# New Algorithm for Evaluation of Electric Fields due to Indirect Lightning Strike

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**Abstract:** Evaluation of electric field due to indirect lightning strike is an interesting subject. Calculation of electric and magnetic fields in time domain with the consideration of ground conductivity effect in the shortest possible time is an important objective. In this paper, using dipole method, Maxwell's equation and Cooray-Rubinstein formula, a new method for calculation of electric field in time domain is proposed. In addition, this proposed algorithm can also be used to evaluate the effect at the far distance cases of observation point from lightning channel.

**Keywords:** Electric fields, Maxwell' equation, Ground conductivity effect

## 1 Introduction

Lightning is an important natural phenomena that can produce electric and magnetic fields. For evaluation of electric and magnetic fields, various methods are presented but each of them have advantages and disadvantages that can affect on time of process and accuracy of analyzes. Dipole and finite difference time domain (FDTD) method are two common methods used for evaluation of electric and magnetic fields due to lightning channel for perfect ground conductivity case. Horizontal electric fields have more influences, considering on finite or non-perfect ground conductivity case (Nucci 1995). Cooray-Rubinstein formula, as discussed in (Nucci 1995) and FDTD method (Yang and Zhou 2004) are two famous methods used for evaluation of ground conductivity effect on electric fields. In the FDTD method, information on lightning current are used for calculating the effect at the observation point (layer by layer process) from the lightning channel position. Whilst the same goes for the other part of calculation i.e. ground conductivity effect from underground position. These processes are time consuming and large memory is needed for data processing. In addition, it can only be validated for a close distance from lightning channel (Liu, Chen et al. 2009). In contrast,

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the Cooray-Rubinstein formula, which is in Fourier domain, applied for only horizontal electric field. This paper proposes the use of dipole method, Maxwell's (Namburu, Mark et al. 2004) equations and Cooray-Rubinstein formula in the development of a new algorithm for evaluation electric fields in time domain. Using proposed method, electric fields which can be calculated directly in time domain, offers low memory and short processing time. Furthermore, it can also be applied for the case of far distances effect from lightning channel. The paper is organized as follow. Section 2 presents the realistic model for lightning return stroke current at ground surface and along channel, while Section 3 presents the magnetic and geometry of problem. Section 4 proposes the new algorithm for the problem discussed and subsequently followed with the results and discussion. Finally, Section 5 concludes the finding of the work.

## 2   Return Stroke Current

Lightning return stroke current at channel base can be predicted using the Heidler model, which has later been improved by Diendorfer and Uman (Diendorfer and Uman 1990; Nucci 1995). This model is presented as in Eq.1.

$$i(0,t) = [\frac{i_{01}}{\eta_1}\frac{\left(\frac{t}{\Gamma_{11}}\right)^{n_1}}{1+\left(\frac{t}{\Gamma_{11}}\right)^{n_1}}\exp\left(\frac{-t}{\Gamma_{12}}\right) + \frac{i_{02}}{\eta_2}\frac{\left(\frac{t}{\Gamma_{21}}\right)^{n_2}}{1+\left(\frac{t}{\Gamma_{21}}\right)^{n_2}}\exp\left(\frac{-t}{\Gamma_{22}}\right)] \qquad (1)$$

Where:
$i_{01}, i_{02}$ are the amplitude of the channel base current,
$\Gamma_{11}, \Gamma_{12}$ are the front time constant,
$\Gamma_{21}, \Gamma_{22}$ are the decay- time constant,
$n_1, n_2$ are exponent (2~10),
$\eta_1, \eta_2$ are the amplitude correction factor.

Alternatively, this model is illustrated in Fig.1 with initial parameters that are tabulated in Tab.1.

