Coupled Evolution of Damage and Fluid Flow in a Mandel-type Problem

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Abstract: Some considerations on the numerical analysis of brittle rocks are presented in this paper. The rock is taken as a poro-elastic domain, in full-saturated condition, based on the Biot's Theory. The solid matrix of this porous medium is considered to be susceptible to isotropic damage occurrence. An implicit boundary element method (BEM) formulation, based on time-independent fundamental solutions, is developed and implemented to couple the fluid flow and two-dimensional elastostatics problems. The integration over boundary elements is evaluated by using a numerical Gauss procedure. A semi-analytical scheme for the case of triangular domain cells is followed to carry out the relevant domain integrals. The non-linear problem is solved by a Newton-Raphson procedure. A geomechanical problem is analyzed in order to illustrate the efficiency of the implemented formulation.

1   Introduction

The study of porous materials is extremely relevant in several areas of knowledge, such as soil and rock mechanics, contaminant diffusion, biomechanics and petroleum engineering. The mechanics of porous media deals with materials where the mechanical behavior is significantly influenced by the presence of fluid phases. The response of the material is highly dependent on the fluids that flow through the pores. Biot (1941) was the first to propose a coupled theory for three-dimensional consolidation, based on the Terzaghi's studies on soil settlement (Terzaghi, 1923). This thermodynamically consistent theory is described in the book by Coussy (2004), who improved significantly the knowledge on poromechanics. Cleary (1977) pre-

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sented the fundamental solutions to porous solids, representing the first contributions on integral equations dedicated to this kind of problems. Among others pioneers BEM works applied to porous media, the ones from Cheng and his collaborators (1984, 1987, 1998) are well-known, using the direct BEM formulation.

In the field of material mechanics, we note the modelling of nonlinear physical processes, as damage and fracture. Processes of energy dissipation and consequent softening have been extensively studied, so that one can count on a wide range of models already developed. Continuum Damage Mechanics (CDM) deals with the load carrying capacity of solids whose material is damaged due to the presence of micro-cracks and micro-voids. CDM was originally conceived by Kachanov (1958), to analyze uniaxial creeping of metals subjected to high-order temperatures. Several authors studied and developed models related to CDM. Lemaitre and colleagues (1985, 1992) contributed significantly to the field. In this work, we use the model of Marigo (1981), who presented a scalar isotropic model for brittle and quasi-brittle materials. The first applications of BEM to damage mechanics reported in the literature are Herding & Kuhn (1996) and Garcia et al (1999). Recently, we can cite the works of Sladek and Sladek (2003), Botta, Venturini and Benallal (2005) and Benallal, Botta and Venturini (2006). These works include non-local formulations to treat strain localization phenomena and associated numerical instabilities. Some aspects on the numerical analysis of porous media experiencing damage are found in Cheng & Dusseault (1993) and Selvadurai (2003).

Due to the increasing complexity of models developed for engineering problems, robust numerical models capable to provide accurate results with the least possible computational effort are looked for. In this scenario, BEM appears as an interesting choice for obtaining numerical solutions in various engineering applications.

In this paper, a non-linear set of transient BEM equations is developed, based on Betti's reciprocity theorem, to deals with isotropic-damaged porous media. The description of porous solid is done in a Lagrangean approach. Marigo's damage model is applied with a local evaluation of the thermodynamic force associated to damage.

Regarding the BEM numerical procedure, the integration over boundary elements is evaluated by using a numerical Gauss procedure. A semi-analytical scheme for the case of triangular domain cells is followed to carry out the relevant domain integrals. A Newton-Raphson procedure is applied to solve the non-linear system, with a consistent tangent operator. This is done in the light of the procedure introduced by Simo and Taylor (1985) for finite elements.

