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Full-Field Analysis of a Functionally Graded Magnetoelectroelastic Nonhomogeneous Layered Half-Plane

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Abstract: In this study, the two-dimensional problem of elastic, electric, and magnetic fields induced by generalized line forces and screw dislocations applied in a functionally graded magnetoelectroelastic layered half-plane is analyzed. It is assumed that the material properties vary exponentially along the thickness direction. The full-field solutions for the transversely isotropic magnetoelectroelastic nonhomogeneous layered half-plane are obtained using the Fourier-transform technique. For the case that material properties are continuous at the interface, it is shown that all magnetoelectroelastic fields are continuous at the interface. Furthermore, this functionally graded layered half-plane has the identical contour slopes for the generalized stress $\boldsymbol{\sigma}_y^{(j)}$ across the interface. Numerical results for the full-field distributions of generalized stresses and strains are presented and discussed in detail.

Keywords: Functionally graded material, magnetoelectroelastic material, layered half-plane, full-field solution, nonhomogeneous, interface

1 Introduction

In the recent decade, magnetoelectroelastic solids have been drawn considerable attentions for their promise potential in various applications such as transducer and sensors due to the coupling effect between mechanical, electric, and magnetic fields. For most engineering applications, multilayered structures are often used in the design of magnetoelectroelastic materials. However, the distinct material properties of each layer in laminated composite structures always induce the discontinuous stresses along the interfaces. The interfacial cracks are usually generated at the layer interfaces during service. Therefore, the analysis of the interfacial fracture

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problem is of great important in multilayered structures. Currently, functionally graded magnetoelectroelastic materials with properties continuously varying in the space are developed to overcome the sharp interface and reduce stress concentrations.

For a layered half-plane problem, the elastic deformations, electric fields, and magnetic fields in magnetoelectroelastic solids are usually produced by generalized line forces or dislocations. Guan and He (2005) obtained expressions of elastic displacement, stress, electric displacement, electric potential, magnetic induction, and magnetic potential for a two-dimensional plane problem of a transversely isotropic magnetoelectroelastic half-plane medium subjected to a point force. Hao and Liu (2006) investigated the interaction between a screw dislocation and a semi-infinite interfacial crack in a transversely isotropic magnetoelectroelastic bimaterial. Li and Kardomateas (2006) investigated the mode III interface crack problem for dissimilar piezoelectromagnetoelastic bimaterial media. The problem for an antiplane interface crack between two dissimilar magnetoelectroelastic layers was analyzed by Wang and Mai (2006). For a magnetoelectroelastic layer sandwiched between dissimilar half spaces, the antiplane crack problem subjected to longitudinal loadings was carried out by Hu, Qin and Kang (2007). Analytical full-field solutions of a magnetoelectroelastic layered half-plane subjected to generalized concentrated forces and screw dislocations were presented by Lee and Ma (2007). The solution obtained was then used to derive image forces of screw dislocations in a layered half-plane by Ma and Lee (2007).

With the help of the development in high-tech electronic devices, the important concept of the functionally graded material with position-dependent properties have been widely used to reduce internal stresses and increase its reliability. A meshless method based on the local Petrov-Galerkin approach was proposed by Sladek, Sladek, Tan and Atluri (2008) to investigate the steady-state and transient heat conduction problems in a functionally graded anisotropic material. Ou and Chue (2006) studied two mode III internal cracks located within two bonded functionally graded piezoelectric half planes. The electroelastic behavior of an antiplane shear crack in a functionally graded piezoelectric strip was investigated by Kwon (2003) and Ma, Wu, Zhou and Guo (2005). The mixed-mode problem of a finite crack embedded in a semi-infinite strip of a nonhomogeneous piezoelectric material under uniform electrical displacement loadings was considered by Ueda (2006). For the sandwich structural system, the dynamic and static antiplane shear problems for a cracked functionally graded piezoelectric interlayer bonded between two dissimilar homogeneous piezoelectric half planes were analyzed by Chen, Liu and Zou (2003) and Hu, Zhong and Jin (2005), respectively. Recently, magnetoelectroelastic structures with functionally graded material properties have attracted many researchers' attentions. The magnetoelectroelastic behavior of a crack in functionally graded piezoelectric/piezomagnetic materials subjected to an antiplane shear stress loading was investigated by Zhou, Wu and Wang (2005). Recently, Zhou and Wang (2008) investigated an interface crack between two functionally graded piezoelectric/piezomagnetic materials subjected to harmonic antiplane shear stress waves. The fracture problem for an embedded crack perpendicular to the boundary of a functionally graded magnetoelectroelastic strip was studied by Feng and Su (2006) and Ma, Li, Abdelmoula and Wu (2007). Pan and Han (2005) derived an exact solution for the simple supported laminated plates made of anisotropic and functionally graded magnetoelectroelastic materials. By semi-analytical finite element method, static analysis of functionally graded magnetoelectroelastic plates have been carried out by Bhangale and Ganesan (2006). It was shown that the advantage of functionally graded material model over the layered model is that there is no discontinuity between the electric potential, magnetic potential, electric displacements, and magnetic inductions between the layers. The two-dimensional fracture problem of nonhomogeneous magneto-electro-thermo-elastic materials under dynamically thermal loading was investigated by Feng, Han and Li (2009) using the meshless local Petrov-Galerkin method.

In this study, an effective mathematical method is developed to construct the analytical full-field solutions for the two-dimensional functionally graded magnetoelectroelastic layered half-plane problem. This layered half-plane is subjected to generalized antiplane forces and screw dislocations applied either in the thin layer or the half-plane. Because of the mathematical difficulties, the solution for the functionally graded magnetoelectroelastic layered half-plane subjected antiplane loadings is not easy to obtain. We assume that all the material properties considered in this study have the same exponential variations. By using the Fourier integral transform technique, the full-field solutions of this nonhomogeneous magnetoelectroelastic layered half-plane problem are presented. Based on the analytical solutions, four different combinations of functionally graded parameters will be used to investigate the interesting phenomenon near the interface. For the special case of a nonhomogeneous layered half-plane with continuous material constants at the interface, it is shown in this study that all magnetoelectroelastic fields are continuous through the interface. Furthermore, it is also proved that the contour curves for the generalized stress $\boldsymbol{\sigma}_y^{(j)}$ at the interface have the same slopes (first derivative). For the computational results, the full-field distributions of the generalized stresses and strains in the nonhomogeneous layered half-plane subjected to line forces or screw dislocations are presented with different functionally graded parameters.

2 Formulations and Governing Equations

For the two-dimensional functionally graded magnetoelectroelastic layered half-plane problem, the solid is assumed to be antiplane deformation in conjunction with the in-plane electric field and in-plane magnetic field as shown in Fig. 1. The thin layer $(0 \leq y \leq h)$ with thickness h is occupied by material 1 and is perfectly bonded to the half-plane $(h \leq y < \infty)$ occupied by material 2. The x axis is taken to be the free surface of the thin layer. In the case of a transversely isotropic magnetoelectroelastic medium with the polarized axis in the z direction, the out-of-plane displacement $w^{(j)}$ is coupled with the in-plane electric potential $\varphi^{(j)}$ and in-plane magnetic potential $\phi^{(j)}$. These field quantities depend on x and y only and are expressed as

$$
\begin{aligned}
&w^{(j)} = w^{(j)}(x,\, y), \quad j = 1,\, 2,\\
&\varphi^{(j)} = \varphi^{(j)}(x,\, y),\\
&\phi^{(j)} = \phi^{(j)}(x,\, y).
\end{aligned}
\tag{1}
$$

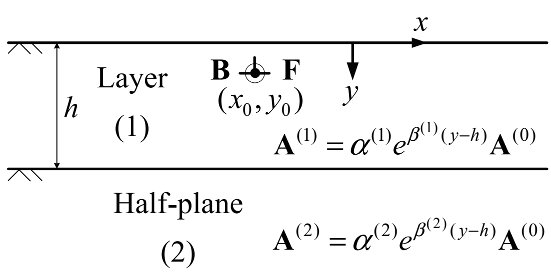

Figure 1: Configuration and coordinate system of a functionally graded magnetoelectroelastic layered half-plane.

The superscripts (j) on the field quantities are employed to label materials j. The shear strains ($\gamma_{xz}^{(j)}$ and $\gamma_{yz}^{(j)}$), in-plane electric fields ($E_x^{(j)}$ and $E_y^{(j)}$), and in-plane magnetic fields ($H_x^{(j)}$ and $H_y^{(j)}$) are defined as follows:

$$
\begin{aligned}
&\gamma_{xz}^{(j)} = \frac{\partial w^{(j)}}{\partial x}, \quad \gamma_{yz}^{(j)} = \frac{\partial w^{(j)}}{\partial y},\\
&E_x^{(j)} = -\frac{\partial \varphi^{(j)}}{\partial x}, \quad E_y^{(j)} = -\frac{\partial \varphi^{(j)}}{\partial y},
\end{aligned}
\tag{2}
$$

$$H_x^{(j)} = -\frac{\partial \phi^{(j)}}{\partial x}, \quad H_y^{(j)} = -\frac{\partial \phi^{(j)}}{\partial y}.$$

For the nonhomogeneous magnetoelectroelastic problem, the material properties are assumed to vary continuously along the y direction, and the constitutive relations are expressed as

$$\begin{bmatrix} \tau_{xz}^{(j)} \\ D_x^{(j)} \\ B_x^{(j)} \end{bmatrix} = \begin{bmatrix} c_{44}^{(j)}(y) & e_{15}^{(j)}(y) & q_{15}^{(j)}(y) \\ e_{15}^{(j)}(y) & -k_{11}^{(j)}(y) & -d_{11}^{(j)}(y) \\ q_{15}^{(j)}(y) & -d_{11}^{(j)}(y) & -\mu_{11}^{(j)}(y) \end{bmatrix} \begin{bmatrix} \gamma_{xz}^{(j)} \\ -E_x^{(j)} \\ -H_x^{(j)} \end{bmatrix}, \tag{3a}$$

$$\begin{bmatrix} \tau_{yz}^{(j)} \\ D_y^{(j)} \\ B_y^{(j)} \end{bmatrix} = \begin{bmatrix} c_{44}^{(j)}(y) & e_{15}^{(j)}(y) & q_{15}^{(j)}(y) \\ e_{15}^{(j)}(y) & -k_{11}^{(j)}(y) & -d_{11}^{(j)}(y) \\ q_{15}^{(j)}(y) & -d_{11}^{(j)}(y) & -\mu_{11}^{(j)}(y) \end{bmatrix} \begin{bmatrix} \gamma_{yz}^{(j)} \\ -E_y^{(j)} \\ -H_y^{(j)} \end{bmatrix}, \tag{3b}$$

where $\tau_{xz}^{(j)}$ (and $\tau_{yz}^{(j)}$), $D_x^{(j)}$ (and $D_y^{(j)}$), and $B_x^{(j)}$ (and $B_y^{(j)}$) denote the shear stresses, electrical displacements, and magnetic inductions, respectively. $c_{44}^{(j)}(y)$, $k_{11}^{(j)}(y)$, $\mu_{11}^{(j)}(y)$, $e_{15}^{(j)}(y)$, $q_{15}^{(j)}(y)$, and $d_{11}^{(j)}(y)$ are the elastic modulus, dielectric permittivity, magnetic permeability, piezoelectric stress coefficient, piezomagnetic stress coefficient, and magnetoelectric coefficient, respectively.

In this study, it is convenient to introduce the following vectors for the generalized displacement ($\mathbf{u}^{(j)}$), stresses ($\boldsymbol{\sigma}_x^{(j)}$ and $\boldsymbol{\sigma}_y^{(j)}$), and strains ($\boldsymbol{\gamma}_x^{(j)}$ and $\boldsymbol{\gamma}_y^{(j)}$) as

$$\mathbf{u}^{(j)} = \begin{bmatrix} w^{(j)} \\ \varphi^{(j)} \\ \phi^{(j)} \end{bmatrix}, \quad \boldsymbol{\sigma}_x^{(j)} = \begin{bmatrix} \tau_{xz}^{(j)} \\ D_x^{(j)} \\ B_x^{(j)} \end{bmatrix}, \quad \boldsymbol{\sigma}_y^{(j)} = \begin{bmatrix} \tau_{yz}^{(j)} \\ D_y^{(j)} \\ B_y^{(j)} \end{bmatrix},$$

$$\boldsymbol{\gamma}_x^{(j)} = \begin{bmatrix} \gamma_{xz}^{(j)} \\ -E_x^{(j)} \\ -H_x^{(j)} \end{bmatrix}, \quad \boldsymbol{\gamma}_y^{(j)} = \begin{bmatrix} \gamma_{yz}^{(j)} \\ -E_y^{(j)} \\ -H_y^{(j)} \end{bmatrix}. \tag{4}$$

Then, the constitutive equations (3a) and (3b) can be rewritten as

$$\boldsymbol{\sigma}_x^{(j)} = \mathbf{A}^{(j)} \boldsymbol{\gamma}_x^{(j)}, \quad \boldsymbol{\sigma}_y^{(j)} = \mathbf{A}^{(j)} \boldsymbol{\gamma}_y^{(j)}, \tag{5}$$

where

$$\mathbf{A}^{(j)} = \begin{bmatrix} c_{44}^{(j)}(y) & e_{15}^{(j)}(y) & q_{15}^{(j)}(y) \\ e_{15}^{(j)}(y) & -k_{11}^{(j)}(y) & -d_{11}^{(j)}(y) \\ q_{15}^{(j)}(y) & -d_{11}^{(j)}(y) & -\mu_{11}^{(j)}(y) \end{bmatrix}. \tag{6}$$

In addition, we consider that the variation of the material properties has an exponential form in the y direction, and the material coefficients in the nonhomogeneous magnetoelectroelastic layered half-plane can be described by

$$\mathbf{A}^{(j)} = \alpha^{(j)} e^{\beta^{(j)}(y-h)} \mathbf{A}^{(0)}, \tag{7}$$

where

$$\mathbf{A}^{(0)} = \begin{bmatrix} c_{44}^{(0)} & e_{15}^{(0)} & q_{15}^{(0)} \\ e_{15}^{(0)} & -k_{11}^{(0)} & -d_{11}^{(0)} \\ q_{15}^{(0)} & -d_{11}^{(0)} & -\mu_{11}^{(0)} \end{bmatrix}.$$

Here $\mathbf{A}^{(0)}$ is a 3×3 real and symmetric matrix, which includes six independent magnetoelectroelastic material constants (i.e., $c_{44}^{(0)}$, $k_{11}^{(0)}$, $\mu_{11}^{(0)}$, $e_{15}^{(0)}$, $q_{15}^{(0)}$, and $d_{11}^{(0)}$). In which $\alpha^{(j)}$ is a positive constant ($\alpha^{(j)} > 0$ for positive define) and $\beta^{(j)}$ is the functionally graded factor which represents the degree of the material gradient in the y direction.