Table 1: Typical values for Diendorfer and Uman channel base current (Nucci 1995; Rachidi, Janischewskyj et al. 2001)

| $n_2$ | $n_1$ | $\Gamma_{22}$ ($\mu$s) | $\Gamma_{21}$ ($\mu$s) | $\Gamma_{12}$ ($\mu$s) | $\Gamma_{11}$ ($\mu$s) | $i_{02}$ (kA) | $i_{01}$ (kA) |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 230 | 2.1 | 2.5 | 0.25 | 6.5 | 10.7 |

On the other hand, Modified Transmission Line Exponential decay model (MTLE) is used for estimation of current along lightning channel (Nucci 1995; Rakov and
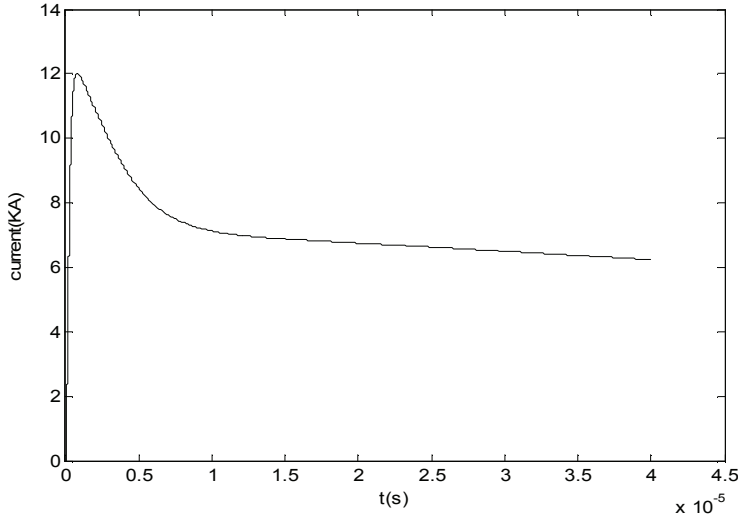
Figure 1: Improvement of Diendorfer and Uman on the Hiedler model for channel base current

Uman 1998; Djalel, Ali et al. 2007). This model is expressed by Eqs. 2 and 3.

$$i(y',t) = i(0, t - y'/v)exp(-z'/\lambda) \quad y' \leq vt \tag{2}$$

$$i(y',t) = 0 y' > vt \tag{3}$$

Where:
$\lambda$ is the decay constant which allows the current to reduce its amplitude with height,
$v$ is the lightning return stroke wave-front velocity,
$y'$ is the vertical space variable.

## 3 Magnetic field

Magnetic field is evaluated by dipole method (Nucci 1995). This method is presented by Eq. 4.

$$H_\varphi(r,z,t) = \left(\frac{1}{4\pi}\right) \int_{-H1}^{H1} \left(\frac{r}{R^3} i\left(y', t - \frac{R}{c}\right) + \frac{r}{cR^2} \frac{\partial i\left(y', t - \frac{R}{c}\right)}{\partial t}\right) dy' \tag{4}$$

Where:
y is height of observation point,

c is light speed in free space,

$$R = \sqrt{(y' - y)^2 + r^2}$$

$$\beta = v/c$$

$$\chi = \sqrt{\frac{1}{1 - \beta^2}}$$

$$A1 = \sqrt{(\beta ct - y)^2 + \left(\frac{r}{\chi}\right)^2}$$

$$H1 = \beta \chi^2 \{-(\beta z - ct) - A1\}$$

Fig. 2 illustrates the geometry of problem. Lightning channel is along y-axis with the observation point P at x, z position. The radial distance from lightning channel to observation point is equal to r.
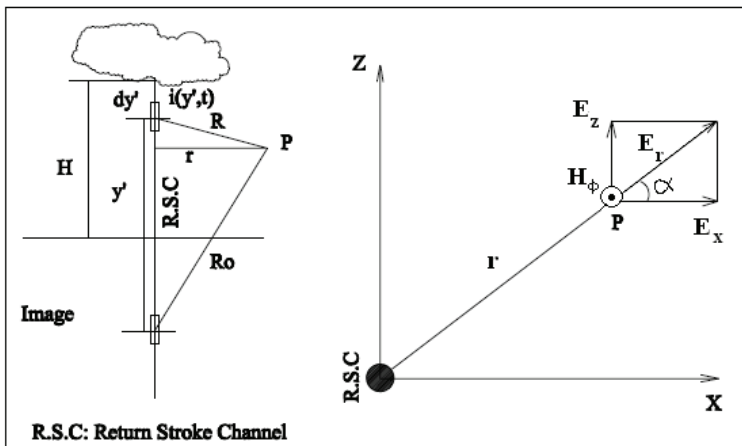


