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Homotopy Analysis of Natural Convection Flows with Effects of Thermal and Mass Diffusion

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Abstract: Both buoyancy effects of thermal and mass diffusion in the natural convection flow about a vertical plate are considered in this paper. The non-linear coupled differential governing equations for velocity, temperature and concentration fields are solved by using the homotopy analysis method. Without the need of iteration, the obtained solution is in the form of an infinite power series which indicates those series have high accuracy when comparing it with other-generated by the traditional method. The impact of the Prandtl number, Schmidt number and the buoyancy parameter on the flow are widely discussed in detail.

Keywords: homotopy analysis method, boundary layer flow, natural convection, heat and mass transfer

1 Introduction

Convective heat and mass transfer problems in the boundary layer flow which have been widely discussed since several decades ago. They have many practical applications in manufacturing processes such as extrusion, melt-spinning, food processing, and other fields. There is a mathematical model describes the phenomena of the boundary layer flow is strong non-linear partial differential equations. There are several techniques to investigate those problems. In the past, the perturbation method and the traditional finite difference method are normally applied to solve those problems. Using similarity transformations on unbounded domain to convert the governing equations into a system of ordinary differential equations is the most common way of solving the non-linear equations. [Ostrach(1953)] analyzed free-convection flow and heat transfer about a vertical flat plate. The results are in good agreement with experimental data in the working fluid is air. [Hellums and Churchill (1962)] presented the transient velocity and temperature fields along

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a semi-infinite vertical plate. The numerical solutions for steady state agree well with previous research [Ostrach (1953)]. [Saville and Churchill (1970)] studied the simultaneous heat and mass transfer in free convection boundary layer flow with a wide range of Prandtl and Schmidt numbers. For prescribed plate temperature and surface heat flux, [Merkin (1985)] found out that the similarity solutions for free convection on a vertical plate was possible only for some special cases. [Ingham (1986)] concerned the free convection boundary layer flow about a vertical flat impermeable surface which is in continuous upward motion with constant speed U_0. Moreover, [Hussain, Hossain, and Wilson (2000)] applied three distinct methodologies to show how the presence of non-uniform species concentration affects the natural convection boundary layer flow from a non-uniformly heated permeable surface with uniform withdrawal (or suction) of fluid. [Lin and Wu (1995, 1997)] solved the problem of combined heat and mass transfer in laminar free convection from a vertical plate with uniform wall temperature and heat flux by usingKeller's Box method. The results indicated that for larger values of Prandtl number as well as Lewis number, mass transfer in a thermal-buoyancy-driven flow is very significant.

Researches mentioned above are mostly solved by traditional schemes. However, non-linear equations are difficult to solve analytically. In the recent year, some efficient and modern methods have been proposed, such as Adomian's decomposition method (ADM) [Adomian (1994)], homotopy analysis method (HAM) [Liao (1992, 2003)] and variational iteration method (VIM) [He (1999)]. Among those new techniques, unlike all other analytic methods, the obtained solution by HAM offers many advantages over other methods. It provides us with a simple way to ensure the convergence of solution series by introducing a convergence-control parameter $\hbar$ [Liao (2010)]. Also, it provides us with great freedom to choose proper base functions to approximate various non-linear problems in science and engineering. Both [Ghotbi, Bararnia, Domairry, and Barari (2009)] and [Motsa, Shateyi, and Makukula (2011)] applied the HAM in a free convection boundary layer flow problem. The HAM approximate solution was found to be in excellent agreement with the numerical result. Besides, the HAM has already been proven for successfully applying to a broad class of problems [Odibat, Momani, and Xu (2010); Tien, Yang, and Chen (2012)].

In this paper, the HAM is applied to solve the non-linear coupled differential equations which govern the combined buoyancy effects of thermal and mass diffusion in natural convection flow about a vertical plate. The obtained results are in good agreement with [Ostrach (1953)].

2 Mathematical Model

As shown in Figure 1, considering a two-dimensional laminar boundary layer flow of an incompressible viscous fluid over a vertical flat plate which maintained with uniform surface temperature T_S and concentration C_S is placed in an ambient fluid of temperature T_∞ and concentration C_∞. Introducing a cartesian coordinate system, x-axis is chosen along the plate in the direction of flow and y-axis normal to it.

