 CMES, vol.85, no.4, pp.329-345, 2012

Numerical Investigation of a Vibroacoustic Analysis with Different Formulations

M. Amdi[1], **M. Souli**[1], **J. Hargreaves**[2] **and F. Erchiqui**[3]

Abstract: Simulation of vibroacoustic problems becomes more and more the focus of engineering in the last decades for acoustic comfort in automotive industry to reduce noise and vibration inside a cabin and also in sport industry to analyze sound produced by a club impacting a golf ball to avoid unexpected noise problems during the design process. Traditionally, Finite element and Boundary element methods are used in frequency domain to model pressure noise from structure vibration in low and mid frequency range. These methods require velocity in frequency domain on the vibrating structure as boundary conditions. To analyze pressure noise from impact analysis like in golf problem for instance, time domain analysis of nonlinear finite element method using explicit or implicit time integration, needs to be performed first, to supply velocity boundary conditions for the acoustic problem. In this paper a combined time domain and frequency domain analysis is performed to solve acoustic problems of vibrating structure. In this paper we use the state of the art in LSDYNA code that combine both analysis to analyze pressure noise deduced from a short time impact on a deformable structure. To validate numerical results from our simulation, different formulations are performed and validated to simulate pressure sound at different locations.

Keywords: BEM, vibroacoustic, low-rank approximation.

1 Introduction

Full scale experimental tests for analyzing a pressure sound level from a vibrating structure are costly. Numerical simulations help to minimize the number of experimental tests required. Once simulations are validated by test results, engineers can use them as tool design for improvement of the system structure involved.

[1] LML, Boulvard Paul Langevin, Cité scientifique, 59655 Villeneuve d'Ascq Cedex, France
[2] The Arup Campus, Blythe Gate, Blythe Valley Park, Solihull, West Midlands, B90 8AE
[3] Université du Québec en Abtitibi-Témiscaminque, 445Boul. de l'Université, Rouyn-Noranda QC, Que., Canada J9X5E4

These analysis have been extensively used in automotive industry to reduce noise in interior car from vibration and more recently in sport industry to analyze sound level during a golf ball impact, which is an important issue in golf industry. In the analysis of pressure noise level form impact problem or vibration on a non linear structure, two steps need to be performed. The first step is the nonlinear analysis of the impact problem, this can be done by nonlinear finite element analysis in time domain. Analysis of impact problems are usually short time analysis and can be performed using explicit time integration method. The finite element analysis provides velocity at the nodes of the structure at each time step of the computation. Using Fast Fourier Transform (FFT), these data can be converted into frequency domain for each node of the structure mesh. Once the structure analysis is performed and provide structure velocity, an acoustic analysis can be performed using BEM (Boundary Element method). To solve the acoustic problem, using velocity on the structure that is related to the normal derivative of the pressure in frequency domain by:

$$\frac{\partial p}{\partial n} = -i\rho\omega v_n$$

where v_n is the normal velocity and ρ is the density of the fluid.

The Helmholtz equation governing acoustic pressure propagation in frequency domain:

$$\Delta p + k^2 p = 0$$

is used in its boundary integral form, $k = \omega/c$ denotes the wave number c is the speed of sound and $\omega = 2\pi f$ is the pulsation, where the solution inside the domain is represented through an integral involving the solution and the normal derivatives of the solution at the boundary. The BEM method has been developed in LSDYNA code to solve level pressure sound in frequency domain for external and internal problems. Unlike Finite Element methods that generate sparse matrix, and require low memory storage, in BEM method a dense matrix is generated. Low Rank method is used to accelerate the solution of the linear system and reduce computational time, and also to minimize RAM memory storage. The main idea of the Low Rank is to employ iterative solvers, such as GMRES, to solve the BEM system of equations and use representative modes to accelerate the matrix-vector multiplication in each iteration step, without forming the full size matrix. The main characteristic of the method is that only a mesh of the structure is required, and external or internal air mesh is not needed. Hence, the method is easier to apply than classical finite element method, which requires a finite element mesh for the air domain. Since the ultimate objective is a design of structure that reduce the noise for acoustic comfort, numerical simulations can be included in shape design optimization with shape optimal design techniques, Souli et al. [Souli and Zolesio (1993)], and material optimisation, Souli et al. [Erchiqui, Souli, and Yedder (2007)], Ozdemir et al. [Ozdemir, Souli, and Fahjan (2010)]. Once simulations are validated by test results, it can be used as design tool for the improvement of the system structure involved.

