

A Time-Domain FEM-BEM Iterative Coupling Algorithm to Numerically Model the Propagation of Electromagnetic Waves

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Abstract: In this work, a time-domain finite element – boundary element iterative coupling technique is presented in order to analyse electromagnetic scattering from two-dimensional inhomogeneous objects. In the iterative coupling algorithm, the domain of the original problem is subdivided into sub-domains and each sub-domain is analysed independently (as an uncoupled model) taking into account successive renewals of variables at common interfaces. In order to improve the effectiveness of the iterative coupling approach, the evaluation of an optimised relaxation parameter is discussed, taking into account the minimisation of a square error functional. The algorithm that arises is efficient, accurate and flexible. At the end of the paper, numerical examples are presented, illustrating the potentialities of the proposed methodology.

Keyword: Finite Elements; Boundary Elements; Time-Domain Analysis; Iterative Coupling; Electromagnetic Waves.

1 Introduction

In engineering analysis there is no single numerical method that can most properly handle all problems. The Finite Element Method (FEM), for instance, is well suited for modelling inhomogeneous and anisotropic media, as well as for dealing with non-linear behaviour. For systems with infinite extension and regions of high local concentrations and/or field variations, however, the use of the Boundary Element Method (BEM) is more advantageous. In fact, it did not take long until some researchers started to com-

bine the FEM and the BEM in order to profit from their respective advantages and try to avoid their disadvantages. Up to now, although a considerable amount of publications is available concerning FEM-BEM coupled analysis, few publications concentrate on the topic when time-domain electromagnetic modelling is focused.

According to Jiao *et al.* (2001), the first works on the theme seems to date from the beginning of the decade. Jiao *et al.* (2001) presented a time-domain finite element – boundary integral method to analyse electromagnetic scattering from two-dimensional inhomogeneous objects. Later on, alternative approaches have been proposed and three-dimensional analyses have been considered (e.g., Jiao *et al.*, 2002; McCowen *et al.*, 2003; Qiu *et al.*, 2007 etc.). Taking into account transformed-domain analyses (especially frequency-domain analyses), FEM-BEM coupling techniques are well established, and several works are currently available considering electromagnetic modelling, namely: Stupfel (2001), Liu and Jin (2001, 2002), Volakis *et al.* (2004), Tzoulis and Eibert (2005), Botha and Jin (2005), Eibert (2007) etc. (for further related topics, the reader is referred to Edelvik and Ledfelt, 2002; Bleszynski *et al.*, 2004; Jose *et al.*, 2004; Young *et al.*, 2005; Frangi *et al.*, 2006 etc.).

For most FEM-BEM coupling procedures, a coupled system of equations is established, which afterwards has to be solved at each time-step of the time-domain analysis. Due to the coupling of usually large FEM matrices with fully populated non-symmetric BEM matrices, the matrices involved in these coupled analyses are large and not banded or sparsely populated. This presents a combination of drawbacks, which drastically increases the complexity and the computational costs of the

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methodology (optimised solvers can no longer be employed etc.). In order to avoid this severe disadvantage of standard FEM-BEM coupled formulations, iterative coupling procedures have been developed.

Taking into account time-domain hyperbolic applications, an iterative FEM-BEM coupling algorithm was introduced by Soares *et al.* (2004), in order to analyse dynamic two-dimensional elastoplastic models. Later on, the procedure was extended, being applied to model solid-fluid interaction (Soares *et al.*, 2005), as well as to simulate three-dimensional and axisymmetric mechanical applications (von Estorff and Hagen, 2005; Warszawski *et al.*, 2008). As reported in these previous works, iterative FEM-BEM coupling algorithms exhibit several advantages when compared to standard coupling schemes, as for instance: (i) FEM and BEM sub-domains can be analysed separately, leading to smaller and better-conditioned systems of equations (different solvers, suitable for each sub-domain, may be employed); (ii) only interface routines are required when one wishes to use existing codes to build coupling algorithms; (iii) non-linearities can be taken into account in the same iterative loop needed for the coupling, thus, consideration of non-linear models (within the FE sub-domains) does not introduce a relevant CPU time increase; etc..