Figure 2: Geometry of problem

On the other hand, magnetic flux density can be expressed by Eq. 5 (Soares Jr, Vinagre et al. 2008).

$$B_\varphi = \mu_0 H_\varphi \tag{5}$$

Where:

$$\mu_0 = 4\pi \times 10^{-7}$$

## 4 Electric fields

According to Maxwell's equations, relation between electric and magnetic fields can be expressed by Eqs. 6 to 8 (Yee 1966; Sartori and Cardoso 2000; Jose, Kanapady et al. 2004; Johnson, Owen et al. 2007; Soares Jr, Vinagre et al. 2008).

$$\nabla \times H = J - \frac{\partial D}{\partial t} \tag{6}$$

$$D = \varepsilon E \tag{7}$$

$$J = \sigma E \tag{8}$$

Where:
$D$ is the electric flux density,
$J$ is the current density vector,
$\sigma$ is the conductivity (conductivity in free space is equal to zero),
$\varepsilon$ is the permittivity ($\varepsilon = \varepsilon_0 \, \varepsilon_r$ and permittivity in free space is equal to $\varepsilon_0 = 8.85 \times 10^{-12}$).

So, by solving Eq. 6 using Eqs. 5, 7 and 8 and FDTD method, electric fields for an observation point in the free space can be expressed by Eqs. 9, 10 and 11 for an observation point overhead ground and perfect ground conductivity case (Sartori and Cardoso 2000; M'ziou, Mokhnache et al. 2009).

$$
\begin{aligned}
E_z^{k+\frac{1}{2}}(m,n,p) = {} & E_z^{k-\frac{1}{2}}(m,n,p) \\
& + \frac{c^2 \triangle t}{\partial l} \left[ B_y^k(m,n+1,p) - B_y^k(m,n-1,p) + B_x^k(m,n,p-1) - B_x^k(m,n,p+1) \right]
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
E_x^{k+\frac{1}{2}}(m,n,p) = {} & E_x^{k-\frac{1}{2}}(m,n,p) \\
& + \frac{c^2 \triangle t}{\partial l} \left[ B_z^k(m,n,p+1) - B_z^k(m,n,p-1) + B_y^k(m-1,n,p) - B_y^k(m+1,n,p) \right]
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
E_y^{k+\frac{1}{2}}(m,n,p) = {} & E_y^{k-\frac{1}{2}}(m,n,p) \\
& + \frac{c^2 \triangle t}{\partial l} \left[ B_x^k(m+1,n,p) - B_x^k(m-1,n,p) + B_z^k(m,n-1,p) - B_z^k(m,n+1,p) \right]
\end{aligned}
\tag{11}
$$

Note that, the observation point is at $(m\Delta x, n\Delta y, p\Delta z)$ and $k$ is training time ($t = k\Delta t$). Also, by assuming $\Delta x = \Delta y = \Delta y$ then $\partial l = 2 \times x$ and $\Delta t$ should be less than $\frac{\partial l}{2c}$ .

On the other hand, for this problem, relation between $B_\varphi$ and $B_x$, $B_y$, $B_z$ can be expressed by Eqs. 12, 13 and 14.

$$B_z = -sin\gamma\, B_\varphi \tag{12}$$

$$B_x = \cos\gamma\, B_\varphi \tag{13}$$

$$B_y = 0 \tag{14}$$

Where:

$$sin\gamma = \frac{x - x_0}{r}$$

$$cos\gamma = \frac{z - z_0}{r}$$

Whilst for evaluation of ground conductivity effect on the electric field by using Cooray-Rubinstein formula (Nucci 1995; Rubinstein 1996) as in Eq.15, it is necessary to convert and solve the problem in time domain (Caligaris, Delfino et al. 2008). This can be done using Eqs. 16, 17, 18 and 19.

