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In-Plane Vibration of a Beam Picking and Placing a Mass Along Arbitrary Curved Tracking

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Abstract: In this study, examine the in-plane vibration of a robot arm picking and placing a mass along arbitrary curved tracking. This mathematical model is established. It is a moving mass problem. Due to the effect of movement along arbitrary curved tracking, the corresponding differential equation is nonlinear with the time-dependent coefficients and non-homogenous boundary conditions. So far, a few literatures devoted to investigate this system due to its complexity. The solution method procedure for this system is presented. It integrates several methods as the transform of variable, the subsection method, the mode superposition method, and the Green function method. Meanwhile, the shift function for the transform of variable is derived. The orthogonality condition for the mode superposition method is proved. For suppressing vibration and overshoot the boundary control method is design. The method is verified to be very effective. The dynamic behavior of a robot arm placing a mass is investigated. It is found that the effect of placing a mass during the way of movement on the vibration is significant. Finally, the effects of several parameters on the overshoot and residual vibration are investigated.

Keywords: moving mass problem; picking and placing a mass; arbitrary track; vibration; semi-analytical solution; structure control

1 Introduction

In engineering, the moving mass problem is important. The conveyed mass may be solid [Lin, 2009] or fluid [Lin *et al.*, 2008^a]. In general, there are four kinds of moving solid mass problems. The first is the dynamic behavior of beam structures, such as bridges on railways, subjected to moving loads or masses. Mostly, a uniform beam is simply supported and carried a moving load [Fryba 1996; Nikkhoo et al., 2007]. The second is the vibration characteristics of a rotating shaft subjected to a moving load or mass [Gu and Cheng, 2004]. This model can simulate dynamic

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behavior of a ball screw and a nut moving along it, which are the key components of a feed drive system for a machine tool. The third is the axially moving string and beam problem. Chao and Lai (2003) investigated the boundary control of an axially moving string. Lee and Jang (2007) investigated the vibration of an axially moving and simply-support beam. Ghayesh and Balar (2008) and Chang *et al.* (2010) investigated the vibration and instability of an axially moving viscoelastic Rayleigh beam. The fourth is the transverse moving beam problem. It can be used to simulate a manipulator arm, a moving scanning probe and a transversely moving spindle [Gu and Piedboeuf, 2003; Brusa1 et al., 2009]. This mathematical model is different to the previous ones. Park and Youm (2001) investigated the vibration of a linear moving beam. The boundary conditions were homogeneous. Lin (2009[a]) investigated the vibration control of the linear moving beam. The model of the linear transverse moving beam is composed of a governing differential equation and a time-dependent boundary condition due to the tip mass inertia force. The end static electric force is used to successfully suppress the overshoot. Moreover, the dynamic positioning of a long-distance moving beam is investigated.

The special case of the curvedly moving beam problem is one with constant rotation [Lin, 2008; Lin *et al.*, 2008[b]]. The coefficients of a governing differential equation and four boundary conditions of this system are independent to time variable and these equations are homogeneous. Another special case is a rotating beam with non-constant speed. Lin (2009[b]) investigated the vibrations of the inward and outward reciprocating rotating beams. So far, no literature devoted to the vibration of a beam picking and placing a mass along arbitrary curved tracking.

It is well known that moving mass in high speed induces the overshoot and residual vibration. The structural control is helpful for suppressing these defaults [Sadek *et al.*, 2009]. In general, there are two control methods: (1) the passive one such as the damping suppression of vibration and (2) the active one such as the external force control suppression. The later is effective but complicated especially for the distributed system. The relevant literatures are as follows: Meirovitch (1997), Gawronski (1998) and Do and Pan (2009) presented the modal control approach. Weaver and Silverberg (1992) investigated the node control theorem. Sadek *et al.* (2009) investigated the boundary control method. In practice, the boundary control method is simple and effective. This method is usually with the time-dependent boundary condition. Some literatures [Lee and Lin, 1996 and 1998; Lin, 1998; Lin and Lee, 2002] investigated the system with the time-dependent boundary condition. However, these studies are neither for a long-distance moving beam or the dynamic positioning.

In this study, the mathematical in-plane vibration model of a beam picking and placing a mass along arbitrary curved tracking is established. The semi-analytical solution for this system is derived. The overshoot and residual vibration problems are investigated here.

2 Governing equations and associated conditions

The governing differential equation of a beam moving in arbitrary curved tracking, as shown in Figure 1, is derived as follows:

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 W}{\partial x^2}\right]-\frac{\partial}{\partial x}\left[N\frac{\partial W}{\partial x}\right]+C\frac{\partial W}{\partial t}+m(x)\frac{\partial^2 W}{\partial t^2}-\left(\frac{d\varphi}{dt}\right)^2 m(x)W=p(x,t) \tag{1}$$

where $p(x,t)=-\rho A\left[-\frac{d^2X}{dt^2}\sin\phi+\frac{d^2Y}{dt^2}\cos\phi+x\frac{d^2\phi}{dt^2}\right]$. The root displacement $\vec{R}_0 = Xe_1+Ye_2$, as shown in Figure 1a. C is the damping coefficient. The mass per unit length $m=\rho A$ where ρ and A are the density and cross-sectional area of the beam. $W(x,t)$ is the flexural displacement, E is the Young's modulus. x is the coordinate along the beam, t is time and L is the length of the beam. I denotes the area moment of inertia. N is the centrifugal force. ϕ is the angle of rotation.

