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Prandtl Number Signature on Flow Patterns of Electrically Conducting Fluid in Square Enclosure

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Abstract: We present in this study a numerical investigation of unsteady two-dimensional natural convection of an electrically conducting fluid in a square cavity under an externally imposed magnetic field. A temperature gradient is applied between the two opposing side walls parallel to y-direction, while the floor and ceiling parallel to x-direction are adiabatic. The flow is characterized by the Rayleigh number *Ra* raged in 10^3-10^6, the Prandtl number *Pr* ranged in 0.01-10, the Hartman number *Ha* determined by the strength of the imposed magnetic field ranged in 0-100 and its tilting angle from x-axis ranging from 0° to 90°. The coupled momentum and energy equations associated with the Lorentz retarding force as well as the buoyancy force terms are solved using the single relaxation lattice Boltzmann (LB) approach. The changes in the buoyant flow patterns and temperature contours due to the effects of varying the controlling parameters and associated heat transfer are examined. It was found that the developed thermal LB model gives excellent results by comparison with former experimental and numerical findings. Starting from the values 10^5 of the Rayleigh number *Ra* and *Ha*=0, the flow is unsteady multicellular for low Prandtl number typical of liquid metal. Increasing gradually *Pr*, the flow undergoes transition to steady bicellular, the transition occurs at a threshold value between *Pr*=0.01 and 0.1. Increasing more the Prandtl number, the flow structure is distorted due to the viscous forces which outweigh the buoyancy forces and a thermal stratification is clearly established. For high Hartman number, the damping effects suppress the unsteady behaviour and results in steady state with extended unicellular pattern in the direction of Lorentz force and diminishes considerably the heat transfer rate.

Keywords: Lattice Boltzmann computer modeling, Prandtl number effect, electrically conducting fluid, heat transfer, thermal convection.

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Nomenclature

$\mathbf{B}$	Magnetic field	$\overline{Nu}_0$	Nusselt number at the hot wall
$\mathbf{e}_k$	Discrete lattice velocity	Ra	Rayleigh number$g\beta\Delta T H^3/\nu\alpha$
$\mathbf{g}$	Gravity field		**Greek symbols**
$\mathbf{u}$	Velocity vector (u,v)	w_k	Weighting factors
$\mathbf{x}$	Lattice node in (x,y) coordinates	ρ	Fluid density
c_s	Lattice sound speed	υ	Kinetic viscosity
f_k, g_k	Discrete distribution functions.	α	Thermal diffusivity
H	Height of the enclosure	τ_υ, τ_α	Relaxation times for f_k and g_k
p	Ideal gas pressure ρ_s^2	β	Thermal expansion coefficient
T	Temperature field	σ	Electrical conductivity of the fluid
ΔT	Horizontal temperature gradient $T_h - T_c$	γ	Magnetic field inclination angle
Δt	Time step		**Subscripts Suscripts**
Δx	Lattice spacing units (=Δ)	eq	Equilibrium part
Ha	Hartmann number	i,j	Lattice vector components
Pr	Prandtl number $\upsilon/\ \alpha$	k	Discrete velocity direction

1 Introduction

Study of magnetohydrodynamic (MHD) flows, has been the subject of a great number of numerical investigations [Gelfgat and Bar-Yosaf (2001), Roussellet, Niu et al. (2011) and Hadavand and Sousa (2011)] Moreover, flows under external magnetic field are of practical interest such as crystal growth in liquids, cooling of nuclear reactors, electronic packages, micro electronic devices; and have been the subject of many earlier and recent studies for free fluid flows [Ece and Büyük (2006), Hasanpour, Farhadi et al. (2012) and Jina and Zhang (2013))]. Furthermore, the study of flow and heat transfer in electrically conducting fluid has received considerable attention and renewed interest due to the many applications in engineering problems such as MHD generators, plasma studies, nuclear reactors, and because of the effect on magnetic fields on the performance of many systems including liquid metals and alloys, mercury amalgams, and blood, known as "low-Prandtl number flows".

Natural convection in enclosures depends strongly on many parameters monitoring the flow behaviour in special industrial situations, namely the Rayleigh and the Prandtl number, the medium tilting angle and its configuration.