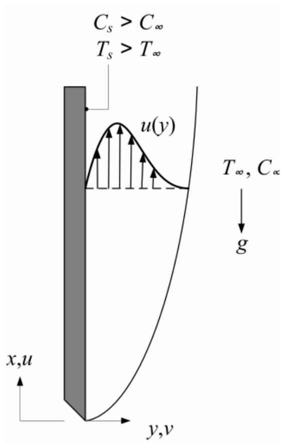

Figure 1: Schematic representation of boundary layer flow about vertical plate

Under the Boussinesq approximation and neglecting the energy dissipation, the govern equations of the steady, laminar, two-dimensional boundary layer flow problem can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T\left(T - T_\infty\right) + g\beta_c\left(C - C_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where β_T and β_C are the volumetric coefficients of thermal and concentration expansion, α and D are thermal diffusivity and molecular diffusivity, respectively.

The boundary conditions of this problem can be regarded as:

$$u = v = 0, \quad T = T_S, \quad C = C_S \text{ at } y = 0 \tag{5}$$

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ as } y \to \infty \tag{6}$$

The continuity equation (1) is satisfied by the stream function $\psi(x,y)$ which defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

Following [Gebhart and Pera (1971)], we introduce the following similarity transformations:

$$\eta = \frac{y}{x}\left(\frac{Gr_{x,T} + Gr_{x,c}}{4}\right)^{1/4}$$

$$\psi(x,y) = 4\nu\left(\frac{Gr_{x,T} + Gr_{x,c}}{4}\right)^{1/4} f(\eta)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_S - T_\infty}$$

$$\varphi(\eta) = \frac{C - C_\infty}{C_S - C_\infty}$$

where $Gr_{x,T} = \frac{g\beta_T(T_S - T_\infty)x^3}{\nu^2}$ and $Gr_{x,c} = \frac{g\beta_c(C_S - C_\infty)x^3}{\nu^2}$ are Grashof numbers of temperature and concentration, respectively. Substituting the above transformations into Eqs. (1)-(4), we obtain the similarity equations:

$$f''' + 3ff'' - 2(f')^2 + \frac{\theta + N\phi}{N+1} = 0 \tag{8}$$

$$\theta'' + 3Pr \cdot f\theta' = 0 \tag{9}$$

$$\varphi'' + 3S_c \cdot f\varphi' = 0 \tag{10}$$

Along with the boundary conditions

$$f(0) = f'(0) = 0, \quad \theta(0) = \varphi(0) = 1 \tag{11}$$

$$f'(\infty) = \theta(\infty) = \varphi(\infty) = 0 \tag{12}$$

where $N = \frac{Gr_{x,c}}{Gr_{x,T}}$ is the quantity which measures the relative importance of mass and thermal diffusion in causing the density difference which drives the fluid.

3 Homotopy analysis method solution

By means of HAM, we first define the non-linear operators as

$$\mathrm{N}_f[F,\Theta,\Gamma] = \frac{\partial^3 F}{\partial\eta^3} + 3F\frac{\partial^2 F}{\partial\eta^2} - 2\left(\frac{\partial F}{\partial\eta}\right)^2 + \frac{\Theta+N\Gamma}{N+1} \tag{13}$$

$$\mathrm{N}_\theta[F,\Theta,\Gamma] = \frac{\partial^2\Theta}{\partial\eta^2} + 3Pr\cdot F\frac{\partial\Theta}{\partial\eta} \tag{14}$$

$$\mathrm{N}_\varphi[F,\Theta,\Gamma] = \frac{\partial^2\Gamma}{\partial\eta^2} + 3S_c\cdot F\frac{\partial\Gamma}{\partial\eta} \tag{15}$$