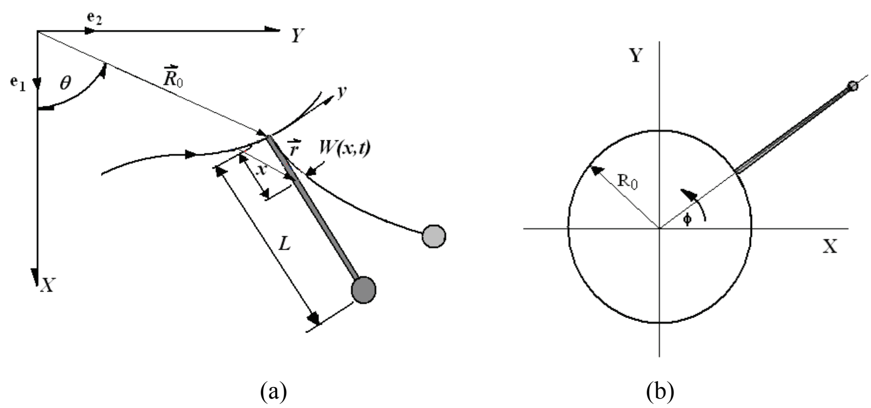


Figure 1: Geometry and coordinate system of a moving beam.

The associated boundary conditions are

At $x=0$:

$$W=0 \tag{2}$$

$$\frac{\partial W}{\partial x}=0 \tag{3}$$

At $x = L$:

$$\frac{\partial^2 W}{\partial x^2} = 0 \tag{4}$$

$$-EI\frac{\partial^3 W}{\partial x^3} + M_{tip}\frac{\partial^2 W}{\partial t^2} = F\left(t, W(L,t)\right).$$

where

$$F\left(t, W(L,t)\right) = -M_{tip}\left(-\frac{d^2X}{dt^2}\sin\phi + \frac{d^2Y}{dt^2}\cos\phi + L\frac{d^2\phi}{dt^2} - \left(\frac{d\phi}{dt}\right)^2 W(L,t)\right) + F_{control} \tag{5}$$

in which $F_{control}$ is the external control force to be designed for suppressing the overshoot and residual vibration. This force depends on the tip deflection $W(L,t)$. Moreover, the motion of picking and placing the tip mass can be simulated by considering the tip mass M_{tip} time-dependent. The corresponding investigations are made later. In addition, the centrifugal force is

$$N = -\int_x^L N_c dm - M_{tip}N_c(L) \tag{6a}$$

where the overall axial acceleration is

$$N_c = \left[\frac{d^2X}{dt^2}\cos\phi + \frac{d^2Y}{dt^2}\sin\phi - \left(\frac{d\phi}{dt}\right)^2 x - 2\frac{d\phi}{dt}\frac{\partial W(x,t)}{\partial t} - \frac{d^2\phi}{dt^2}W(x,t)\right]. \tag{6b}$$

Obviously, the last two accelerations of Eq. (6b) are greatly smaller than the first three terms due to small displacement, (6b) becomes

$$N_c = \frac{d^2X}{dt^2}\cos\phi + \frac{d^2Y}{dt^2}\sin\phi - \left(\frac{d\phi}{dt}\right)^2 x \tag{7}$$

It should be noted that the coefficients are time-dependent and the system is nonlinear. So far, no literature is devoted to derive the solution and study its physical phenomenon.

The corresponding initial conditions are expressed as

$$W(x,0) = W_0(x), \quad \frac{\partial W(x,0)}{\partial t} = \dot{W}_0(x) \tag{8}$$

For clarity and without the loss of generality, a uniform beam with a tip mass is considered here. In terms of the following dimensionless quantities

$$c = \frac{CL^2}{\sqrt{EI\rho A}}, \quad m_{tip} = \frac{M_{tip}}{\rho AL}, \quad r_0 = \frac{R_0}{L},$$

$$\bar{x} = \frac{X}{L}, \quad \bar{y} = \frac{Y}{L}, \quad w(\xi, \tau) = \frac{W(x,t)}{L},$$

$$\xi = \frac{x}{L}, \quad \tau = \frac{t}{L^2}\sqrt{\frac{EI}{\rho A}},$$

$$n(\xi, \tau) = -\int_{\xi}^{1} n_c d\xi - m_{tip} n_c(1, \tau),$$

$$n_c = \frac{d^2\bar{x}}{d\tau^2}\cos\phi + \frac{d^2\bar{y}}{d\tau^2}\sin\phi - \left(\frac{d\phi}{d\tau}\right)^2 \xi, \tag{9}$$

the dimensionless governing differential equation is

$$\frac{\partial^4 w}{\partial \xi^4} + \frac{\partial}{\partial \xi}\left(n\frac{\partial w}{\partial \xi}\right) + c\frac{\partial w}{\partial \tau} + \frac{\partial^2 w}{\partial \tau^2} - \left(\frac{d\phi}{d\tau}\right)^2 w = p(\xi, \tau) \tag{10}$$

where $p(\xi, \tau) = -\left[-\frac{d^2\bar{x}}{d\tau^2}\sin\phi + \frac{d^2\bar{y}}{d\tau^2}\cos\phi + \xi\frac{d^2\phi}{d\tau^2}\right]$.
The associated dimensionless boundary conditions are
At $\xi = 0$:

$$w = 0 \tag{11}$$

$$\frac{\partial w}{\partial \xi} = 0 \tag{12}$$

At $\xi = 1$:

