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Flow Simulations in a Liquid Ring Pump Using a Particle Method

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Abstract: The application of the MPS (Moving Particle Semi-implicit) scheme to incompressible viscous fluid flow problem in the liquid ring vacuum pump with rotating impeller is presented. The rotating impeller in the pump is attached to a center hub and located in off-set from the center of a cylindrical body. For such flow problem there are some interesting phenomena including the formation of the liquid ring by rotating impeller, the interface dynamics between gas and liquid, and so forth. The MPS scheme is widely utilized as a particle strategy for the free surface flow, the problem of moving boundary, and multi-physics/multi-scale ones. Numerical results demonstrate the workability and the validity of the present approach through incompressible viscous fluid flow in the pump with rotating impeller blades.

Keywords: particle method, MPS, liquid ring vacuum pump, rotating impeller.

1 Introduction

From a simulation-based practical point of view, it is important to compute efficiently multi-physics problem and moving boundary/obstacle one in the wide fields of science and engineering. Heretofore, we have proposed a finite element scheme based on the Petrov-Galerkin weak formulation using exponential weighting functions for solving effectively and in a stable manner the incompressible Navier-Stokes equations up to high Reynolds number regimes [Kakuda and Tosaka (1992);Kakuda, Tosaka and Nakamura (1996);Kakuda (2004);Kakuda, Miura and Tosaka (2006)]. The Navier-Stokes equations are semi-explicitly integrated in time by using a fractional step strategy [Donea, Giuliani, Laval and Quartapelle (1982)], and hence split into the convection-diffusion equation and linear Euler-type equations using an auxiliary velocity vector. As the time-marching scheme, we adopt

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the second-order accurate Adams-Bashforth explicit differencing for both convection and diffusion terms.

It is not easy to simulate such problems by using the finite element-based schemes. There are various meshless-based methods, such as SPH (Smoothed Particle Hydrodynamics) method [Lucy (1977);Gingold and Monaghan (1977)], MPS (Moving Particle Semi-implicit) one [Koshizuka and Oka (1996)], and MLPG (Meshless Local Petrov-Galerkin) one [Atluri and Zhu (1998)], to simulate effectively such problems. The SPH methods for solving compressible fluid flows with gravity have been firstly developed in the field of astrophysics [Lucy (1977);Gingold and Monaghan (1977)], and applied successfully to a wide variety of complicated problems, including free surface incompressible flows [Monaghan (1994);Sakai, Yang and Jung (2004)] involving breaking dam, wave propagation, and so forth, thermal conduction with heat flux across discontinuities in material properties [Cleary and Monaghan (1999)], impact fracture in solids [Swegle, Hicks and Attaway (1995);Hoover (2006)], and the behaviors of arctic sea ice in oceanography [Lindsay and Stern (2004)]. The MPS method [Koshizuka and Oka (1996)] as an incompressible fluid flow solver has been widely applied to the problem of breaking wave with large deformation [Koshizuka, Nobe and Oka (1998)], the fluid-structure interaction problem [Chikazawa, Koshizuka and Oka (2001)], and the micro multiphase flow one [Harada, Suzuki, Koshizuka, Arakawa and Shoji (2006)]. Atluri and Zhu [Atluri and Zhu (1998)] have developed the MLPG approach based on the local symmetric weak form and the moving least squares [Lancaster and Salkauskas (1981)] for solving accurately potential problems, and the approach was extended to deal with the problems for convection-diffusion equation [Lin and Atluri (2000)] and incompressible Navier-Stokes equations [Lin and Atluri (2001)] in fluid dynamics.

The purpose of this paper is to present the application of the MPS scheme to incompressible viscous fluid flow in a pump with rotating impeller. The liquid ring vacuum pump has an impeller with blades attached to a center hub, located by the decentering in a cylindrical body. The phenomena in the pump require the multiphysics problem including the moving interface boundary between gas and liquid, and the rotating impeller with blades. It is particularly indispensable to catch the interface boundary between the gas and the liquid to design the impeller which is off-set from the center of the body. The workability and validity of the present approach are demonstrated through flow in the liquid ring pump, and compared with experimental data and other numerical ones.

Throughout this paper, the summation convention on repeated indices is employed. A comma following a variable is used to denote partial differentiation with respect to the spatial variable.

