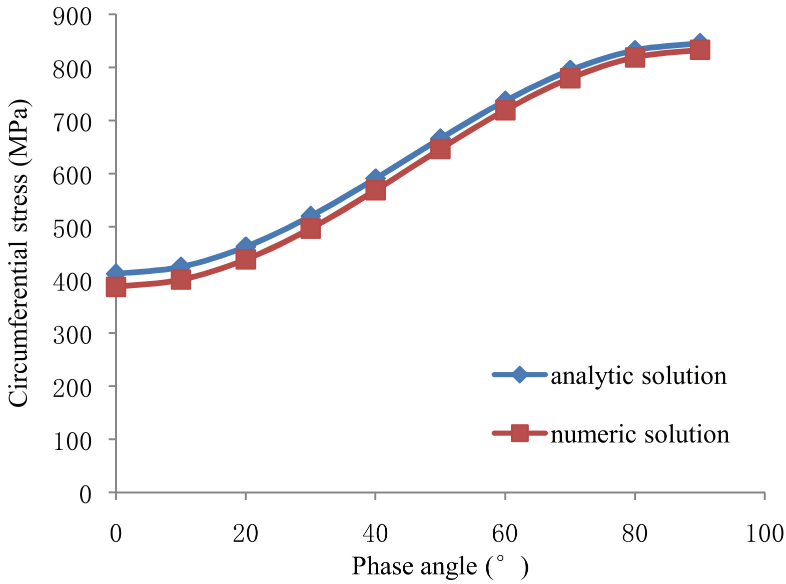

Figure 7: The comparison of circumferential stresses at the inner wall of a casing, obtained by two methods

(2) The normal and shear tractions exerted on casing, under the non-uniform remote crustal stress-field, are respectively of oval and four-leaf clover shapes.

(3) The radial stress and shear stress within the casing are so small that they may be ignored in designs for casing-strength. The circumferential stress at the inner wall of the casing is a kind of a "peanut" shape, in which the maximum value is in direction of the minimum horizontal crustal stress, so that the initial yield failure of the casing occurs in this direction generally.

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