2   Governing equations

Let us assume the free energy potential per unit volume of a saturated porous medium subject to damage, as,

$$\rho\psi(\varepsilon_{jk}, D, \phi - \phi_0) = \frac{1}{2}(1 - D)\varepsilon_{jk}E^d_{jklm}\varepsilon_{lm} + \frac{1}{2}b^2 M\left[Tr\left(\varepsilon_{jk}\right)\right]^2$$
$$+ \frac{1}{2}M(\phi - \phi_0)^2 - bM(\phi - \phi_0)Tr\left(\varepsilon_{jk}\right) \tag{1}$$

where the constants M and b represent the Biot modulus and Biot coefficient of effective stress, respectively. In full-saturated condition, the lagrangian porosity ϕ measures the variation of fluid content per unit volume of porous material. The bulk density is described by ρ. The tensor ε_{jk} denotes the strains in the solid skeleton. Assuming isotropic case, the damage is represented by the scalar-valued internal variable D, which defines the internal state of the material, taking values between zero (sound material) and one (complete degradation). The initial porosity field is indicated by ϕ_0. E^d_{jklm} represents the isotropic elastic tensor in drained condition, defined as

$$E^{dr}_{kjlm} = \left(K^{dr} - \frac{2G}{3}\right)\delta_{kj}\delta_{lm} + 2GI_{kjlm} \tag{2}$$

The bulk modulus K^{dr} and the shear modulus G refer to the drained material and can be obtained experimentally. The fourth order identity tensor is represented by I_{kjlm}. It can be observed that one of the possible sets of parameters for the characterization of porous material is formed by M, b, K^{dr} and G.

The derivatives of free energy potential with respect to the internal variables lead to the associate variables, that are the total stress σ_{jk}, the pore-pressure p and the thermodynamical force Y conjugated to D.

$$\sigma_{jk} = \rho\frac{\partial\psi}{\partial\varepsilon_{jk}} = (1 - D)E^d_{jklm}\varepsilon_{lm} + bM\left[bTr\left(\varepsilon_{jk}\right) - (\phi - \phi_0)\right]\delta_{jk} \tag{3}$$

$$(p - p_0) = \rho\frac{\partial\psi}{\partial(\phi - \phi_0)} = M\left[(\phi - \phi_0) - bTr\left(\varepsilon_{jk}\right)\right] \tag{4}$$

$$Y = -\rho\frac{\partial\psi}{\partial D} = \frac{1}{2}\varepsilon_{jk}E^d_{jklm}\varepsilon_{lm} \tag{5}$$

Using equations (3) and (4) the total stress tensor is written as

$$\sigma_{jk} = E_{jklm}\varepsilon_{lm} - DE_{jklm}\varepsilon_{lm} - b(p - p_0)\delta_{jk} \tag{6}$$

from which it is seen that it includes three different contributions, being the first one the effective stress σ_{jk}^{ef}, acting on the grains of the solid matrix, and the second one the stress due to damage σ_{jk}^{d}.

In addition to the state laws given above, it is necessary to define a damage criterion. In Marigo's model it takes the form:

$$F(Y,D) = Y - \kappa(D) \tag{7}$$

The term $\kappa(D)$ represents the maximum value of Y reached during the loading history, and is adopted here in its simple linear form $\kappa(D) = Y_0 + AD$, where parameters Y_0 and A are material dependent. The damage evolution becomes from the consistency condition $\dot{F}(Y,D) = 0$, resulting in:

$$\dot{D} = \dot{Y}/A \tag{8}$$

The fluid flow through the porous space can be described by Darcy's law. Assuming a laminar flow, this law considers a linear relationship between the flow rate and the pressure gradient:

$$v_k = k[-p_{,k} + f_k] \tag{9}$$

In this simple version, it is assumed isotropic, with $k = \frac{k}{\mu}$ the scalar permeability coefficient, defined as a function of the intrinsic permeability k and the fluid viscosity μ. The fluid body force is represented by f_k.

The fluid mass balance equation, assuming no external fluid sources, is written as:

$$\frac{d(\rho_f \phi)}{dt} + (\rho_f v_k)_{,k} = 0 \tag{10}$$

The following equilibrium and compatibility relations, added to appropriate boundary conditions complete the set of equations that describes the poro-elasto-damage problem, in quasi-static conditions:

$$\sigma_{jk,k} + b_j = 0 \tag{11}$$

$$\varepsilon_{jk} = \frac{1}{2}(u_{k,j} + u_{j,k}) \tag{12}$$

3   Integral equations

In order to couple the behaviour of the solid and fluid phases, two sets of integral equations are derived. The first one is related to the elastostatics problem, for which

a pore-pressure field is distributed over the domain, while the other equation refers to the pore-pressure itself.