2 Statement of the problem

Let Ω be a bounded domain in Euclidean space with a piecewise smooth boundary Γ. The unit outward normal vector to Γ is denoted by $\boldsymbol{n}$. Also, $\Im$ denotes a closed time interval.

The motion of an incompressible viscous fluid flow is governed by the following Navier-Stokes equations :

$$\frac{Du_i}{Dt} = -\frac{1}{\rho} p_{,i} + \nu u_{i,jj} + f_i \qquad in\, \Im \times \Omega \tag{1}$$

$$\frac{D\rho}{Dt} = 0 \qquad in\, \Im \times \Omega \tag{2}$$

where u_i is the velocity vector component, ρ is the density, p is the pressure, f_i is the external force, ν is the kinematic viscosity, and D/Dt denotes the Lagrangian differentiation.

In addition to Eq. 1 and Eq. 2, we prescribe the initial condition $u_i(\boldsymbol{x},0) = u_i^0$, where u_i^0 denotes the given initial velocity, and the Dirichlet and Neumann boundary conditions.

3 MPS formulation

Let us briefly describe the MPS proposed by Koshizuka [Koshizuka and Oka (1996)]. The MPS method is one of the particle methods. The particle interaction models as illustrated in Fig. 1 are prepared with respect to differential operators, namely, gradient, divergence and Laplacian. The incompressible viscous fluid flow is calculated by a semi-implicit algorithm, such as SMAC (Simplified MAC) scheme [Amsden and Harlow (1970)].

The particle number density n at particle i with the neighboring particles j is defined as

$$n_i = \sum_{j \neq i} w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{3}$$

in which the weighting function $w(r)$ is

$$w(r) = \begin{cases} \dfrac{r_e}{r} - 1 & (r < r_e) \\ 0 & (r \geq r_e) \end{cases} \tag{4}$$

where r_e is the radius of the interaction area as shown in Fig. 1.

The model of the gradient vectors at particle i between particles i and j are weighted with the kernel function and averaged as follows :

$$<\nabla\phi>_i = \frac{d}{n^0}\sum_{j\neq i}\left[\frac{\phi_j-\phi_i}{|\boldsymbol{r}_j-\boldsymbol{r}_i|^2}(\boldsymbol{r}_j-\boldsymbol{r}_i)w(|\boldsymbol{r}_j-\boldsymbol{r}_i|)\right] \tag{5}$$

where d is the number of spatial dimensions, ϕ_i and ϕ_j denote the scalar quantities at coordinates $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$, respectively, and n^0 is the constant value of the particle number density.

The Laplacian model at particle i is also given by

$$<\nabla^2\phi>_i = \frac{2d}{n^0\lambda}\sum_{j\neq i}(\phi_j-\phi_i)w(|\boldsymbol{r}_j-\boldsymbol{r}_i|) \tag{6}$$

where λ is an ad hoc coefficient.

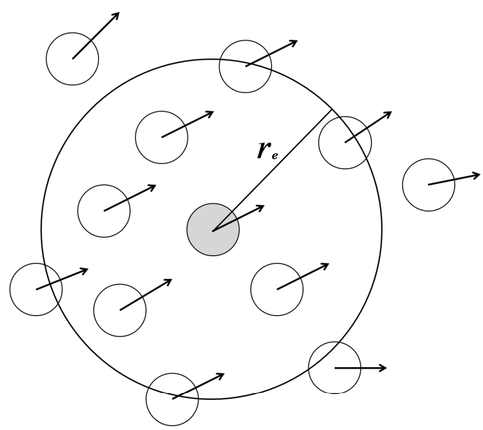
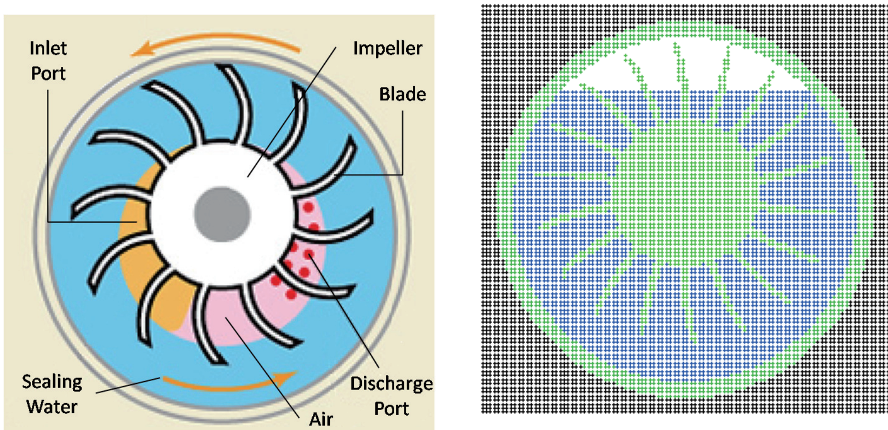