$$\frac{\partial^2 w}{\partial \xi^2} = 0 \tag{13}$$

$$-\frac{\partial^3 w}{\partial \xi^3} + m_{tip}\frac{\partial^2 w}{\partial \tau^2} = f(\tau, w(1, \tau)) \tag{14}$$

where

$$f = f_a(\tau) + f_w(\tau, w(1,\tau)) + f_{control}(\tau, w(1,\tau)),$$

$$f_a(\tau) = -m_{tip}\left(-\frac{d^2\bar{x}}{d\tau^2}\sin\phi + \frac{d^2\bar{y}}{d\tau^2}\cos\phi + \frac{d^2\phi}{d\tau^2}\right),$$

$$f_w(\tau, w(1,\tau)) = m_{tip}\left(\frac{d\phi}{d\tau}\right)^2 w(1,\tau).$$

The corresponding initial conditions are expressed as

$$w(\xi,0) = w_0(\xi),$$

$$\frac{\partial w(\xi,0)}{\partial \tau} = \dot{w}_0(\xi). \tag{15}$$

3 Solution method

3.1 Change of variable

By taking a change of dependent variable with a shifting function the original system can be transformed to be one composed of one non-homogeneous governing differential equation and four homogeneous boundary conditions. The relation among variables is assumed to be

$$w(\xi,\tau) = \bar{w}(\xi,\tau) + g(\xi)\, f(\tau, w(1,\tau)) \tag{16}$$

where $g(x)$ is the shifting function and chosen to satisfy the following conditions

$$g(0) = 0, \quad \frac{dg(0)}{d\xi} = 0, \quad g(1) = 0,$$

$$\frac{d^2g(1)}{d\xi^2} = 0, \quad \frac{d^3g(1)}{d\xi^3} = -1, \quad \frac{d^4g(1)}{d\xi^4} = 0$$

If the shifting function is

$$g(\xi) = \alpha_0 + \alpha_1\xi + \alpha_2\xi^2 + \alpha_3\xi^3 + \alpha_4\xi^4 + \alpha_5\xi^5, \tag{17}$$

Based on the conditions (17), the shifting function is found

$$g(\xi) = -\frac{1}{3}\xi^2 + \frac{2}{3}\xi^3 - \frac{5}{12}\xi^4 + \frac{1}{12}\xi^5. \tag{18}$$

Substituting Eqs. (9) and (12) into Eqs. (2-8), the transformed differential equation and corresponding boundary conditions are, respectively

$$\frac{\partial^4 \bar{w}}{\partial \xi^4} + \frac{\partial}{\partial \xi}\left(n(\xi,\tau)\frac{\partial \bar{w}}{\partial \xi}\right) + c\frac{\partial \bar{w}}{\partial \tau} + \frac{\partial^2 \bar{w}}{\partial \tau^2} - \left(\frac{d\phi}{d\tau}\right)^2 \bar{w} = \bar{p}(\xi,\tau,w(1,\tau)) \tag{19}$$

where $\bar{p} = p - \frac{d^4 g}{d\xi^4} f - cg\frac{df}{d\tau} - g\frac{d^2 f}{d\tau^2} - \frac{\partial}{\partial \xi}\left(n(\xi,\tau)\frac{dg}{d\xi}\right) f.$

At $\xi = 0$:

$$\bar{w}(0,\tau) = 0, \tag{20}$$

$$\frac{\partial \bar{w}(0,\tau)}{\partial \xi} = 0, \tag{21}$$

At $\xi = 1$:

$$\frac{\partial^2 \bar{w}(1,\tau)}{\partial \xi^2} = 0, \tag{22}$$

$$-\frac{\partial^3 \bar{w}(1,\tau)}{\partial \xi^3} + m_{tip}\frac{\partial^2 \bar{w}(1,\tau)}{\partial \tau^2} = 0. \tag{23}$$

The transformed initial condition (15) becomes

$$\bar{w}(\xi,0) = w_0(\xi) - g(\xi) f(0,\bar{w}(1,0)),$$
$$\frac{\partial \bar{w}(\xi,0)}{\partial \tau} = \dot{w}_0(\xi) - g(\xi)\frac{df(0,\bar{w}(1,0))}{d\tau}. \tag{24}$$

3.2 Subsection method

Obviously, it is very difficult to solve these implicit differential equations. In this study, the subsection method is presented as follows:

The displacement is divided into a lot of section and expressed as

$$w(\xi,\tau) = \begin{cases} w_1(\xi,\tau), & 0 < \tau < \tau_1 \\ w_2(\xi,\tau), & \tau_1 < \tau < \tau_2 \\ \vdots & \\ w_j(\xi,\tau), & \tau_{j-1} < \tau < \tau_j \\ \vdots & \\ w_n(\xi,\tau), & \tau_{n-1} < \tau < \tau_n \end{cases} \tag{25}$$

$$w(\xi,\tau)=\begin{cases}\bar{w}_1(\xi,\tau)+g(\xi)f_1(\tau,w(1,\tau)), & 0<\tau<\tau_1\\ \bar{w}_2(\xi,\tau)+g(\xi)f_2(\tau,w(1,\tau)), & \tau_1<\tau<\tau_2\\ \vdots & \\ \bar{w}_j(\xi,\tau)+g(\xi)f_j(\tau,w(1,\tau)), & \tau_{j-1}<\tau<\tau_j\\ \vdots & \\ \bar{w}_n(\xi,\tau)+g(\xi)f_n(\tau,w(1,\tau)), & \tau_{n-1}<\tau<\tau_n\end{cases} \tag{26}$$

where

$$f_j=f_{aj}(\tau)+f_{ej}(\tau)+f_{wj}(\tau,w(1,\tau)),\ \tau_{j-1}<\tau<\tau_{j+1}$$

$$f_{wj}(\tau,w(1,\tau))=M\left(\frac{d\varphi}{d\tau}\right)^2 w_j(1,\tau) \tag{27}$$