In the absence of body force, electric charge density, and electric current density, the generalized equilibrium equation is given as

$$\boldsymbol{\sigma}_{x,x}^{(j)} + \boldsymbol{\sigma}_{y,y}^{(j)} = \mathbf{0}. \tag{8}$$

Substitution of Eqs. (5) and (7) into Eq. (8), the governing equation for an antiplane functionally graded magnetoelectroelastic problem can be expressed as

$$\nabla^2 \mathbf{u}^{(j)} + \beta^{(j)} \frac{\partial \mathbf{u}^{(j)}}{\partial y} = \mathbf{0}, \tag{9}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ represents the two-dimensional Laplace operator for the variables x and y. This problem will be solved by the Fourier transform technique. Take the Fourier transform pairs of the generalized displacement $\mathbf{u}^{(j)}(x,y)$ defined as

$$\tilde{\mathbf{u}}^{(j)}(\omega,y) = \int_{-\infty}^{\infty} \mathbf{u}^{(j)}(x,y) e^{-i\omega x} dx, \quad \mathbf{u}^{(j)}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\mathbf{u}}^{(j)}(\omega,y) e^{i\omega x} d\omega, \tag{10}$$

where ω is the transform parameter and $i = \sqrt{-1}$. Then, the governing differential equation (9) becomes an ordinary differential equation of order two as follows:

$$\frac{\partial^2 \tilde{\mathbf{u}}^{(j)}}{\partial y^2} + \beta^{(j)} \frac{\partial \tilde{\mathbf{u}}^{(j)}}{\partial y} - \omega^2 \tilde{\mathbf{u}}^{(j)} = \mathbf{0}. \tag{11}$$

The general solutions in the Fourier-transform domain for materials 1 (thin layer) and 2 (half-plane) can be expressed as

$$\tilde{\mathbf{u}}^{(j)}(\omega,\ y) = \mathbf{c}^{(j)} e^{s_1^{(j)} y} + \mathbf{d}^{(j)} e^{s_2^{(j)} y}, \tag{12}$$

$$\tilde{\boldsymbol{\sigma}}_y^{(j)}(\omega,\ y) = \alpha^{(j)} e^{\beta^{(j)}(y-h)} \mathbf{A}^{(0)} [s_1^{(j)} \mathbf{c}^{(j)} e^{s_1^{(j)} y} + s_2^{(j)} \mathbf{d}^{(j)} e^{s_2^{(j)} y}], \tag{13}$$

where

$$\mathbf{c}^{(j)} = \begin{bmatrix} c_1^{(j)} \\ c_2^{(j)} \\ c_3^{(j)} \end{bmatrix}, \quad \mathbf{d}^{(j)} = \begin{bmatrix} d_1^{(j)} \\ d_2^{(j)} \\ d_3^{(j)} \end{bmatrix}.$$

Here $\mathbf{c}^{(j)}$ and $\mathbf{d}^{(j)}$ are undetermined coefficients and can be obtained by applying the jump, continuity, and boundary conditions. By substituting the general solutions into Eq. (11) yields a characteristic equation for $s_k^{(j)}$ $(k = 1,\ 2)$, the characteristic roots are readily known to be

$$s_1^{(j)} = -\frac{\beta^{(j)}}{2} + Q^{(j)}, \quad s_2^{(j)} = -\frac{\beta^{(j)}}{2} - Q^{(j)}, \tag{14}$$

with

$$Q^{(j)} = \sqrt{(\frac{\beta^{(j)}}{2})^2 + \omega^2}. \tag{15}$$

Because the positive nature of the quantity $Q^{(j)}$ in the transform domain, it is noted that the roots $s_1^{(j)}$ and $s_2^{(j)}$ must be positive and negative, respectively.

3 Solutions of a Functionally Graded Magnetoelectroelastic Layered Half-Plane

Consider a functionally graded magnetoelectroelastic layer half-plane subjected to a generalized line force $\mathbf{F} = [f_z, -q, -j]^T$ and a generalized screw dislocation $\mathbf{B} = [b_z, b_\varphi, b_\phi]^T$ as shown in Fig. 1, where f_z, q, and j denote the strength of a line force, a line electric charge, and a line electric current, respectively, and b_z, b_φ, and b_ϕ represent a screw dislocation, an electric-potential dislocation, and a magnetic-potential dislocation, respectively. A generalized line force $\mathbf{F}$ and a generalized screw dislocation $\mathbf{B}$ are applied at $(x, y) = (x_0, y_0)$, $y_0 \geq 0$, which may be located in the thin layer$(0 \leq y_0 \leq h)$or the half-plane$(h \leq y_0 < \infty)$ of the layered half-plane.

The full-field solutions for the thin layer and the half-plane will be presented in this section. The boundary conditions on the top surface ($y=0$) of the thin layer are

$$\boldsymbol{\sigma}_y^{(1)}(x,0)=\mathbf{0}. \tag{16}$$

From Eq. (16), $\mathbf{c}^{(1)}$ and $\mathbf{d}^{(1)}$ in Eq. (13) are related as

$$s_1^{(1)}\mathbf{c}^{(1)}+s_2^{(1)}\mathbf{d}^{(1)}=\mathbf{0}. \tag{17}$$

The continuity conditions of the generalized displacement and stress at the interface $y=h$ are

$$\mathbf{u}^{(1)}(x,h)=\mathbf{u}^{(2)}(x,h), \quad \boldsymbol{\sigma}_y^{(1)}(x,h)=\boldsymbol{\sigma}_y^{(2)}(x,h). \tag{18}$$

From Eq. (18), the relations between $\mathbf{c}^{(1)}$, $\mathbf{d}^{(1)}$, $\mathbf{c}^{(2)}$, and $\mathbf{d}^{(2)}$ can be established as

$$\mathbf{c}^{(1)}e^{s_1^{(1)}h}+\mathbf{d}^{(1)}e^{s_2^{(1)}h}=\mathbf{c}^{(2)}e^{s_1^{(2)}h}+\mathbf{d}^{(2)}e^{s_2^{(2)}h}, \tag{19}$$

$$\alpha^{(1)}[s_1^{(1)}e^{s_1^{(1)}h}\mathbf{c}^{(1)}+s_2^{(1)}e^{s_2^{(1)}h}\mathbf{d}^{(1)}]=\alpha^{(2)}[s_1^{(2)}e^{s_1^{(2)}h}\mathbf{c}^{(2)}+s_2^{(2)}e^{s_2^{(2)}h}\mathbf{d}^{(2)}]. \tag{20}$$

For $\omega>0$, the bounded conditions of the generalized stresses in the infinite of the half-plane require that

$$\mathbf{c}^{(2)}=\mathbf{0}. \tag{21}$$

There are two situations for this nonhomogeneous problem, i.e., the applied loadings are located in the interior of the thin layer or in the half-plane will be discussed in detail.

3.1 The loadings are applied in the thin layer $(0\le y_0\le h)$

Consider a nonhomogeneous magnetoelectroelastic layered half-plane subjected to a generalized line force and screw dislocation applied at $(x,y)=(x_0,y_0)$ in the thin layer. If we take the positive x axis as the slip plane $(x>x_0,\ y=y_0)$ of the dislocations, the jumps of the generalized displacement and stress across the slip plane are

$$\mathbf{u}^{(1-)}(x,y_0^-)-\mathbf{u}^{(1+)}(x,y_0^+)=\mathbf{B}H(x-x_0), \tag{22}$$

$$\boldsymbol{\sigma}_y^{(1-)}(x,y_0^-)-\boldsymbol{\sigma}_y^{(1+)}(x,y_0^+)=\mathbf{F}\delta(x-x_0), \tag{23}$$

where $H(\ )$ is the Heaviside function and $\delta(\ )$ is the Delta function. In Eqs. (22) and (23), superscripts '–' and '+' indicate the field quantities above and below the plane of the applied generalized force and screw dislocation in the thin layer. From Eqs. (22) and (23), $\mathbf{c}^{(1)}$ and $\mathbf{d}^{(1)}$ have the relations

$$(\mathbf{c}^{(1-)}-\mathbf{c}^{(1+)})e^{s_1^{(1)}y_0}+(\mathbf{d}^{(1-)}-\mathbf{d}^{(1+)})e^{s_2^{(1)}y_0}=\frac{1}{i\omega}\mathbf{B}e^{-i\omega x_0}, \tag{24}$$

$$e^{\beta^{(1)}(y_0-h)}[s_1^{(1)}(\mathbf{c}^{(1-)}-\mathbf{c}^{(1+)})e^{s_1^{(1)}y_0}+s_2^{(1)}(\mathbf{d}^{(1-)}-\mathbf{d}^{(1+)})e^{s_2^{(1)}y_0}] \\ =\frac{1}{\alpha^{(1)}}(\mathbf{A}^{(0)})^{-1}\mathbf{F}e^{-i\omega x_0}, \tag{25}$$

With the aid of boundary conditions (17) and (21) and continuity conditions (19) and (20), the constants $\mathbf{c}^{(j)}$ and $\mathbf{d}^{(j)}$ can be determined as follows:

$$\mathbf{d}^{(1+)}=\frac{1}{s_1^{(1)}-s_2^{(1)}}e^{-i\omega x_0}(1-\Omega_1\Omega_2e^{-(s_1^{(1)}-s_2^{(1)})h})^{-1} \\ \times\left[\frac{1}{\alpha^{(1)}}e^{-\beta^{(1)}(y_0-h)}(e^{-s_2^{(1)}y_0}-\Omega_1e^{-s_1^{(1)}y_0})(\mathbf{A}^{(0)})^{-1}\mathbf{F}-\frac{1}{i\omega}s_1^{(1)}(e^{-s_2^{(1)}y_0}-e^{-s_1^{(1)}y_0})\mathbf{B}\right],$$

$$\mathbf{c}^{(1+)}=-\Omega_2e^{-(s_1^{(1)}-s_2^{(1)})h}\mathbf{d}^{(1+)},$$

$$\mathbf{c}^{(1-)}=\frac{1}{\alpha^{(1)}(s_1^{(1)}-s_2^{(1)})}e^{-i\omega x_0-\beta^{(1)}(y_0-h)} \\ \times\left[e^{-s_1^{(1)}y_0}-\Omega_2e^{-(s_1^{(1)}-s_2^{(1)})h}(1-\Omega_1\Omega_2e^{-(s_1^{(1)}-s_2^{(1)})h})^{-1}(e^{-s_2^{(1)}y_0}-\Omega_1e^{-s_1^{(1)}y_0})\right] \\ \times(\mathbf{A}^{(0)})^{-1}\mathbf{F}-\frac{1}{i\omega(s_1^{(1)}-s_2^{(1)})}e^{-i\omega x_0} \\ \times\left[s_2^{(1)}e^{-s_1^{(1)}y_0}-s_1^{(1)}\Omega_2e^{-(s_1^{(1)}-s_2^{(1)})h}(1-\Omega_1\Omega_2e^{-(s_1^{(1)}-s_2^{(1)})h})^{-1}(e^{-s_2^{(1)}y_0}-e^{-s_1^{(1)}y_0})\right]\mathbf{B},$$

$$\mathbf{d}^{(1-)}=-\Omega_1\mathbf{c}^{(1-)},\quad \mathbf{d}^{(2)}=(1-\Omega_2)e^{(s_2^{(1)}-s_2^{(2)})h}\mathbf{d}^{(1+)}, \tag{26}$$

where

$$\Omega_1=\frac{s_1^{(1)}}{s_2^{(1)}}=\frac{\beta^{(1)}-2Q^{(1)}}{\beta^{(1)}+2Q^{(1)}},$$

$$\Omega_2 = \frac{\alpha^{(1)} s_2^{(1)} - \alpha^{(2)} s_2^{(2)}}{\alpha^{(1)} s_1^{(1)} - \alpha^{(2)} s_2^{(2)}} = \frac{\alpha^{(1)}(\beta^{(1)} + 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)})}{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)})}.$$