$$E_{rF}(r, y, j\omega) = E_{rP}(r, y, j\omega) - H_\varphi(r, y = 0, j\omega) \cdot \frac{\sqrt{\mu_0}}{\sqrt{\varepsilon_{rg} + \sigma_g/(j\omega)}} \tag{15}$$

$$S_k = \int_{t_{k-1}}^{t_k} -\sqrt{\frac{\mu_0}{\varepsilon}} \frac{\sigma}{2\varepsilon} e^{-(\frac{\sigma}{2\varepsilon})t} [i_1\left(\left(\frac{\sigma}{2\varepsilon}\right)t\right) - i_0\left(\left(\frac{\sigma}{2\varepsilon}\right)t\right)] dt \tag{16}$$

$$E_{rF}(r, y, t) = \int_0^t H_{\varphi il}(r, 0, \tau)[S(t - \tau)]d\tau + E_{rP}(r, y, t) - \sqrt{\frac{\mu_0}{\varepsilon}} H_\varphi(r, 0, t) \tag{17}$$

$$\int_0^t H_\varphi(r, 0, \tau)[S(t - \tau)]d\tau = \int_1^k h_i S_{k-i+1} \tag{18}$$

So

$$E_{rF}(r, z, k\triangle t) = \int_1^k h_i S_{k-i+1} + E_{rP}(r, z, k\triangle t) - \sqrt{\frac{\mu_0}{\varepsilon}} H_\varphi(r, 0, k\triangle t) \tag{19}$$

Where:

$S_k$ is $S(k\Delta t)$

$h_i = H_\varphi(r, 0, i\triangle t)$

$E_{rF}$ is the horizontal electric field at finite ground conductivity,

$i_0$, $i_1$ are modified Bessel functions,
$\varepsilon$ is the relative ground permittivity,
$E_{rP}$ is the horizontal electric field at perfectly ground conductivity,
$H_\varphi$ is the magnetic field,
$\sigma$ is the ground conductivity.

The relation between horizontal electric field and electric fields along x, z axis is given by Eqs. 20 and 21, respectively.

$$E_x = E_r \times \cos \alpha \tag{20}$$

$$E_z = E_r \times \sin \alpha \tag{21}$$

Where:

$$\alpha = \arccos((\frac{x - x_0}{r})$$

So, using Eqs. 9, 10, 16, 19, 20 and 21, electric field values along x, z axis, considering on finite ground conductivity case, can be expressed by Eqs. 22 and 23, respectively.

$$E_z^{k+\frac{1}{2}}(m,n,p) = E_z^{k-\frac{1}{2}}(m,n,p) + \left[ \int_{i=\frac{1}{2}}^{k+\frac{1}{2}} h_i S_{k+\frac{1}{2}-i+\frac{1}{2}} - \int_{i=\frac{1}{2}}^{k-\frac{1}{2}} h_i S_{k-\frac{1}{2}-i+\frac{1}{2}} \right] \times \sin \alpha$$

$$- \sqrt{\frac{\mu_0}{\varepsilon}} \left[ H_\varphi \left( r, 0, (k + \frac{1}{2})\triangle t \right) - H_\varphi \left( r, 0, (k - \frac{1}{2})\triangle t \right) \right] \times \sin \alpha$$

$$+ \frac{c^2 \triangle t}{\partial l} \left[ B_y^k(m, n+1, p) - B_y^k(m, n-1, p) + B_x^k(m, n, p-1) - B_x^k(m, n, p+1) \right]$$

$$\tag{22}$$

$$E_x^{k+\frac{1}{2}}(m,n,p) = E_x^{k-\frac{1}{2}}(m,n,p) + \left[ \int_{i=\frac{1}{2}}^{k+\frac{1}{2}} h_i S_{k+\frac{1}{2}-i+\frac{1}{2}} - \int_{i=\frac{1}{2}}^{k-\frac{1}{2}} h_i S_{k-\frac{1}{2}-i+\frac{1}{2}} \right] \times \cos \alpha$$