Recently, Yılmaz *et al.* (2007) presented a time-domain FEM-BEM coupling methodology based on an outer-inner iterative scheme to analyse electromagnetic waves. In that work, the so-called “outer” iterative process was adopted in order to more efficiently deal with the FEM-BEM coupled system of equations, uncoupling it, and the “inner” iterative process was related to the solution of the uncoupled systems of equations that arose by iterative solvers. As reported by the authors, the technique provided an efficient and stable methodology. The current work is an advance on the topic: here, optimised iterative procedures are proposed, rendering a more effective and robust technique.

In the present work, a time-domain FEM-BEM iterative coupling algorithm to analyse the propaga-

tion of electromagnetic waves through inhomogeneous media is described. Two-dimensional models are focused and the suggested procedures can be easily extended to more general applications (e.g., three-dimensional analyses etc.). In the text that follows, first, the electromagnetic wave propagation governing equations are presented and, next, the basic FEM and BEM discretization techniques are discussed. In the sequence, the iterative FEM-BEM coupling methodology is described and an expression for an optimal relaxation parameter is introduced. At the end of the paper, numerical applications are considered, illustrating the accuracy and potentialities of the proposed technique.

2 Governing equations

Maxwell’s equations in differential form can be written as follows:

$$e_{ijk}E_{k,j} = -\dot{B}_i \quad (1a)$$

$$e_{ijk}H_{k,j} = \dot{D}_i + J_i \quad (1b)$$

$$D_{i,i} = \rho \quad (1c)$$

$$B_{i,i} = 0 \quad (1d)$$

where indicial notation for Cartesian axes is considered and e_{ijk} stands for the permutation symbol (also known as alternator tensor). Inferior commas and overdots indicate partial space and time derivatives, respectively (i.e., $V_{i,j} = \partial V_i / \partial x_j$ and $\dot{V}_i = \partial V_i / \partial t$, where $V_i(X, t)$ stands for a generic vector field representation and X and t denote its spatial and temporal arguments, respectively).

In equations (1), E_i and H_i are the electric and magnetic field intensity components, respectively; D_i and B_i represent the electric and magnetic flux density, respectively; and J_i and ρ stand for the electric current and electric charge density, respectively. The constitutive relations between the field quantities are specified as follows:

$$D_i = \varepsilon E_i \quad (2a)$$

$$B_i = \mu H_i \quad (2b)$$

$$J_i = \sigma E_i \quad (2c)$$

where the parameters ε , μ and σ denote, respectively, the permittivity, permeability and conductivity of the medium.

Combining equations (1) and (2), vectorial wave equations describing the electric and the magnetic field can be obtained, as is indicated below:

$$e_{mni}(\mu^{-1}e_{ijk}E_{k,j})_{,n} + \varepsilon \ddot{E}_m = -\dot{J}_m \quad (3a)$$

$$e_{mni}(\varepsilon^{-1}e_{ijk}H_{k,j})_{,n} + \mu \ddot{H}_m = e_{mni}(\varepsilon^{-1}J_i)_{,n} \quad (3b)$$

where the wave propagation velocity of the medium is specified as $c = (\varepsilon\mu)^{-1/2}$.

Taking into account two-dimensional applications, equations (3) can be simplified and written in a unique general form:

$$(\kappa^{-1}\phi_{,i})_{,i} - \nu \ddot{\phi} = \gamma \quad (4)$$

where ϕ is a generic representation for an electric (E_k) or magnetic (H_k) field intensity component (e.g., $i = 1, 2$ and $k = 3$) and γ stands for a generic source term. κ and ν represent μ or ε , according to the case of analysis.

Once the governing differential equation is established, temporal and spatial boundary conditions must be defined. The spatial boundary conditions for the model in focus are:

$$\phi = \bar{\phi} \quad (5a)$$

$$\theta = \phi_{,i}n_i = \bar{\theta} \quad (5b)$$

where equation (5a) stands for essential (or Dirichlet) boundary conditions and equation (5b) stands for natural (or Neumann) boundary conditions (n_i represents an outward unit vector normal to the boundary). In equations (5), overbars indicate prescribed values.