Let $q\in[0,1]$ denote an embedding parameter and L_f, L_θ, L_ϕ auxiliary linear operators and $f_0(\eta)$, $\theta_0(\eta)$, $\phi_0(\eta)$ the initial guesses which satisfy the boundary conditions (11)-(12), respectively. Here, the auxiliary linear operator L has the properties

$$\mathrm{L}[0] = 0$$

and

$$\mathrm{L}[\alpha_1(q)\mathscr{W}_1(\eta,q)+\alpha_2(q)\mathscr{W}_2(\eta,q)] = \alpha_1(q)\mathrm{L}[\mathscr{W}_1(\eta,q)]+\alpha_2(q)\mathrm{L}[\mathscr{W}_2(\eta,q)]$$

where $\alpha_1(q)$, $\alpha_2(q)$, $\mathscr{W}_1(\eta,q)$ and $\mathscr{W}_2(\eta,q)$ are any real functions. Then, we construct the so-called zeroth-order deformation equations

$$(1-q)\mathrm{L}_f[F-f_0] = q\hbar_f\mathrm{N}_f[F,\Theta,\Gamma] \tag{16}$$

$$(1-q)\mathrm{L}_\theta[\Theta-\theta_0] = q\hbar_\theta\mathrm{N}_\theta[F,\Theta,\Gamma] \tag{17}$$

$$(1-q)\mathrm{L}_\phi[\Gamma-\phi_0] = q\hbar_\phi\mathrm{N}_\phi[F,\Theta,\Gamma] \tag{18}$$

Subject to the boundary conditions

$$F(0;q) = \left.\frac{\partial F(\eta;q)}{\partial\eta}\right|_{\eta=0} = \left.\frac{\partial F(\eta;q)}{\partial\eta}\right|_{\eta=\infty} = 0 \tag{19}$$

$$\Theta(0;q) = 1, \quad \Theta(\infty;q) = 0 \tag{20}$$

$$\Gamma(0;q) = 1, \quad \Gamma(\infty;q) = 0 \tag{21}$$

where $\hbar_f$, $\hbar_\theta$ and $\hbar_\phi$ denote the convergence-control parameters. With the properties of L, it's easily to realize that when $q=0$ and $q=1$, we have the solutions of Eqs. (16)-(18) read

$$F(\eta;0) = f_0(\eta), \quad \Theta(\eta;0) = \theta_0(\eta), \quad \Gamma(\eta;0) = \phi_0(\eta) \tag{22}$$

and

$$F(\eta;1) = f(\eta), \quad \Theta(\eta;1) = \theta(\eta), \quad \Gamma(\eta;1) = \phi(\eta) \tag{23}$$

Thus, as the embedding parameter q increases from 0 to 1, $F(\eta;q)$, $\Theta(\eta;q)$ and $\Gamma(\eta;q)$ varies continuously from the initial guesses to the exact solutions of Eqs. (8)-(12). According to Taylor's theorem, expanding $F(\eta;q)$, $\Theta(\eta;q)$ and $\Gamma(\eta;q)$ with respect to q and using (22), we have the following power series

$$F(\eta;q) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta)\, q^m \tag{24}$$

$$\Theta(\eta;q) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)\, q^m \tag{25}$$

$$\Gamma(\eta;q) = \varphi_0(\eta) + \sum_{m=1}^{\infty} \phi_m(\eta)\, q^m \tag{26}$$

where

$$f_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m F(\eta;q)}{\partial q^m} \right|_{q=0}$$

$$\theta_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \Theta(\eta;q)}{\partial q^m} \right|_{q=0}$$

$$\varphi_m(\eta) = \frac{1}{m!} \left. \frac{\partial^m \Gamma(\eta;q)}{\partial q^m} \right|_{q=0}$$

The convergence of the series (24)-(26) depends on the initial guesses, the auxiliary linear operators and the convergence-control parameters. Assuming that all of them are appropriate selected such that the above series converge at $q = 1$. Thus, we have due to (23) the relationship between the initial guesses and the exact solutions

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \tag{27}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \tag{28}$$

$$\varphi(\eta) = \phi_0(\eta) + \sum_{m=1}^{\infty} \varphi_m(\eta) \tag{29}$$

Next, we define the vectors

$$\overrightarrow{f}_k = \{f_0, f_1, f_2, \ldots, f_k\}, \quad \overrightarrow{\theta}_k = \{\theta_0, \theta_1, \theta_2, \ldots, \theta_k\}, \quad \overrightarrow{\varphi}_k = \{\varphi_0, \varphi_1, \varphi_2, \ldots, \varphi_k\} \tag{30}$$