For $\omega < 0$, the derivation and results are similar. After substituting the coefficients $\mathbf{c}^{(j)}$ and $\mathbf{d}^{(j)}$ into Eqs. (12) and (13) and perform the inverse Fourier transformation, the analytical full-field solutions of the generalized displacement and stresses for the functionally graded magnetoelectroelastic layered half-plane are presented as follows:

$$\begin{bmatrix} w^{(1-)} \\ \varphi^{(1-)} \\ \phi^{(1-)} \end{bmatrix} = \frac{1}{2\pi} e^{-\frac{\beta^{(1)}}{2}(y+y_0-2h)} \int_0^\infty \frac{1}{\alpha^{(1)} Q^{(1)}} \cos\omega(x-x_0) (\mathbf{A}^{(0)})^{-1} \mathbf{F}$$

$$\times \left\{ \begin{array}{l} \left[e^{Q^{(1)}(y-y_0)} - \Omega_1 e^{-Q^{(1)}(y+y_0)} \right] - \Omega_2 (1 - \Omega_1 \Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times \left[(e^{Q^{(1)}(y+y_0-2h)} - \Omega_1 e^{Q^{(1)}(y-y_0-2h)}) - \Omega_1 (e^{-Q^{(1)}(y-y_0+2h)} \right. \\ \left. -\Omega_1 e^{-Q^{(1)}(y+y_0+2h)}) \right] \end{array} \right\} d\omega$$

$$+ \frac{1}{4\pi} e^{-\frac{\beta^{(1)}}{2}(y-y_0)} \int_0^\infty \frac{1}{\omega} \frac{\beta^{(1)} + 2Q^{(1)}}{Q^{(1)}} \sin\omega(x-x_0) \mathbf{B}$$

$$\times \left\{ \begin{array}{l} \left[e^{Q^{(1)}(y-y_0)} - \Omega_1 e^{-Q^{(1)}(y+y_0)} \right] - \Omega_1 \Omega_2 (1 - \Omega_1 \Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times \left[(e^{Q^{(1)}(y+y_0-2h)} - e^{Q^{(1)}(y-y_0-2h)}) \right. \\ \left. -\Omega_1 (e^{-Q^{(1)}(y-y_0+2h)} - e^{-Q^{(1)}(y+y_0+2h)}) \right] \end{array} \right\} d\omega, \qquad (27a)$$

$$\begin{bmatrix} \tau_{xz}^{(1-)} \\ D_x^{(1-)} \\ B_x^{(1-)} \end{bmatrix} = (\alpha^{(1)} e^{\beta^{(1)}(y-h)}) \mathbf{A}^{(0)} \begin{bmatrix} \gamma_{xz}^{(1-)} \\ -E_x^{(1-)} \\ -H_x^{(1-)} \end{bmatrix}$$

$$= -\frac{1}{2\pi} e^{\frac{\beta^{(1)}}{2}(y-y_0)} \int_0^\infty \frac{\omega}{Q^{(1)}} \sin\omega(x-x_0) \mathbf{F}$$

$$\times \left\{ \begin{array}{l} \left[e^{Q^{(1)}(y-y_0)} - \Omega_1 e^{-Q^{(1)}(y+y_0)} \right] - \Omega_2 (1 - \Omega_1 \Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times \left[(e^{Q^{(1)}(y+y_0-2h)} - \Omega_1 e^{Q^{(1)}(y-y_0-2h)}) - \Omega_1 (e^{-Q^{(1)}(y-y_0+2h)} \right. \\ \left. -\Omega_1 e^{-Q^{(1)}(y+y_0+2h)}) \right] \end{array} \right\} d\omega$$

$$+\frac{1}{4\pi}e^{\frac{\beta^{(1)}}{2}(y+y_0-2h)}\int_0^\infty \frac{\alpha^{(1)}(\beta^{(1)}+2Q^{(1)})}{Q^{(1)}}\cos\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times\left\{\begin{array}{l}\left[e^{Q^{(1)}(y-y_0)}-\Omega_1 e^{-Q^{(1)}(y+y_0)}\right]-\Omega_1\Omega_2(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}\\ \times\Big[\big(e^{Q^{(1)}(y+y_0-2h)}-e^{Q^{(1)}(y-y_0-2h)}\big)\\ -\Omega_1\big(e^{-Q^{(1)}(y-y_0+2h)}-e^{-Q^{(1)}(y+y_0+2h)}\big)\Big]\end{array}\right\}d\omega, \tag{27b}$$

$$\begin{bmatrix}\tau_{yz}^{(1-)}\\ D_y^{(1-)}\\ B_y^{(1-)}\end{bmatrix}=(\alpha^{(1)}e^{\beta^{(1)}(y-h)})\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{yz}^{(1-)}\\ -E_y^{(1-)}\\ -H_y^{(1-)}\end{bmatrix}$$

$$=-\frac{1}{4\pi}e^{\frac{\beta^{(1)}}{2}(y-y_0)}\int_0^\infty \frac{\beta^{(1)}-2Q^{(1)}}{Q_1}\cos\omega(x-x_0)\mathbf{F}$$

$$\times\left\{\begin{array}{l}\left[e^{Q^{(1)}(y-y_0)}-e^{-Q^{(1)}(y+y_0)}\right]-\Omega_2(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}\\ \times\Big[\big(e^{Q^{(1)}(y+y_0-2h)}-\Omega_1 e^{Q^{(1)}(y-y_0-2h)}\big)-\big(e^{-Q^{(1)}(y-y_0+2h)}\\ -\Omega_1 e^{-Q^{(1)}(y+y_0+2h)}\big)\Big]\end{array}\right\}d\omega$$

$$-\frac{1}{8\pi}e^{\frac{\beta^{(1)}}{2}(y+y_0-2h)}\int_0^\infty \frac{\alpha^{(1)}}{\omega}\frac{(\beta^{(1)})^2-4(Q^{(1)})^2}{Q^{(1)}}\sin\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times\left\{\begin{array}{l}\left[e^{Q^{(1)}(y-y_0)}-e^{-Q^{(1)}(y+y_0)}\right]-\Omega_1\Omega_2(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}\\ \times\Big[\big(e^{Q^{(1)}(y+y_0-2h)}-e^{Q^{(1)}(y-y_0-2h)}\big)-\big(e^{-Q^{(1)}(y-y_0+2h)}\\ -e^{-Q^{(1)}(y+y_0+2h)}\big)\Big]\end{array}\right\}d\omega, \tag{27c}$$

$$\begin{bmatrix}w^{(1+)}\\ \varphi^{(1+)}\\ \phi^{(1+)}\end{bmatrix}=\frac{1}{2\pi}e^{-\frac{\beta^{(1)}}{2}(y+y_0-2h)}\int_0^\infty \frac{1}{\alpha^{(1)}Q^{(1)}}\cos\omega(x-x_0)(\mathbf{A}^{(0)})^{-1}\mathbf{F}$$

$$\times(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}$$

$$\times\Big[\big(e^{-Q^{(1)}(y-y_0)}-\Omega_1 e^{-Q^{(1)}(y+y_0)}\big)-\Omega_2\big(e^{Q^{(1)}(y+y_0-2h)}-\Omega_1 e^{Q^{(1)}(y-y_0-2h)}\big)\Big]d\omega$$

$$+\frac{1}{4\pi}e^{-\frac{\beta^{(1)}}{2}(y-y_0)}\int_0^\infty \frac{1}{\omega}\frac{\beta^{(1)}-2Q^{(1)}}{Q^{(1)}}\sin\omega(x-x_0)\mathbf{B}$$

$$\times(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}$$

$$\times\left[\left(e^{-Q^{(1)}(y-y_0)}-e^{-Q^{(1)}(y+y_0)}\right)-\Omega_2\left(e^{Q^{(1)}(y+y_0-2h)}-e^{Q^{(1)}(y-y_0-2h)}\right)\right]d\omega, \tag{28a}$$

$$\begin{bmatrix}\tau_{xz}^{(1+)}\\ D_x^{(1+)}\\ B_x^{(1+)}\end{bmatrix}=(\alpha^{(1)}e^{\beta^{(1)}(y-h)})\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{xz}^{(1+)}\\ -E_x^{(1+)}\\ -H_x^{(1+)}\end{bmatrix}$$

$$=-\frac{1}{2\pi}e^{\frac{\beta^{(1)}}{2}(y-y_0)}\int_0^\infty\frac{\omega}{Q^{(1)}}\sin\omega(x-x_0)\mathbf{F}$$

$$\times(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}$$

$$\times\left[\left(e^{-Q^{(1)}(y-y_0)}-\Omega_1e^{-Q^{(1)}(y+y_0)}\right)-\Omega_2\left(e^{Q^{(1)}(y+y_0-2h)}-\Omega_1e^{Q^{(1)}(y-y_0-2h)}\right)\right]d\omega$$

$$+\frac{1}{4\pi}e^{\frac{\beta^{(1)}}{2}(y+y_0-2h)}\int_0^\infty\frac{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})}{Q^{(1)}}\cos\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}$$

$$\times\left[\left(e^{-Q^{(1)}(y-y_0)}-e^{-Q^{(1)}(y+y_0)}\right)-\Omega_2\left(e^{Q^{(1)}(y+y_0-2h)}-e^{Q^{(1)}(y-y_0-2h)}\right)\right]d\omega, \tag{28b}$$

$$\begin{bmatrix}\tau_{yz}^{(1+)}\\ D_y^{(1+)}\\ B_y^{(1+)}\end{bmatrix}=(\alpha^{(1)}e^{\beta^{(1)}(y-h)})\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{yz}^{(1+)}\\ -E_y^{(1+)}\\ -H_y^{(1+)}\end{bmatrix}$$

$$=-\frac{1}{4\pi}e^{\frac{\beta^{(1)}}{2}(y-y_0)}\int_0^\infty\frac{\beta^{(1)}+2Q^{(1)}}{Q_1}\cos\omega(x-x_0)\mathbf{F}$$

$$\times(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}$$

$$\times\left[\left(e^{-Q^{(1)}(y-y_0)}-\Omega_1e^{-Q^{(1)}(y+y_0)}\right)-\Omega_1\Omega_2\left(e^{Q^{(1)}(y+y_0-2h)}-\Omega_1e^{Q^{(1)}(y-y_0-2h)}\right)\right]d\omega$$

$$+\frac{1}{8\pi}e^{\frac{\beta^{(1)}}{2}(y+y_0-2h)}\int_0^\infty\frac{\alpha^{(1)}}{\omega}\frac{4(Q^{(1)})^2-(\beta^{(1)})^2}{Q^{(1)}}\sin\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}$$

$$\times\left[\left(e^{-Q^{(1)}(y-y_0)}-e^{-Q^{(1)}(y+y_0)}\right)-\Omega_1\Omega_2\left(e^{Q^{(1)}(y+y_0-2h)}-e^{Q^{(1)}(y-y_0-2h)}\right)\right]d\omega, \tag{28c}$$

$$\begin{bmatrix} w^{(2)} \\ \varphi^{(2)} \\ \phi^{(2)} \end{bmatrix} = -\frac{2}{\pi} e^{-\frac{1}{2}(\beta^{(2)} y + \beta^{(1)} y_0 - (\beta^{(1)} + \beta^{(2)}) h)}$$

$$\times \int_0^\infty \frac{1}{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)})} \cos\omega(x - x_0) (\mathbf{A}^{(0)})^{-1} \mathbf{F}$$

$$\times (1 - \Omega_1 \Omega_2 e^{-2Q^{(1)} h})^{-1}$$

$$\times \left[e^{-(Q^{(2)} y - Q^{(1)} y_0 + (Q^{(1)} - Q^{(2)}) h)} - \Omega_1 e^{-(Q^{(2)} y + Q^{(1)} y_0 + (Q^{(1)} - Q^{(2)}) h)} \right] d\omega$$

$$-\frac{1}{\pi} e^{-\frac{1}{2}(\beta^{(2)} y - \beta^{(1)} y_0 + (\beta^{(1)} - \beta^{(2)}) h)}$$

$$\times \int_0^\infty \frac{1}{\omega} \frac{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)})}{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)})} \sin\omega(x - x_0) \mathbf{B}$$

$$\times (1 - \Omega_1 \Omega_2 e^{-2Q^{(1)} h})^{-1}$$

$$\times \left[e^{-(Q^{(2)} y - Q^{(1)} y_0 + (Q^{(1)} - Q^{(2)}) h)} - e^{-(Q^{(2)} y + Q^{(1)} y_0 + (Q^{(1)} - Q^{(2)}) h)} \right] d\omega, \tag{29a}$$

$$\begin{bmatrix} \tau_{xz}^{(2)} \\ D_x^{(2)} \\ B_x^{(2)} \end{bmatrix} = (\alpha^{(2)} e^{\beta^{(2)}(y-h)}) \mathbf{A}^{(0)} \begin{bmatrix} \gamma_{xz}^{(2)} \\ -E_x^{(2)} \\ -H_x^{(2)} \end{bmatrix}$$

$$= \frac{2}{\pi} e^{\frac{1}{2}(\beta^{(2)} y - \beta^{(1)} y_0 + (\beta^{(1)} - \beta^{(2)}) h)}$$

$$\times \int_0^\infty \frac{\alpha^{(2)} \omega}{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)})} \sin\omega(x - x_0) \mathbf{F}$$

$$\times (1 - \Omega_1 \Omega_2 e^{-2Q^{(1)} h})^{-1}$$

$$\times \left[e^{-(Q^{(2)} y - Q^{(1)} y_0 + (Q^{(1)} - Q^{(2)}) h)} - \Omega_1 e^{-(Q^{(2)} y + Q^{(1)} y_0 + (Q^{(1)} - Q^{(2)}) h)} \right] d\omega$$

$$-\frac{1}{\pi} e^{\frac{1}{2}(\beta^{(2)} y + \beta^{(1)} y_0 - (\beta^{(1)} + \beta^{(2)}) h)}$$

$$\times \int_0^\infty \frac{\alpha^{(1)} \alpha^{(2)} (\beta^{(1)} - 2Q^{(1)})}{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)})} \cos\omega(x - x_0) \mathbf{A}^{(0)} \mathbf{B}$$

$$\times (1 - \Omega_1 \Omega_2 e^{-2Q^{(1)} h})^{-1}$$

$$\times\left[e^{-(Q^{(2)}y-Q^{(1)}y_0+(Q^{(1)}-Q^{(2)})h)}-e^{-(Q^{(2)}y+Q^{(1)}y_0+(Q^{(1)}-Q^{(2)})h)}\right]d\omega, \tag{29b}$$

$$\begin{bmatrix}\tau_{yz}^{(2)}\\ D_y^{(2)}\\ B_y^{(2)}\end{bmatrix}=(\alpha^{(2)}e^{\beta^{(2)}(y-h)})\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{yz}^{(2)}\\ -E_y^{(2)}\\ -H_y^{(2)}\end{bmatrix}$$

$$=\frac{1}{\pi}e^{\frac{1}{2}(\beta^{(2)}y-\beta^{(1)}y_0+(\beta^{(1)}-\beta^{(2)})h)}$$

$$\times\int_0^\infty\frac{\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})}{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})}\cos\omega(x-x_0)\mathbf{F}$$

$$\times(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}$$

$$\times\left[e^{-(Q^{(2)}y-Q^{(1)}y_0+(Q^{(1)}-Q^{(2)})h)}-\Omega_1e^{-(Q^{(2)}y+Q^{(1)}y_0+(Q^{(1)}-Q^{(2)})h)}\right]d\omega$$

$$+\frac{1}{2\pi}e^{\frac{1}{2}(\beta^{(2)}y+\beta^{(1)}y_0-(\beta^{(1)}+\beta^{(2)})h)}$$

$$\times\int_0^\infty\frac{1}{\omega}\frac{\alpha^{(1)}\alpha^{(2)}(\beta^{(1)}-2Q_1)(\beta^{(2)}+2Q^{(2)})}{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})}\sin\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}$$

$$\times\left[e^{-(Q^{(2)}y-Q^{(1)}y_0+(Q^{(1)}-Q^{(2)})h)}-e^{-(Q^{(2)}y+Q^{(1)}y_0+(Q^{(1)}-Q^{(2)})h)}\right]d\omega. \tag{29c}$$