At the interface between two media, field continuity conditions are defined as follows:

$$(\phi)_+ = (\phi)_- \quad (6a)$$

$$(\kappa^{-1}\theta)_+ = -(\kappa^{-1}\theta)_- \quad (6b)$$

which are of great importance in a FEM-BEM coupling context.

In the sections that follow, the numerical discretization of the above-presented governing equations is briefly discussed, taking into account finite element and boundary element techniques. In the sequence, the FEM-BEM coupling algorithm is presented.

3 Finite element modelling

In a finite element approach, the incognita field is spatially interpolated within the element, as indicated below:

$$\phi(X, t) = N_\alpha(X)\phi_\alpha(t) \quad (7)$$

where N represents element spatial interpolation functions and greek subscripts stand for an element internal numeration (element nodes or edges).

Taking into account electromagnetic wave propagation phenomena, the time-domain system of equations that arises, once finite element spatial discretization is considered (equation (7)), is given by:

$$\mathbf{M}\ddot{\Phi}^n + \mathbf{K}\Phi^n = \mathbf{F}^n \quad (8)$$

where Φ is a generic vector describing electric or magnetic field components and \mathbf{F} is a vector of generalized applied sources. The superscript n stands for the current time of analysis. The matrix and vector entries involved in equation (8) are defined, at element level, as follows:

$$M_{\alpha\beta} = \int_{\Omega_e} \nu N_\alpha N_\beta d\Omega \quad (9a)$$

$$K_{\alpha\beta} = \int_{\Omega_e} \kappa^{-1} (N_{,i})_\alpha (N_{,i})_\beta d\Omega \quad (9b)$$

$$F_\alpha = \int_{\Gamma_e} N_\alpha \kappa^{-1} \bar{\theta} d\Gamma - \int_{\Omega_e} N_\alpha \gamma d\Omega \quad (9c)$$

where Γ_e and Ω_e stand for the boundary and the domain of the element, respectively.

In order to discretize equation (8) in the time domain, the Newmark method is here employed. In the Newmark method, the following relations are considered:

$$\ddot{\Phi}^n = (1/(\eta_1\Delta t^2))(\Phi^n - \Phi^{n-1}) - (1/(\eta_1\Delta t))\dot{\Phi}^{n-1} - (1/(2\eta_1) - 1)\ddot{\Phi}^{n-1} \quad (10a)$$

$$\dot{\Phi}^n = \dot{\Phi}^{n-1} + (\Delta t(1 - \eta_2))\ddot{\Phi}^{n-1} + (\eta_2\Delta t)\ddot{\Phi}^n \quad (10b)$$

where $\eta_2 \geq 0.5$ and $\eta_1 \geq 0.25(0.5 + \eta_2)^2$ are the newmark parameters and Δt is the selected time-step.

After introducing equations (10) into the system of equations (8), as well as considering the boundary conditions of the problem, the following system of equations arises, which enables the computation of the transient FEM response at time t^n :

$$\mathbf{A}\mathbf{X}^n = \mathbf{B}^n \quad (11)$$

where \mathbf{A} and \mathbf{B} are the FEM effective matrix and vector, respectively, and the entries of \mathbf{X} are the unknown variables (one should observe that vector \mathbf{B} accounts for boundary prescribed conditions and domain sources, as well as some previous step contributions).

4 Boundary element modelling

In a boundary element approach, the incognita fields (mixed formulation) are temporally and spatially interpolated within the element, as indicated below:

$$\phi(X, t) = N_\alpha(X) M^m(t) \phi_\alpha^m \quad (12a)$$

$$\theta(X, t) = N_\alpha(X) M^m(t) \theta_\alpha^m \quad (12b)$$

where, once again, N represents element spatial interpolation functions and greek subscripts stand for an element internal numeration. M represents temporal interpolation functions (one should keep in mind that different interpolation functions may be considered regarding the ϕ and θ incognita fields).