In this paper, we perform an analysis of sound pressure level at arbitrary locations, issued from vibration of a clamped structural shell plate subjected to an impulsive loading. To validate numerical results, different formulations are performed and compared to the simulation from SYSNOISE a well established acoustic code. This paper is structured as follows, in section 2, the mathematical and numerical description of the finite element model is described. Section 3 is devoted to the mathematical formulation and description of the Boundary Element formulation deduced from Helmholtz equation that describes acoustic wave propagation in frequency domain. The Low Rank method used to reduce computational time and memory storage when solving the linear system with GMERS iterative method is described in section 4. In section 5, we compare different formulations using their accuracy and computational time.

2 Structural dynamic problem

In this paper, the interaction of an elastic structure with a compressible, isotropic, homogeneous and non-viscid fluid is considered. Let's consider an isotropic structure occupying a volume Ω_S [Fig. 1]. When it is subjected to a body force f, the equation governing its vibratory behavior is given by the following momentum equation:

$$\rho \frac{d^2 \overrightarrow{u}}{dt^2} = div(\overline{\overline{\sigma}}) + \overrightarrow{f} \tag{1}$$

where $\overrightarrow{u}$ is the displacement, $\overline{\overline{\sigma}}$ is the Cauchy stress, ρ is the density.

Let Γ_{S_0} and Γ_S denote the boundaries subjected to a displacement and traction, respectively. The boundary conditions associated to the structure can be written in Γ_s as:

$$\begin{cases} u(x,0) = 0 \\ \frac{\partial u(x,0)}{\partial t} = 0 \end{cases} \tag{2}$$

In almost all studies, the structure simulations have been done using explicit Finite Element Method. The solution is advanced in time using centered second order

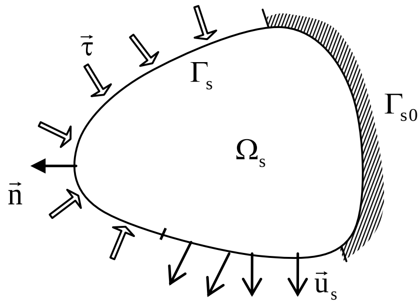

Figure 1: Structural problem

scheme. The resolution is advanced in time with the central difference method, which provides a second order accuracy for time integration. For each node, the velocity and displacement are updated using centered second order finite difference method in time:

$$\dot{u}^{n+\frac{1}{2}} = \dot{u}^{n-\frac{1}{2}} + \Delta t M^{-1}(F_{ext} + F_{int}) \tag{3}$$

$$u^{n+1} = u^n + \Delta t\, \dot{u}^{n+\frac{1}{2}} \tag{4}$$

Where F_{int} is the internal vector force and F_{ext} the external vector force associated with body forces, coupling forces, and pressure boundary conditions, M is a diagonal lumped mass matrix and $\dot{u}$ is the velocity. At each node, the internal force is computed as follows:

$$F_{int} = \sum_{k=1}^{Nelem} \int_k B^t \sigma dv \tag{5}$$

Where B is the gradient matrix and Nelem is the number of elements. Using the mass matrix would require the solution of a system of linear equations for the displacements at each time step, which would be costly, therefore a lumped diagonal mass matrix is commonly used [Belytschko (2000)] for explicit time integration.

The time step size, Δt, is limited by the courant stability condition [Benson (1992)], which may be expressed as

$$\Delta t \leq \frac{l}{c}$$

Where l is the characteristic length of the element and c is the speed of sound through the material in the element. For solid material, the speed of sound is defined as

$$c = \sqrt{\frac{K}{\rho}}$$

Where ρ is the material density and K is the modulus of compressibility.

To perform the Boundary Element Method, nodal velocities in time domain are stored at each time step and will be converted to frequency domain at the end of the structural analysis. The FFT applied to the nodal velocity constitutes the boundary condition of the BEM.

3 Boundary Element Method

Once the structural dynamic problem is solved in time domain; and velocity converted from time domain to frequency domain using Fast Fourier Transform we use the velocity as boundary conditions for the acoustic analysis. The BEM is used to evaluate the pressure response in the acoustic domain from the structure velocity results deduced from the first analysis. Consider a boundary surface S enclosing a volume Ω_s filled and surrounded by an ideal and homogeneous fluid medium [Fig. 2].

