Parametric Study of a Pitching Flat Plate at Low Reynolds Numbers

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Abstract: In this paper we simulate the unsteady, incompressible, and laminar flow behavior over a flat plate with round leading and trailing edges. A pressure-Poisson method is used to solve the incompressible Navier-Stokes equations. Both convection and diffusion terms are discretized using a second-order accurate central difference method. A second-order accurate split-step scheme with an Adam's predictor corrector time-stepping method is adopted for the time integration. An overlapping moving grid approach is employed to dynamically update the grid due to the plate motion. The effects of the pitch rate, Reynolds number, location of pitch axis, and computational domain size are investigated. Comparisons are made with experimental results.

Keywords: Micro air vehicles, flapping wing.

Nomenclature

- C_L = Lift coefficient per unit span C_T = thrust coefficient per unit span c = chord length Ω = pitch rate in rad/s
- Ω^+ = nondimensional pitch rate, $\Omega c/U_0$
- t = time
- t^+ = dimensionless time,
- U_0 = reference velocity, typically freestream velocity
- θ = pitching angle of attack
- θ_{max} = maximum amplitude of pitching angle

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1 Introduction

Micro air vehicles (MAVs) are envisioned to play critical roles in many civilian and military missions. As the vehicle size continues to decrease, a flapping wing design becomes promising. Compared with fixed and rotary wings, a flapping wing has many desirable characteristics. First, given the wing size and the operating Reynolds number, a flapping wing has a higher lift coefficient than a rigid wing [Dickinson (1994); Ellingto, Berg, Willmott, and Thomas (1996)]. Second, a flapping wing can simultaneously generate both lift and thrust. Third, a flapping wing can effectively alleviate gust fluctuations to stabilize flight [Lian and Shyy (2007)]. The study of flapping wing has spurred a new wave of interests in the unsteady low Reynolds number flows [Shyy, Lian, Tang, Viieru, and Lou. (2008)]. Researchers have investigated the effects of wing kinematics [Ol (2007); Young and Lai (2004); Wang (2000)], wing geometry [Kesel (2000)], flexibility [Aono, Chimakurthi, Cesnik, Liu, and Shyy (2009); Hu, Kumar, Abate, and Albertani (2009)], and Reynolds number. However, there lack canonical cases for which researchers can cross-exam their results to identify the sources of discrepancies.

The AIAA Fluid Dynamics Technical Committee recently suggested such a canonical case [Ol (2009)]. The case is to study the aerodynamics of a flat-plate of 2%thickness with round leading and trailing edges and infinite-span. The plate starts at angle of attack (AoA), , of zero degree and linearly pitches up to the maximum pitching angle, . Once reaching , the plate holds its position for a dimensionless time, , of 0.05. Then it linearly pitches back down to its starting position.

When an airfoil rapidly pitches up beyond its static stall angle, it generates a dynamic stall vortex which causes the lift coefficient to increase beyond its maximum value for unstalled conditions [Morgan and Visbal (2001)]. This phenomenon is called dynamic stall and is an unsteady flow phenomenon. Dynamic stall is desirable for helicopter blades and other highly maneuverable flight machines such as MAVs as it delays the onset of a stall and sustains lift. Erricson and Reding (1971) used a quasi-steady theory to analytically examine the dynamic stall of a helicopter blade section. They found the quasi-steady theory gave reasonably good results when the dimensionless pitch rate was less than 0.5. At high Reynolds numbers, an experimental study was conducted and the results were reported by McCroskey (1976) who found that the reduced frequency and the leading edge profile of the airfoil were more important in determining the aerodynamic forces than the Reynolds number. At low Reynolds numbers, Ohmi, Coutanceau, Phuoc Loc, and Dulieu. (1991) examined the flow over an NACA0012 airfoil and identified that the Reynolds number was less important than other parameters. The analysis and progress of dynamic stall was reviewed by Carr (1988). Visbal (1986) used a Navier-Stokes flow solver to study dynamic stall on a 2D airfoil. Later Visbal and

Shang (1989) examined the impact of pitch rate, pitch axis, and initial acceleration on the vertical structures and aerodynamic forces of a 2D airfoil. Simulations of unsteady 3D separation on a pitching wing were reported by Morgan and Visbal (2001).