Taking into account electromagnetic wave propagation phenomena, the system of equations that arises, once time-domain boundary element spatial and temporal discretization is considered (equation (12)), is given by:

$$\mathbf{C}\Phi^n = \mathbf{G}^{nm}\Theta^m - \mathbf{H}^{nm}\Phi^m + \mathbf{S}^n \quad (13)$$

where $m = 1, \dots, n$; \mathbf{C} is a geometric matrix and \mathbf{G} and \mathbf{H} are influence matrices. once again, equation (13) stands for a general expression: Φ is a generic vector describing electric or magnetic field components and Θ is related to the spatial derivatives of these components. \mathbf{S} is a vector accounting for generalized source terms.

The entries of the influence matrices involved in equation (13), as well as of the source vector, are given by:

$$G_{\alpha\beta}^{nm} = \int_{\Gamma} N_\beta \int_0^{t^n} \Phi_\alpha^n M^m d\tau d\Gamma \quad (14a)$$

$$H_{\alpha\beta}^{nm} = \int_{\Gamma} N_\beta \int_0^{t^n} \Theta_\alpha^n M^m d\tau d\Gamma \quad (14b)$$

$$S_\alpha^n = \int_{\Omega} \int_0^{t^n} \Phi_\alpha^n \gamma d\tau d\Omega \quad (14c)$$

where Φ and Θ are the fundamental solutions of the time-domain two-dimensional model. Φ is defined as follows ($\Theta = \Phi_{,i}n_i$):

$$\begin{aligned} \Phi_\alpha^n &= \Phi(X, t^n; X_\alpha, \tau) \\ &= \frac{c}{2\pi} (c^2(t^n - \tau)^2 - r^2)^{-1/2} H[c(t^n - \tau) - r] \end{aligned} \quad (15)$$

where $r = |X - X_\alpha|$ is the distance between the observation and the collocation point and H stands for the heaviside function.

After considering the boundary conditions of the problem, the following system of equations arises from expression (13), which enables the computation of the transient BEM response at time t^n :

$$\mathbf{A}\mathbf{X}^n = \mathbf{B}^n \quad (16)$$

where \mathbf{A} and \mathbf{B} are the BEM effective matrix and vector, respectively, and the entries of \mathbf{X} are the unknown variables (one should observe that vector \mathbf{B} accounts for boundary prescribed conditions, domain discretized terms and time convolution contributions).

5 Finite element – boundary element coupling

In the present work, the finite element – boundary element coupling is carried out by means of an iterative procedure. In this iterative approach, each sub-domain of the model is analysed independently and a successive renewal of the variables at the common interfaces is performed, until convergence is achieved. In order to maximize

the efficiency and robustness of the proposed iterative coupling algorithm, the evaluation of an optimised relaxation parameter is introduced, taking into account the minimisation of a square error functional.

The notation ${}_{F/B}^{(k)}\mathbf{V}$ is here adopted in order to better explain the iterative coupling method: according to this notation, a variable \mathbf{V} , computed by FEM or BEM techniques (${}_{F/B}\mathbf{V}$), is at a common interface, at an iterative-step k (${}^{(k)}\mathbf{V}$).

Initially, in the FEM-BEM iterative coupling, the FEM sub-domains are analysed and the FEM electromagnetic field components at the common interfaces are evaluated (${}_{F}^{(k+\varphi)}\Phi^n$), as described in section 3. In the sequence, a relaxation parameter φ is introduced in order to ensure and/or to speed up convergence, such that:

$${}_{F}^{(k+1)}\Phi^n = (\varphi)_{F}^{(k+\varphi)}\Phi^n + (1-\varphi)_{F}^{(k)}\Phi^n \quad (17)$$

Once ${}_{F}^{(k+1)}\Phi^n$ is computed, the field continuity equation (equation (6a)) is considered, providing the following relation:

$${}_{B}^{(k+1)}\overline{\phi}^n = {}_{F}^{(k+1)}\phi^n \quad (18)$$

which enables the computation of the BEM essential prescribed boundary conditions at the common interfaces. In the sequence, the BEM sub-domains are analysed, as described in section 4, and vector ${}_{B}^{(k+1)}\Theta^n$ is evaluated at the common interfaces.