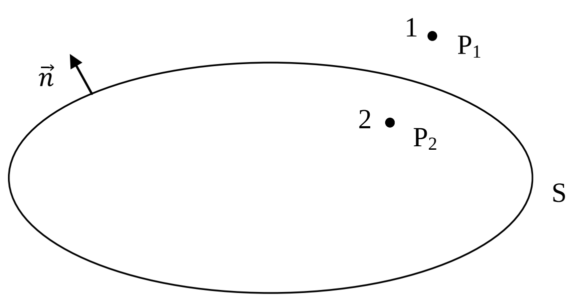

Figure 2: Acoustical problem

For a harmonic disturbance of frequency f without any source or loss mechanisms, the pressure p satisfies the Helmholtz equation:

$$\Delta p + k^2 p = 0 \tag{6}$$

For Neumann boundary condition with implies that the velocity is continuous across the surface:

$$\frac{\partial p}{\partial n} = -i\rho\omega v_n \tag{7}$$

By using Green's theorem, the corresponding integral equation can be written as:

$$C(r)p(r) = \int_{S_y} (G(r,r_y)\frac{\partial p(r_y)}{\partial n_y} - p(r_y)\frac{\partial G(r,r_y)}{\partial n_y})ds_y \tag{8}$$

This equation allows the calculation of sound pressure at any point of the acoustic domain. In equation (6) and (8) $k = \frac{\omega}{c}$ denotes the wave number, c is the speed of sound, $\omega = 2\pi f$ is the pulsation, $p(r)$ is the pressure at any field point r,$G(r,r_y) = \frac{e^{-ik\|r-r_y\|}}{4\pi\|r-r_y\|}$ is the Green's function. Where r is the position vector of any field point, r_y is the position vector of a source point located at acoustic domain boundary and C is the jump term resulting from the treatment of the singular integral involving Green's function.

The indirect Boundary Element Method defines the primary variables as the jump in the pressure $\mu = p_1 - p_2$ and the jump in the normal gradient of the pressure $\sigma = \frac{\partial p(r_{y1})}{\partial n_y} - \frac{\partial p(r_{y2})}{\partial n_y} = i\omega\rho(V_n(r_{y1}) - V_n(r_{y2}))$ between the two sides of the boundary element method [Z. Zhang and Zhang (2000)].

Due to the definition of the primary variables, there is no differentiation between interior and exterior acoustic domain. Therefore, the Indirect BEM approach is suitable for simulation of general geometries involving multi-connections as well as free edges of non-closed objects [Wu (2001)]. In addition, the Indirect BEM approach takes into account the fluid on both sides, which makes it suitable for noise transmission problems through elastic structures. However, care must be taken when modeling free edges where the primary variables must be forced to zero [Wu (2001)].

For Neumann problem that involve velocities prescribed on the acoustic boundary, the integral form in Eq. (5) is obtained:

$$p(r) = \int_{S_y} \frac{\partial G(r,r_y)}{\partial n_y}\mu(r_y)ds_y \tag{9}$$

Hence, equation (9) can be written as following:

$$-i\rho\omega V_n(r_x) = \int_{S_y} \frac{\partial^2 G(r,r_y)}{\partial n_x \partial n_y} \mu(r_y) ds_y \tag{10}$$

Equation (10) can be solved using the variational principle to the integral equation. In fact, it permits to reduce the hypersingular integrals to a less singular form. In addition, the variational indirect boundary element method yields to symmetric fully populated matrices. By using the variational method, the last equation can be written as:

$$-i\rho\omega \int_{S_x} V_n(r_x)\mu(r_x) ds_x = \int_{S_x}\int_{S_y} \frac{\partial^2 G(r,r_y)}{\partial n_x \partial n_y} \mu(r_x)\mu(r_y) ds_x ds_y \tag{11}$$

where $\mu(r_x)$ represents the test function of the variational method.

This method has been widely used despite the hypersingularity [Hamdi (1982)] which can be reduced to a less singular form more suitable for numerical calculations [Hamdi (1982)]. The solution of Eq. (11) can be obtained by dividing the surface into boundary elements. Therefore, the discretized form of the integral equation leads to a linear system given by $A\mu = B$.

From equation (11), the double potential layer is calculated. Finally, the pressure at any point of the field can be computed via equation (9). It is to be emphasized that in BEM the linear system depends on the frequency via Green's function. For each frequency, the system has to be solved. For this reason, we have used an iterative solver like GMRES which is more efficient for this kind of problems than the direct solver. GMRES is an algorithm for solving non-symmetric linear system based on Hessenberg process. GMRES iterative method accesses the matrix A through a matrix-vector product at each iteration and does not need to use the matrix coefficient explicitly. The number of required iterations of the GMRES which are equal to the number of required matrix-vector products is crucial.