In order to obtain the integral equations one can use Betti's reciprocity theorem, which can only be applied to fields that keep a linear and proportional relationship between them. Thus, in the case of elasticity, assuming the effective stress definition:

$$\int_\Omega \sigma_{jk}^{ef}(q)\varepsilon_{ijk}^*(s,q)d\Omega = \int_\Omega \varepsilon_{jk}(q)\sigma_{ijk}^*(s,q)d\Omega \tag{13}$$

$$\int_\Omega \left(\sigma_{jk}(q) + \sigma_{jk}^d(q) + b\delta_{jk}p(q)\right)\varepsilon_{ijk}^*(s,q)d\Omega = \int_\Omega \varepsilon_{jk}(q)\sigma_{ijk}^*(s,q)d\Omega \tag{14}$$

where s and q represent the source and field points, and X^* is the fundamental solution for the variable X, from now on. The direction i refers to the application of the unit load on the source point into the fundamental domain. In elastostatics, one applies the well-known Kelvin fundamental solutions. By applying the divergence theorem to equation (14), and considering the transient nature of the problem, one obtains the following integral equation for displacements on the boundary points S:

$$C_{ik}\dot{u}_k(S) = \int_\Gamma \dot{T}_k(Q)u_{ik}^*(S,Q)d\Gamma - \int_\Gamma T_{ik}^*(S,Q)\dot{u}_k(Q)d\Gamma$$
$$+ \int_\Omega b\delta_{jk}\dot{p}(q)\varepsilon_{ijk}^*(S,q)d\Omega + \int_\Omega \dot{\sigma}_{jk}^d(q)\varepsilon_{ijk}^*(S,q)d\Omega \tag{15}$$

The stresses at internal points are obtained by differentiating equation (15), now written for internal points, and applying Hooke's law, which leads to

$$\dot{\sigma}_{ij}(s) = -\int_\Gamma S_{ijk}(s,Q)\dot{u}_k(Q)d\Gamma + \int_\Gamma D_{ijk}(s,Q)\dot{T}_k(Q)d\Gamma + \int_\Omega R_{ijkl}(s,q)\dot{\sigma}_{kl}^d(q)d\Omega$$
$$+ TL_{ij}\left[\dot{\sigma}_{kl}^d(s)\right] + \int_\Omega b\delta_{kl}R_{ijkl}(s,q)\dot{p}(q)d\Omega + TL_{ij}\left[b\delta_{kl}\dot{p}(s)\right] \tag{16}$$

where S_{ijk}, D_{ijk} and R_{ijkl} are the derivatives of the fundamental solutions, and TL_{ij} are the free-terms coming from differentiation.

The integral equation for the pore-pressure can be obtained in a similar way, defining the proportional flow vector $v_k^{pr} = v_k - kf_k = -kp_{,k}$ in order to apply Betti's

Theorem

$$\int_{\Omega} [v_k - kf_k] \, p^*_{,k}(s,q) d\Omega = \int_{\Omega} v^*_k(s,q) p_{,k}(q) d\Omega \tag{17}$$

from what the divergence theorem leads to write:

$$
\begin{aligned}
p(s) = &- \int_{\Gamma} v^*_\eta(s,Q) p(Q) d\Gamma + \int_{\Gamma} p^*(s,Q) v_\eta(Q) d\Gamma \\
&- \int_{\Omega} p^*(s,q) v_{k,k}(q) d\Omega - \int_{\Omega} p^*_{,k}(s,q) kf_k(q) d\Omega
\end{aligned}
\tag{18}
$$

η indicates the outward normal direction to the boundary. Assuming $v_{k,k} = -\dot{\phi}$ (see (10)) and, neglecting the body force f_k, we get:

$$
p(s) = - \int_{\Gamma} v^*_\eta(s,Q) p(Q) d\Gamma + \int_{\Gamma} p^*(s,Q) v_\eta(Q) d\Gamma + \int_{\Omega} p^*(s,q) \dot{\phi}(q) d\Omega \tag{19}
$$