Based on [Liao (2003)], we differentiate the zeroth-order deformation Eqs. (16)-(18) m times with respect to q then dividing by $m!$ and finally setting $q = 0$, we obtain the following mth-order deformation equations

$$\mathrm{L}_f\left[f_m - \chi_m f_{m-1}\right] = \hbar_f \mathscr{R}_m^f\left(\overrightarrow{f}_{m-1}, \overrightarrow{\theta}_{m-1}, \overrightarrow{\varphi}_{m-1}\right) \tag{31}$$

$$\mathrm{L}_\theta\left[\theta_m - \chi_m \theta_{m-1}\right] = \hbar_\theta \mathscr{R}_m^\theta\left(\overrightarrow{f}_{m-1}, \overrightarrow{\theta}_{m-1}, \overrightarrow{\varphi}_{m-1}\right) \tag{32}$$

$$\mathrm{L}_\varphi\left[\varphi_m - \chi_m \varphi_{m-1}\right] = \hbar_\varphi \mathscr{R}_m^\varphi\left(\overrightarrow{f}_{m-1}, \overrightarrow{\theta}_{m-1}, \overrightarrow{\varphi}_{m-1}\right) \tag{33}$$

Subject to the boundary conditions

$$f_m(0) = f_m'(0) = f_m'(\infty) = 0 \tag{34}$$

$$\theta_m(0) = 0, \quad \theta_m(\infty) = 0 \tag{35}$$

$$\varphi_m(0) = 0, \quad \varphi_m(\infty) = 0 \tag{36}$$

where

$$\mathscr{R}_m^f\left(\overrightarrow{f}_{m-1}, \overrightarrow{\theta}_{m-1}, \overrightarrow{\varphi}_{m-1}\right) = f_{m-1}''' + 3\sum_{j=0}^{m-1} f_j f_{m-1-j}'' - 2\sum_{i=0}^{m-1} f_i' f_{m-1-i}' + \frac{\theta_{m-1} + N\varphi_{m-1}}{N+1} \tag{37}$$

$$\mathscr{R}_m^\theta\left(\overrightarrow{f}_{m-1}, \overrightarrow{\theta}_{m-1}, \overrightarrow{\varphi}_{m-1}\right) = \theta_{m-1}'' + 3\cdot Pr\sum_{j=0}^{m-1} f_j \theta_{m-1-j}' \tag{38}$$

$$\mathscr{R}_m^\varphi\left(\overrightarrow{f}_{m-1}, \overrightarrow{\theta}_{m-1}, \overrightarrow{\varphi}_{m-1}\right) = \varphi_{m-1}'' + 3\cdot S_c\sum_{j=0}^{m-1} f_j \varphi_{m-1-j}' \tag{39}$$

and

$$\chi_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1. \end{cases} \tag{40}$$

It should be emphasized that the high-order deformation equations (31)-(33) are linear. In the frame of the HAM, a system of non-linear PDEs can be transferred into an infinite number of linear ODEs. Now, let us choose the initial guesses and the auxiliary linear operators. It is well-known that most viscous flows decay exponentially at infinity, especially the velocity of Blasius' similarity boundary layer flows [Blasius (1908)] decays like $\exp(-\eta^2)$ far from the wall. So, even though we do not know the details of the solutions, we are quite sure that the solutions should be in such a form

$$f(\eta) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} a_{m,n}\eta^n exp(-2m\eta) \tag{41}$$

$$\theta(\eta) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} b_{m,n}\eta^n exp(-2m\eta) \tag{42}$$

$$\phi(\eta) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} c_{m,n}\eta^n exp(-2m\eta) \tag{43}$$

where $a_{m,n}$, $b_{m,n}$, and $c_{m,n}$ are coefficients to be determined. The above expression, namely *the solution expression*, is very important in the frame of the HAM. Considering the solution expression and the boundary conditions (11)-(12), we choose