4 Low Rank Method

Low-rank matrix factorization is one of the most useful tools in scientific computing and data analysis. The goal of low-rank factorization is to decompose a matrix into a product of two smaller matrices of lower rank that approximates the original matrix well.

$$\underset{m\times n}{A} = \underset{m\times k}{B}\,\underset{k\times n}{C} \tag{12}$$

A universal benefit of such low-rank decomposition is that fewer elements are required to represent the matrix ($km+kn$ "versus" mn), requiring less storage and less operations to perform matrix-vector multiplication. The A matrix can be well approximated by a low rank matrix. The easiest way to show this is to consider the QR Houshoulder decomposition of A which decomposes a matrix A into two matrices:

$$\underset{m\times n}{A} = \underset{m\times m}{Q}\,\underset{m\times n}{R} \tag{13}$$

such that Q has orthonormal columns, and R is upper triangular. The basis formed by the columns of Q are an orthogonalisation of the columns of A. The full QR decomposition is computed by a sequence of special projections, the Householder projections of the form $H_q = I - qq^t$, which are reflections about a line determined by q. Vector q_i can always be chosen to transform $H_{q_{i-1}} \dots H_{q_1}A$ into a similar matrix which zeros out the elements of the i^{th} column below the diagonal. After n iterations, we obtain $H_{q_n} \dots H_{q_1}A = R$ so $Q = H_{q_1} \dots H_{q_n}$.

The rank-revealing QR, or $RRQR$ method, can be used for partial decomposition [Gu and Eisenstat (July 1996)]. $RRQR$ works much the same as standard QR with Housholder reflections, but also determines a pivoting of the columns to terminate when the Frobenius norm of the unprocessed columns is determined to be negligible.

Contrarily to the FEM matrices, the BEM matrix is full dense and cannot be stored as dense array since the memory requirement would grow up very quickly with the size of the system. In order to limit the memory requirement, a domain decomposition is done on the BEM mesh, which splits the BEM matrices into submatrices as shown in figure 3. On the off-diagonal submatrices, a low rank approximation based on a rank revealing QR decomposition is performed [Golub and van Loan (1996)-Businger and Golub (1965)]. For submatrices corresponding to far away domains, the rank can be significantly smaller than the size of the submatrix, thus reducing the storage of the submatrix [Huang, Ashcraft, and L'Eplattenier (2008)]. We typically see reductions of the full dense matrix and the block matrix with low rank approximations. This low rank approximation also speeds up the matrix-vector operation used intensively in the iterative method to solve the BEM system.

Modeling reduces to solving linear system $A\mu = b$, where A is a dense matrix that has special block structure:

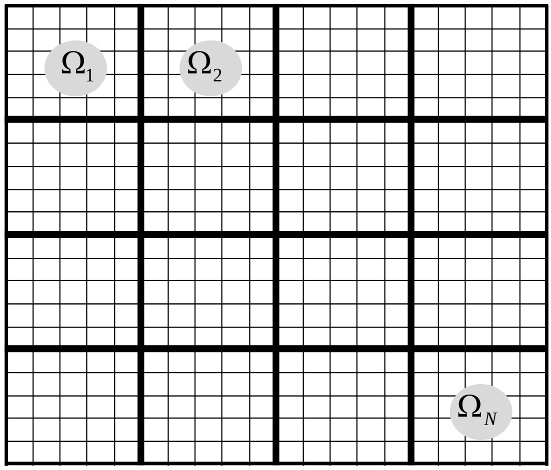

Figure 3: Partitioning of the domain

$$A = \begin{pmatrix} A_{1,1} & A_{1,2} & \dots & A_{1,p} \\ A_{2,1} & A_{2,2} & \dots & A_{2,p} \\ \vdots & \vdots & \ddots & \vdots \\ A_{p,1} & A_{p,2} & \dots & A_{p,p} \end{pmatrix} \tag{14}$$

The off-diagonal submatrices $A_{i,j}$ are numerically deficient to some degree, depending on discretisation accuracy requested. They represent the operator between subdomains Ω_i and Ω_j [Fig.3].