Figure 1: Particle interaction models

4 Numerical example

In this section we present numerical results obtained from applications of the above-mentioned numerical methods to incompressible viscous flow problems in a liquid ring pump with rotating impeller blades from a practical point of view. The initial velocities are assumed to be zero everywhere in the interior domain.

Fig. 2 shows the geometry and the initial state of particles for flow in a liquid ring pump with rotating impeller. In Fig. 2(a) the blades near the top of the pump are

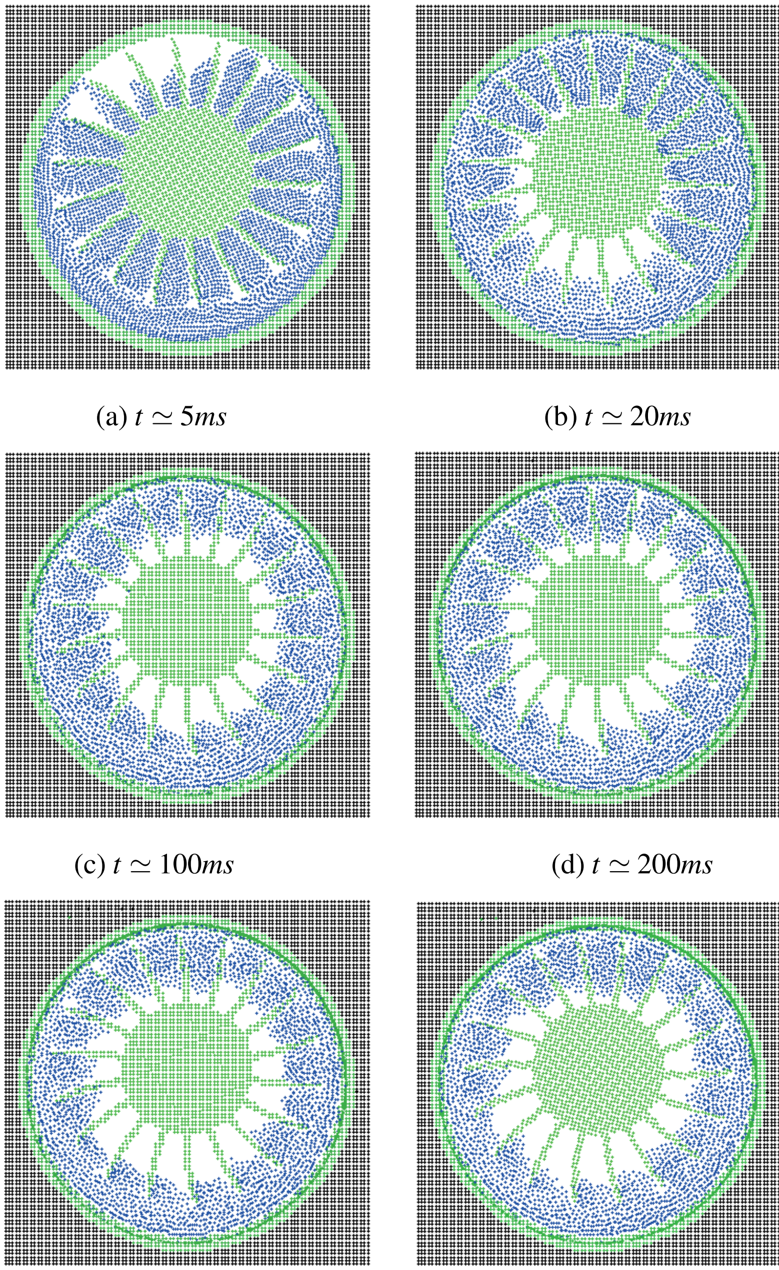


(a) Geometry (b) Initial state of particles

Figure 2: Geometrical configuration and initial state of particles

very closer to the outside wall than at the side and bottom of the pump. The impeller with blades is attached to a center hub and located in off-set from the center of the cylindrical body. In this two-dimensional simulation, we set $9,527$ particles in the initial configuration, $2,400$rpm and $1,200$rpm as the speed of the rotating impeller, and the CFL condition $u_{max}\Delta t/l_{min} \leq C$, where C is the Courant number $(= 0.1)$. The kernel size for the particle number density and the gradient/Laplacian models is $r_e = 4.0l_0$ in which l_0 is the distance between two neighboring particles in the initial state. In this case, we set $l_0 = 0.002333$.