In this paper, we will report our results from the simulation of the suggested canonical case mentioned above. The effects of pitch rate, position of pitch axis, Reynolds number, and computational domain size are investigated.

2 Numerical Methods

2.1 Governing Equations

We simulate the flow field by solving the incompressible Navier-Stokes equations in curvilinear coordinates. For clarity, we present the governing equations in Cartesian coordinates

$$\Delta(\vec{u}_t + \vec{u}.\nabla\vec{u}) = -\nabla p + \nabla . \nu \nabla \vec{u} \tag{1}$$

$$\nabla . \vec{u} = 0 \tag{2}$$

Where \vec{u} is the velocity vector, ρ is the density, p is the pressure, and v is the kinematic viscosity. The equations were discretized on an overlapping grid using a second order accurate central difference approximation of the velocity and pressure equations. Time stepping was accomplished using a second-order accurate split-step scheme with an Adam's predictor corrector time-stepping method [Hen-shaw and Petersson (2003)]. For the Reynolds number considered, 10,000, the flow was assumed to be accurately modeled as a direct numerical simulation and no turbulence model was employed.

2.2 Moving Overlapping Grid

The wing flapping motion leads to a moving boundary problem. We need a moving grid approach to dynamically update the computational grid to accommodate the wing motion. The overlapping moving grid is adopted for this numerical simulation [Henshaw (1994)]. This method uses boundary-conforming structured grids to achieve high-quality representations of boundaries. It employs Cartesian grids as the background grids so that the efficiencies inherent with such grids can be exploited. The irregular boundary associated with standard Cartesian grid methods takes the form of the interpolation boundary between overlapping grids. The use of overlapping grids is desirable for moving bodies because it is computationally less expensive than most other conventional approaches. Interpolation points are

located in the overlap region between different grids and are used to couple the solutions. As the body moves, the grid moves with it, meaning that only the interpolation points between grids must be recalculated as opposed to the need to regenerate the whole mesh, as may be necessary with other methods. A sample overlapping grid is shown in Figure 1. The background Cartesian grid and body-fitting curvilinear grid are represented by blue and green colors respectively. At the overlapping boundary, the interpolation points are marked with solid square symbols. The geometric conservation law [Thomas and Lombard (1979)] is enforced to update the Jacobin matrix. Other methods for moving boundary problem include the meshless methods which can be found in the work of Atluri and coworkers [Atluri and Zhu (1998, 2000); Atluri and Shen (2002); Atluri, Han, and Shen (2003); Atluri, Han, and Rajendran (2004); Atluri (2004)]. Recently Avila and Atluri studied a rigid flapping plate using meshless methods [Avila and Atluri (2009)]. In their study the plate was assumed to have zero thickness and experienced harmonic oscillation. Unsteady flow due to structural flexibility was also investigated in their paper.

3 Results and Discussion

We simulated the flat plate with round leading and trailing edges and 2% thickness. A sample overlapping grid is shown in Figure 1. The unsmoothed pitching motion started from the minimum incidence of zero degree and linearly ramped up to the maximum pitching angle, , of 40 degrees. After pausing for a dimensionless time = of 0.05, it linearly pitched downward to the minimum incidence of zero degree.

3.1 Code validation

Before we studied the physical aspects of the flow, we first validated our code. The study was based on the flat plate which was pitched at quarter chord. The dimensionless pitch rate was 0.2. In the experiment the flat plate was mounted wall-to-wall to approximate the situation of infinite aspect ratio and in the simulation a 2D flat plate was simulated [OI (2009)]. The boundary conditions followed the experimental setup: the plate was in the middle of the computational domain and 2-chord lengths away from the top and bottom boundaries which used the non-slip condition. A uniform velocity boundary condition was specified at the inlet and pressure was fixed at the outlet. A nonslip boundary condition was specified on the flat plate. For a prescribed motion, the grid velocity can be calculated either numerically or analytically. In this work the grid velocity was calculated analytically. The Reynolds number based on the flow remained laminar.

To ensure a smooth acceleration, the pitching motion suggested by Visbal and



Figure 1: Sample overlapping grid. Blue color represents the background Cartesian grid and green color represents the body-fitting curvilinear grid. The solid square symbols represent the interpolation points. Leading edge of the plate is at origin.