For convenience, it is possible to take the derivative $\dot{\phi}(q)$ from (4), so that the pore-pressure is given by the following equation:

$$
\begin{aligned}
p(s) = &- \int_{\Gamma} v^*_\eta(s,Q) p(Q) d\Gamma + \int_{\Gamma} p^*(s,Q) v_\eta(Q) d\Gamma \\
&+ \int_{\Omega} p^*(s,q) \left[\frac{1}{M} \dot{p}(q) + bTr(\dot{\varepsilon}(q)) \right] d\Omega
\end{aligned}
\tag{20}
$$

Considering a finite time step $\Delta t_n = t_{n+1} - t_n$ and a corresponding variable increment $\Delta X = X_{n+1} - X_n$, one can integrate equations (15), (16) and (20) along the interval Δt, leading to the following set of equations, in terms of the variable increments:

$$
\begin{aligned}
C_{ik} \Delta u_k(S) = &\int_{\Gamma} \Delta T_k(Q) u^*_{ik}(S,Q) d\Gamma - \int_{\Gamma} T^*_{ik}(S,Q) \Delta u_k(Q) d\Gamma \\
&+ \int_{\Omega} b\delta_{jk} \Delta p(q) \varepsilon^*_{ijk}(S,q) d\Omega + \int_{\Omega} \Delta\sigma^d_{jk}(q) \varepsilon^*_{ijk}(S,q) d\Omega
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\Delta\sigma_{ij}(s) = &- \int_{\Gamma} S_{ijk}(s,Q) \Delta u_k(Q) d\Gamma + \int_{\Gamma} D_{ijk}(s,Q) \Delta T_k(Q) d\Gamma \\
&+ \int_{\Omega} R_{ijkl}(s,q) \Delta\sigma^d_{kl}(q) d\Omega + TL_{ij} \left[\Delta\sigma^d_{kl}(s) \right] \\
&+ \int_{\Omega} b\delta_{kl} R_{ijkl}(s,q) \Delta p(q) d\Omega + TL_{ij} [b\delta_{kl} \Delta p(s)]
\end{aligned}
\tag{22}
$$

$$c(s)p(s) = -\int_\Gamma v_\eta^*(s,Q)p(Q)d\Gamma + \int_\Gamma p^*(s,Q)v_\eta(Q)d\Gamma$$

$$+\frac{1}{\Delta t}\int_\Omega \frac{1}{M}p^*(s,q)\Delta p(q)d\Omega + \frac{1}{\Delta t}\int_\Omega bp^*(s,q)Tr\left(\Delta\varepsilon(q)\right)d\Omega \tag{23}$$

4 Algebraic Equations and Solution Procedure

The numerical solution of the boundary value problem requires both the time and space discretizations. It should represent the system of equations in a discrete way along the linear boundary elements and into the triangular domain cells in order to obtain the approximate values of the variables of interest. One defines the number of boundary points by N_n and the number of internal nodes by N_i. The appropriate discretization of the integrals on (21)-(23), followed by some algebraic manipulations inherent to BEM, leads to the following system:

$$[H]\{\Delta u\} = [G]\{\Delta T\} + [Q]\left\{\Delta\sigma^d\right\} + b[Q][IK]\{\Delta p\} \tag{24}$$

$$\{\Delta\sigma\} = -[HL]\{\Delta u\} + [GL]\{\Delta T\} + [QL]\left\{\Delta\sigma^d\right\} + b[QL][IK]\{\Delta p\} \tag{25}$$

$$\{p_{(i)}\} = -\left[HP_{(i)}\right]\{p\} + \left[GP_{(i)}\right]\{V\}$$
$$+\frac{1}{M\Delta t}\left[QP_{(i)}\right]\left\{\Delta p_{(i)}\right\} + \frac{b}{\Delta t}\left[QP_{(i)}\right][Tr]\{\Delta\varepsilon\} \tag{26}$$