The matrix A is split into the sum of two matrices M and N $A = M + N$, with M having dense submatrices (diagonal bloks):

$$M = \begin{pmatrix} A_{1,1} & 0 & \dots & 0 \\ 0 & A_{2,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & A_{p,p} \end{pmatrix} \tag{15}$$

and N contains low rank submatrices:

$$N = \begin{pmatrix} 0 & Q_{1,2}R_{1,2} & \cdots & Q_{1,p}R_{1,p} \\ Q_{2,1}R_{2,1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & Q_{p-1,p}R_{p-1,p} \\ Q_{p,1}R_{p,1} & \cdots & Q_{p,p-1}R_{p,p-1} & 0 \end{pmatrix} \tag{16}$$

For each off-diagonal submatrices, we compute a partial *RRQR* factorization:

$$Q \begin{pmatrix} R_{1,1} & R_{1,2} \\ 0 & R_{2,2} \end{pmatrix} = A_{i,j}P \tag{17}$$

The factorization stops when [Huang, Ashcraft, and L'Eplattenier (2008)]:

$$\|R_{2,2}\|_F \leq \tau \sqrt{\|A_{i,i}\|_F \cdot \|A_{j,j}\|_F} \tag{18}$$

Where the Frobenius norm is written as:

$$\|A\|_F = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{i,j}|^2} \tag{19}$$

Then finally the matrix approximation:

$$\widehat{A}_{i,j} = Q \begin{pmatrix} R_{1,1} & R_{1,2} \end{pmatrix} \simeq A_{i,j} \tag{20}$$

As we can see in the figure 4 the approximated matrix seems to the symbolic matrix.

5 Numerical Application

We modeled a vibrating plate of Aluminum with dimensions of $0.9m$ in width, $0.6m$ in height and $0.001m$ in thickness. The material model of the plate was taken as elastic material with the following mechanical properties: Young Modulus $E = 210GPa$, Density $\rho = 7800Kg/m^3$, Poisson's Ratio $\nu = 0.3$. The plate was considered to be surrounded by air with following physical properties: speed

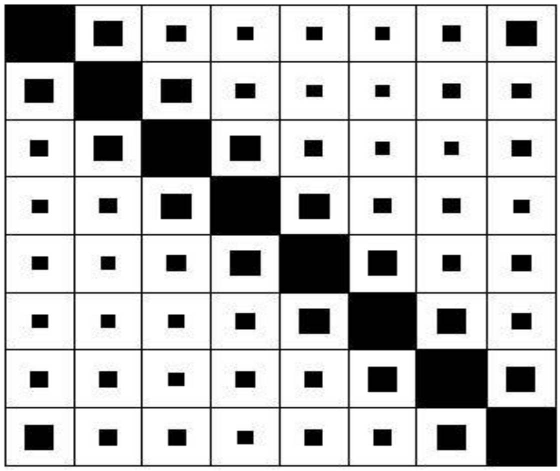

Figure 4: Symbolic matrix factorized

of sound $= 340m/s$ and mean air density $= 1.21Kg/m^3$.The BEM model was built by 600 shell elements. The constraints were applied to the edges of the modeled plate for no displacements and rotations in all directions. The model was excited by applying a nodal force at node $(0.33, 0.45, 0)$ as shown in figure 5. The application of the nodal force versus time is illustrated in figure 6. The pressure fluctuations caused by the structural response of the plate was noted at a field point located at a distance of $1m$ away from the plate, as shown in figure 5.

In order to calculate the sound pressure, several numerical applications are presented in this section; the predicted pressure from the simulation is validated using numerical results from SYSNOISE a well established acoustic code worldwide used for different industrial and academic applications. Our simulation using different Boundary Element formulation is compared to numerical pressure level from SYSNOISE used as reference solution.

5.1 Boundary element method

In figure 7 the computed sound pressure by the BEM described above and SYSNOISE curves for the plate in motion are given. The presented numerical result shows good correlation with SYSNOISE result.