The inverse term $(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}$ in Eqs. (27a)–(29c) control the convergence of the solutions. It can be shown that $\Omega_1\Omega_2e^{-2Q^{(1)}h}<1$ for $\omega<0$ and $\omega>0$. If $\alpha^{(1)}=\alpha^{(2)}$ and $\beta^{(1)}=\beta^{(2)}\neq 0$, i.e., $\Omega_2=0$, the solutions presented in Eqs. (27a)–(29c) for materials 1 and 2 are reduced to the results for the nonhomogeneous half-plane medium with the applied loadings located at $(x,y)=(x_0,y_0)$.

3.2 The loadings are applied in the half-plane $(h\leq y_0<\infty)$

If the applied loadings are located in material 2 (half-plane), then the jump conditions are given as

$$\mathbf{u}^{(2-)}(x,y_0^-)-\mathbf{u}^{(2+)}(x,y_0^+)=\mathbf{B}H(x-x_0), \tag{30}$$

$$\boldsymbol{\sigma}_y^{(2-)}(x,y_0^-)-\boldsymbol{\sigma}_y^{(2+)}(x,y_0^+)=\mathbf{F}\delta(x-x_0), \tag{31}$$

In terms of the unknown constants $\mathbf{c}^{(2)}$ and $\mathbf{d}^{(2)}$, we have

$$(\mathbf{c}^{(2-)}-\mathbf{c}^{(2+)})e^{s_1^{(2)}y_0}+(\mathbf{d}^{(2-)}-\mathbf{d}^{(2+)})e^{s_2^{(2)}y_0}=\frac{1}{i\omega}\mathbf{B}e^{-i\omega x_0}, \tag{32}$$

$$e^{\beta^{(2)}(y_0-h)}[s_1^{(2)}(\mathbf{c}^{(2-)}-\mathbf{c}^{(2+)})e^{s_1^{(2)}y_0}+s_2^{(2)}(\mathbf{d}^{(2-)}-\mathbf{d}^{(2+)})e^{s_2^{(2)}y_0}]$$
$$=\frac{1}{\alpha^{(2)}}(\mathbf{A}^{(0)})^{-1}\mathbf{F}e^{-i\omega x_0}, \tag{33}$$

With the boundary conditions expressed in Eqs. (17) and (21) and the continuity conditions in Eqs. (19) and (20), the constants $\mathbf{c}^{(j)}$ and $\mathbf{d}^{(j)}$ are obtained as

$$\mathbf{c}^{(1)}=\frac{\alpha^{(2)}}{\alpha^{(1)}s_1^{(1)}-\alpha^{(2)}s_2^{(2)}}(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}e^{-i\omega x_0-(s_1^{(1)}-s_1^{(2)})h-s_1^{(2)}y_0}$$
$$\left[\frac{1}{\alpha^{(2)}}e^{\beta^{(2)}(h-y_0)}(\mathbf{A}^{(0)})^{-1}\mathbf{F}-\frac{1}{i\omega}s_2^{(2)}\mathbf{B}\right],$$

$$\mathbf{d}^{(1)}=-\Omega_1\mathbf{c}^{(1)},$$

$$\mathbf{c}^{(2-)}=\frac{1}{s_1^{(2)}-s_2^{(2)}}e^{-i\omega x_0-s_1^{(2)}y_0}\left[\frac{1}{\alpha^{(2)}}e^{\beta^{(2)}(h-y_0)}(\mathbf{A}^{(0)})^{-1}\mathbf{F}-\frac{1}{i\omega}s_2^{(2)}\mathbf{B}\right],$$

$$\mathbf{d}^{(2-)}=\frac{\alpha^{(1)}(s_1^{(1)}-s_2^{(1)})}{\alpha^{(1)}s_1^{(1)}-\alpha^{(2)}s_2^{(2)}}e^{(s_2^{(1)}-s_2^{(2)})h}\mathbf{d}^{(1)}-\Omega_4e^{(s_1^{(2)}-s_2^{(2)})h}\mathbf{c}^{(2-)},$$

$$\mathbf{d}^{(2+)}=\mathbf{d}^{(2-)}+\frac{1}{s_1^{(2)}-s_2^{(2)}}e^{-i\omega x_0-s_2^{(2)}y_0}\left[\frac{1}{\alpha^{(2)}}e^{\beta^{(2)}(h-y_0)}(\mathbf{A}^{(0)})^{-1}\mathbf{F}-\frac{1}{i\omega}s_1^{(2)}\mathbf{B}\right], \tag{34}$$

where

$$\Omega_3=\frac{s_2^{(2)}}{s_1^{(2)}}=\frac{\beta^{(2)}+2Q^{(2)}}{\beta^{(2)}-2Q^{(2)}},$$

$$\Omega_4=\frac{\alpha^{(1)}s_1^{(1)}-\alpha^{(2)}s_1^{(2)}}{\alpha^{(1)}s_1^{(1)}-\alpha^{(2)}s_2^{(2)}}=\frac{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}-2Q^{(2)})}{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})}.$$

Then, the full-field solutions of field variables for the functionally graded magnetoelectroelastic layered half-plane are expressed explicitly as follows:

$$\begin{bmatrix} w^{(1)} \\ \varphi^{(1)} \\ \phi^{(1)} \end{bmatrix}=-\frac{2}{\pi}e^{-\frac{1}{2}(\beta^{(1)}y+\beta^{(2)}y_0-(\beta^{(1)}+\beta^{(2)})h)}$$

$$\times \int_0^\infty \frac{1}{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})} \cos\omega(x-x_0)(\mathbf{A}^{(0)})^{-1}\mathbf{F}$$
$$\times(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}$$
$$\times\left[e^{(Q^{(1)}y-Q^{(2)}y_0-(Q^{(1)}-Q^{(2)})h)}-\Omega_1 e^{-(Q^{(1)}y+Q^{(2)}y_0+(Q^{(1)}-Q^{(2)})h)}\right]d\omega$$
$$-\frac{1}{\pi}e^{-\frac{1}{2}(\beta^{(1)}y-\beta^{(2)}y_0-(\beta^{(1)}-\beta^{(2)})h)}$$
$$\times \int_0^\infty \frac{1}{\omega}\frac{\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})}{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})} \sin\omega(x-x_0)\mathbf{B}$$
$$\times(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}$$
$$\times\left[e^{(Q^{(1)}y-Q^{(2)}y_0-(Q^{(1)}-Q^{(2)})h)}-\Omega_1 e^{-(Q^{(1)}y+Q^{(2)}y_0+(Q^{(1)}-Q^{(2)})h)}\right]d\omega, \tag{35a}$$

$$\begin{bmatrix}\tau_{xz}^{(1)}\\ D_x^{(1)}\\ B_x^{(1)}\end{bmatrix} = (\alpha^{(1)}e^{\beta^{(1)}(y-h)})\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{xz}^{(1)}\\ -E_x^{(1)}\\ -H_x^{(1)}\end{bmatrix}$$
$$=\frac{2}{\pi}e^{\frac{1}{2}(\beta^{(1)}y-\beta^{(2)}y_0-(\beta^{(1)}-\beta^{(2)})h)}$$
$$\times \int_0^\infty \frac{\alpha^{(1)}\omega}{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})} \sin\omega(x-x_0)\mathbf{F}$$
$$\times(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}$$
$$\times\left[e^{(Q^{(1)}y-Q^{(2)}y_0-(Q^{(1)}-Q^{(2)})h)}-\Omega_1 e^{-(Q^{(1)}y+Q^{(2)}y_0+(Q^{(1)}-Q^{(2)})h)}\right]d\omega$$
$$-\frac{1}{\pi}e^{\frac{1}{2}(\beta^{(1)}y+\beta^{(2)}y_0-(\beta^{(1)}+\beta^{(2)})h)}$$
$$\times \int_0^\infty \frac{\alpha^{(1)}\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})}{\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})} \cos\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$
$$\times(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}$$
$$\times\left[e^{(Q^{(1)}y-Q^{(2)}y_0-(Q^{(1)}-Q^{(2)})h)}-\Omega_1 e^{-(Q^{(1)}y+Q^{(2)}y_0+(Q^{(1)}-Q^{(2)})h)}\right]d\omega, \tag{35b}$$

$$\begin{bmatrix}\tau_{yz}^{(1)}\\ D_y^{(1)}\\ B_y^{(1)}\end{bmatrix} = (\alpha^{(1)}e^{\beta^{(1)}(y-h)})\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{yz}^{(1)}\\ -E_y^{(1)}\\ -H_y^{(1)}\end{bmatrix}$$

$$
\begin{aligned}
&= \frac{1}{\pi} e^{\frac{1}{2}(\beta^{(1)}y - \beta^{(2)}y_0 - (\beta^{(1)} - \beta^{(2)})h)} \\
&\times \int_0^\infty \frac{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)})}{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)})} \cos\omega(x - x_0)\mathbf{F} \\
&\times (1 - \Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\
&\times \left[e^{(Q^{(1)}y - Q^{(2)}y_0 - (Q^{(1)} - Q^{(2)})h)} - e^{-(Q^{(1)}y + Q^{(2)}y_0 + (Q^{(1)} - Q^{(2)})h)} \right] d\omega \\
&+ \frac{1}{2\pi} e^{\frac{1}{2}(\beta^{(1)}y + \beta^{(2)}y_0 - (\beta^{(1)} + \beta^{(2)})h)} \\
&\times \int_0^\infty \frac{1}{\omega} \frac{\alpha^{(1)}\alpha^{(2)}(\beta^{(1)} - 2Q^{(1)})(\beta^{(2)} + 2Q^{(2)})}{\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)})} \sin\omega(x - x_0)\mathbf{A}^{(0)}\mathbf{B} \\
&\times (1 - \Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\
&\times \left[e^{(Q^{(1)}y - Q^{(2)}y_0 - (Q^{(1)} - Q^{(2)})h)} - e^{-(Q^{(1)}y + Q^{(2)}y_0 + (Q^{(1)} - Q^{(2)})h)} \right] d\omega, \qquad (35c)
\end{aligned}
$$

$$
\begin{aligned}
&\begin{bmatrix} w^{(2-)} \\ \varphi^{(2-)} \\ \phi^{(2-)} \end{bmatrix} = \frac{1}{\pi} e^{-\frac{\beta^{(2)}}{2}(y + y_0 - 2h)} \int_0^\infty \frac{1}{2\alpha^{(2)}Q^{(2)}} \cos\omega(x - x_0)(\mathbf{A}^{(0)})^{-1}\mathbf{F} \\
&\times \begin{bmatrix} e^{Q^{(2)}(y - y_0)} - \Omega_4 e^{-Q^{(2)}(y + y_0 - 2h)} \\ -\frac{16\Omega_1\alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)}))^2}(1 - \Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y + y_0) + 2(Q^{(1)} - Q^{(2)})h)} \end{bmatrix} d\omega \\
&+ \frac{1}{2\pi} e^{-\frac{\beta^{(2)}}{2}(y - y_0)} \int_0^\infty \frac{1}{\omega} \frac{\beta^{(2)} + 2Q^{(2)}}{2Q^{(2)}} \sin\omega(x - x_0)\mathbf{B} \\
&\times \begin{bmatrix} e^{Q^{(2)}(y - y_0)} - \Omega_4 e^{-Q^{(2)}(y + y_0 - 2h)} \\ -\frac{16\Omega_1\alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)} - 2Q^{(1)}) - \alpha^{(2)}(\beta^{(2)} + 2Q^{(2)}))^2}(1 - \Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y + y_0) + 2(Q^{(1)} - Q^{(2)})h)} \end{bmatrix} d\omega, \qquad (36a)
\end{aligned}
$$