The subscript (i) refers to internal points. The influence matrices represented by $[\,]$ come from the integration of the fundamental solutions and its derivatives. The variables represented by $\{\}$ are prescribed or unknown variables along the boundary or over the domain. After some arrangements, the system given above is written as

$$[E]\{\Delta\varepsilon\} = \{\Delta Ns\} + [[QS]+[I]]\left\{\Delta\sigma^d\right\} + b[[QS]+[I]][IK]\left\{\Delta p_{(i)}\right\} \tag{27}$$

$$\left[[I]-\frac{1}{M\Delta t}\left[\overline{QP}_{(i)}\right]\right]\left\{\Delta p_{(i)}\right\} = \{\overline{Np}\} + \frac{b}{\Delta t}\left[\overline{QP}_{(i)}\right][Tr]\{\Delta\varepsilon\} \tag{28}$$

where $\{\Delta Ns\}$ and $\{\overline{Np}\}$ are vectors containing prescribed values and $[E]$ the drained elastic tensor. Finally, arranging the two equations in a single one, in terms of $\{\Delta\varepsilon\}$ only, leads to

$$\left[\overline{E}\right]\{\Delta\varepsilon\} = [\Delta Ns] + \left\{\overline{\overline{Np}}\right\} + \left[\overline{QS}\right]\left\{\Delta\sigma^d\right\} \tag{29}$$

which contains the new terms:

$$\left\{ \overline{\overline{Np}} \right\} = b \left[\overline{QS} \right] [IK] \left[[I] - \frac{1}{M\Delta t} \left[\overline{QP}_{(i)} \right] \right]^{-1} \left\{ \overline{Np} \right\} \tag{30}$$

$$\left[\overline{E} \right] = \left[[E] - \frac{b^2}{\Delta t} \left[\overline{QS} \right] [IK] \left[[I] - \frac{1}{M\Delta t} \left[\overline{QP}_{(i)} \right] \right]^{-1} \left[\overline{QP}_{(i)} \right] [Tr] \right] \tag{31}$$

Due to the presence of correction terms associated with damage, equation (29) is non-linear at each time increment, and can be written:

$$\left\{ Y \left(\{ \Delta \varepsilon_n \} \right) \right\} = - \left[\overline{E} \right] \{ \Delta \varepsilon_n \} + [\Delta Ns] + \left\{ \overline{\overline{Np}} \right\} + \left[\overline{QS} \right] \left\{ \Delta \sigma_n^d \right\} = 0 \tag{32}$$

The solution is carried out by a Newton-Raphson's scheme. An iterative process is required to reach equilibrium. Then, from iteration i, the next try $i+1$ is given by $\left\{ \Delta \varepsilon_n^{i+1} \right\} = \left\{ \Delta \varepsilon_n^i \right\} + \left\{ \delta \Delta \varepsilon_n^i \right\}$. The correction $\left\{ \delta \Delta \varepsilon_n^i \right\}$ is calculated from the first term of the Taylor expansion, as follows:

$$\left\{ Y \left(\{ \Delta \varepsilon_n^i \} \right) \right\} + \frac{\partial \left\{ Y \left(\{ \Delta \varepsilon_n^i \} \right) \right\}}{\partial \left\{ \Delta \varepsilon_n^i \right\}} \left\{ \delta \Delta \varepsilon_n^i \right\} = 0 \tag{33}$$

where the derivative $\frac{\partial \left\{ Y \left(\{ \Delta \varepsilon_n^i \} \right) \right\}}{\partial \{ \Delta \varepsilon_n^i \}}$ is the consistent tangent operator.

5   Numerical example

The validation of the implemented formulation was presented in Lima Junior (2011) and Lima Junior, Venturini and Benallal (2010), based on benchmark cases concerning poroelasticity and damage evolution. It is proposed in this paper the analysis of a plane problem, as shown in Fig. 1. It consists of a rectangular area, with 2 m wide and 1 m in height. A load of 20 MN is applied monotonically over 2 s, on impermeable plates placed on the top and bottom faces. The flow occurs only through the lateral faces. The boundary conditions of the problem are inspired by the problem of consolidation proposed by Mandel (1953). The constituent material is Berea sandstone whose properties are defined in Tab. 1. The discretization used contains 24 boundary elements and 32 domain cells. The four possible material behaviors are considered, being the uncoupled elasto-damage and poroelastic cases and the coupled poro-damage regime.