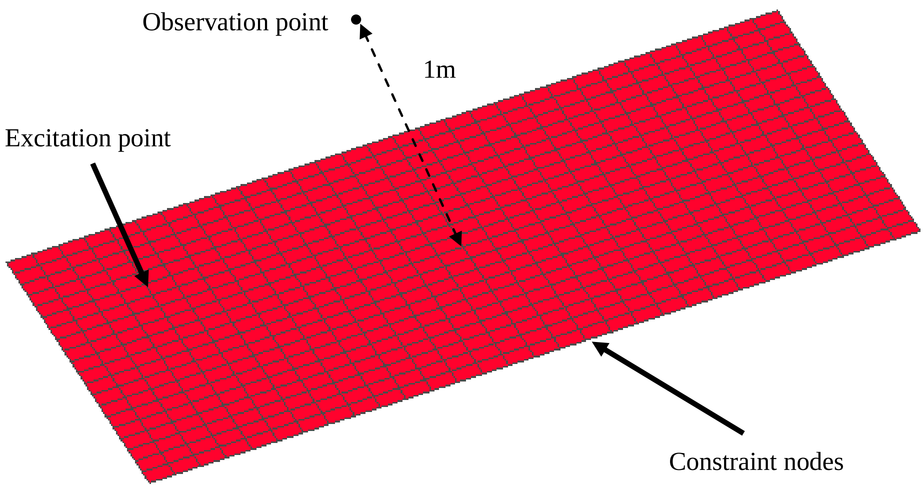

Figure 5: Boundary conditions applied to the BEM mesh

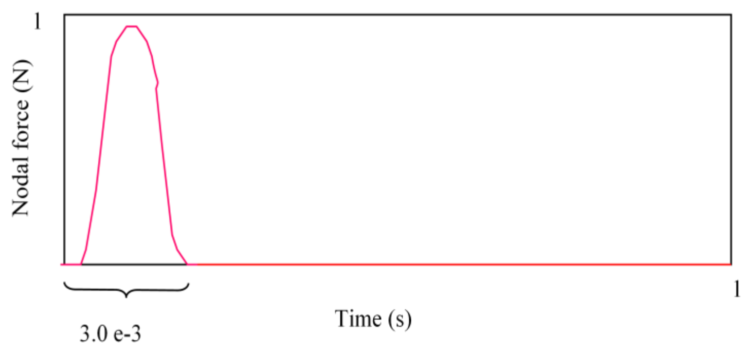

Figure 6: Nodal force load curve plotted against time

5.2 *Rayleigh method*

The BEM solver is very computationally intensive and time consuming therefore; the simple Rayleigh method based on plane wave approximation was used instead for the study. The acoustic pressure at the structure is given by:

$$P = \rho c v \tag{21}$$

With c is the speed of sound $= 340m/s$, ρ is the mean air density $= 1.21kg/m^3$ and v is the normal velocity on the plate.

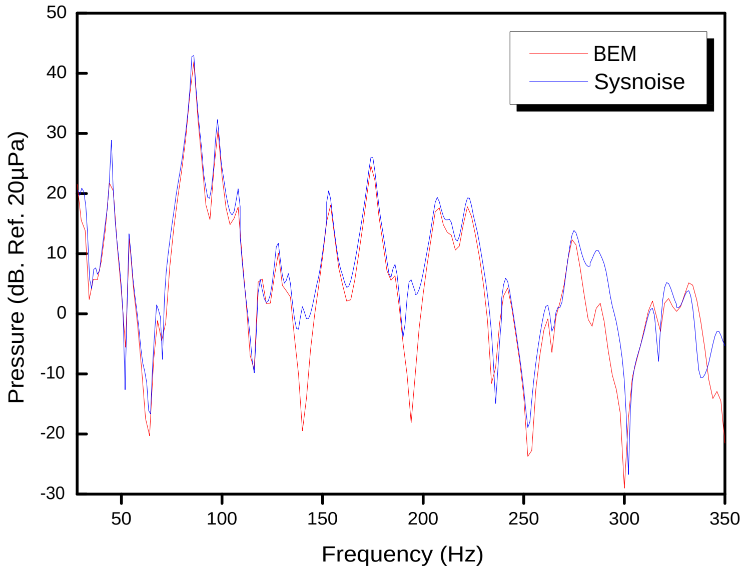

Figure 7: Comparison between BEM method and SYSNOISE

Rayleigh method assumes that the radiating structure is a plane surface clamped into an infinite rigid plane. The Rayleigh integral [Rayleigh (1974)] directly relates the sound pressure in the acoustic domain to the velocity distribution on the plate. This model can be applied to a range of acoustic problems. For example it can model the acoustic field around the near-flat surface of an object. From the computational point of view, Rayleigh method is fastest among all methods since no linear system is being solved, the acoustic pressure at the structure is known analytically through equation 21. In figures 8, computed Rayleigh and SYSNOISE sound pressures in the hearing point of the plate are compared. The results appear to be in good agreement.

5.3 Kirchhoff method

The other alternative is the Kirchhoff method; where a layer of finite element using acoustic material shearing nodes with the structure is added to the problem, to compute the acoustic pressure at the stucture. The radiating boundary condition at infinity is satisfied by prescribing a non-reflecting boundary condition. The mesh