$$f_0(\eta) = \frac{1}{2} - \frac{1}{2}e^{-2\eta} - \eta e^{-2\eta} \tag{44}$$

$$\theta_0(\eta) = e^{-2\eta} \tag{45}$$

$$\varphi_0(\eta) = e^{-2\eta} \tag{46}$$

as the initial guesses of $f(\eta)$, $\theta(\eta)$ and $\varphi(\eta)$, respectively. According to the HAM, we have great freedom to choose the auxiliary operators only restricting by the solution expression (41)-(43) and the boundary conditions (11)-(12). Therefore, we choose the auxiliary operators

$$\mathrm{L}_f = \frac{\partial^3}{\partial\eta^3} + a_0(\zeta)\frac{\partial^2}{\partial\eta^2} + a_1(\zeta)\frac{\partial}{\partial\eta} + a_2(\zeta) \tag{47}$$

$$\mathrm{L}_\theta = \frac{\partial^2}{\partial\eta^2} + b_0(\zeta)\frac{\partial}{\partial\eta} + b_1(\zeta) \tag{48}$$

$$\mathrm{L}_\varphi = \frac{\partial^2}{\partial\eta^2} + c_0(\zeta)\frac{\partial}{\partial\eta} + c_1(\zeta) \tag{49}$$

where $a_0(\zeta)$, $a_1(\zeta)$, $a_2(\zeta)$, $b_0(\zeta)$, $b_1(\zeta)$, $c_0(\zeta)$ and $c_1(\zeta)$ are real functions to be determined. Let $\mathscr{W}_1^*(\eta)$, $\mathscr{W}_2^*(\eta)$ and $\mathscr{W}_3^*(\eta)$ denote the three non-zero solutions of $\mathrm{L}_f\mathscr{W} = 0$, i. e. ,

$$\mathrm{L}_f[\mathscr{W}_1^*(\eta)] = \mathrm{L}_f[\mathscr{W}_2^*(\eta)] = \mathrm{L}_f[\mathscr{W}_3^*(\eta)] = 0 \tag{50}$$

Due to the solution expression and the boundary conditions, we can choose

$$\mathscr{W}_1^* = 1, \quad \mathscr{W}_2^* = \eta, \quad \mathscr{W}_3^* = exp(-2\eta) \tag{51}$$

Substituting (51) into (50) with the definition (47), we have the auxiliary linear operator

$$\mathrm{L}_f F = \frac{\partial^3 F}{\partial \eta^3} + 2\frac{\partial^2 F}{\partial \eta^2} \tag{52}$$

In the similar way, we have

$$\mathrm{L}_\theta \Theta = \frac{\partial^2 \Theta}{\partial \eta^2} + 2\frac{\partial \Theta}{\partial \eta} \tag{53}$$

$$\mathrm{L}_\varphi \Gamma = \frac{\partial^2 \Gamma}{\partial \eta^2} + 2\frac{\partial \Gamma}{\partial \eta} \tag{54}$$

with the properties

$$\mathrm{L}_f \left[\mathrm{C}_1 + \mathrm{C}_2\eta + \mathrm{C}_3 e^{-2\eta}\right] = 0 \tag{55}$$

$$\mathrm{L}_\theta \left[\mathrm{C}_4 + \mathrm{C}_5 e^{-2\eta}\right] = 0 \tag{56}$$

$$\mathrm{L}_\varphi \left[\mathrm{C}_6 + \mathrm{C}_7 e^{-2\eta}\right] = 0 \tag{57}$$

Then, the general solution of the high-order deformation equations (31)-(33)are given by

$$f_m(\eta) = f_m^*(\eta) + \mathrm{C}_1 + \mathrm{C}_2\eta + \mathrm{C}_3 e^{-2\eta} \tag{58}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + \mathrm{C}_4 + \mathrm{C}_5 e^{-2\eta} \tag{59}$$

$$\varphi_m(\eta) = \varphi_m^*(\eta) + \mathrm{C}_6 + \mathrm{C}_7 e^{-2\eta} \tag{60}$$

where $f_m^*(\eta)$, $\theta_m^*(\eta)$ and $\varphi_m^*(\eta)$ are special solutions of (31)-(33), and C_1 to C_7 are constants which can be determined by the boundary conditions.