Besides, several numerical simulations have been conducted in the past using conventional numerical method based on discretization of macroscopic equations. Recently, the Lattice Boltzmann Method (LBM) has met with significant success for numerical simulation and modeling of many classical and complex flows [Chen and Doolen (1998), Seta, Takegoshi et al. (2006), Semma, El Ganaoui et al. (2008), Djebali, ElGanaoui et al. (2012)]. The LBM have been used recently to investigate flows under external magnetic field. For citation, Hao, Xinhua and Yongzhi (2011) simulated a multicomponent system formed by fluid and magnetic particles using a multiphase LB model, under external magnetic field. Authors concluded that the LB method is so helpful to explore and understand the chainlike particles behavior when applying external magnetic field on a random distribution of particles. Chatterjee and Amiroudine (2011) used a non-isothermal LB model to predict the thermo-fluidic phenomena in a direct current MHD micropump. It was remarked that flow and heat transfer characteristics depend strongly on Hartmann, Prandtl and Eckert numbers and channel aspect ratios. An excellent agreement is also observed between LB results and experimental, analytical and other available numerical results in the literature. Han (2009) used the FV approach to investigate MHD natural convection flow for a tilted square cavity. It was concluded that for high magnetic field strength, the velocities are suppressed and the convective heat transfer rate is reduced and that the effect of a magnetic field is found to decrease the Nusselt number considerably, regardless of the inclination angle. Ece and Büyük (2006) have investigated the steady laminar natural-convection flow in an inclined square enclosure heated and cooled from adjacent sides in the presence of a magnetic field. The governing equations based on the stream function, vorticity and temperature have been solved using the Differential Quadrature Method for various Grashof and Hartman numbers and aspect ratios, inclination of the cavity and magnetic field orientations. Its has been observed that the flow characteristics, therefore the heat transfer rate are affected significantly by the variation of Hartman number, the aspect ratio and the inclination of the enclosure. Zhang, Jin et al. (2010) introduced a LB model to investigate a thermo-sensitive magnetic fluid in porous medium. Authors obtained excellent agreement with previous results and concluded that the LB method is a promising tool for understanding magnetic fluid non-isothermal behavior in porous media.

Through this literature review, one can state that flow patterns and temperature field exhibit distinctly different behavior in differentially heated enclosures by varying the monitoring parameters: Rayleigh number and Hartmann number (Magnetic field strength). Additionally, no / very little works has been reported on this topic with regard to the Prandtl number effects. In the present paper, a thermal lattice Boltzmann model is developed and used to investigate the dynamic and thermal behavior in electrically conducting fluid in a square enclosure. The effects of the Rayleigh number, the Prandtl number and the magnetic field strength and its orientation -in wide ranges- on flow and heat transfer are analyzed and tabulated for benchmarking purposes.

2 Problem statement

The investigated problem is a two-dimensional square enclosure of edge $H(=L)$ filled with a viscous, incompressible and electrically conducting fluid. The non-slip boundary conditions hold on the four walls. A temperature difference Δ=$T_h - T_c$ is applied between walls parallel to y-direction ($T = T_h$for$x$ =0 and$T = T_c$ for$x = H$) and zero heat flux is imposed to walls parallel to x-direction. The fluid is permeated by a uniform magnetic field of strength B tilted by an angle γ with respect to x-axis (Fig. 1). The gravity field reigns in the vertical descendant direction. It is assumed that the induced magnetic field can be neglected in comparison with the imposed magnetic field.

We assume all fluid properties including the electrical conductivity to be constant, except the density which is linearly temperature-dependent, so that the *Boussinesq approximation* is used: $\rho = \rho_0 (1 - \beta (T - T_\infty))$ where T_∞ is the reference temperature taken here the cold temperature. Neglecting viscous heat dissipation and compression work done by the pressure, the unsteady state governing equations can be summarized as follows:

Continuity

$$\nabla.\mathbf{U} = 0 \tag{1}$$

Momentum

$$\partial_t \mathbf{U} + (\mathbf{U}.\nabla)\mathbf{U} = -\nabla P + \nabla.(\upsilon \nabla \mathbf{U}) + \rho \mathbf{F}/\rho_0 \tag{2}$$

Energy

$$\partial_t T + (\mathbf{U}.\nabla)T = \nabla.(\alpha \nabla T) \tag{3}$$

Where $F_i = \beta(T - T_\infty) g \delta_{i2} + \sigma \left[(B_j u_j) B_i - B^2 u_i \right]$ and B_j denotes the magnetic fields components B_x=B $cos(\gamma)$ and B_y=B $sin(\gamma)$. We assume the Joule heating can be neglected since Ra>10^3 and Ha<200.