$$- \sqrt{\frac{\mu_0}{\varepsilon}} \left[ H_\varphi \left( r, 0, (k + \frac{1}{2})\triangle t \right) - H_\varphi \left( r, 0, (k - \frac{1}{2})\triangle t \right) \right] \times \cos \alpha$$

$$+ \frac{c^2 \triangle t}{\partial l} \left[ B_z^k(m, n, p+1) - B_z^k(m, n, p-1) + B_y^k(m-1, n, p) - B_y^k(m+1, n, p) \right]$$

$$\tag{23}$$

Where:
$r$ is equal to $\sqrt{(m\Delta x)^2 + (p\Delta z)^2}$

$$\alpha = \arccos \Delta (\frac{m\Delta x}{\sqrt{(m\Delta x)^2 + (p\Delta z)^2}})$$

## 5 Result and Discussion

Figs. 3, 4 and 5 show magnetic field $E_x$ and $E_z$ due to the lightning channel, respectively. Initial parameters are tabulated in Table 1 and with other parameters of the return stroke velocity that is equal to $1.9 \times 10^8$ m/s and $\lambda$=2000 m, $\sigma$=0.001S/m and $\varepsilon_r = 10$. Note that, lightning channel is at y-axis.
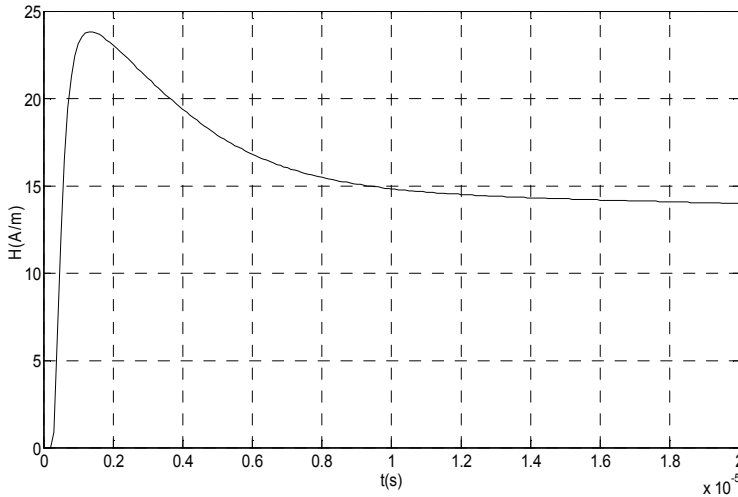


Figure 3: Magnetic field due to lightning channel for an observation point at position x=50m, z=55m, y=10m

Horizontal electric field can be evaluated by $E_x$ or $E_z$ as per Eqs 20 and 21, respectively. Fig. 6 illustrates comparison between horizontal electric fields for perfect and finite ground conductivity cases.

Whilst Fig. 7 shows the vertical electric field obtained under the same condition highlighted previously.