4 Result and analysis

As was pointed out by Liao [Liao (2003)], the series solutions obtained by (58)-(60) contain the convergence-control parameters $\hbar_f$, $\hbar_\theta$ and $\hbar_\varphi$ which provide us a simple way to adjust and control the convergence region and rate of series solutions. Therefore, to choose proper values of $\hbar$, we regard $\hbar$ as a variable and plot

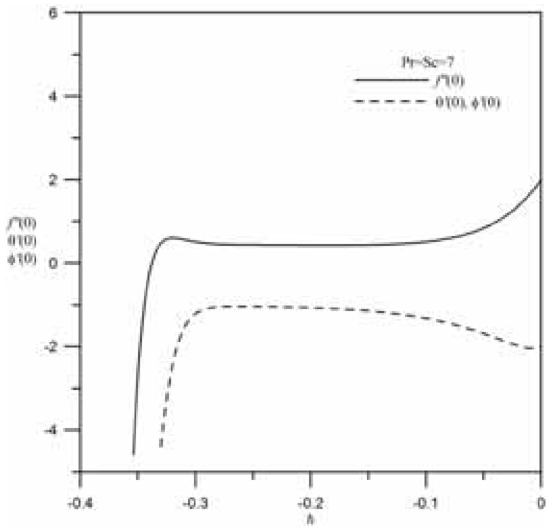

Figure 2: The $\hbar$-curve of $f''(0)$, $\theta'(0)$ and $\varphi'(0)$ for 18^{th}-order HAM approximation when $Pr = S_c = 7$, $N = 0$ and $\hbar_f = \hbar_\theta = \hbar_\varphi$.

the so-called $\hbar$-curve, as shown in Fig. 2. Since $f''(0)$ is closely related to the local coefficient of skin friction and the local coefficient of skin friction is unique. Mathematically, all convergent series solutions of $f''(0)$ given by different values of convergence-control parameters should converge to the same value.

From Fig. 2, we note that for $Pr = S_c = 7$ and $N = 0$, the stable region for $f''(0)$, $\theta'(0)$ and $\varphi'(0)$ are about 3/10< $\hbar$ ≤1/5. Similarly, for the other set of parameters, we have different valid regions of $\hbar$. Based on the theory of HAM, setting $\hbar$ any value in the valid region, we are quite sure that the corresponding solution series converge. Figure 3 shows the dimensionless temperature, concentration and velocity distributions for $Pr = 0.7$ and 7 with $N = 0$. Note that, the buoyancy parameter N measures the relative importance of mass and thermal diffusion in causing the density difference. N is zero for no species diffusion, infinite for no thermal diffusion, positive for both effects combining to derive the flow. Comparing the working fluid between air $(Pr = 0.7)$ and water $(Pr = 7)$, the maximum dimensionless velocity of air is faster than that of water and the mean dimensionless temperature of air is also higher than water. This is due to the thermal boundary layer thickness of air is thicker than water. On the other hand, the velocity boundary layer of water is thicker than air. Besides, it is worth to point out that if $Pr = S_c$ and $N = 0$, the results of the temperature profile are the same with the concentration.

Considering the same conditions in [Ostrach (1953)] that are defined as $S_c = N = 0$.

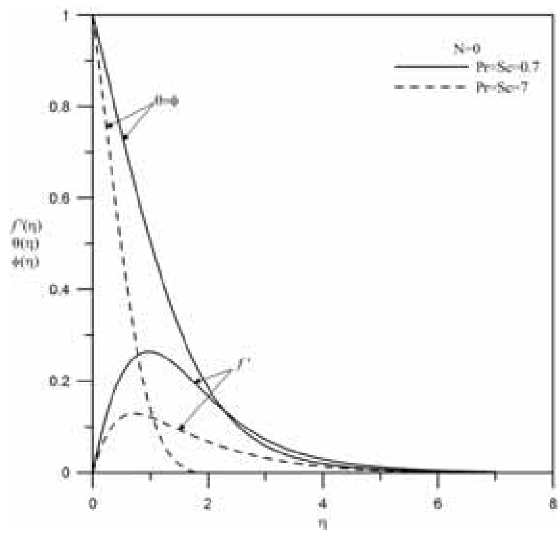

Figure 3: Dimensionless temperature, concentration and velocity distributions for $Pr = 0.7$ and 7 with $N = 0$.