For the sake of comparison with previous studies, all predicted quantities are presented in a non-dimensional form. The reference scales $l_0 = H$, U_0=α/H $t_0 = H^2/\alpha$, p_0=$\rho_0 U_0^2$ and ΔT=$T_h - T_c$ are used for length, velocity, time pressure and relative temperature respectively. The reference temperature is chosen to be T_c.

The flow is characterized by the Rayleigh number ($10^3 \leq Ra \leq 10^6$), the Prandtl number ($0.01 \leq \Pr \leq 10$), the Hartmann number ($0 \leq Ha \leq 100$) and the magnetic field orientation ($0° \leq \gamma \leq 90°$), defined as:

$$Ra = g\beta \, \Delta T \, H^3 / \nu\alpha, \Pr = \frac{\nu}{\alpha} \text{ and } Ha = H\,B\sqrt{\sigma/\mu} \tag{4}$$

The convective heat transfer is described using the average Nusselt number $\overline{Nu}_0$ along the hot wall:

$$\overline{Nu} = \frac{1}{\Delta T/H} \frac{1}{H} \int_0^H - \left. \frac{\partial T}{\partial x} \right|_{x=0} dy \tag{5}$$

The convergence criterion for steady state is defined as follows:

$$\left| \frac{\overline{Nu}(t+5000\Delta t) - \overline{Nu}(t)}{\overline{Nu}(t)} \right| \leq 10^{-4} \tag{6}$$

3 Computational method

The lattice Boltzmann is considered in this study for simulating the fluid flow and heat transfer. In the LBM approach, the fluid is modeled by fictitious particle modeled by distribution functions that occupy nodes and transit to neighboring nodes in a streaming phase. In traditional CFD solvers, the Poisson equation is time consuming and its solution takes typically 80–90% of the CPU time [Madabhushi and Vanka (1991)], its absence in LBM means that codes are comparatively fast based on time step per grid point. In our previous works [Djebali, Pateyron and El Ganaoui (2011), Djebali and El Ganaoui (2011), Djebali, El Ganaoui, Pateyron and Sammouda (2011), Djebali, El Ganaoui and Pateyron (2012) and Djebali, El Ganaoui and Naffouti (2012)], we have found that a D2Q9-D2Q4 lattice is a suitable model for simulating thermal flows, for its stability compared to the D2Q9-D2Q9 thermal model (see Fig. 2), it preserves the computational efforts, since the collision step takes around 70% of the CPU time.

The evolution of the distribution functions in the D2Q9-D2Q4 lattice model in the presence of source term S_k is written as follows:

$$\begin{cases} f_k(\mathbf{x}',t') - f_k(\mathbf{x},t) = -\left(f_k(\mathbf{x},t) - f_k^{eq}(\mathbf{x},t)\right)/\tau_\upsilon + \Delta t\, S_k, & k = 0,8 \\ g_k(\mathbf{x}',t') - g_k(\mathbf{x},t) = -\left(g_k(\mathbf{x},t) - g_k^{eq}(\mathbf{x},t)\right)/\tau_\alpha, & k = 1,4 \end{cases} \tag{7}$$

Where $\mathbf{x}' = \mathbf{x} + \mathbf{e}_k\Delta t$, $t' = t + \Delta t$, $\mathbf{x}$ is the lattice site, Δt is the time step, Δx is the lattice grid spacing unit (=Δy=1), $e_k$discrete lattice velocity, and f_k and g_k are the density and temperature distribution functions. The correspondent equilibrium parts f_k^{eq} and $g_k^{eq}$are defined as:

$$\begin{cases} f_k^{eq}(\mathbf{x},t) = \omega_k \rho \left[1 + 3\,\mathbf{e}_k.\mathbf{u} + 4.5\,(\mathbf{e}_k.\mathbf{u})^2 - 1.5\,\mathbf{u}^2\right] \\ g_k^{eq}(\mathbf{x},t) = T\left[1 + 2\,\mathbf{e}_k.\mathbf{u}\right]/4 \end{cases} \tag{8}$$