$$
\begin{aligned}
&\begin{bmatrix} \tau_{xz}^{(2-)} \\ D_x^{(2-)} \\ B_x^{(2-)} \end{bmatrix} = (\alpha^{(2)} e^{\beta^{(2)}(y - h)})\mathbf{A}^{(0)} \begin{bmatrix} \gamma_{xz}^{(2-)} \\ -E_x^{(2-)} \\ -H_x^{(2-)} \end{bmatrix} \\
&= -\frac{1}{\pi} e^{\frac{\beta^{(2)}}{2}(y - y_0)} \int_0^\infty \frac{\omega}{2Q^{(2)}} \sin\omega(x - x_0)\mathbf{F}
\end{aligned}
$$

$$\times \left[\begin{array}{l} e^{Q^{(2)}(y-y_0)} - \Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1 \alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array} \right] d\omega$$

$$+\frac{1}{2\pi}e^{\frac{\beta^{(2)}}{2}(y+y_0-2h)}\int_0^\infty \frac{\alpha^{(2)}(\beta^{(2)}+2Q^{(2)})}{2Q^{(2)}}\cos\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times \left[\begin{array}{l} e^{Q^{(2)}(y-y_0)} - \Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1 \alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array} \right] d\omega, \tag{36b}$$

$$\begin{bmatrix} \tau_{yz}^{(2-)} \\ D_y^{(2-)} \\ B_y^{(2-)} \end{bmatrix} = (\alpha^{(2)}e^{\beta^{(2)}(y-h)})\mathbf{A}^{(0)} \begin{bmatrix} \gamma_{yz}^{(2-)} \\ -E_y^{(2-)} \\ -H_y^{(2-)} \end{bmatrix}$$

$$= -\frac{1}{\pi}e^{\frac{\beta^{(2)}}{2}(y-y_0)}\int_0^\infty \frac{\beta^{(2)}-2Q^{(2)}}{4Q^{(2)}}\cos\omega(x-x_0)\mathbf{F}$$

$$\times \left[\begin{array}{l} e^{Q^{(2)}(y-y_0)} - \Omega_3\Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1\Omega_3 \alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array} \right] d\omega$$

$$-\frac{1}{2\pi}e^{\frac{\beta^{(2)}}{2}(y+y_0-2h)}\int_0^\infty \frac{\alpha^{(2)}}{\omega}\frac{(\beta^{(2)})^2-4(Q^{(2)})^2}{4Q^{(2)}}\sin\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times \left[\begin{array}{l} e^{Q^{(2)}(y-y_0)} - \Omega_3\Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1\Omega_3 \alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array} \right] d\omega, \tag{36c}$$

$$\begin{bmatrix} w^{(2+)} \\ \varphi^{(2+)} \\ \phi^{(2+)} \end{bmatrix} = \frac{1}{\pi}e^{-\frac{\beta^{(2)}}{2}(y+y_0-2h)}\int_0^\infty \frac{1}{2\alpha^{(2)}Q^{(2)}}\cos\omega(x-x_0)(\mathbf{A}^{(0)})^{-1}\mathbf{F}$$

$$\times \left[\begin{array}{l} e^{-Q^{(2)}(y-y_0)} - \Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1 \alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array} \right] d\omega$$

$$+\frac{1}{\pi}e^{-\frac{\beta^{(2)}}{2}(y-y_0)}\int_0^\infty \frac{1}{\omega}\frac{\beta^{(2)}-2Q^{(2)}}{4Q^{(2)}}\sin\omega(x-x_0)\mathbf{B}$$

$$\times\left[\begin{array}{l} e^{-Q^{(2)}(y-y_0)}-\Omega_3\Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1\Omega_3\alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array}\right] d\omega, \tag{37a}$$

$$\begin{bmatrix}\tau_{xz}^{(2+)}\\ D_x^{(2+)}\\ B_x^{(2+)}\end{bmatrix}=(\alpha^{(2)}e^{\beta^{(2)}(y-h)})\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{xz}^{(2+)}\\ -E_x^{(2+)}\\ -H_x^{(2+)}\end{bmatrix}$$

$$=-\frac{1}{\pi}e^{\frac{\beta^{(2)}}{2}(y-y_0)}\int_0^\infty \frac{\omega}{2Q^{(2)}}\sin\omega(x-x_0)\mathbf{F}$$

$$\times\left[\begin{array}{l} e^{-Q^{(2)}(y-y_0)}-\Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1\alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array}\right] d\omega$$

$$+\frac{1}{\pi}e^{\frac{\beta^{(2)}}{2}(y+y_0-2h)}\int_0^\infty \frac{\alpha^{(2)}(\beta^{(2)}-2Q^{(2)})}{4Q^{(2)}}\cos\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times\left[\begin{array}{l} e^{-Q^{(2)}(y-y_0)}-\Omega_3\Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1\Omega_3\alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array}\right] d\omega, \tag{37b}$$

$$\begin{bmatrix}\tau_{yz}^{(2+)}\\ D_y^{(2+)}\\ B_y^{(2+)}\end{bmatrix}=(\alpha^{(2)}e^{\beta^{(2)}(y-h)})\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{yz}^{(2+)}\\ -E_y^{(2+)}\\ -H_y^{(2+)}\end{bmatrix}$$

$$=-\frac{1}{2\pi}e^{\frac{\beta^{(2)}}{2}(y-y_0)}\int_0^\infty \frac{\beta^{(2)}+2Q^{(2)}}{2Q^{(2)}}\cos\omega(x-x_0)\mathbf{F}$$

$$\times\left[\begin{array}{l} e^{-Q^{(2)}(y-y_0)}-\Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1\alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array}\right] d\omega$$

$$-\frac{1}{2\pi}e^{\frac{\beta^{(2)}}{2}(y+y_0-2h)}\int_0^\infty \frac{\alpha^{(2)}}{\omega}\frac{(\beta^{(2)})^2-4(Q^{(2)})^2}{4Q^{(2)}}\sin\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}$$

$$\times\left[\begin{array}{l} e^{-Q^{(2)}(y-y_0)}-\Omega_3\Omega_4 e^{-Q^{(2)}(y+y_0-2h)} \\ -\frac{16\Omega_1\Omega_3\alpha^{(1)}\alpha^{(2)}Q^{(1)}Q^{(2)}}{(\alpha^{(1)}(\beta^{(1)}-2Q^{(1)})-\alpha^{(2)}(\beta^{(2)}+2Q^{(2)}))^2}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} \\ \times e^{-(Q^{(2)}(y+y_0)+2(Q^{(1)}-Q^{(2)})h)} \end{array}\right] d\omega. \tag{37c}$$

It is interesting to note that when $\alpha^{(1)} = \alpha^{(2)}$ and $\beta^{(1)} = \beta^{(2)} \neq 0$, i.e., $\Omega_1\Omega_3 = 1$ and $\Omega_2 = \Omega_4 = 0$, the solutions presented in Eqs. (35a)–(37c) for materials 1 and 2 are reduced to the results for the nonhomogeneous half-plane problem. The solutions for the degenerate case of a homogeneous half-plane can be obtained by setting $\alpha^{(1)} = \alpha^{(2)}$ and $\beta^{(1)} = \beta^{(2)} = 0$, i.e., $\Omega_1 = \Omega_3 = -1$ and $\Omega_2 = \Omega_4 = 0$.

If the loadings are applied on the free surface ($y_0 = 0$) of material 1 or at the interface ($y_0 = h$), their corresponding solutions can also be obtained directly from the results presented in this section. If the loadings are applied at the free surface of material 1, the complete solutions can be obtained from Eqs. (27a)–(29c) by setting $y_0 = 0$. However, when the loadings are applied at the interface, the solutions can be constructed either from case A (the loadings are applied in the thin layer, i.e., Eqs. (27a)–(29c)) or case B (the loadings are applied in the half-plane, i.e., Eqs. (35a)–(37c)) by setting $y_0 = h$.

4 The Characteristics of Magnetoelectroelastic Fields at the Interface for Continuous Material Constants

For the magnetoelectroelastic composites with homogeneous material properties subjected to the mechanical or electromagnetic loadings, the discontinuous stresses are always generated at the layer interfaces due to the jump magnetoelectroelastic properties between the component materials. Therefore, the development of the functionally graded material is a need to increase fracture toughness at interface. In addition, the full-field solutions for a nonhomogeneous magnetoelectroelastic layered half-plane subjected generalized loadings applied in the thin layer are presented in Eqs. (27a)–(29c). In this section, we will show that the functionally graded magnetoelectroelastic layered half-plane has a general feature that all magnetoelectroelastic fields are continuous at the interface if material constants are continuous at the interface. There are four cases for different functionally graded parameters $\alpha^{(j)}$ and $\beta^{(j)}$ will be investigated and discussed in detail.

Case (A), the material constants are continuous at the interface (i.e., $\alpha^{(1)} = \alpha^{(2)}$). We consider a nonhomogeneous functionally graded magnetoelectroelastic layer bonded to a dissimilar nonhomogeneous half-plane (i.e., $\beta^{(1)} \neq \beta^{(2)} \neq 0$). From the solutions presented in Eqs. (27a)–(29c), it is found that the magnetoelectroelastic fields are all continuous at the interface ($y = h$) even for the generalized stress $\boldsymbol{\sigma}_x^{(j)}$, i.e.,

$$\boldsymbol{\sigma}_x^{(1)}(x,h) = \boldsymbol{\sigma}_x^{(2)}(x,h)$$

$$= \frac{1}{\pi}\int_0^\infty \frac{1}{(\beta^{(1)} - \beta^{(2)}) - 2(Q^{(1)} + Q^{(2)})}(1 - \Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1} e^{-Q^{(1)}(y_0+h)}$$

$$\times\Big[2\omega e^{-\frac{1}{2}\beta^{(1)}(y_0-h)}(e^{2Q^{(1)}y_0}-\Omega_1)\sin\omega(x-x_0)\mathbf{F}$$
$$+\alpha^{(1)}e^{\frac{1}{2}\beta^{(1)}(y_0-h)}(e^{2Q^{(1)}y_0}-1)(2Q^{(1)}-\beta^{(1)})\cos\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}\Big]\,d\omega, \tag{38a}$$

$$\boldsymbol{\sigma}_y^{(1)}(x,h)=\boldsymbol{\sigma}_y^{(2)}(x,h)$$
$$=\frac{1}{\pi}\int_0^\infty\frac{\beta^{(2)}+2Q^{(2)}}{(\beta^{(1)}-\beta^{(2)})-2(Q^{(1)}+Q^{(2)})}(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}e^{-Q^{(1)}(y_0+h)}$$
$$\times\Big[e^{-\frac{1}{2}\beta^{(1)}(y_0-h)}(e^{2Q^{(1)}y_0}-\Omega_1)\cos\omega(x-x_0)\mathbf{F}$$
$$-\frac{\alpha^{(1)}}{2\omega}e^{\frac{1}{2}\beta^{(1)}(y_0-h)}(e^{2Q^{(1)}y_0}-1)(2Q^{(1)}-\beta^{(1)})\sin\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}\Big]\,d\omega. \tag{38b}$$

It is indicated in Eq. (18) for the continuity conditions that only the generalized stress $\boldsymbol{\sigma}_y^{(j)}$ is required to be continuous at the interface. However, both $\boldsymbol{\sigma}_x^{(j)}$ and $\boldsymbol{\sigma}_y^{(j)}$ are found to be continuous at the interface. Furthermore, the first derivative of the generalized stress $\sigma_y^{(j)}$ is continuous at the interface. We have the following interesting results

$$\frac{\boldsymbol{\sigma}_{x,x}^{(1)}(x,h)}{\boldsymbol{\sigma}_{x,y}^{(1)}(x,h)}\neq\frac{\boldsymbol{\sigma}_{x,x}^{(2)}(x,h)}{\sigma_{x,y}^{(2)}(x,h)}, \tag{39a}$$

$$\frac{\boldsymbol{\sigma}_{y,x}^{(1)}(x,h)}{\boldsymbol{\sigma}_{y,y}^{(1)}(x,h)}=\frac{\boldsymbol{\sigma}_{y,x}^{(2)}(x,h)}{\sigma_{y,y}^{(2)}(x,h)}, \tag{39b}$$

where

$$\boldsymbol{\sigma}_{y,x}^{(1)}(x,h)=\boldsymbol{\sigma}_{y,x}^{(2)}(x,h)$$
$$=-\frac{1}{\pi}\int_0^\infty\frac{\beta^{(2)}+2Q^{(2)}}{(\beta^{(1)}-\beta^{(2)})-2(Q^{(1)}+Q^{(2)})}(1-\Omega_1\Omega_2e^{-2Q^{(1)}h})^{-1}e^{-Q^{(1)}(y_0+h)}$$
$$\times\Big[\omega e^{-\frac{1}{2}\beta^{(1)}(y_0-h)}(e^{2Q^{(1)}y_0}-\Omega_1)\sin\omega(x-x_0)\mathbf{F}$$
$$+\frac{\alpha^{(1)}}{2}e^{\frac{1}{2}\beta^{(1)}(y_0-h)}(e^{2Q^{(1)}y_0}-1)(2Q^{(1)}-\beta^{(1)})\cos\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}\Big]\,d\omega, \tag{40a}$$

$$\boldsymbol{\sigma}_{y,y}^{(1)}(x,h)=\boldsymbol{\sigma}_{y,y}^{(2)}(x,h)$$

$$= -\frac{1}{\pi}\int_0^\infty \frac{1}{(\beta^{(1)}-\beta^{(2)})-2(Q^{(1)}+Q^{(2)})}(1-\Omega_1\Omega_2 e^{-2Q^{(1)}h})^{-1}e^{-Q^{(1)}(y_0+h)}$$

$$\times\Big[2\omega^2 e^{-\frac{1}{2}\beta^{(1)}(y_0-h)}(e^{2Q^{(1)}y_0}-\Omega_1)\cos\omega(x-x_0)\mathbf{F}$$

$$-\alpha^{(1)}\omega e^{\frac{1}{2}\beta^{(1)}(y_0-h)}(e^{2Q^{(1)}y_0}-1)(2Q^{(1)}-\beta^{(1)})\sin\omega(x-x_0)\mathbf{A}^{(0)}\mathbf{B}\Big]\,d\omega. \tag{40b}$$

For case (B), one nonhomogeneous magnetoelectroelastic thin layer is bonded to a homogeneous magnetoelectroelastic half-plane, i.e., $\alpha^{(1)}=\alpha^{(2)}$, $\beta^{(1)}\neq 0$, and $\beta^{(2)}=0$, is considered. From the expressions (38a)–(40b), it is interesting to find that the generalized stresses $\boldsymbol{\sigma}_x^{(j)}$ and $\sigma_y^{(j)}$ are both continuous at the interface. Furthermore, the first derivative of the generalized stress $\boldsymbol{\sigma}_y^{(j)}$ is continuous at the interface. Similar interfacial continuous characteristics are also found in case (C) that one homogeneous magnetoelectroelastic thin layer bonded to a nonhomogeneous magnetoelectroelastic half-plane (i.e., $\alpha^{(1)}=\alpha^{(2)}$, $\beta^{(1)}=0$, and $\beta^{(2)}\neq 0$).