Both Fig. 4 and Fig. 5 show the comparison of the HAM solution with [Ostrach (1953)] for different values of Prandtl numbers. Obviously, the present method gives quick and accurate results instead of complicated numerical integration and iteration procedures.

A good agreement between the two results is observed, which confirms the validity of the homotopy analysis method. Also, we realize that the maximum value of the dimensionless velocity distributions occurs at larger values of the argument η as the Prandtl number decreasing. As can be clearly seen in Fig. 6, the dimensionless velocity decreases with increasing the buoyancy parameter N. This is because when increasing the value of the buoyancy ratio N, the effect of species diffusion has more advantages over thermal diffusion which means the concentration species is dispersed farther away. And coupled with the effect of gravity, the whole velocity distribution reduces in all domains. Meanwhile, decreasing the Schmidt number increases the velocity level and its extent.

Since the effect of $Gr_{x,c}$ increases while increasing the buoyancy parameter, and the definition of Grashof number is the ratio of the buoyancy to viscous force. In Fig. 7, we observe that the dimensionless concentration distribution increases as N increases. In addition, Schmidt number is defined as the ratio of momentum diffusivity and mass diffusivity. This in turn leads to the gradient of dimensionless concentration near the wall increases as the Schmidt number increasing. Fig. 8 not

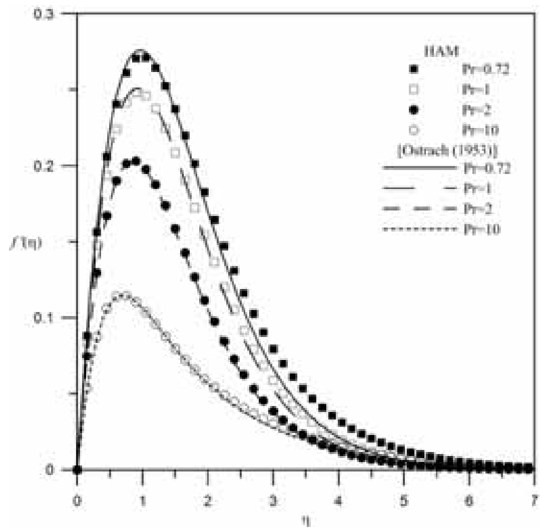

Figure 4: Comparison of the HAM solution (symbols) with [Ostrach (1953)] (line) of the velocity distributions with different values of Prandtl number as$S_c = N = 0$.

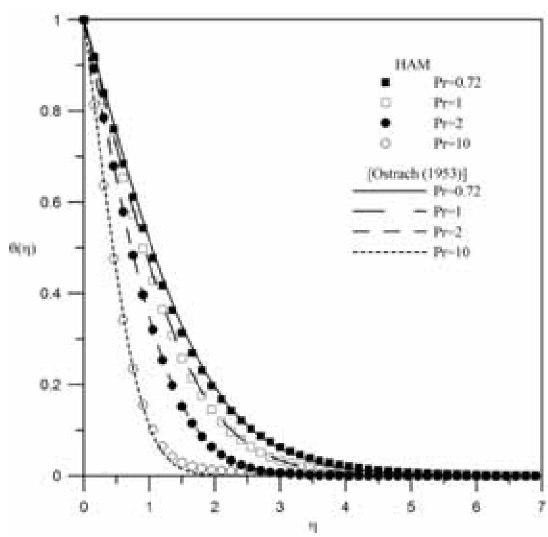

Figure 5: Comparison of the HAM solution (symbols) with [Ostrach (1953)] (line) of the temperature distributions with different values of Prandtl number as$S_c = N = 0$.