Because it is implicit and very difficult to solve, the approximate solution is considered as follows:

$$\begin{aligned}f_{wj}(\tau,w(1,\tau))&=M\left(\frac{d\varphi}{d\tau}\right)^2 w_j(1,\tau)\\ &\approx M\left(\frac{d\varphi}{d\tau}\right)^2\left[w_{j-1}(1,\tau_{j-1})+(\tau-\tau_{j-1})\frac{dw_{j-1}(1,\tau_{j-1})}{d\tau}\right]\triangleq\tilde{f}_{wj}(\tau)\end{aligned} \tag{28}$$

Finally,

$$\begin{aligned}\bar{p}_j(\xi,\tau)\approx\tilde{p}_j(\xi,\tau)\triangleq p_j(\xi,\tau)-\frac{d^4g}{d\xi^4}\tilde{f}_j(\tau)-cg(\xi)\frac{d\tilde{f}_j(\tau)}{d\tau}-g(\xi)\frac{d^2\tilde{f}_j(\tau)}{d\tau^2}\\ -\frac{\partial}{\partial\xi}\left(n(\xi,\tau_{j-1})\frac{dg}{d\xi}\right)\tilde{f}_j(\tau)\end{aligned} \tag{29}$$

The corresponding approximated equation becomes

$$\frac{\partial^4\bar{w}_j}{\partial\xi^4}+\frac{\partial}{\partial\xi}\left(n(\xi,\tau_{j-1})\frac{\partial\bar{w}_j}{\partial\xi}\right)+c\frac{\partial\bar{w}_j}{\partial\tau}+\frac{\partial^2\bar{w}_j}{\partial\tau^2}-\left(\frac{d\phi}{d\tau}\right)^2\bar{w}_j=\tilde{p}_j(\xi,\tau),$$

$$0<\xi<1;\quad \tau_{j-1}<\tau<\tau_j \tag{30}$$

At $\xi=0$:

$$\bar{w}_j(0,\tau)=0, \tag{31}$$

$$\frac{\partial \bar{w}_j(0,\tau)}{\partial \xi} = 0, \tag{32}$$

At $\xi = 1$:

$$\frac{\partial^2 \bar{w}_j(1,\tau)}{\partial \xi^2} = 0, \tag{33}$$

$$-\frac{\partial^3 \bar{w}_j(1,\tau)}{\partial \xi^3} + m_{tip}\frac{\partial^2 \bar{w}_j(1,\tau)}{\partial \tau^2} = 0. \tag{34}$$

The corresponding initial conditions are

$$\bar{w}_{j-1}(\xi,\tau_{j-1}) = w_{j-1}(\xi,\tau_{j-1}) - g(\xi)\, f_{j-1}(\tau_{j-1},\bar{w}(1,\tau_{j-1})),$$

$$\frac{\partial \bar{w}_{j-1}(\xi,\tau_{j-1})}{\partial \tau} = \dot{w}_{j-1}(\xi,\tau_{j-1}) - g(\xi)\,\frac{d f_{j-1}(\tau_{j-1},\bar{w}(1,\tau_{j-1}))}{d\tau}. \tag{35}$$

3.3 Mode superposition method

The solution for the equations (31-36) of the j-th section motion can be obtained by using the mode superposition method. The free vibration mathematical model of the j-th section motion is listed in Appendix A. One can derive the orthogonality condition of the eigenfunctions of this model as follows:

$$\int_0^1 W_i W_j d\xi + m_{tip} W_i(1)\, W_j(1) = \begin{cases} 0, & i \neq j \\ \varepsilon_{ii}, & i = j \end{cases} \tag{36}$$

where W_i is the ith mode shape. The displacement solution of the j-th section motion can be expressed in the following eigenfunction expansion form:

$$\bar{w}_j = \sum_{i=0}^{\infty} W_i(\xi) T_{ji}(\tau), \tag{37}$$

Substituting it back the transformed governing equation (31) and the initial conditions (36), multiplying by '$W_k(\xi)\,[1 + m_{tip}\delta(\xi - 1)]$' and integrating in accordance with the orthogonalty condition (37), one obtains

$$\frac{d^2 T_{jk}(\tau)}{d\tau^2} + c\frac{dT_{jk}(\tau)}{d\tau} + \left[\omega_k^2 - \left(\frac{d\phi}{d\tau}\right)^2\right] T_{jk}(\tau) = \bar{F}_{jk}(\tau), \quad \tau_{j-1} < \tau < \tau_j \tag{38}$$

where $\bar{F}_{jk}(\tau) = \frac{1}{\varepsilon_{kk}} \int_0^1 W_k(\xi)\,[1 + m_{tip}\delta(\xi - 1)]\tilde{p}(\xi,\tau) d\xi$.