The central point of the domain is taken as reference for the analysis of the problem. Initially, we observe the behavior in the vertical direction along which the load is applied. Based on the graphs concerning to damage and porodamage regimes in
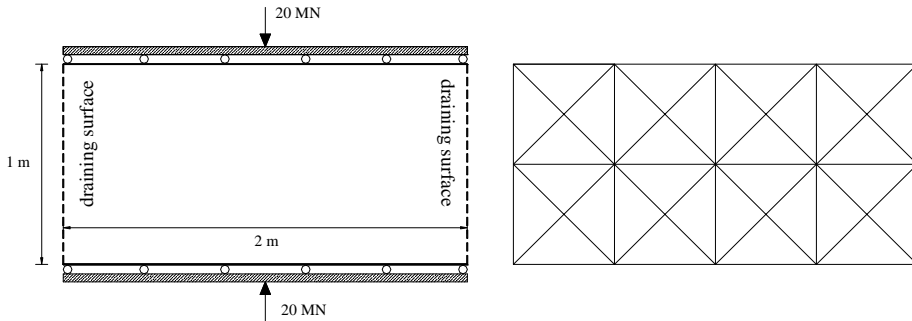
Figure 1: Problem definition, adopted cells mesh

Table 1: Parameters of the Berea sandstone

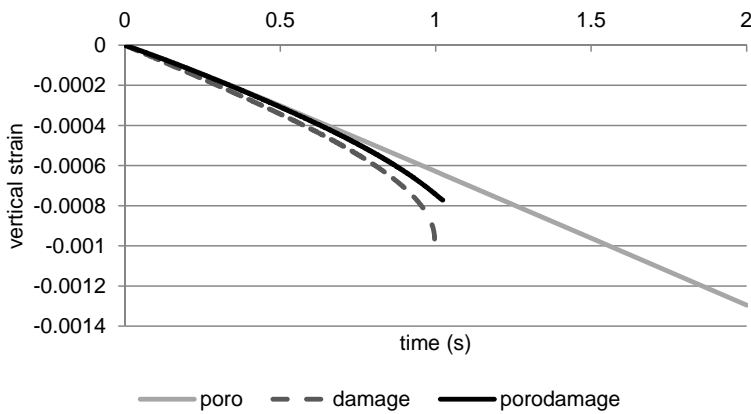| Parameter	Value
G	6000 MPa
υ	0.2
υ^u	0.33
K_s	36000 MPa
φ_0	0.19
k	1.9×10^{-13} m^2
μ	1×10^{-9} MPa.s

Figure 2: Vertical strain evolution at the central point
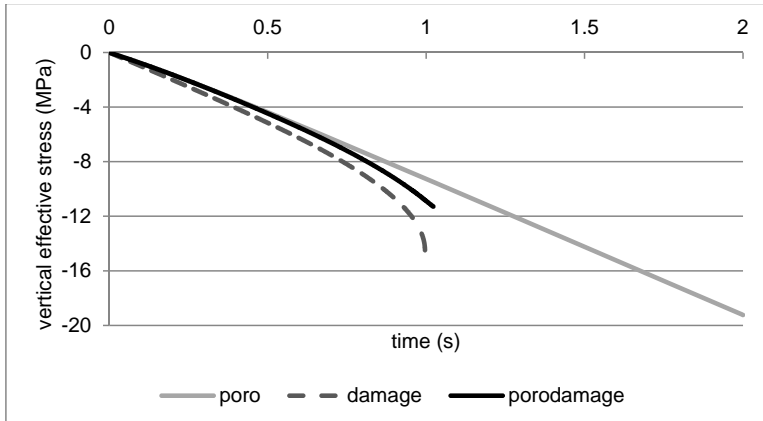
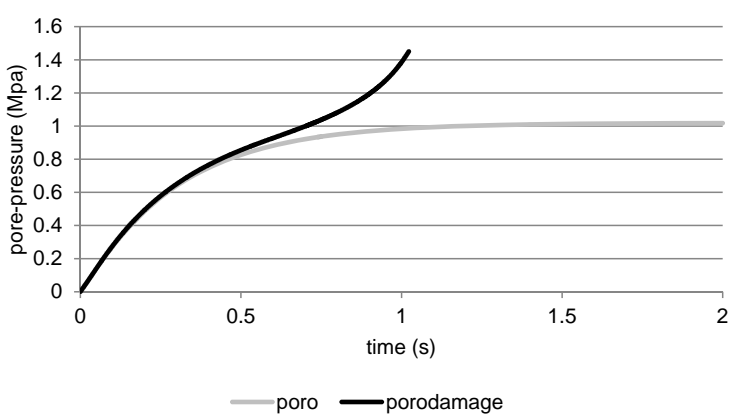Figure 3: Vertical effective stress evolution at the central point