Using Eqs. 4, 5, 12, 13, 16, 22 and 23, electric field values can be calculated directly in time domain, making use of the low memory and short processing time. In addition, the proposed model offers evaluation for the far distance from lightning channel because of the use of this model, which is based on magnetic fields that is evaluated by dipole method and Maxwell's equation. Good results obtained using this method for a far lateral distances from lightning channel, in contrast to the FDTD method, which was only validated for close distances from lightning channel (less than100m) (Yang and Zhou 2004; Liu, Chen et al. 2009).
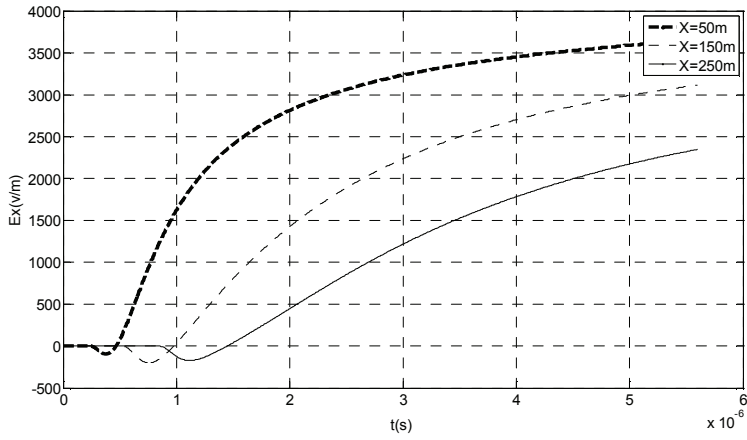
Figure 4: $E_X$ values due to lightning channel for different observation points at position z=55m, y=10m
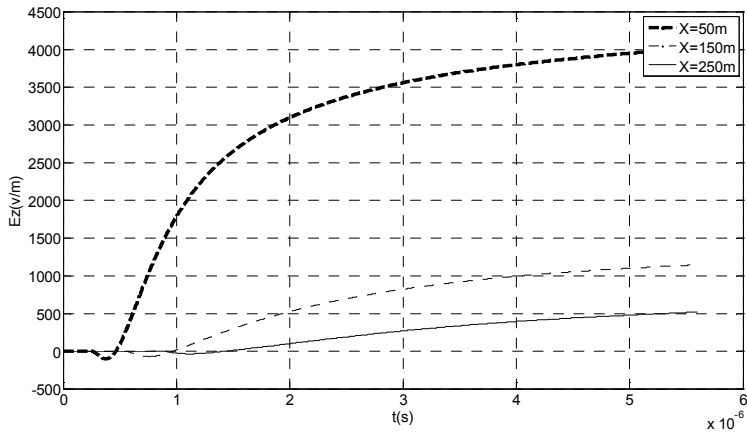


Figure 5: $E_z$ values due to lightning channel for different observation points at position z=55m, y=10m
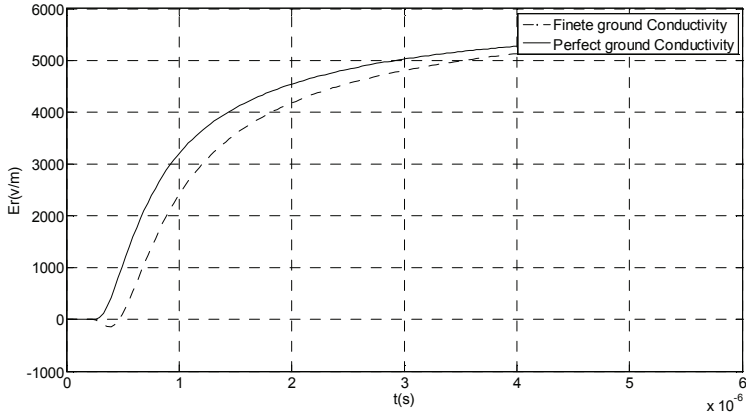
Figure 6: Horizontal electric field in finite ground conductivity case in comparison with the perfect ground conductivity case for an observation point at position x=50m, z=55m, y=10m



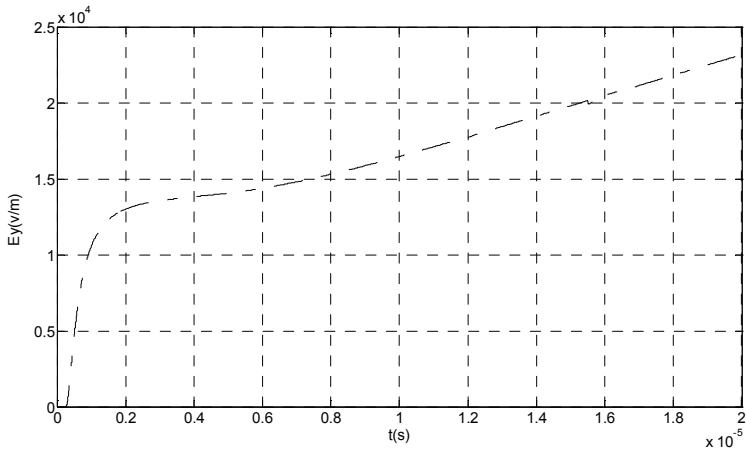Figure 7: Vertical electric field due to lightning channel for the observation point at position x=55m, z=50m, y=10m

## 6 Conclusion

In this study, electric fields due to lightning channel is calculated by other method using dipole method, Maxwell's equation and also ground conductivity effect, which is predicted in this method using Cooray-Rubinstein formula. There are two equations proposed for calculating of electric fields with the consideration of ground conductivity effect.