The solution of Eq. (39) is expressed as

$$T_{jk}(\tau) = \sum_{i=1}^{2} C_i V_{ji}(\tau) + V_{pjk}(\tau), \tag{39}$$

where V_{pjk} is the particular solution and $\{V_{j1}, V_{j2}\}$ are the fundamental solutions of Eq. (39), which are assumed to satisfy the following normalized condition

$$\begin{bmatrix} V_{1jk}(\tau_{j-1}) & V_{2jk}(\tau_{j-1}) \\ \frac{dV_{1jk}(\tau_{j-1})}{d\tau} & \frac{dV_{2jk}(\tau_{j-1})}{d\tau} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{40}$$

Further, considering the continuity conditions, the solution (40) is expressed as

$$\begin{aligned} T_{jk}(\tau) &= \left[T_{jk}(\tau_{j-1}) - V_{pjk}(\tau_{j-1})\right] V_{1jk}(\tau) \\ &+ \left[\frac{dT_{jk}(\tau_{j-1})}{d\tau} - \frac{dV_{pjk}(\tau_{j-1})}{d\tau}\right] V_{2jk}(\tau) + V_{pjk}(\tau),\ \tau_{j-1} < \tau < \tau_j \end{aligned} \tag{41}$$

where the particular solution is derived and listed in Appendix B.

4 Numerical results and discussion

To demonstrate efficiency and convergence of the proposed numerical method, the dynamic positioning of an outward rotating beam is determined, as shown in Figure 1b. The forced term of equation (10) reduces to

$$p(\xi, \tau) = -(\xi + r_0)\frac{d^2\phi}{d\tau^2} \tag{42}$$

Assume that the initial displacement and velocity of beam are zero. The beam is rotated from the origin position to some specified one. The angular acceleration is expressed as a unit function

$$\frac{d^2\phi}{d\tau^2} = \begin{cases} 0, & \tau < 0 \\ \alpha, & 0 < \tau < T/2 \\ -\alpha, & T/2 < \tau < T \\ 0, & \tau > T \end{cases} \tag{43}$$

where α is the acceleration and T is the period of rotation. High-speed mass movement results in the overshoot and the residual vibration phenomenon as shown in Figure 2. Moreover, it is observed that the displacement solution determined by the

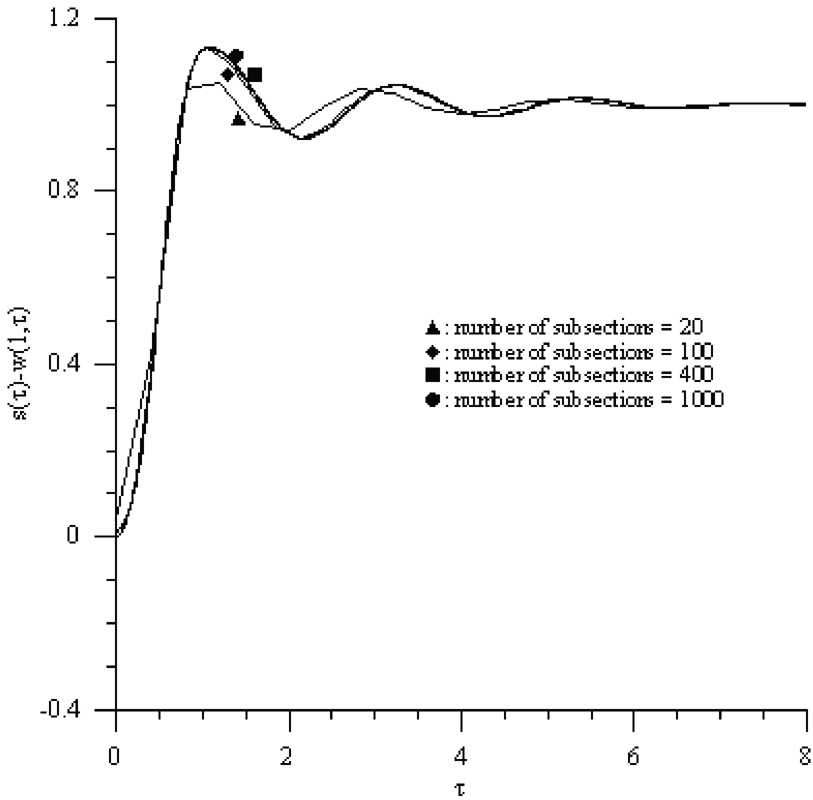


Figure 2: Convergence pattern of the presented method. [outward rotating, s(τ) is the distance of tip track; c=1, r_0 = 1, T=1].

proposed method converge very rapidly. Even when the number of subsections is only 100, the difference between the corresponding one and the converged one is very small.

Figure 3 shows the influence of the radius of root on the overshoot and the residual vibration. It is found that the larger the radius of root is, the smaller the overshoot. In other words, for a fixed tip moving distance, the smaller the root radius is, the larger the rotating angle and overshoot.