Finally, the degenerate case (D) of a homogeneous magnetoelectroelastic layered half-plane, i.e., $\alpha^{(1)}\neq\alpha^{(2)}$ and $\beta^{(1)}=\beta^{(2)}=0$, subjected to the generalized loadings applied at (x_0,y_0) in the thin layer (material 1) is investigated. The analytical full-field solutions are explicitly presented as follows (Lee and Ma (2007)):

$$\begin{bmatrix} w^{(1)} \\ \varphi^{(1)} \\ \phi^{(1)} \end{bmatrix} = -\frac{1}{2\pi}$$

$$\times\left\{\begin{array}{l} \frac{1}{\alpha^{(1)}}\left[\ln((x-x_0)^2+(y-y_0)^2)^{1/2}+\ln((x-x_0)^2+(y+y_0)^2)^{1/2}\right](\mathbf{A}^{(0)})^{-1}\mathbf{F} \\ -\left[\tan^{-1}\frac{y-y_0}{x-x_0}-\tan^{-1}\frac{y+y_0}{x-x_0}\right]\mathbf{B} \end{array}\right\}$$

$$-\frac{1}{2\pi}\sum_{n=0}^{\infty}Q^{n+1}\left\{\begin{array}{l} \frac{1}{\alpha^{(1)}}\left[\begin{array}{c}\ln((x-x_0)^2+(y-y_0\pm 2(n+1)h)^2)^{1/2} \\ +\ln((x-x_0)^2+(y+y_0\pm 2(n+1)h)^2)^{1/2}\end{array}\right](\mathbf{A}^{(0)})^{-1}\mathbf{F} \\ -\left[\tan^{-1}\frac{y-y_0\pm 2(n+1)h}{x-x_0}-\tan^{-1}\frac{y+y_0\pm 2(n+1)h}{x-x_0}\right]\mathbf{B} \end{array}\right\}, \tag{41a}$$

$$\begin{bmatrix} \tau_{xz}^{(1)} \\ D_x^{(1)} \\ B_x^{(1)} \end{bmatrix} = \alpha^{(1)}\mathbf{A}^{(0)}\begin{bmatrix} \gamma_{xz}^{(1)} \\ -E_x^{(1)} \\ -H_x^{(1)} \end{bmatrix}$$

$$
= -\frac{1}{2\pi}\left\{\begin{array}{l}\left[\frac{x-x_0}{(x-x_0)^2+(y-y_0)^2}+\frac{x-x_0}{(x-x_0)^2+(y+y_0)^2}\right]\mathbf{F} \\ +\alpha^{(1)}\left[\frac{y-y_0}{(x-x_0)^2+(y-y_0)^2}-\frac{y+y_0}{(x-x_0)^2+(y+y_0)^2}\right]\mathbf{A}^{(0)}\mathbf{B}\end{array}\right\}
$$

$$
-\frac{1}{2\pi}\sum_{n=0}^{\infty}Q^{n+1}\left\{\begin{array}{l}\left[\frac{x-x_0}{(x-x_0)^2+(y-y_0\pm 2(n+1)h)^2}+\frac{x-x_0}{(x-x_0)^2+(y+y_0\pm 2(n+1)h)^2}\right]\mathbf{F} \\ +\alpha^{(1)}\left[\frac{y-y_0\pm 2(n+1)h}{(x-x_0)^2+(y-y_0\pm 2(n+1)h)^2}-\frac{y+y_0\pm 2(n+1)h}{(x-x_0)^2+(y+y_0\pm 2(n+1)h)^2}\right]\mathbf{A}^{(0)}\mathbf{B}\end{array}\right\}, \tag{41b}
$$

$$
\begin{bmatrix}\tau_{yz}^{(1)}\\ D_y^{(1)}\\ B_y^{(1)}\end{bmatrix} = \alpha^{(1)}\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{yz}^{(1)}\\ -E_y^{(1)}\\ -H_y^{(1)}\end{bmatrix}
$$

$$
= -\frac{1}{2\pi}\left\{\begin{array}{l}\left[\frac{y-y_0}{(x-x_0)^2+(y-y_0)^2}+\frac{y+y_0}{(x-x_0)^2+(y+y_0)^2}\right]\mathbf{F} \\ -\alpha^{(1)}\left[\frac{x-x_0}{(x-x_0)^2+(y-y_0)^2}-\frac{x-x_0}{(x-x_0)^2+(y+y_0)^2}\right]\mathbf{A}^{(0)}\mathbf{B}\end{array}\right\}
$$

$$
-\frac{1}{2\pi}\sum_{n=0}^{\infty}Q^{n+1}\left\{\begin{array}{l}\left[\frac{y-y_0\pm 2(n+1)h}{(x-x_0)^2+(y-y_0\pm 2(n+1)h)^2}+\frac{y+y_0\pm 2(n+1)h}{(x-x_0)^2+(y+y_0\pm 2(n+1)h)^2}\right]\mathbf{F} \\ -\alpha^{(1)}\left[\frac{x-x_0}{(x-x_0)^2+(y-y_0\pm 2(n+1)h)^2}-\frac{x-x_0}{(x-x_0)^2+(y+y_0\pm 2(n+1)h)^2}\right]\mathbf{A}^{(0)}\mathbf{B}\end{array}\right\}, \tag{41c}
$$

$$
\begin{bmatrix}w^{(2)}\\ \varphi^{(2)}\\ \phi^{(2)}\end{bmatrix} =
$$

$$
-\frac{1}{2\pi}\sum_{n=0}^{\infty}(1+Q)Q^n\left\{\begin{array}{l}\frac{1}{\alpha^{(1)}}\left[\begin{array}{l}\ln((x-x_0)^2+(y-y_0+2nh)^2)^{1/2}\\ +\ln((x-x_0)^2+(y+y_0+2nh)^2)^{1/2}\end{array}\right](\mathbf{A}^{(0)})^{-1}\mathbf{F} \\ -\left[\tan^{-1}\frac{y-y_0+2nh}{x-x_0}-\tan^{-1}\frac{y+y_0+2nh}{x-x_0}\right]\mathbf{B}\end{array}\right\}, \tag{42a}
$$

$$
\begin{bmatrix}\tau_{xz}^{(2)}\\ D_x^{(2)}\\ B_x^{(2)}\end{bmatrix} = \alpha^{(2)}\mathbf{A}^{(0)}\begin{bmatrix}\gamma_{xz}^{(2)}\\ -E_x^{(2)}\\ -H_x^{(2)}\end{bmatrix}
$$

$$= -\frac{\alpha^{(2)}}{2\pi}\sum_{n=0}^{\infty}(1+Q)Q^n \left\{ \begin{array}{l} \frac{1}{\alpha^{(1)}}\left[\frac{x-x_0}{(x-x_0)^2+(y-y_0+2nh)^2}+\frac{x-x_0}{(x-x_0)^2+(y+y_0+2nh)^2}\right]\mathbf{F} \\ +\left[\frac{y-y_0+2nh}{(x-x_0)^2+(y-y_0+2nh)^2}-\frac{y+y_0+2nh}{(x-x_0)^2+(y+y_0+2nh)^2}\right]\mathbf{A}^{(0)}\mathbf{B} \end{array} \right\}, \tag{42b}$$

$$\begin{bmatrix} \tau_{yz}^{(2)} \\ D_y^{(2)} \\ B_y^{(2)} \end{bmatrix} = \alpha^{(2)}\mathbf{A}^{(0)} \begin{bmatrix} \gamma_{yz}^{(2)} \\ -E_y^{(2)} \\ -H_y^{(2)} \end{bmatrix}$$

$$= -\frac{\alpha^{(2)}}{2\pi}\sum_{n=0}^{\infty}(1+Q)Q^n \left\{ \begin{array}{l} \frac{1}{\alpha^{(1)}}\left[\frac{y-y_0+2nh}{(x-x_0)^2+(y-y_0+2nh)^2}+\frac{y+y_0+2nh}{(x-x_0)^2+(y+y_0+2nh)^2}\right]\mathbf{F} \\ -\left[\frac{x-x_0}{(x-x_0)^2+(y-y_0+2nh)^2}-\frac{x-x_0}{(x-x_0)^2+(y+y_0+2nh)^2}\right]\mathbf{A}^{(0)}\mathbf{B} \end{array} \right\}, \tag{42c}$$

where $Q = (\alpha^{(1)} - \alpha^{(2)})/(\alpha^{(1)} + \alpha^{(2)})$ is the reflection coefficient and $1+Q = 2\alpha^{(1)}/(\alpha^{(1)} + \alpha^{(2)})$ is the refraction coefficient for the homogeneous magnetoelectroelastic layered half-plane. It is noted that the generalized stress $\boldsymbol{\sigma}_y^{(j)}$ is continuous at the interface due to the continuity condition; however, the generalized stress $\sigma_x^{(j)}$ is discontinuous at the interface, and the results are

$$\boldsymbol{\sigma}_x^{(1)}(x,h) \neq \boldsymbol{\sigma}_x^{(2)}(x,h), \tag{43a}$$

$$\boldsymbol{\sigma}_y^{(1)}(x,h) = \boldsymbol{\sigma}_y^{(2)}(x,h) = -\frac{1}{2\pi}\sum_{n=0}^{\infty}\alpha^{(2)}(1+Q)Q^n$$

$$\times \left\{ \begin{array}{l} \frac{1}{\alpha^{(1)}}\left[\frac{(2n+1)h-y_0}{(x-x_0)^2+((2n+1)h-y_0)^2}+\frac{(2n+1)h+y_0}{(x-x_0)^2+((2n+1)h+y_0)^2}\right]\mathbf{F} \\ -\left[\frac{x-x_0}{(x-x_0)^2+((2n+1)h-y_0)^2}-\frac{x-x_0}{(x-x_0)^2+((2n+1)h+y_0)^2}\right]\mathbf{A}^{(0)}\mathbf{B} \end{array} \right\}. \tag{43b}$$

In short, if the material constants are continuous at the interface, then both generalized stresses $\boldsymbol{\sigma}_x^{(j)}$ and $\sigma_y^{(j)}$ are continuous at the interface, moreover, the first derivative of $\boldsymbol{\sigma}_y^{(j)}$ is also continuous at the interface. According to the results presented in this section, it is noted that the interfacial continuous characteristics of the generalized stresses can reduce the mismatch of stresses at the interface which can significantly prevent the interfacial fracture problem in more complicated multilayered structures.

For numerical illustrations, the functionally graded magnetoelectroelastic layered half-plane subjected to a concentrated force or a screw dislocation will be presented. Furthermore, the influence of functionally graded factor of the nonhomogeneous magnetoelectroelastic material is demonstrated, which will be useful for the future design and manufacturing of the functionally graded structures. The full-field distributions of the generalized stresses and strains with different functionally graded parameters will be presented and discussed in detail in the next section.

5 Numerical Results and Discussions

This section presents the full-field distributions of field quantities in the nonhomogeneous magnetoelectroelastic layered half-plane subjected to a line force or a screw dislocation. The interfacial continuous characteristics of the magnetoelectroelastic fields will be discussed. A computational program for the numerical calculation is conducted by using the analytical formulation of the solutions presented in previous sections. The magnetoelectroelastic material constants [Lee and Ma (2007)] for the numerical calculation are indicated as follows :

$$\mathbf{A}^{(0)} = \begin{bmatrix} 4.5\times10^{10}\ \mathrm{N/m} & 1.16\ \mathrm{C\,/\,m^2} & 496\ \mathrm{N\,/\,Am} \\ 1.16\ \mathrm{C\,/\,m^2} & -1.19\times10^{-9}\ \mathrm{C^2/Nm^2} & -5\times10^{-12}\ \mathrm{Ns/VC} \\ 496\ \mathrm{N\,/\,Am} & -5\times10^{-12}\ \mathrm{Ns/VC} & -5.3\times10^{-4}\ \mathrm{Ns^2/C^2} \end{bmatrix}$$

The contours of normalized shear stresses τ_{xz} and τ_{yz} for the nonhomogeneous layered half-plane subjected to a line force f_z at $(x,y)=(0,0.7h)$ with the functionally graded factors $\alpha^{(1)}=1$, $\alpha^{(2)}=2$, $\beta^{(1)}=3$, and $\beta^{(2)}=-1$ are indicated in Figs. 2(a) and 2(b), respectively. From Fig. 2(b), the continuity condition along the interface for τ_{yz} is satisfied, and the contour is symmetric with respect to the y axis. This figure also shows that τ_{yz} is zero on the free surface and the traction free boundary condition is satisfied. As we can see that the normalized contour τ_{xz} is zero along the y axis and is discontinuous along the interface due to the fact that material constants are discontinuous along the interface, i.e., $\alpha^{(1)} \neq \alpha^{(2)}$. Figures 2(c) and 2(d) show the contours of normalized shear stresses $\tau_{xz}h/c_{44}^{(0)}b_z$ and $\tau_{yz}h/c_{44}^{(0)}b_z$ for the nonhomogeneous layered half-plane subjected to a screw dislocation b_z at $(x,y)=(0,0.7h)$ with the functionally graded factors $\alpha^{(1)}=1$, $\alpha^{(2)}=2$, $\beta^{(1)}=3$, and $\beta^{(2)}=-1$, respectively.