Fig. 3 shows the instantaneous particle behaviors for the rotational speed 2,400rpm of the impeller blades. When the pump starts, the impeller slings the water sealant by centrifugal force, to the outside walls of the body, forming a water ring at the outside walls of the body with passage in the time. As you can see in these figures, some of the blades are fully immersed in water, and some are almost out of the water, because of the decentering impeller in the body. Fig. 4 shows the corresponding instantaneous velocity vector fields at different time using the MPS scheme. With passage in the time, you can see the extension of high velocity vector fields near the bottom wall of the body. The present results at the same time for different rotational speed of the impeller are shown in Fig. 5. As the results, it is clear that the width of the water ring for high rotating speed is considerably narrower than that of low rotating speed by centrifugal force. The pump with an impeller of the high rotating speed expands also the high velocity vector field to the neighborhood of

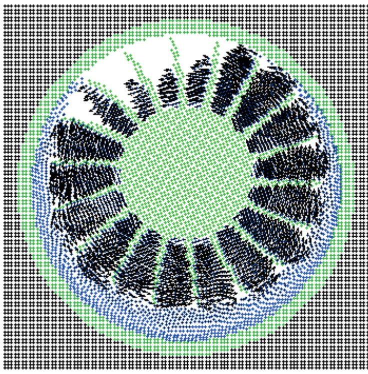
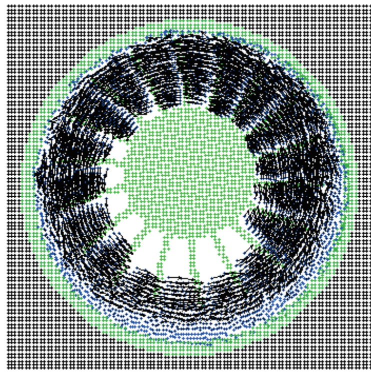


(a) $t \simeq 5ms$ (b) $t \simeq 20ms$

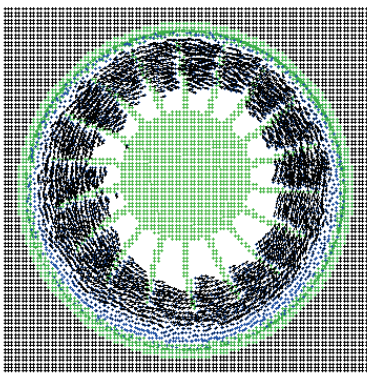
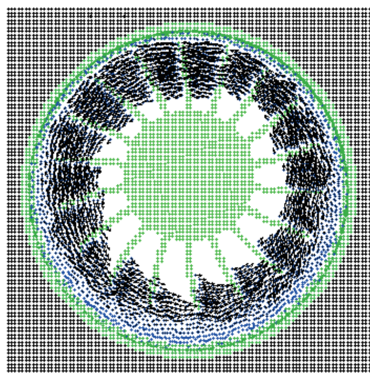
(c) $t \simeq 100ms$ (d) $t \simeq 200ms$

(e) $t \simeq 300ms$ (f) $t \simeq 449ms$

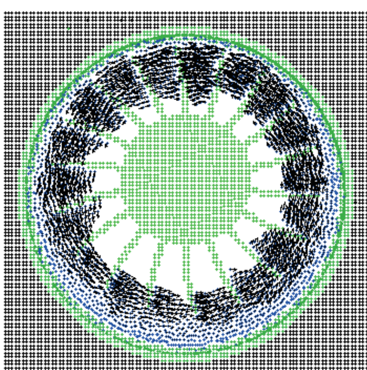
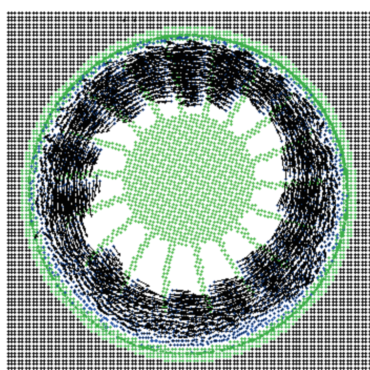
Figure 3: Instantaneous particle behaviors