The forcing term S_k for incompressible fluid flow is chosen as:

$$S_k = \mathbf{F}.\frac{(\mathbf{e} - \mathbf{u})}{c_s^2} f_k^{eq} \tag{9}$$

The single-relaxation-times τ_υ and τ_α are linked to the kinematic viscosity and the heat diffusivity as

$$\upsilon = \frac{2\tau_\upsilon - 1}{6}\frac{\Delta x^2}{\Delta t}, \quad \alpha = \frac{2\tau_\alpha - 1}{4}\frac{\Delta x^2}{\Delta t} \tag{10}$$

Where ω_k are weighting factors and e_k are the lattice velocity vectors.

In LB heat and flow modeling philosophy, the macroscopic variables: density, velocity and temperature are computed as $[\rho\,(\mathbf{x},t),\,\rho\,\mathbf{u}(\mathbf{x},t),T] = \sum\,[f^{eq},\,\mathbf{e}.f^{eq},\,g^{eq}]$.

Since it affects the accuracy of the computations, implementation of boundaries conditions is a very important issue in LBM. The second-order bounce back boundary rule for the non-equilibrium distribution function proposed by Zou and He (1997) is used to account for the no-slip boundary condition along the four walls as: $(f - f^{eq})^< = (f - f^{eq})^>$, where the asterisk "<" and ">" denote for inner (unknown) and outer (known) particles respectively at the wall node. For the temperature field, the temperature distribution functions at the isothermal walls obey: $g^< = -g^> + 0.5\,T_{wall}$. The adiabatic boundary condition is transferred to Dirichlet-type condition using the conventional second-order finite difference approximation as: $g_{wall} = (4g_1 - g_2)/3$.

We have to mention that the grid size sensitivity has been tested previously for the same range of the Rayleigh number Ra, a uniform grid size 150×150 will be adopted for the following. To establish the credibility of the thermal lattice Boltzmann developed code, we made two test cases involving different situations of the above mentioned monitoring parameters.

First a square cavity counter-clockwise tilted to 0° or 20° from horizontal is considered, the Prandtl number is set to 0.733 and Ra=2×10^5. The streamline traces of the present calculations are plotted in Fig. 3 side-by-side with former numerical and experimental findings reported by Linthorst, Schinkel, et al. (1981) and Han (2009). Qualitatively, the present streamlines distributions are consistent with the literature results. At the same case, the dimensionless y-velocity calculated at the cavity mid-plane for Ra=1.3$\times$10^5 when the enclosure is 0° or 50° tilted, are plotted in Fig. 4. As one can see, the LB method present a good level of predictability compared to the previous experimental results (symbolized in Fig. 4) and numerical predictions of finite volume method using a refined mesh near walls 51$\times$51 or either using non-uniform control volumes 201$\times$201. In the second test case, the LB present predictions are compared to the ADI (Alternating Direction Implicit) and FE methods for natural convection flow in square cavity under external magnetic field effects for different Hartmann numbers. The present LB results are tabulated in Table 1 and gathered with the references findings. As one can remark, our results are in excellent agreement with predictions of conventional approaches. In the following the Rayleigh number Ra is set to 10^5, Ha=0 or 50 and Pr=0.01, 0.1, 1 or 10 and the left hot wall of the cavity is kept parallel to y-direction.

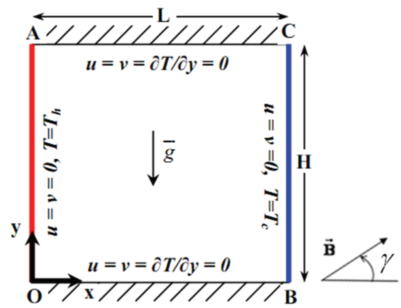

Figure 1: Configuration model.