It is well known that the effect of overshoot and residual vibration on the positioning is significant. These will occur due to too short acceleration time. For overcoming this fault an active control for suppressing vibration is studied. In the closed-loop control for suppressing vibration, the control electrostatic force is

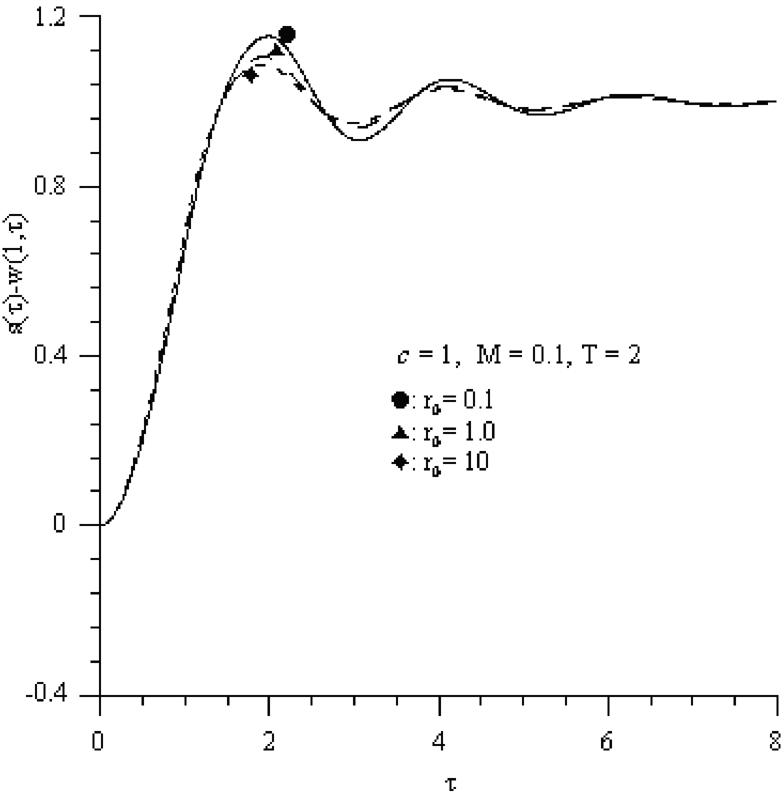


Figure 3: Influence of the root radius r_0 on the overshoot and the residual oscillation of an outward rotating beam.

designed by the uniform control law and expressed as '$F_{control} = QE_e$' where the concentrate electric charge Q is constant and the electric intensity is

$$E_e(\tau) = -sign(w(1,\tau_0))E_0, \ \tau_0 < \tau < \tau_0 + \Delta\tau, \tag{44}$$

in which the E_0 is constant and the direction of the electric field is adjusted against the direction of beam displacement. $\Delta\tau$ is the sampling time of the piezoelectric sensor measuring the tip displacement. Therefore, the dimensionless electric force can be expressed as

$$f_{control}(\tau) = -k_e sign(w(1,\tau_0)), \ \tau_0 < \tau < \tau_0 + \Delta\tau. \tag{45}$$

$F_{control}$ is the dimensionless electrostatic force, F_eL^2/EI [Lin, 2007]. Figure 4 shows that the overshooting is greatly decreased by using this control law, especially for the case with the gain factor $k_e = 0.3$.

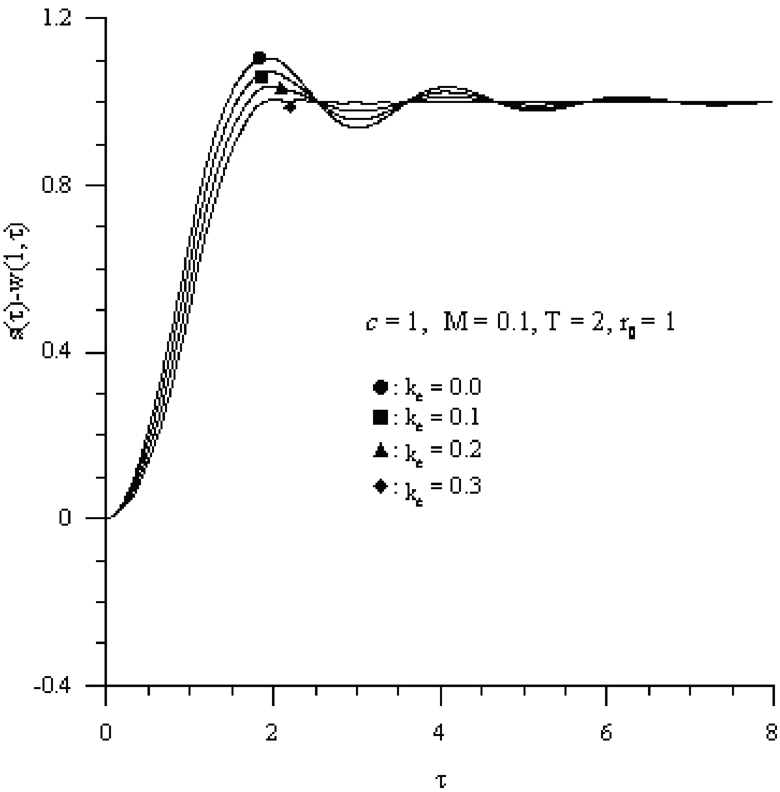
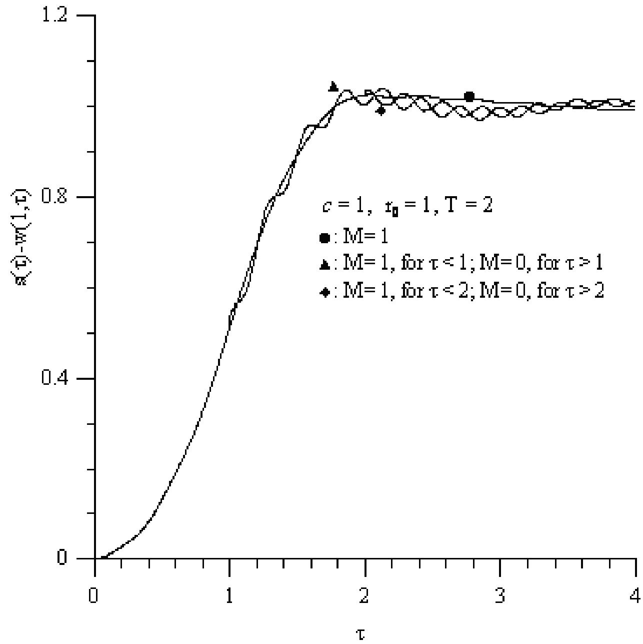