Once ${}_{B}^{(k+1)}\Theta^n$ is computed, the field derivative continuity equation (equation (6b)) is considered, providing the following relation:

$${}_{F}^{(k+1)}\overline{\theta}^n = -{}_{B}^{(k+1)}\theta^n ({}_B\mathbf{K}^{-1}/{}_F\mathbf{K}^{-1}) \quad (19)$$

which enables the computation of the FEM natural prescribed boundary conditions at the common interfaces. In the sequence, the FEM sub-domains are once again analysed, repeating the whole process until convergence is achieved.

As can be observed, the iterative coupling approach allows BEM and FEM sub-domains to be analysed separately, leading to smaller and better-conditioned systems of equations (different solvers, suitable for each sub-domain, may be

employed). Moreover, the proposed procedure is easy to implement and only interface routines are required when one wishes to use existing codes to build coupling algorithms.

The effectiveness of the iterative coupling methodology is intimately related to the relaxation parameter selection: an inappropriate selection for φ can drastically increase the number of iterations in the analysis or, even worse, make convergence unfeasible. Once appropriate φ values are considered, convergence is usually achieved in quite few iterative steps, providing an efficient and robust FEM-BEM coupling technique. In the next sub-section, an expression for an optimal relaxation parameter is deduced.

5.1 Optimal relaxation parameter

In order to evaluate an optimal relaxation parameter, the following square error functional is here minimized:

$$f(\varphi) = \left\| {}_{B}^{(k+1)}\Phi^n(\varphi) - {}_{B}^{(k)}\Phi^n(\varphi) \right\|^2 \quad (20)$$

Taking into account the relaxation of the electromagnetic field components for the $(k+1)$ and (k) iterations, equations (21a) and (21b) may be written, regarding equations (17)-(18):

$${}_{B}^{(k+1)}\Phi^n = (\varphi)_{F}^{(k+\varphi)}\Phi^n + (1-\varphi)_{F}^{(k)}\Phi^n \quad (21a)$$

$${}_{B}^{(k)}\Phi^n = (\varphi)_{F}^{(k+\varphi-1)}\Phi^n + (1-\varphi)_{F}^{(k-1)}\Phi^n \quad (21b)$$

Substituting equations (21) into equation (20) yields:

$$\begin{aligned} f(\varphi) &= \left\| \varphi^{(k+\varphi)}\mathbf{W} + (1-\varphi)_{F}^{(k)}\mathbf{W} \right\|^2 \\ &= \varphi^2 \left\| {}_{F}^{(k+\varphi)}\mathbf{W} \right\|^2 + 2\varphi(1-\varphi) \langle {}_{F}^{(k+\varphi)}\mathbf{W}, {}_{F}^{(k)}\mathbf{W} \rangle \\ &\quad + (1-\varphi)^2 \left\| {}_{F}^{(k)}\mathbf{W} \right\|^2 \end{aligned} \quad (22)$$

where the inner product definition is employed (e.g., $\langle \mathbf{W}, \mathbf{W} \rangle = \|\mathbf{W}\|^2$) and new variables, as defined in equation (23), are considered.

$${}_{F}^{(k+\lambda)}\mathbf{W} = {}_{F}^{(k+\lambda)}\Phi^n - {}_{F}^{(k+\lambda-1)}\Phi^n \quad (23)$$

To find the optimal φ that minimizes the functional $f(\varphi)$, equation (22) is differentiated with