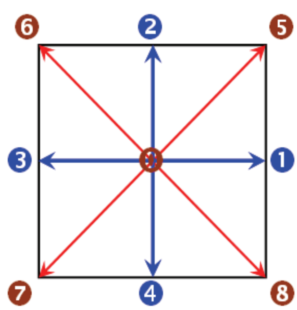

Figure 2: The nine particle speeds used in the hydrodynamic of lattice Boltzmann equation. Only the four blue velocities are necessary for the temperature field.

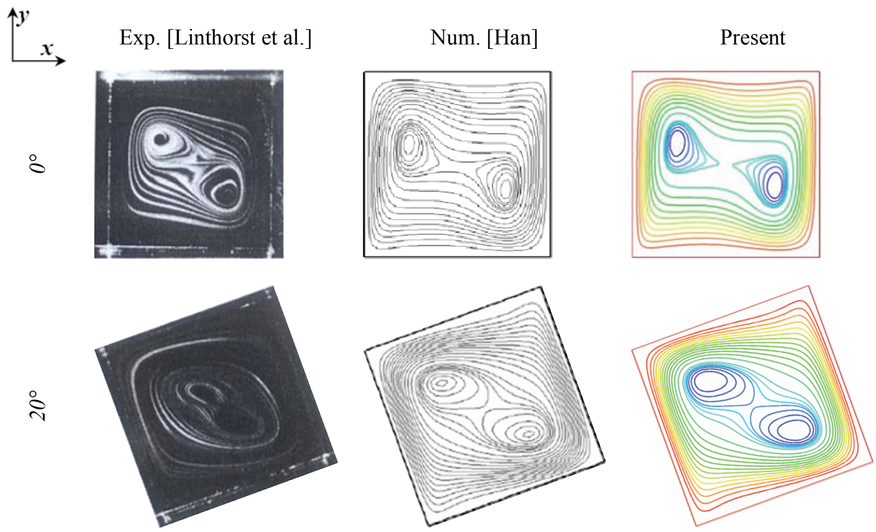

Figure 3: Comparison of present streamlines calculation with former experimental and numerical findings for Pr=0.733, Ra = 2×10^5 and different cavity inclinations.

Table 1: Comparison of the calculated Nusselt number on the left wall of the cavity with reference to former works using different methods.

Ra/Pr	Ha	Rudraiah et al. (1995): ADI	Sathiyamoorthy et al. (2011): FEM	Present LBM
2×10^4	0	2.5188	2.5439	2.5516
	10	2.2234	2.2385	2.2458
	100	1.0110	1.0066	1.0128
2×10^5	0	4.9198	5.0245	5.1276
	10	4.8053	4.9136	4.9252
	100	1.4317	1.4292	1.4467

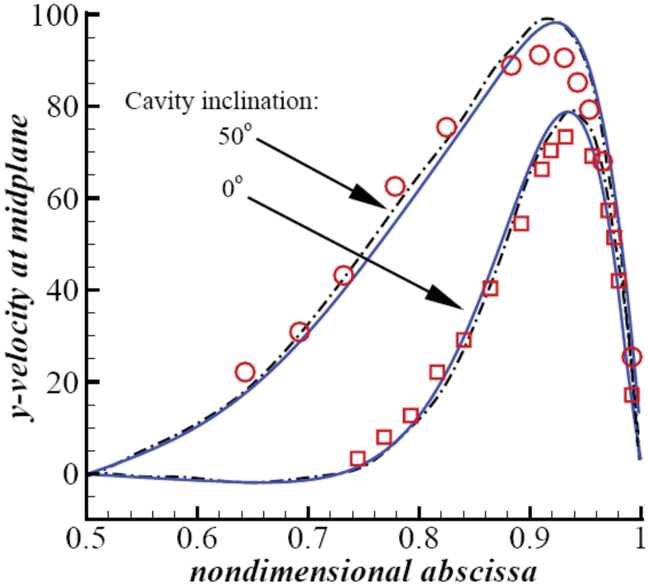

Figure 4: Comparison of dimensionless y-velocities at cavity mid-plane for Pr=0.733, Ra = $1.3\times10^5$and different cavity inclinations. Symbols: experimental results of [Linthorst, Schinkel, et al. (1981)], Dashdotted lines: FVM results of [Han (2009)] and blue-solid lines: present predictions.