## References

**Caligaris, C., Delfino,F. et al.** (2008): Cooray–Rubinstein Formula for the Evaluation of Lightning Radial Electric Fields: Derivation and Implementation in the Time Domain. *IEEE Transactions on Electromagnetic Compatibility* 50(1): 194-197.

**Djalel, D, Ali., H. et al.** (2007): Coupling Phenomenon between the Lightning and High Voltage Networks. *Proceedings of Word Aacademy of Science, Engineering and Technology (WASET)*, 21(1307): 95-101.

**Diendorfer, G., Uman, M. A.** (1990): An Improved Return Stroke Model with Specified Channel-Base Current. *Journal of Geophysical Research-Atmospheres* 95: 13,621-613, 644.

**Johnson, J., Owen, J. et al.** (2007): A meshless Local Petrov-Galerkin method for magnetic diffusion in non-magnetic conductors. *CMES: Computer Modeling in Engineering & Sciences* 22(3): 165.

**Jose, P., Kanapady, R. et al.** (2004): Transform Domain Based Hybrid Element Formulations for Transient Electromagnetic Field Computations. *CMES: Computer Modeling in Engineering & Sciences* 5(5): 409-422.

**Liu, J., Chen, D. et al.** (2009): Efficient Analysis of Electromagnetic Fields near Lightning Channel by Using SIBC in FDTD. *20th International Symposium on Electromagnetic Compatibility*: 1-59.

**M'ziou, N., Mokhnache, L. et al.** (2009): Validation of the Simpson-finite-difference time domain method for evaluating the electromagnetic field in the vicinity of the lightning channel initiated at ground level. *IET* 3(3): 279-285.

**Namburu, R., Mark, E. et al.** (2004): Scalable electromagnetic simulation environment. *CMES: Computer Modeling in Engineering & Sciences* 5(5): 443-454.

**Nucci, C.** (1995): Lightning-induced voltages on overhead power lines. Part I: return stroke current models with specified channel-base current for the evaluation of the return stroke electromagnetic fields. *ELECTRA* 161: 75-102.

**Rachidi, F., Janischewskyj, W. et al.** (2001): Current and electromagnetic field

associated with lightning-return strokes to tall towers. *IEEE Transactions on Electromagnetic Compatibility* 43(3): 356-367.

**Rakov, V., Uman, M.** (1998): Review and evaluation of lightning return stroke models including some aspects of their application. *IEEE Transactions on Electromagnetic Compatibility* 40(4): 403-426.

**Rubinstein, M.** (1996): An approximate formula for the calculation of the horizontalelectric field from lightning at close, intermediate, and longrange. *IEEE Transactions on Electromagnetic Compatibility* 38(3): 531-535.

**Sartori, C., Cardoso, J.** (2000): An analytical-FDTD method for near LEMP calculation. *IEEE Transactions on Magnetics* 36(4): 1631-1634.

**Soares Jr, D., Vinagre, M. et al.** (2008): Numerical Computation of Electromagnetic Fields by the Time-Domain Boundary Element Method and the Complex Variable Method. *CMES: Computer Modeling in Engineering & Sciences* 25(1): 1-8.

**Yang, C., Zhou, B.** (2004): Calculation methods of electromagnetic fields very close to lightning. *IEEE Transactions on Electromagnetic Compatibility* 46(1): 133-141.

**Yee, K.** (1966): Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media. *IEEE Trans. Antennas Propagat* 14(3): 302-307.