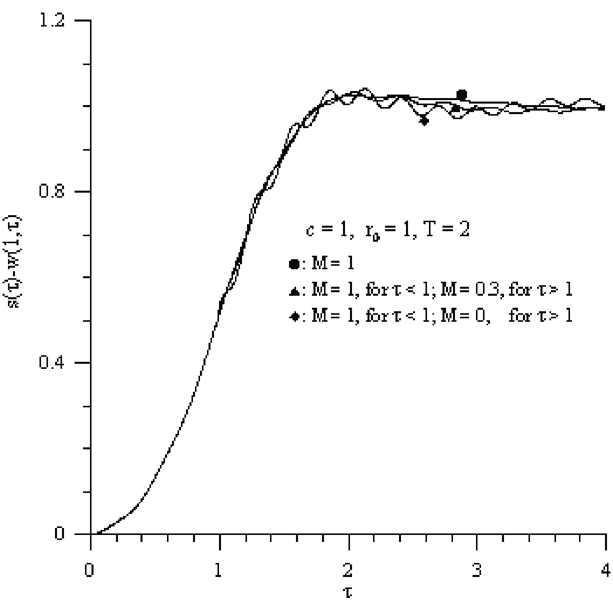
Figure 4: Influence of the uniform electric control law on the vibration suppression of a moving beam.

Figure 5a demonstrates the effect of placing mass at different time on the overshoot and residual oscillation. It is observed from Figure 5a that placing the tip mass immediately results in the oscillation of higher frequency. The reason is that the natural frequency of a beam without a tip mass is higher than that of a beam with one. Moreover, the effect of the placing mass on the overshoot is slight. Figure 5b shows the influence of the placing mass on the residual oscillation. It is found that the more the tip mass is placed, the higher the residual oscillation. It is because the smaller the tip mass attached to the beam is, the higher the natural frequencies.

Figure 6 demonstrates influence of the root radius r_0 of an inward rotating beam



(a)



(b)

Figure 5: Influence of laying down mass at different time on the overshoot and residual oscillation of an outward rotating beam.

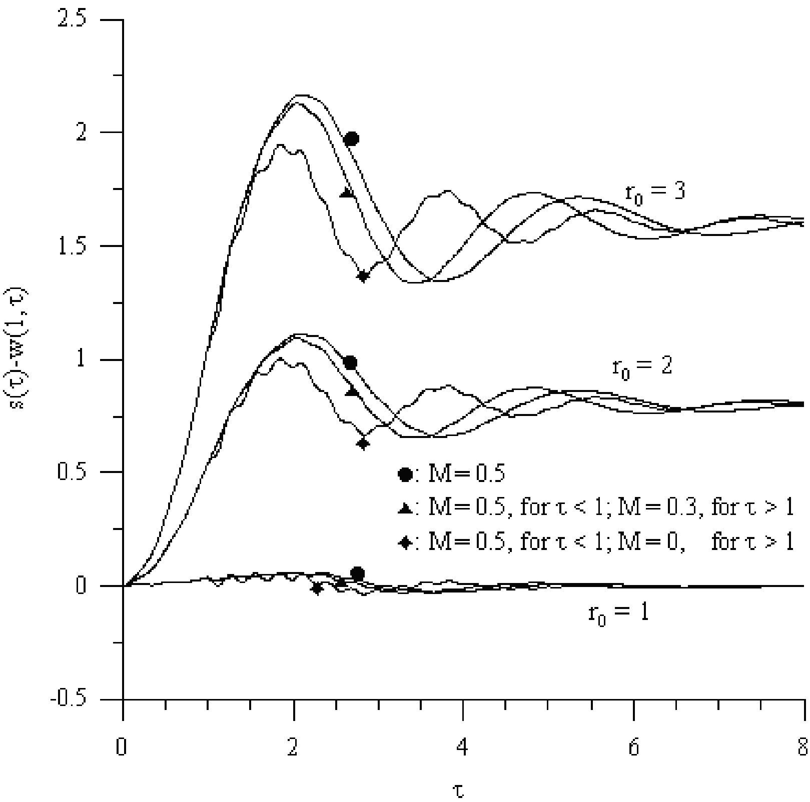


Figure 6: Influence of laying down mass at different time on the overshoot and residual oscillation of an inward rotating beam. [c=1, T=2].

and the placing tip mass on the overshoot and the residual oscillation. It is found that the overshoot of the beam with the root radius $r_0 = 1$, is very small. Moreover, the larger the root radius is, the greater the overshoot. The reason is that for the case with the root radius $r_0 = 1$, the effect of the moving acceleration on the tip mass is almost negligible. It is also discovered that placing the tip mass decreases significantly the overshoot. This phenomenon is greatly different to that of the outward rotating.