4 Results and discussions

Free buoyant flow (Ha=0): The temperature contours and the streamtraces of a fluid flowing for Ra=10^5 without external magnetic field under the effects of varying the Prandtl number are computed. For *Pr*=0.01, the flows is unsteady (chaotic) and the dynamic structure changes continually with time showing two centro-symmetric cells at the core of the cavity; a principle quasi-circle clockwise rotating large cell occupies the enclosure. The flow exhibits also two counter rotating secondary cells at the four corners symmetric by the cavity center. The isotherms are distorted at the cavity core and more stretched and piled near the cavity mid-height. It is worth-noting that this type-flow (low Prandtl number flows) characterizing liquid metal show usually time dependent behaviors and instabilities. The Nusselt number time history is gathered to its spectra of amplitude frequency in Figure 5, the spectra exhibit four principal oscillating frequency close to 46, 567, 1250 and 1822. The time-average Nusselt number is close to 3.19. Increasing the Prandtl number to 0.1 the flows behaviour changes distinctly (see Fig. 6): a thermal stratification appears and the dynamic structure shows only three counter rotating cells, and the flow is also steady. The Nusselt number is close to 3.92 and the absolute maximum stream-function is 7.84. Increasing more the Pr number to 1 and 10, the flow exhibits only two centro-symmetric rotating cells at the cavity core. For Pr=1 the left cell center is the highest, however it is the lower for Pr=10 and the cells are smaller compared to the Pr=1 case. This is certainly due to the fact that a rise in the Pr number leads to a rise in viscosity (against the diffusivity of the fluid) which requires the sufficient buoyancy force to conquer viscous forces. The isotherms are more stretched to the isothermal walls and the thermal stratification is intensified. The Nusselt numbers $\overline{Nu}$ are 4.60 and 4.72 and the maximum stream-function magnitudes $|\psi|_{\max}$ are 10.02 and 11.11 for respectively Pr=1 and Pr=10.

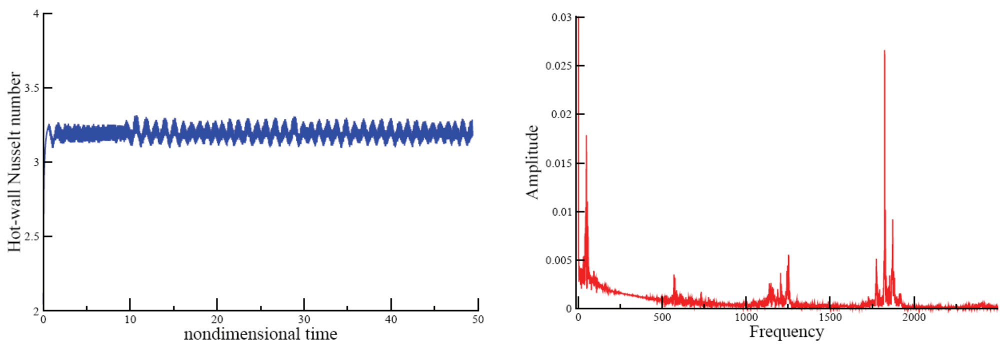

Figure 5: Nusselt time history (left) and its spectra of amplitude frequency (right) for Ra=10^5, Pr=0.01 and Ha=0.

Fluid flow under horizontal magnetic field (γ=0°): A simple scaling procedure leads to the following form of the non-dimensional Lorentz force $F_l \equiv Pr.Ha^2\,[\sin\gamma(-u.\sin\gamma+v.\cos\gamma), \cos\gamma(u.\sin\gamma-v.\cos\gamma)]$ and the buoyancy force $G=Pr.Ra.T.e_y$. When the magnetic field is applied parallel to the x-axis (case γ=0°) the Lorentz force reduces to $F_l=-Pr.Ha^2 v$, so it acts directly against the buoyancy force. Accordingly, the magnitude of ratio buoyancy force by the Lorentz force is proportional to Ra/Ha^2. Furthermore, one knows that the Lorentz force reduces velocities and dumps the convection currents and consequently reduces the heat transfer. As a result, major effects are observed between the cases: with/without applying magnetic field (see Fig. 7): the dynamic and thermal structures of the flow for Ha=0 change entirely for a moderate magnetic field magnitude of Ha=50; a like-elongated core-cell pattern is remarked for Pr $\geq$ 0.1 and a thermal stratification along the first diagonal of the cavity occurs. The stream-function magnitude does not changed much, for