Shang (1989) was adopted for the motion of the plate

$$\Omega(t^+) = \Omega_0(t^+)(1 - \exp(-4.6t^+/t_0)) \quad t^+ \ge 0$$
(3)

where Ω is the pitch rate in rad/s, t_0 is the time at which Ω has reached 99% of the asymptotic pitch rate Ω_0 . The parameter t_0 was found to affect only the early stages of plate motion.

Three different grid systems were chosen. For all three systems a body-fitting O-

grid was used around the plate. The fine grid had 165 points in the circumferential direction and 105 points in the radial direction. The effect of grid size on lift coefficient is shown in Figure 2. The maximum difference in lift coefficient between the fine and medium grids (135x75) was less than 2% in most regions. Large discrepancies were observed, however; when the plate started to pitch up and pitch down. Figure 3 shows the vorticity contours when the plate was pitched up to $0.5\theta_{max}$. Refining grid from coarse (100x50) to medium produced significant changes. However, the difference in the vorticity contours between the fine and medium grids was relatively small. More importantly, both the fine and medium grids captured the detailed vertical structures in the proximity of the plate. The medium grid was deemed to be enough to capture the prominent flow features and was used throughout this work.



Figure 2: Effect of grid size on computed lift coefficient. Dimensionless pitch rate is 1.4.

3.2 Effect of dimensionless pitch rate

Snapshots at five time instants across a range of dimensionless pitch rate from 0.2 through 1.4 from the experimental results [OI (2009)] were compared to the results of the numerical simulation. The pitch axis was at quarter chord and the Reynolds number was 10^4 . The snapshots were taken at the instant when the plate reached on its way up, θ_{max} , $\theta_{max}/2$ on its way down, 0 degree, and one ramp-motion's time after motion cessation. A typical motion time history is shown in Figure 4, which also defines the time snapshots.



Figure 3: Comparison of vorticity contours among different grid sizes at dimensionless pitch rate of 1.4. Plate was pitched upward to reach $0.5\theta_{max}$. The range of vorticity contours is [-10 10] throughout this paper. Blue represents negative value and red represents positive value.



Figure 4: Time-trace of pitch angle when is 0.7 and 1.4, respectively. Snapshots were taken in time denoted by red circles and squares.

The experimental snapshots were based on dye injection, the CFD results were vorticity contours. Figure 5 compares results at different pitch rates. We had the following observations

1. The flow structures indicated from the experimental dye injection matched fairly well with those illustrated from the simulated vortex contours.



Figure 5: Comparison of flow structures at different pitch rates. Experimental results are based on dye injection and CFD results show the vorticity contours. Blue is clockwise and red is counter-clockwise. Contour range is [-10 10].



Figure 6: Comparison of flow structures with different pivot point. Experimental results are based on dye injection and CFD results show the vorticity contours. $\Omega^+=1.4$.



Figure 7: Comparison of flow structures at different Reynolds numbers.

- 2. At all pitch rates a leading edge vortex (LEV) appeared when the plate was pitched up. Increasing the pitch rate caused the formation of the LEV to begin at higher angle of attack (AoA). The LEV became more compact as pitch rate increased.
- 3. At low rates, a Karman vortex structure was shown in the experiment but not in the simulation, possibly due to the numerical dissipation in the sim-



Figure 8: Effect of tunnel size on the lift. Smaller domain leads to higher lift coefficient. Left: $\Omega^+ = 0.2$; Right: $\Omega^+ = 1.4$.



Figure 9: Velocity distributions 0.5-chord after the trailing edge at the instant when the plate is half way up. The pitch rate is 1.4. u is the streamwise velocity component (along x-direction), v is normal velocity component (along y-direction)

ulation. From the vorticity contours we also observed a counter-clockwise (red) trailing edge vortex (TEV) at all pitch rates when the plate reached. The TEV was stronger at higher pitch rates and we did not observe the shear layer vortex reported by Visbal and Shang (1989) in their study of a pitching NACA0015 airfoil.