Figure 4: Pore-pressure evolution at the central point

Fig. 2, it can be seen the influence of the fluid as mitigation in the evolution of the strains on the solid skeleton, in the presence of damage.

The analysis of Fig. 3 allows to visualize that the coupled behavior (porodamage) is governed initially by the poroelastic regime, going to suffer the effects of damage, which starts at around 0.6s analysis (Fig. 5).

From around 0.6 s the pore-pressure starts to evolve coupled to the damage level on the material, as shown in Fig. 4 and Fig. 5 in which it can be seen that the damage initiation, as well as its intensity, are delayed along the time, in the porodamage regime.
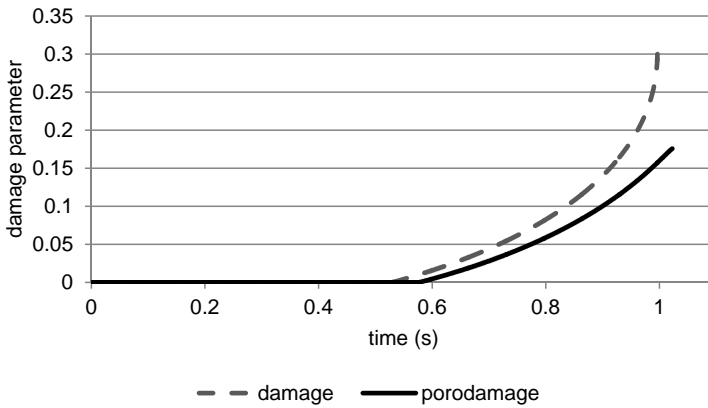
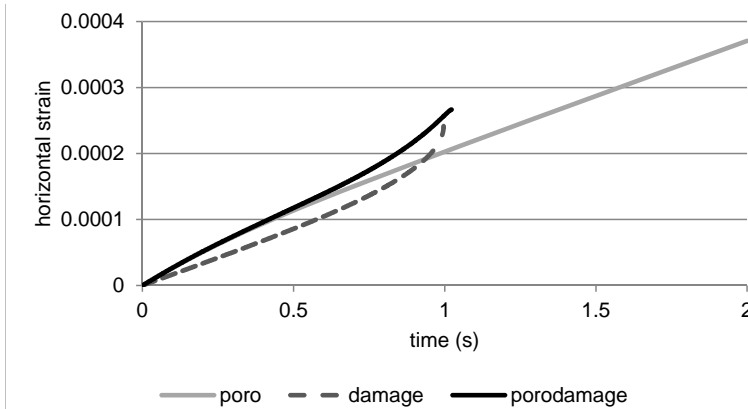Figure 5: Damage parameter evolution at the central point

Figure 6: Horizontal strain evolution at the central point

Consider now the problem response along the horizontal direction, also measured at the center of the domain. Fig. 6 shows the evolution of horizontal strain over time, considering the different behaviors.

Considering that this is not the direction of load application, the effects of loading are manifested only partially in the horizontal direction, due to Poisson's effect. However, the fluid flows preferentially along horizontal direction, due to the imposed boundary conditions.