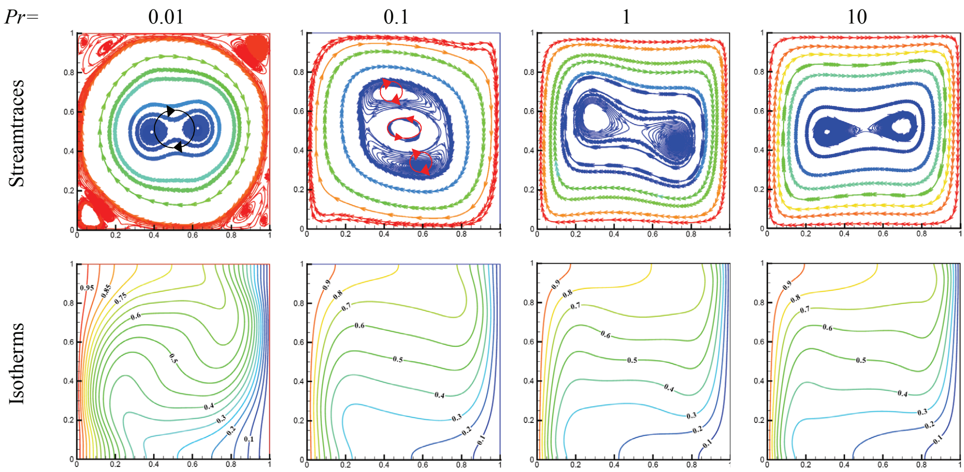

Figure 6: Streamtraces and isotherms plots for Ra= 10^5, Ha=0 and different Prandtl numbers.

Pr=0.01, 0.1, 1 and 10 we have $|\psi|_{\max}$=3.22, 3.48, 3.36 and 3.35 respectively, then a maximal change amount of 8.1%. The same conclusion applies for the heat transfer, the Nusselt number is 2.03, 2.15, 2.16 and 2.17 when increasing the Prandtl number, then a maximal change amount of 15.6% to reference value $\overline{Nu}$=2.03. We should mention that the lower Nusselt number $\overline{Nu}$=2.03 is superior to unit then the convective currents still act, this is due to the fact that the ratio Ra/Ha^2=$10^5/50^2$=40 which is so greater than unit.

Fluid flow under vertical magnetic field (γ=90°): The results of our predictions are shown in Fig. 8. In this case of γ=90° the Lorentz force reduces to $F_l = -Pr.Ha^2u$ and consequently it acts counter the horizontal flow currents. Effectively, for low Prandtl number (Pr=0.01) the core-cell is extended according to the second diagonal with two small cells at the correspondent corners. However, by increasing the Prandtl number (ie increasing the viscous force) the viscous force acts according to "-y"; the resulting force leads to a two cells elongated according to the first diagonal, with alteration rising with increasing the Prandtl number to 10. The Prandtl number effect is, as one can see, more expressed on the dynamic structure more than the thermal structure. The isotherms exhibit a stretching pattern along the first diagonal with slight clockwise rotation under increasing Pr; this behavior is more expressed compared to the case of horizontal magnetic field, but examining the flow pattern under increasing the Prandtl number at vertical mag-