4. As the plate continued to pitch up, the LEV was convected downstream. The vortex core location depended on the pitch rate. At the same pitch angle, the vortex was further downstream at low pitch rate due to longer convection time (Figure 4 shows at the convection time is twice as long as at). At high



Figure 10: Comparison of vorticity contours with different computational domain sizes. Left: airfoil was 2-chord lengths away from the top and bottom boundaries; right: airfoil was 10-chord lengths away from the top and bottom boundaries. The pitch rate is 1.4.

pitch rate of 1.4 the LEV was able to retain its integrity.

- 5. When the plate pitched half way down, the LEV continued to be convected downstream. At pitch rate of 0.2 we observed a shed counter-clockwise vortex marked in red color. This vortex was not evolved from the TEV shown in the second column. This behavior was related to the effective AoA encountered by the trailing edge. The effective AoA can be estimated by subtracting the plate velocity due to pitch motion from the freestream velocity. When the plate started to pitch downward, the trailing edge moved up. At low pitch rate of 0.2, the effective angle of attack encountered by the trailing edge was still positive, which caused a new vortex on the top surface. The vortex structure shown in column 4 at the pitch rate of 0.4 can be explained in the same way.
- 6. When the plate ramped down to its starting position, both the LEV and TEV are clearly seen at high pitch rate of 1.4. At one ramp-motion's time after motion cessation, a Karman vortex street is seen at low pitch rate. At high pitch rate, the LEV and TEV continued to move downstream.

3.3 The effect of pitch point

Figure 6 compares flow structures with same dimensionless pitch rate of 1.4 but different pivot point locations. The Reynolds number was 10^4 . With pivot point further aft, the LEV on the top surface became smaller and weaker. After the pivot point moved pass x/c=0.5, the vortex appeared at the bottom surface when the plate was pitched up. This can also be explained by the effective AoA as discussed

before. When the pivot point moved pass x/c=0.5, the effective AoA became negative, hence the LEV appeared at the lower surface. It was also clear the vortex on the bottom surface became stronger as the pivot point moved further aft. When the plate was pitched downward, a LEV appeared on the upper surface at all pitch rates. Overall, the flow structures show good agreement between the experiment and the simulation.

3.4 The effect of Reynolds number

In Figure 7 we examine the impact of Reynolds number at different pitch rates. The pitch point was at quarter chord. We made the following conclusions

- 1. At low pitch rate of 0.2, the LEV showed strong dependence on the Reynolds number. The vortex structure became stronger and more coherent with the increase of Reynolds number.
- 2. At high pitch rate of 1.4, the flow field became less dependent on the Reynolds number. We do not observe clear difference when the Reynolds number exceeded 500.

3.5 The effect of domain size

Early study showed that the aerodynamic forces of a flapping airfoil could be significantly affected by the distance from the airfoil to the top and bottom walls [Lian (2009)]. The effect of domain size on the lift coefficient is shown in Figure 8. At both a low pitch rate of 0.2 and a high pitch rate of 1.4, the small domain size resulted in a higher lift coefficient during the pitch up process. Figure 9 compared the velocity profiles 0.5 chord after the trailing edge of the plate at the time when the plate was pitched up to . As the left graph in Figure 9 shows, when the computational domain was 4-chord high, flow was squeezed in the downstream direction due to the low clearance, leading to a higher streamwise velocity near the plate. On the other hand, flow was more spread out for the case of larger domain size. As the right graph in Figure 9 shows, we did not observe much difference in the vertical velocity between the two domain sizes. However, no obvious difference was observed in the vorticity contours shown in Figure 10.

4 Conclusion

We studied the flapping airfoil performance in different wind gust conditions. From our numerical simulations we have the following conclusions:

1. Increasing the pitch rate delayed the formation of the LEV on the top surface.

- 2. Increasing the pitch rate made the LEV more compact and stronger.
- 3. Moving the pitch position further aft weakened the LEV on the top surface when the plate was pitched upward. Once the pitch position was moved pass the middle chord, the vortex appeared at the bottom surface. This phenomenon was due to the effective angle of attack.
- 4. The influence of the Reynolds number on the flow structure depended on the pitch rate. The Reynolds number showed strong impact when the pitch rate was low but weak impact when the pitch rate was high.
- 5. The computational domain size affected the aerodynamic forces. A smaller domain increases the lift generation when the plate was pitched up.

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