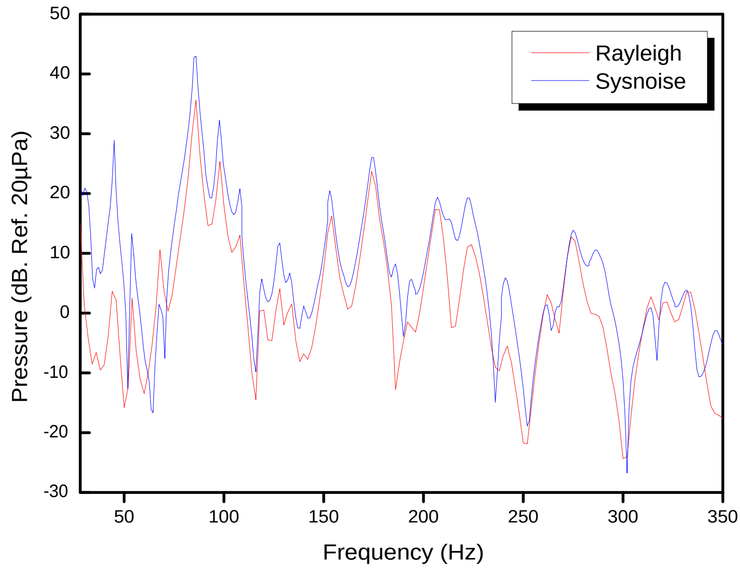

Figure 8: Comparison between Rayleigh method and SYSNOISE

corresponding to Kirchhoff method is shown in figure 9.

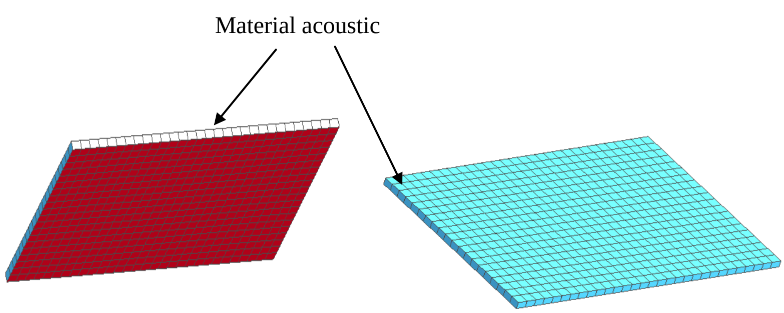

Figure 9: Vibrating plate model used in Kirchhoff method

The sound pressure level (dB) at the observation point is computed using Kirchhoff

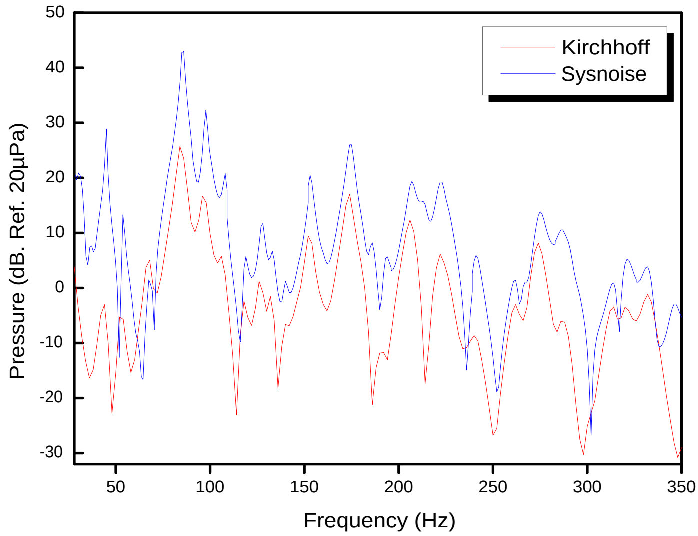

Figure 10: Comparison between Kirchhoff method and SYSNOISE

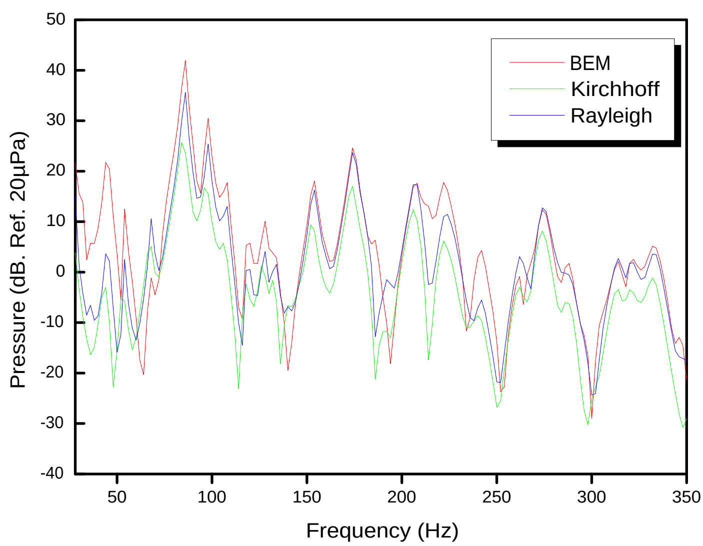

Figure 11: Variation of radiated pressure of plate with respect to frequency

method. The results can be found in Figure 10. We observe the same shape of the curves with a slight difference in the amplitude.