In order to demonstrate the continuous characteristics of magnetoelectroelastic fields at the interface for continuous material constants. We will focus on the cases that the material constants are continuous at the interface, i.e., $\alpha^{(1)}=\alpha^{(2)}=1$. Figures 3(a)–3(d) show the normalized contours of stresses (τ_{xz} and τ_{yz}) and electric fields (E_x and E_y) for the nonhomogeneous layered half-plane subjected to a

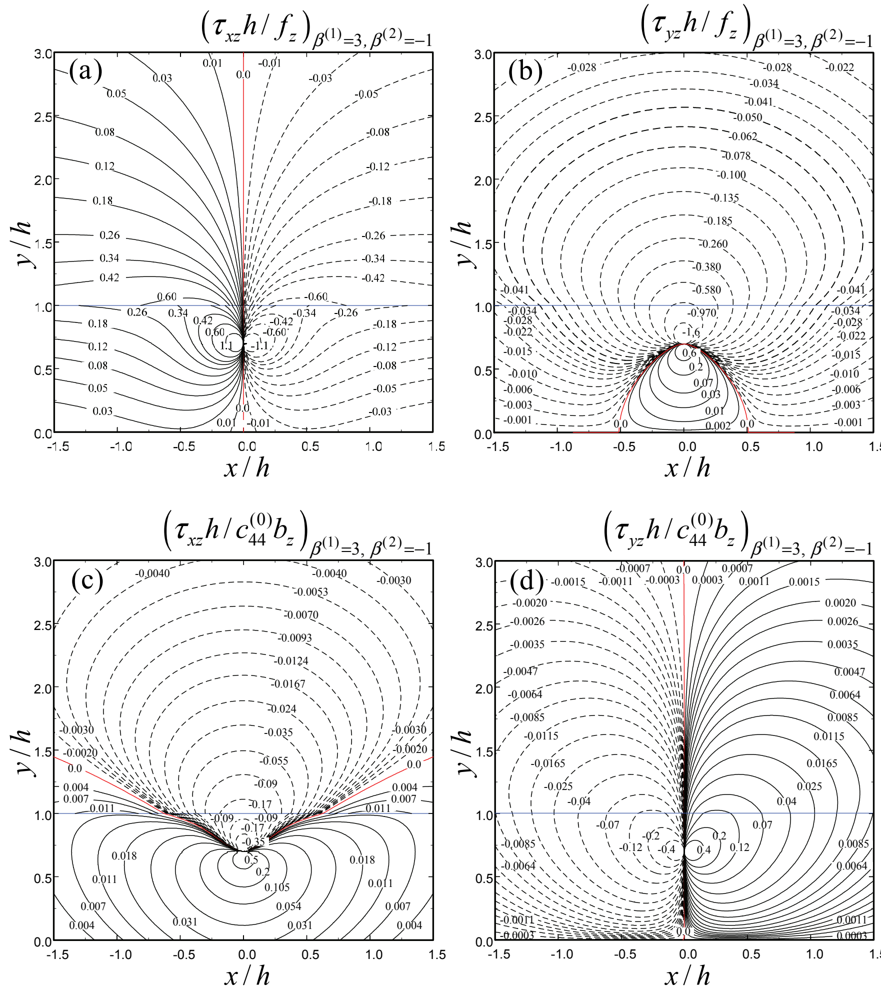

Figure 2: (a): Full-field distribution of shear stress τ_{xz} for the layered half-plane subjected to a concentrated force f_z in the layer with $\alpha^{(1)} = 1$, $\alpha^{(2)} = 2$, $\beta^{(1)} = 3$, and $\beta^{(2)} = -1$. (b): Full-field distribution of shear stress τ_{yz} for the layered half-plane subjected to a concentrated force f_z in the layer with $\alpha^{(1)} = 1$, $\alpha^{(2)} = 2$, $\beta^{(1)} = 3$, and $\beta^{(2)} = -1$. (c): Full-field distribution of shear stress τ_{xz} for the layered half-plane subjected to a screw dislocation b_z in the layer with $\alpha^{(1)} = 1$, $\alpha^{(2)} = 2$, $\beta^{(1)} = 3$, and $\beta^{(2)} = -1$. (d): Full-field distribution of shear stress τ_{yz} for the layered half-plane subjected to a screw dislocation b_z in the layer with $\alpha^{(1)} = 1$, $\alpha^{(2)} = 2$, $\beta^{(1)} = 3$, and $\beta^{(2)} = -1$.

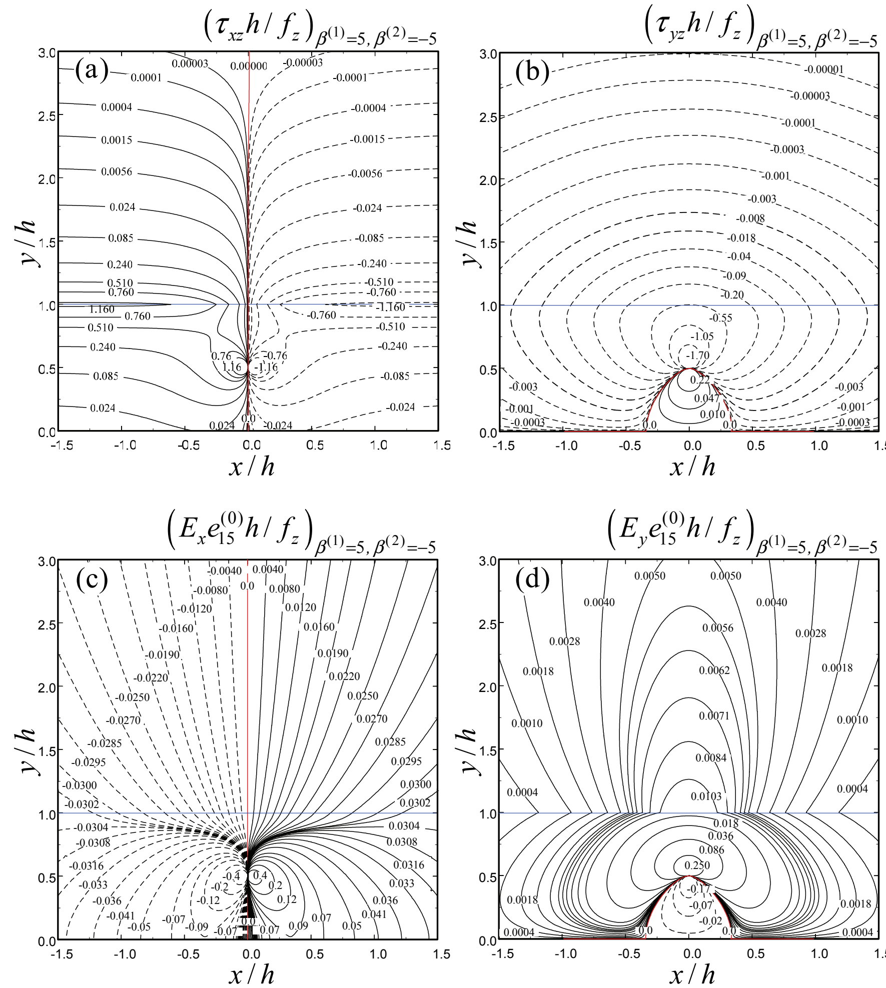

Figure 3: (a): Full-field distribution of shear stress τ_{xz} for the layered half-plane subjected to a concentrated force f_z in the layer with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = -5$. (b): Full-field distribution of shear stress τ_{yz} for the layered half-plane subjected to a concentrated force f_z in the layer with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = -5$. (c): Full-field distribution of electric field E_x for the layered half-plane subjected to a concentrated force f_z in the layer with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = -5$. (d): Full-field distribution of electric field E_y for the layered half-plane subjected to a concentrated force f_z in the layer with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = -5$.

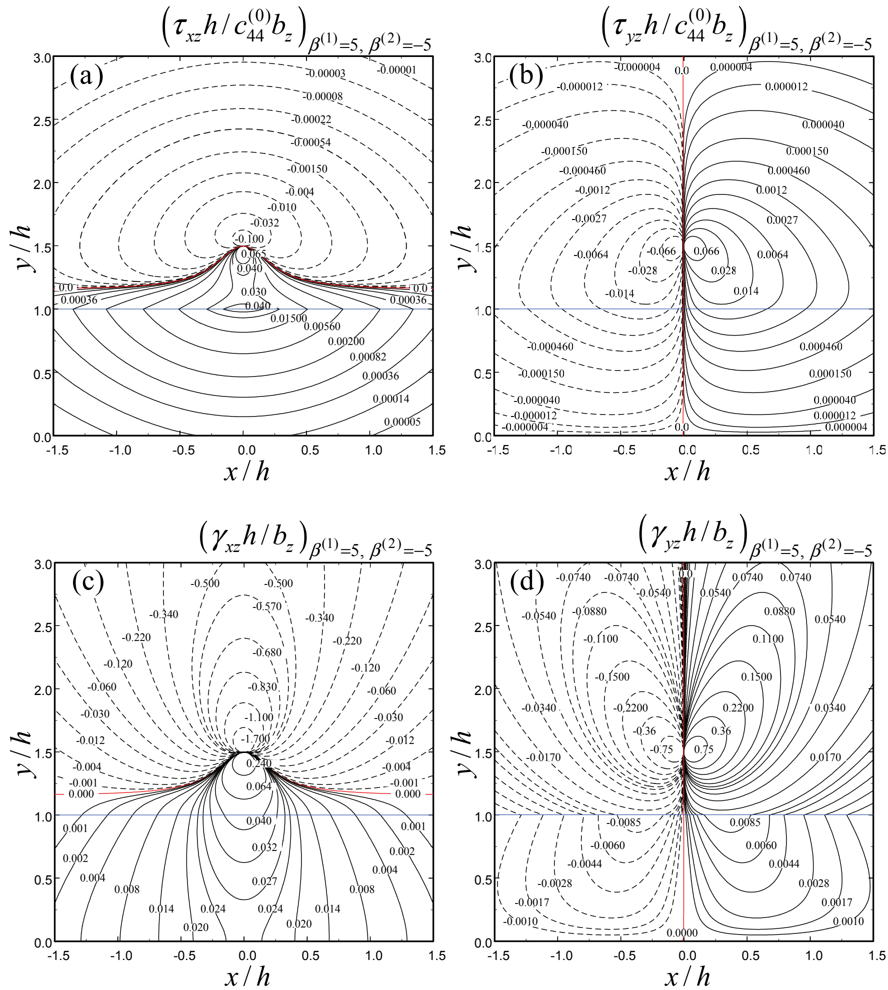

Figure 4: (a): Full-field distribution of shear stress τ_{xz} for the layered half-plane subjected to a screw dislocation b_z in the half-plane with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = -5$. (b): Full-field distribution of shear stress τ_{yz} for the layered half-plane subjected to a screw dislocation b_z in the half-plane with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = -5$. (c): Full-field distribution of shear strain γ_{xz} for the layered half-plane subjected to a screw dislocation b_z in the half-plane with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = -5$. (d): Full-field distribution of shear strain γ_{yz} for the layered half-plane subjected to a screw dislocation b_z in the half-plane with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = -5$.

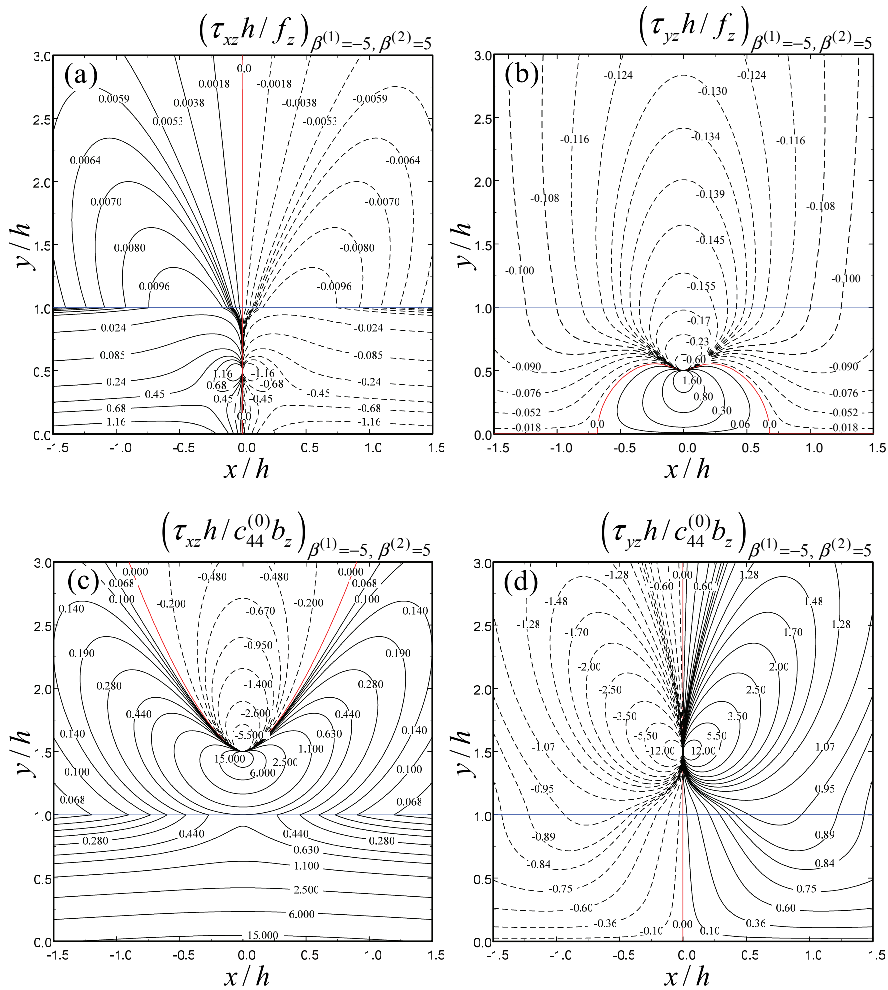

Figure 5: (a): Full-field distribution of shear stress τ_{xz} for the layered half-plane subjected to a concentrated force f_z in the layer with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = -5$, and $\beta^{(2)} = 5$. (b): Full-field distribution of shear stress τ_{yz} for the layered half-plane subjected to a concentrated force f_z in the layer with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = -5$, and $\beta^{(2)} = 5$. (c): Full-field distribution of shear stress τ_{xz} for the layered half-plane subjected to a screw dislocation b_z in the half-plane with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = -5$, and $\beta^{(2)} = 5$. (d): Full-field distribution of shear stress τ_{yz} for the layered half-plane subjected to a screw dislocation b_z in the half-plane with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = -5$, and $\beta^{(2)} = 5$.