The comparison between the strain curves regarding the damage and porodamage regimes in Fig. 6, allows the verification of the predominance of the effects due to the presence of fluid. The horizontal strains induced in the poroelastic case are
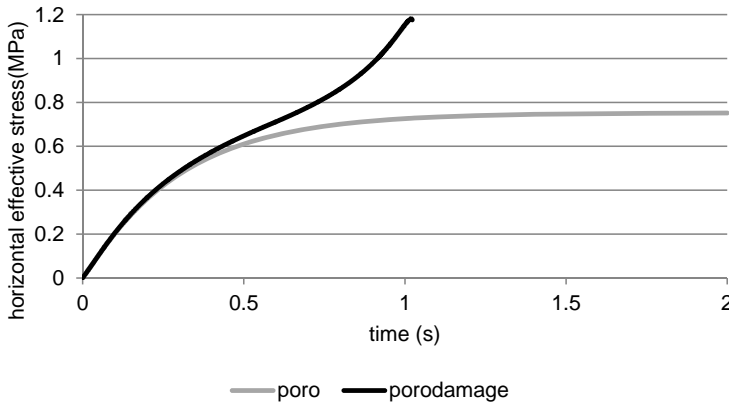
Figure 7: Horizontal effective stress evolution at the central point
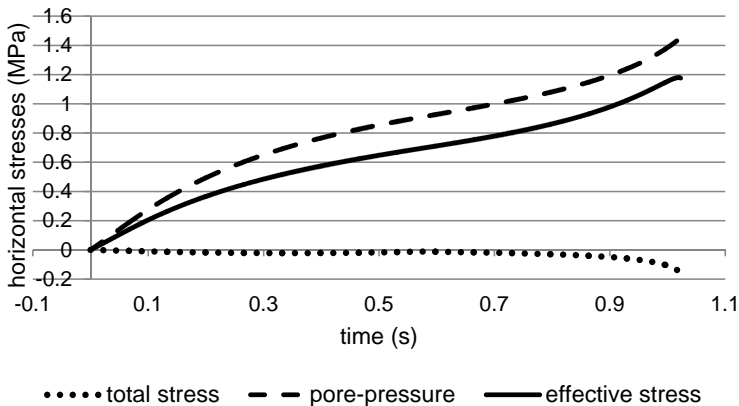

Figure 8: Stress balance in the horizontal direction, at the central point

higher than those caused in the damage case over the major part of the analysis.

The values of effective stress in horizontal direction are negligible, considering the boundary conditions of the problem. From Fig. 7 we observe the increase in effective stress caused by the consideration of the damage in poroelastic problem.

In order to illustrate conclusively the difference between the measured responses in the central point along the two coordinate directions, it is presented in Fig. 8 and Fig. 9 the evolution of the parts of stress tensor, admitting the porodamage coupled regime. The predominance of the poroelastic behavior along the horizontal direction becomes clear.
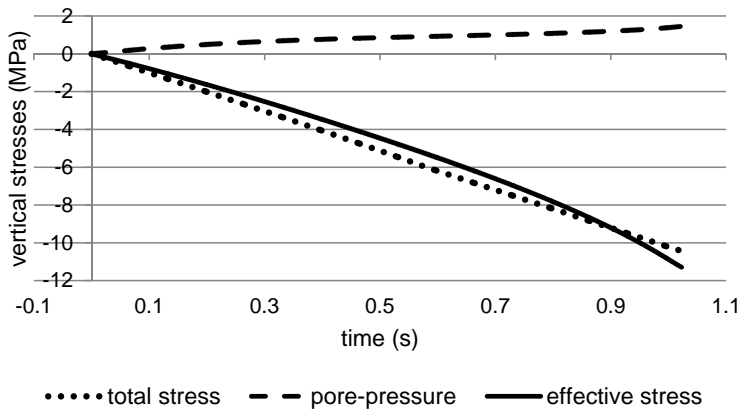
Figure 9: Stress balance in the vertical direction, at the central point

6 Conclusions and perspectives

A BEM formulation to poro-elasto-damaged material was applied to a Mandel-type problem. The model has shown a reasonable level of coupling between the damage and the fluid seepage. The predominance of each process becomes clear in the two different directions. The literature, on theoretical and experimental levels, poses several interesting questions, among which the variations that the damage state imposes on the poro-elastic parameters, specially about the permeability. Some developments in this way are being made in the presented model, in order to improve the solid-fluid interaction.

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