Finally we compared the result from LS-DYNA for three different formulations given as BEM, Kirchhoff and Rayleigh as shown in figure 11. The analysis showed good agreement within the results. The numerical results given by the three methods correlate well. We observe from these analysis that each formulation predicts the resonance frequency that correlate very well with the reference solution from Sysnoise.

The solution by BEM can reach a high accuracy since it solves the singular integral equation and get the primary unknown variables on each node without any assumption. Rayleigh and Kirchhoff methods are each based on some assumptions thus they are less accurate. But Rayleigh and Kirchhoff methods may be employed as the first attack when solving large problems because they are much faster than BEM, 5 seconds for Rayleigh and 7 seconds for Kirchhoff instead of 384 seconds for BEM. One can see for this example of a rectangular plate, the Rayleigh and Kirchhoff methods can still provide satisfactory results. This is because the geometry of the problem is simple and satisfies the assumption of the two approximate methods.

6 Conclusion

In the present work, different formulations have been used to model a simple vibroacoustic problem. The computational solution of a given acoustic radiation problem first involves the selection of an appropriate acoustic radiation model which underlies the choice of method. For example the model of a closed surface in an infinite acoustic medium underlies the boundary element method. In terms of accuracy, the solution by BEM can reach a high accuracy since it solves the singular integral equation and get the primary unknown variables on each node without any assumption. Rayleigh and Kirchhoff methods are each based on some assumptions thus they are less accurate. But Rayleigh and Kirchhoff methods may be used for first estimation of acoustic pressure level when solving large problems because they are much faster than BEM. For external problems as the one described above, Rayleigh and Kirchhoff methods can still provide satisfactory results, this is not true for internal problems where BEM method needs to be used.

References

Belytschko, T. (2000): *Nonlinear Finite Elements for Continua and Structures*. Ed. J. Wiley & Sons.

Benson, D. (1992): Computational methods in lagrangian and eulerian hydrocodes. *Computer Methods in Applied Mechanics and Engineering*, vol. 99(2), pp. 235–394.

Businger, P. A.; Golub, G. H. (1965): Linear least squares solution via householder transformations. *Numer. Math.*, vol. 7, pp. 269–276.

Erchiqui, F.; Souli, M.; Yedder, R. B. (2007): Nonisothermal finite-element analysis of thermoforming of polyethylene terephthalate sheet: Incomplete effect of the forming stage. *Polymer Engineering and Science*, vol. 47(12), pp. 2129–2144.

Golub, G. H.; van Loan, C. F. (1996): *Matrix Computations, Third Edition*. Johns Hopkins University Press, MD, Baltimore.

Gu, M.; Eisenstat, S. C. (July 1996): Efficient algorithms for computing a strong rank-revealing qr factorization. *SIAM Journal on Scientific Computing*, vol. 17, pp. 848–869.

Hamdi, M. (1982): *Formulation variationnelle par équations intégrales pour le calcul de champs acoustiques linéaires proches et lointains*. PhD thesis, UTC, 1982.

Huang, Y.; Ashcraft, C.; L'Eplattenier, P. (2008): A fast collocation boundary element method for vibro-acoustic problems in ls-dyna. In *Proceedings of the Inaugural International Conference of the Engineering Mechanics Institute (EM08), May 18-21, 2008*, Minneapolis, Minnesota. International Institute of Informatics and Systemics.

Ozdemir, Z.; Souli, M.; Fahjan, Y. M. (2010): Application of nonlinear fluid-structure interaction methods to seismic analysis of anchored and unanchored tanks. *Engineering Structures*, vol. 32(2), pp. 409–423.

Rayleigh, L. (1974): *The Theory of Sound*. Dover.

Souli, M.; Zolesio, J. P. (1993): Shape derivative of discretized problems. *Computer Methods in Applied Mechanics and Engineering*, vol. 108, pp. 187–199.

Wu, T. W. (2001): *Boundary Element Acoustics: Fundamentals and Computer Codes, Advances in Boundary Elements*. WITpress, Boston.

Z. Zhang, N. Vlahopoulos, S. R. T. A.; Zhang, K. (2000): A computational acoustic field reconstruction process based on an indirect boundary element formulation. *J Acoust Soc Am*, vol. 108, pp. 2167–78.