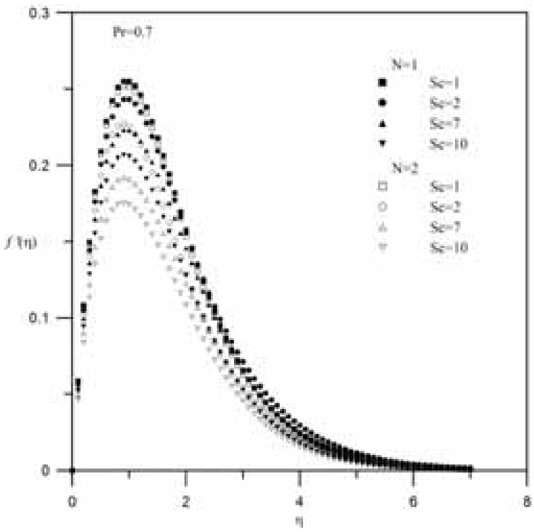

Figure 6: Dimensionless velocity distribution for different Schmidt numbers at given Prandtl number.

only shows the dimensionless temperature distributions for purely thermal effect and the combining of two buoyancy effects but also illustrates the effect of varying Schmidt number. Note that the gradient of dimensionless temperature increase as S_c decreasing. As we know from Fig. 7, the gradient of concentration near the wall increases as the Schmidt number increasing which could enhance the conduction heat transfer in the boundary layer region. So, the gradient of temperature becomes gentle with increasing S_c.

5 Conclusions

The present paper analyzes the effects of the Prandtl number Pr, the Schmidt number S_c, and the buoyancy parameter N, on the laminar nature convection boundary layer flow from a vertical plate. The homotopy analysis method is applied to solve the non-linear coupled differential equations which govern the combined buoyancy effects of thermal and mass diffusion in the boundary layer flow. Different from all other analytic methods, the HAM offers many advantages over other methods. It provides us with a simple way to adjust and control the convergence region of solution series by introducing a convergence-control parameter $\hbar$. According to the results we obtain from the investigations, it may conclude as follows:

1. An increase in the value of Prandtl number Pr, leads to decrease the velocity

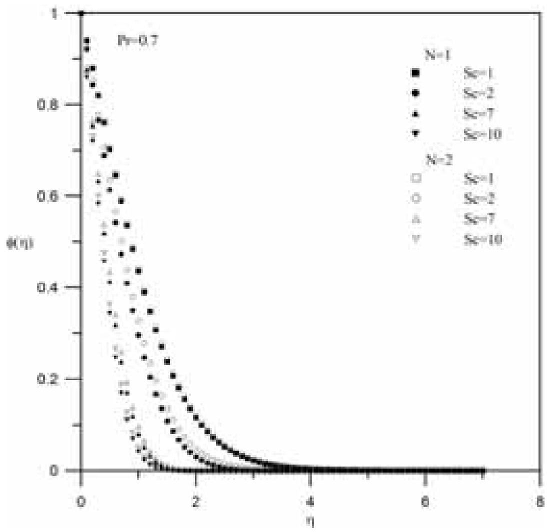

Figure 7: Dimensionless concentration distribution for different Schmidt numbers and buoyancy ratios at given Prandtl number.

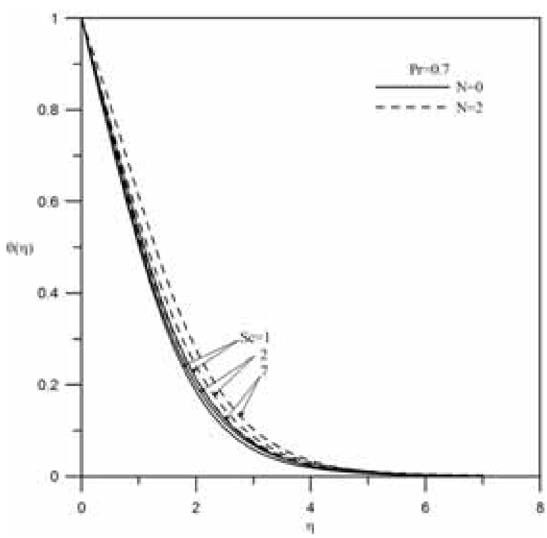

Figure 8: Dimensionless temperature distribution for different Schmidt numbers and buoyancy ratios at given Prandtl number.

distributions and increase the gradient of the temperature distributions.

2. Similar to Prandtl number, the velocity distributions and the gradient of the concentration distributions decrease and the gradient of the temperature distributions increase as increasing the value of Schmidt number S_c.

3. By increasing the buoyancy parameter parameter N, leads to slow down the maximum velocities and decreases both the gradients of the temperature and concentration distributions.

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