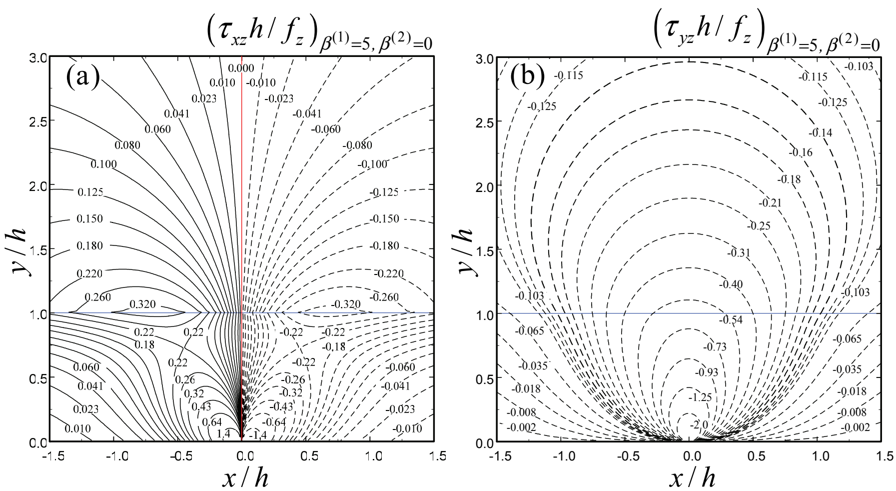

Figure 6: (a): Full-field distribution of shear stress τ_{xz} for the layered half-plane subjected to a concentrated force f_z on the free surface with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = 0$. (b): Full-field distribution of shear stress τ_{yz} for the layered half-plane subjected to a concentrated force f_z on the free surface with $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = 0$.

line force f_z at $(x,y) = (0,0.5h)$ with the functionally graded factors $\beta^{(1)} = 5$ and $\beta^{(2)} = -5$. Figures 4(a)–4(d) show the normalized contours of shear stresses (τ_{xz} and τ_{yz}) and shear strains (γ_{xz} and γ_{yz}) for the nonhomogeneous layered half-plane subjected to a screw dislocation b_z at $(x,y) = (0,1.5h)$ with the functionally graded factors $\beta^{(1)} = 5$ and $\beta^{(2)} = -5$. These figures show that all magnetoelectroelastic fields are continuous at the interface. Furthermore, it is worth noting that the first derivatives of τ_{yz}, E_x, and γ_{xz} are continuous at the interface which agrees with the result presented in Eq. (39b). These continuous characteristics are mainly due to the interfacial continuous material property (i.e., $\alpha^{(1)} = \alpha^{(2)}$). For the case of $\beta^{(1)} = -5$ and $\beta^{(2)} = 5$, Figs. 5(a) and 5(b) show full-field distributions of shear stresses τ_{xz} and τ_{yz} for the nonhomogeneous layered half-plane subjected to a line force f_z at $(x,y) = (0,0.5h)$ in the layer, respectively. Figs. 5(c) and 5(d) show that the full-field normalized contours of τ_{xz} and τ_{yz} for the nonhomogeneous layered half-plane subjected to a screw dislocation b_z at $(x,\ y) = (0,1.5h)$ in the half-plane, respectively. It is noted that the full-field distributions presented in Figs. 3(a) and 3(b) (Figs. 4(a) and 4(b)) are quite different from Figs. 5(a) and 5(b) (Figs. 5(c) and 5(d)) due to the switch of the functionally graded factors.

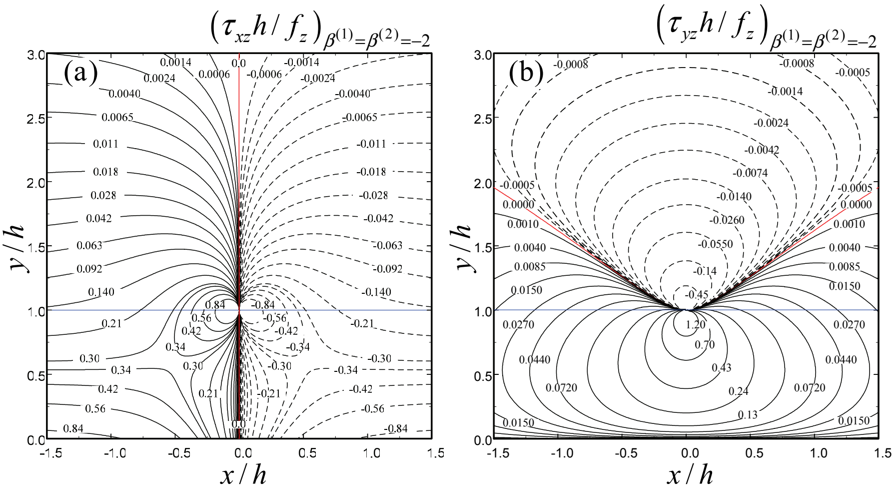

Figure 7: (a): Full-field distribution of shear stress τ_{xz} for the nonhomogeneous half-plane subjected to a concentrated force f_z at the interface with $\alpha^{(1)} = \alpha^{(2)} = 1$ and $\beta^{(1)} = \beta^{(2)} = -2$. (b): Full-field distribution of shear stress τ_{yz} for the nonhomogeneous half-plane subjected to a concentrated force f_z at the interface with $\alpha^{(1)} = \alpha^{(2)} = 1$ and $\beta^{(1)} = \beta^{(2)} = -2$.

Finally, Figs. 6(a) and 6(b) present the normalized contours of shear stresses τ_{xz} and τ_{yz} for a nonhomogeneous magnetoelectroelastic thin layer bonded to a homogeneous magnetoelectroelastic half-plane (i.e., $\alpha^{(1)} = \alpha^{(2)} = 1$, $\beta^{(1)} = 5$, and $\beta^{(2)} = 0$) subjected to a line force f_z applied on the free surface at $(x,y) = (0,0)$, respectively. Figs. 7(a) and 7(b) show the contours of normalized shear stresses τ_{xz} and τ_{yz} of the degenerate case for a nonhomogeneous magnetoelectroelastic half-plane ($\alpha^{(1)} = \alpha^{(2)} = 1$ and $\beta^{(1)} = \beta^{(2)} = -2$) subjected to a line force f_z applied at the interface $(x,y) = (0,h)$, respectively.

6 Concluding Remarks

This study presents full-field solutions for a functionally graded magnetoelectroelastic layered half-plane subjected to generalized concentrated forces and screw dislocations applied either on the layer or on the half-plane. The analytical solutions for the nonhomogeneous magnetoelectroelastic problem are obtained using the Fourier transform technique. It is indicated in this study that if the functionally graded magnetoelectroelastic material properties are continuous at the interface, then the elastic, electric, and magnetic fields are all continuous at the interface.

Furthermore, the first derivative of the generalized stress $\boldsymbol{\sigma}_y^{(j)}$ is also continuous at the interface. If the functionally graded effect is neglected, the results are reduced to the solutions of the transversely isotropic magnetoelectroelastic homogeneous layered half-plane problem. A computational program for numerical calculation of the full field analysis is easily constructed using the analytical solutions. Detailed numerical results of full-field distributions with different functionally graded parameters for applying a concentrated force or a screw dislocation in the nonhomogeneous magnetoelectroelastic layered half-plane are presented and discussed in detail.

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References

Bhangale, R.K.; Ganesan, N. (2006): Static analysis of simply supported functionally graded and layered magneto-electro-elastic plates. *International Journal of Solids and Structures*, vol. 43, pp. 3230-3253.

Chen, J.; Liu, Z.; Zou, Z. (2003): Dynamic response of a crack in a functionally graded interface of two dissimilar piezoelectric half-planes. *Archive of Applied Mechanics*, vol. 72, pp. 686-696.

Feng, W.J.; Su, R.K.L. (2006): Dynamic internal crack problem of a functionally graded magneto-electro-elastic strip. *International Journal of Solids and Structures*, vol. 43, pp. 5196-5216.

Feng, W.J.; Han, X.; Li, Y.S. (2009): Fracture analysis for two-dimensional plane problems of nonhomogeneous magneto-electro-thermo-elastic plates subjected to thermal shock by using the meshless local Petrov-Galerkin method. *CMES: Computer Modeling in Engineering & Sciences*, vol. 48, pp. 1-26.

Guan, Q.; He, S.R. (2005): Two-dimensional analysis of piezoelectric/piezomagnetic and elastic media. *Composite Structures*, vol. 69, pp. 229-237.

Hao, R.J.; Liu, J.X. (2006): Interaction of a screw dislocation with a semi-infinite interfacial crack in a magneto-electro-elastic bi-material. *Mechanics Research Communications*, vol. 33, pp. 415-424.

Hu, K.Q.; Qin, Q.H.; Kang, Y.L. (2007): Anti-plane shear crack in a magneto-electroelastic layer sandwiched between dissimilar half spaces. *Engineering Fracture Mechanics*, vol. 74, pp. 1139-1147.

Hu, K.Q.; Zhong, Z.; Jin, B. (2005): Anti-plane shear crack in a functionally gradient piezoelectric layer bonded to dissimilar half spaces. International Journal of Mechanical Sciences, vol. 47, pp. 82-93.

Kwon, S.M. (2003): Electrical nonlinear anti-plane shear crack in a functionally graded piezoelectric strip. *International Journal of Solids and Structures*, vol. 40, pp. 5649-5667.

Lee, J.M.; Ma, C.C. (2007): Analytical full-field solutions of a magnetoelectroelastic layered half-plane. *Journal of Applied Physics*, vol. 101, 083502.

Li, R.; Kardomateas, G.A. (2006): The mode III interface crack in piezo-electro-magneto-elastic dissimilar bimaterials. *ASME Journal of Applied Mechanics*, vol. 73, pp. 220-227.

Ma, C.C.; Lee, J.M. (2007): Image forces of screw dislocations in a magnetoelectroelastic layered half-plane. Journal of Applied Physics, vol. 101, 123513.

Ma, L.; Li, J.; Abdelmoula, R.; Wu, L.Z. (2007): Mode III crack problem in a functionally graded magneto-electro-elastic strip. *International Journal of Solids and Structures*, vol. 44, pp. 5518-5537.

Ma, L.; Wu, L.Z.; Zhou, Z.G.; Guo, L.C. (2005): Fracture analysis of a functionally graded piezoelectric strip. *Composite Structures*, vol. 69, pp. 294-300.

Ou, Y.L.; Chue, C.H. (2006): Two mode III internal cracks located within two bonded functionally graded piezoelectric half planes respectively. *Archive of Applied Mechanics*, vol. 75, pp. 364-378.

Pan, E.; Han, F. (2005): Exact solution for functionally graded and layered magneto-electro-elastic plates. *International Journal of Engineering Science*, vol. 43, pp. 321-339.

Sladek, J.; Sladek, V.; Tan, C.L.; Atluri, S.N. (2008): Analysis of transient heat conduction in 3D anisotropic functionally graded solids by the MLPG method. *CMES: Computer Modeling in Engineering & Sciences*, vol. 32, pp. 161-174.

Ueda, S. (2006): A finite crack in a semi-infinite strip of a graded piezoelectric material under electric loading. *European Journal of Mechanics A/Solids*, vol. 25, pp. 250-259.

Wang, B.L.; Mai, Y.W. (2006): Closed-form solution for an antiplane interface crack between two dissimilar magnetoelectroelastic layers. *ASME Journal of Applied Mechanics*, vol. 73, pp. 281-290.

Zhou, Z.G.; Wang, B. (2008): An interface crack between two dissimilar functionally graded piezoelectric/piezomagnetic material half infinite planes subjected to the harmonic anti-plane shear stress waves. *International Journal of Applied Electromagnetics and Mechanics*, vol. 27, pp. 117-132.

Zhou, Z.G.; Wu, L.Z.; Wang, B. (2005): The behavior of a crack in functionally graded piezoelectric/piezomagnetic materials under anti-plane shear loading. *Archive of Applied Mechanics*, vol. 74, pp. 526-535.