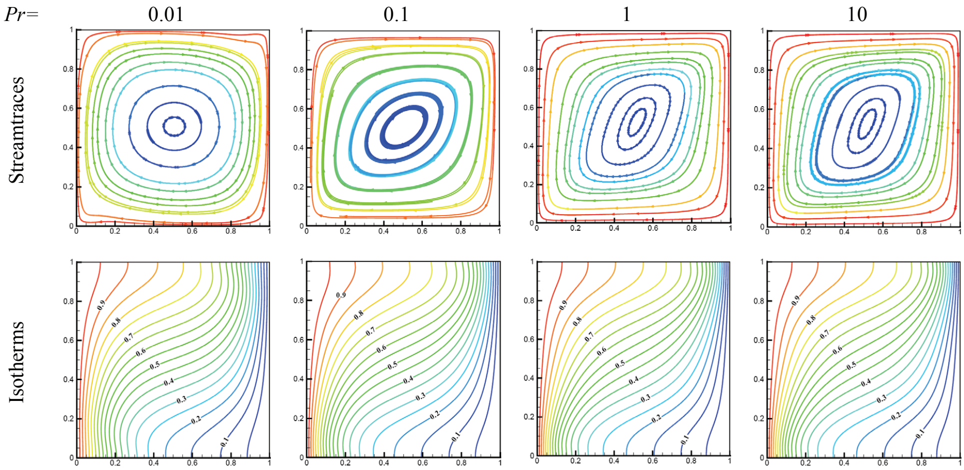

Figure 7: Streamtraces and isotherms plots for Ra= 10^5, Ha=50, γ=0° and different Prandtl numbers.

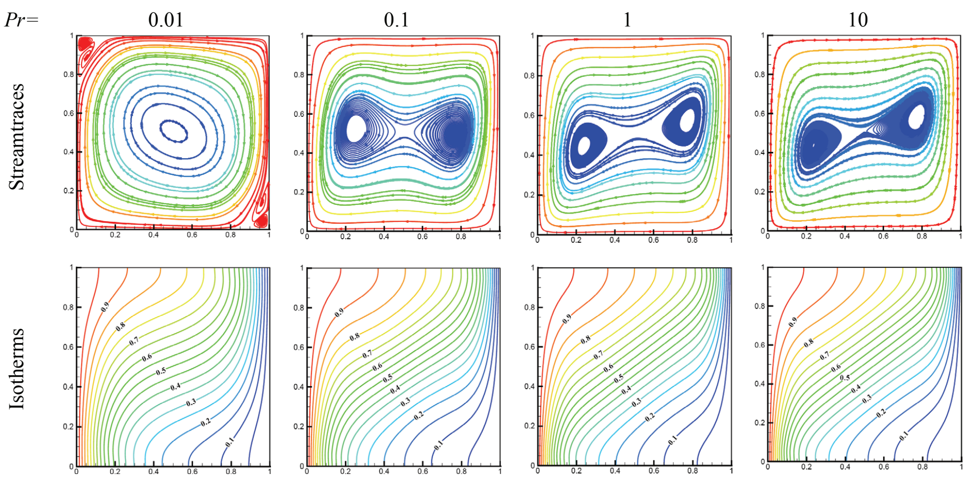

Figure 8: Streamtraces and isotherms plots for Ra= 10^5, Ha=50, γ=90° and different Prandtl numbers.

netic field we can state that Pr has a minor effect at high values, this conclusion remains valid either for the case without external magnetic. The behavior is the same for the stream-function magnitude $|\psi|_{\max}$ which is close 3.09, 3.42, 3.54 and 3.54 and the hot-wall Nusselt number to 2.05, 2.32, 2.37 and 2.37 for respectively Pr=0.01, 0.1, 1 and 10, then a decreased heat transfer rate.

5 Conclusions

We consider in present study an unsteady, two-dimensional MHD natural convection within a electrically conducting filled square enclosure in the presence of inclined magnetic field for different Prandtl number. The governing equations are solved using the lattice Boltzmann method for the differentially heated cavity problem. It was found that for low Prantdl number flow without magnetic field, the cavity hot wall Nusselt number exhibited time dependent behaviour and the the flow is multicellular; however increasing the Prandtl number, a transition to bicellular flow occurs.

Besides, by applying the external magnetic field at low Prandtl number, the convective heat transfer rate is reduced considerably and no unsteady state are observed in the tested monitoring parameters ranges. Furthermore, by increasing the Prandtl number, the flow is always bicellular and the coupled effect of the Lorentz force and viscous force results in dumping the convective currents so that no great changes are observed between Pr=1 and 10 on isotherms and heat transfer quantified using the Nusselt number, however the signature (of varying Pr) is well clear on the streamtraces.

It is found also through this study, that the LB method is a promising tool for investigating MHD convective heat transfer in confined space. Moreover, the LB approach allows more simply accounting for complex physics such as external magnetic force and particularly preserves the computational cost.

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