(a) $t \simeq 5ms$ (b) $t \simeq 20ms$

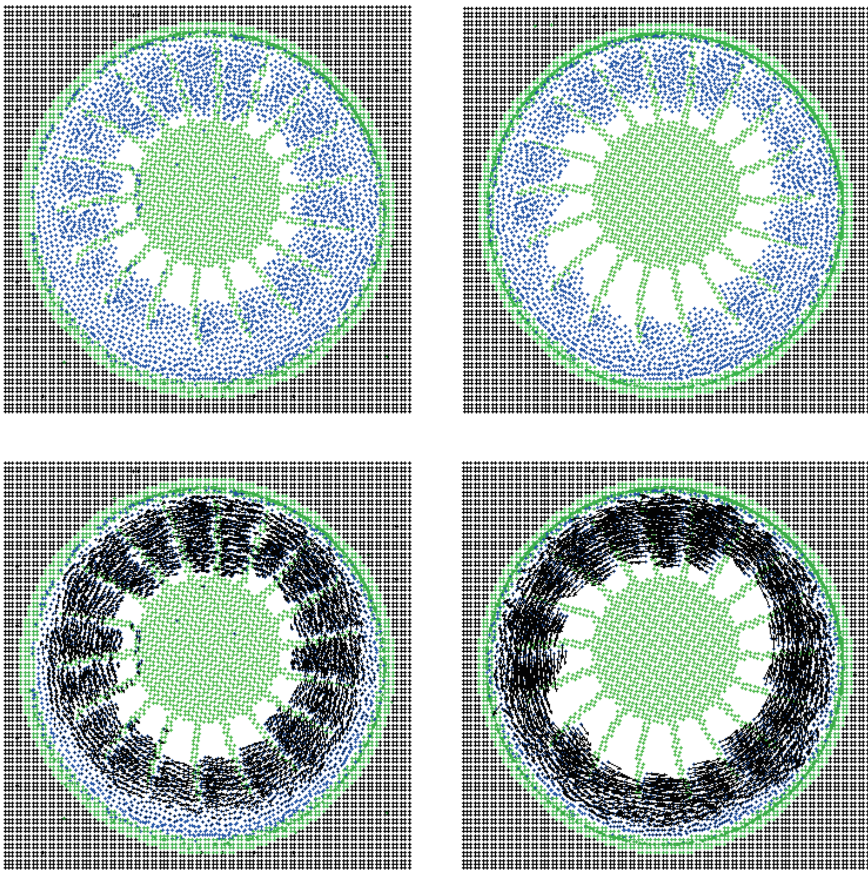
(c) $t \simeq 100ms$ (d) $t \simeq 200ms$

(e) $t \simeq 300ms$ (f) $t \simeq 449ms$

Figure 4: Instantaneous velocity vector fields

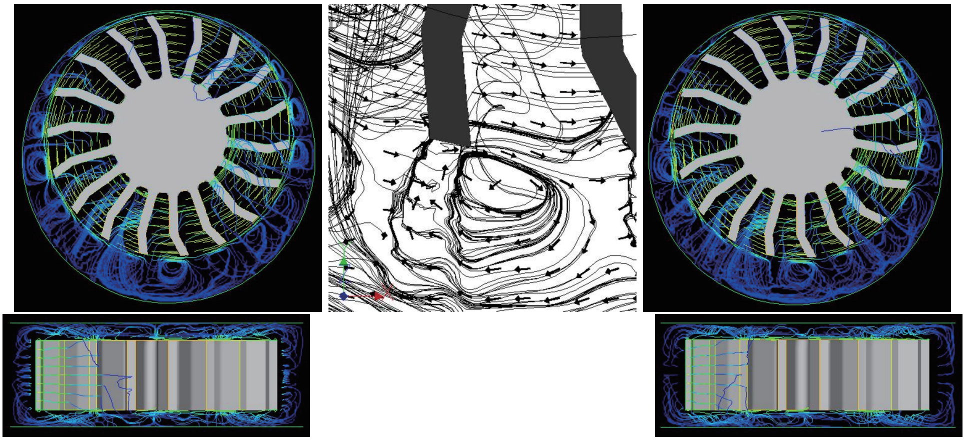
the outside wall.