5 Conclusion

In this study, the mathematical models of the beam moving in arbitrary curved track are established. The coefficients of the equations are time- and position-dependent and nonlinear. The analytical solution for this system is presented. The active control of electric field for suppressing overshoot and vibration of the moving beam is verified to be very effective. Beside, the effects of several important parameters on the vibration of a moving beam are concluded as follows:

- Placing the tip mass immediately results in the oscillation of higher frequency. It is because the natural frequency of a beam without a tip mass is higher than that of a beam with one.
- For inward rotating system placing the tip mass decreases significantly the overshoot. But for outward rotating system this effect on the overshoot is slight.
- The overshoot of the inward rotating beam with the root radius $r_0 = 1$, is very small. Moreover, the larger the root radius is, the greater the overshoot.

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Appendix A

The characteristic subsystem

$$\frac{\partial^4 \bar{w}_j}{\partial \xi^4} + \frac{\partial}{\partial \xi}\left(n(\xi,\tau_{j-i})\frac{\partial \bar{w}_j}{\partial \xi}\right) + \frac{\partial^2 \bar{w}_j}{\partial \tau^2} = 0, \tag{A1}$$

At $\xi = 0$:

$$\bar{w}_j(0,\tau) = 0 \tag{A2}$$

$$\frac{\partial \bar{w}_j(0,\tau)}{\partial \xi} = 0 \tag{A3}$$

At $\xi = 1$:

$$\frac{\partial^2 \bar{w}_j(1,\tau)}{\partial \xi^2} = 0 \tag{A4}$$

$$-\frac{\partial^3 \bar{w}_j(1,\tau)}{\partial \xi^3} + m_{tip}\frac{\partial^2 \bar{w}_j(1,\tau)}{\partial \tau^2} = 0 \tag{A5}$$

Appendix B

Derivation of the particular solution of Eq. (40)

In general, the second-order ordinary differential equation with variable coefficients can be written as

$$q_2(\tau)\frac{d^2V}{d\tau^2} + q_1(\tau)\frac{dV}{d\tau} + q_0(\tau)V = p(\tau) \quad \tau \in (\tau_{j-1}, \tau_j) \tag{B1}$$

where the leading coefficient $q_2(\tau)$ does not vanish anywhere on the closure domain. The equation (B1) in terms of Green's function is

$$q_2(\tau)\frac{d^2E_\zeta}{d\tau^2}+q_1(\tau)\frac{dE_\zeta}{d\tau}+q_0(\tau)E_\zeta=\delta(\tau-\zeta)\quad \tau\&\xi\in(\tau_{j-1},\tau_j) \tag{B2}$$

One assumes

$$E_\zeta(\tau)=G_\zeta(\tau)H(\tau-\zeta)=[C_1(\zeta)V_1(\tau)+C_2(\zeta)V_2(\tau)]H(\tau-\zeta) \tag{B3}$$

and

$$[C_1(\zeta)V_1(\zeta)+C_2(\zeta)V_2(\zeta)]=0 \tag{B4}$$

Where the fundamental solutions satisfy the normalized condition (42) and $H(\tau-\zeta)$ is the Heaviside function. Eq. (B2) becomes

$$\left\{\sum_{i=1}^{2}C_i(\zeta)\left[q_2(\tau)\frac{d^2V_i(\tau)}{d\tau^2}+q_1(\tau)\frac{dV_i(\tau)}{d\tau}+q_0(\tau)V_i(\tau)\right]\right\}H(\tau-\zeta)$$
$$+q_2(\tau)\left[C_1(\zeta)\frac{dV_1(\tau)}{d\tau}+C_2(\zeta)\frac{dV_2(\tau)}{d\tau}\right]\delta(\tau-\zeta)=\delta(\tau-\zeta) \tag{B5}$$

Because $\left[q_2(\tau)\frac{d^2V_i(\tau)}{d\tau^2}+q_1(\tau)\frac{dV_i(\tau)}{d\tau}+q_0(\tau)V_i(\tau)\right]=0$,

$$q_2(\zeta)\left[C_1(\zeta)\frac{dV_1(\zeta)}{d\tau}+C_2(\zeta)\frac{dV_2(\zeta)}{d\tau}\right]=1 \tag{B6}$$

Based on Eqs. (B4) and (B6), the coefficients of the Green function (B3) is obtained

$$\begin{bmatrix}C_1(\zeta)\\C_2(\zeta)\end{bmatrix}=\begin{bmatrix}V_1(\zeta) & V_2(\zeta)\\ \frac{dV_1(\zeta)}{d\tau} & \frac{dV_2(\zeta)}{d\tau}\end{bmatrix}^{-1}\begin{bmatrix}0\\1/q_2(\zeta)\end{bmatrix} \tag{B7}$$

Further, the particular solution is obtains

$$V_{pk}(\tau)=\sum_{i=1}^{2}V_i(\tau)\int_{\tau_j}^{\tau}\varepsilon_i(\chi)\bar{F}_k(\chi)\,d\chi, \tag{B8}$$

where

$$\begin{bmatrix}\varepsilon_1(\zeta)\\\varepsilon_2(\zeta)\end{bmatrix}=\frac{1/q_2(\zeta)}{V_1(\zeta)V_2'(\zeta)-V_2(\zeta)V_1'(\zeta)}\begin{bmatrix}-V_1'(\zeta)\\V_1(\zeta)\end{bmatrix}.$$