(a) 1,200rpm (b) 2,400rpm

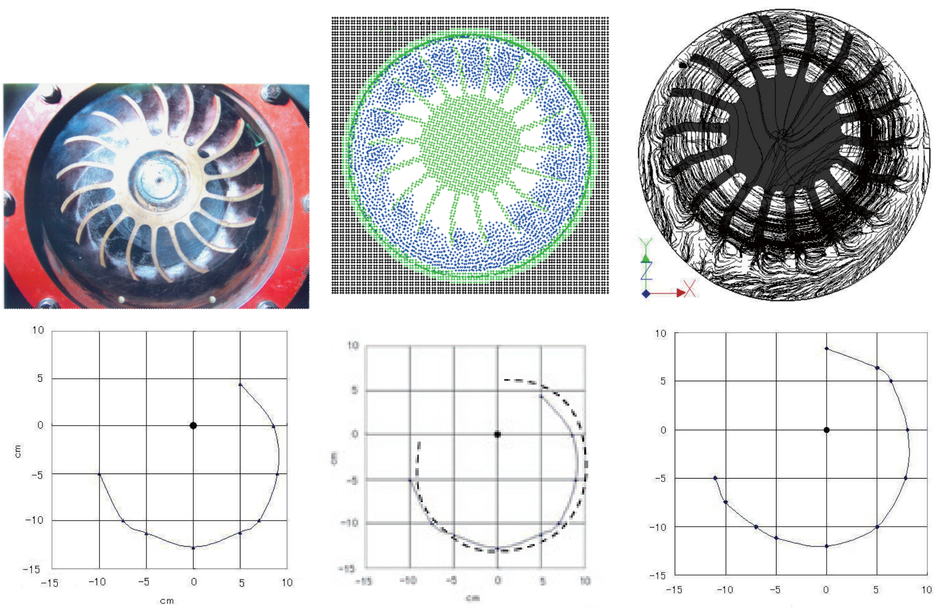
Figure 5: Comparisons with different rotating impeller speed at $t \simeq 449ms$

Fig. 6 shows the instantaneous streamlines for different Reynolds number using the Petrov-Galerkin FE scheme[Kakuda, Toyotani, Matsuda, Tanaka and Katagiri (2010)]. The finite element-based parameters are summarized in Tab. 1. In this case, we adopt the lowest interpolation functions in which the velocity vector and the scalar potential are piecewise tri-linear, and the pressure is constant over each element. As you can see there exist the vortexes from some edges of the impeller blades. Fig. 7 shows the 2D simulation using the MPS (see Fig. 7(b)) and the Petrov-Galerkin FE results (see Fig. 7(c)) through comparison with experimental



(a) $Re = 10^3$ (b) $Re = 10^5$

Figure 6: Instantaneous streamlines fields



(a) Experiment (b) MPS method (2D) (c) P-G method

Figure 7: Comparisons with the experimental data

photograph (see Fig. 7(a)). The air-water interface line obtained from the experiment is shown in Fig. 7(a) by a solid line. The maximum streamline using the Petrov-Galerkin method (see Fig. 7(c)) seem to be overestimated near the top and underestimated slightly at others in comparison with the air-water interface line obtained from the experiment, while the agreement between the present results, namely a dot line, using the MPS scheme and the experimental data appears satisfactory as shown in Fig. 7(b). In the left area of the impeller blades, the instantaneous interface behaviors are qualitatively similar to the experimental ones.

Table 1: A summary of the parameters using Petrov-Galerkin FEM

Re	Nodes	Elements	$\triangle t$	α_i
$10^3, 10^5$	105,948	90,784	0.001	0.5

5 Conclusions

We have presented the MPS approach for solving numerically incompressible viscous fluid flow in a liquid ring pump with rotating impeller blades. The MPS scheme has been widely utilized as a particle strategy for free surface flow, the problem of moving boundary, and multi-physics/multi-scale ones. As the numerical example, flow in a liquid ring pump with rotating impeller is carried out and compared with experimental data and other numerical ones. The numerical results obtained herein are summarized as follows:

(1) It is confirmed that the pump forms a water ring at the outside walls of the body with passage in the time.

(2) The qualitative agreements between the present results and the experimental data appear satisfactory.

(3) The numerical results demonstrate that the approach is capable of solving qualitatively and in a stable manner the complicated flow phenomena.

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