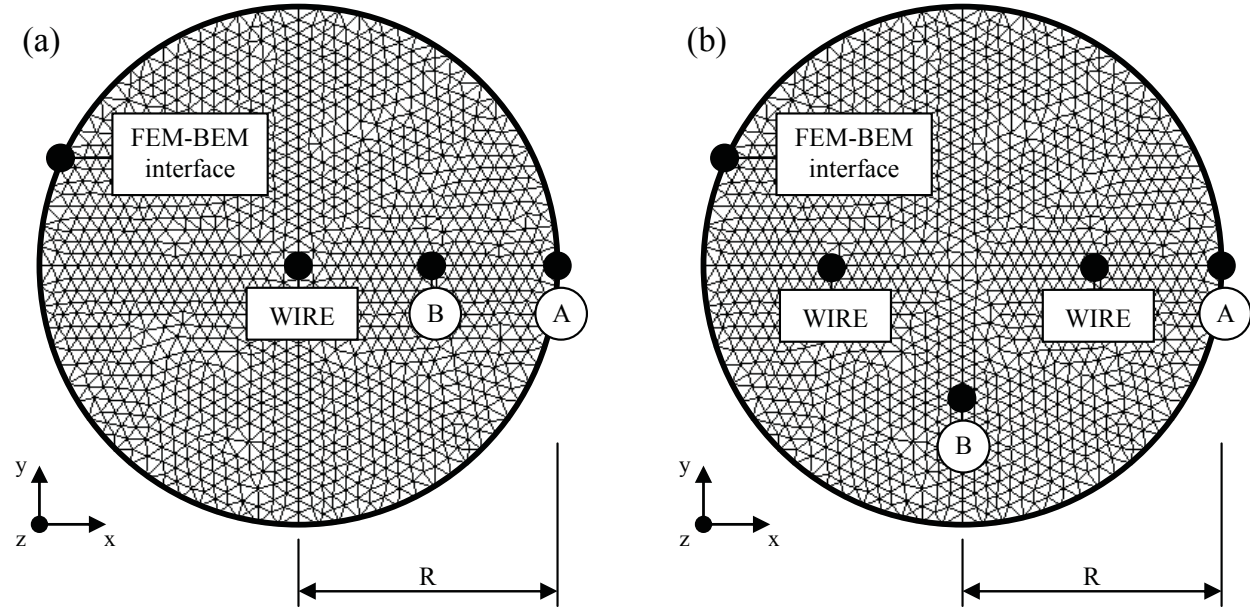


Figure 1: Sketch of the model and adopted spatial discretization for the homogeneous medium analysis: (a) case 1 – one wire; (b) case 2 – two wires.

respect to φ and the result is set to zero, as described below:

$$\varphi \|(k+\varphi)\mathbf{W}\|^2 + (1-2\varphi) \langle (k+\varphi)\mathbf{W}, (k)\mathbf{W} \rangle + (\varphi-1) \|(k)\mathbf{W}\|^2 = 0 \quad (24)$$

Re-arranging the terms in equation (24) yields:

$$\varphi = \langle (k)\mathbf{W}, (k)\mathbf{W} - (k+\varphi)\mathbf{W} \rangle / \|(k)\mathbf{W} - (k+\varphi)\mathbf{W}\|^2 \quad (25)$$

which is an easy to implement expression that provides an optimal value for the relaxation parameter φ , at each iterative step.

It is important to note that the relation $0 < \varphi \leq 1$ must hold. In the present work, the optimal relaxation parameter is evaluated according to equation (25) and if $\varphi \notin (0.01; 1.00)$ the previous iterative-step relaxation parameter is adopted. For the first iterative step, $\varphi = 0.5$ is selected.

6 Numerical Applications

In the next sub-sections, some numerical applications are presented, illustrating the potentialities of the proposed methodology. In the first application, the electromagnetic fields associated to in-

finitely long wires carrying polynomial-like time-dependent currents are analysed. In the second example, a three media model is considered and the electromagnetic wave propagation through the different materials, due to a time-sinusoidal current, is analysed.

For all the applications that follow, within the FEM sub-domains, the trapezoidal rule ($\eta_1 = 0.25$ and $\eta_2 = 0.50$) is considered for time integration and linear finite elements are adopted. For the BEM sub-domains, spatial discretization based on linear boundary elements is adopted and linear and piecewise constant time interpolation functions are considered for the electromagnetic field components and their directional derivatives, respectively. The convergence of the iterative coupling process is analysed based on the FEM computed fields and residual norms (in the present work, a tolerance error of 10^{-3} is selected).

6.1 Homogeneous medium

In the present application, the electromagnetic fields surrounding infinitely long wires are studied (Soares and Vinagre, 2008). Two cases of analysis are focused, namely: (a) case 1, where one wire is considered; (b) case 2, where two

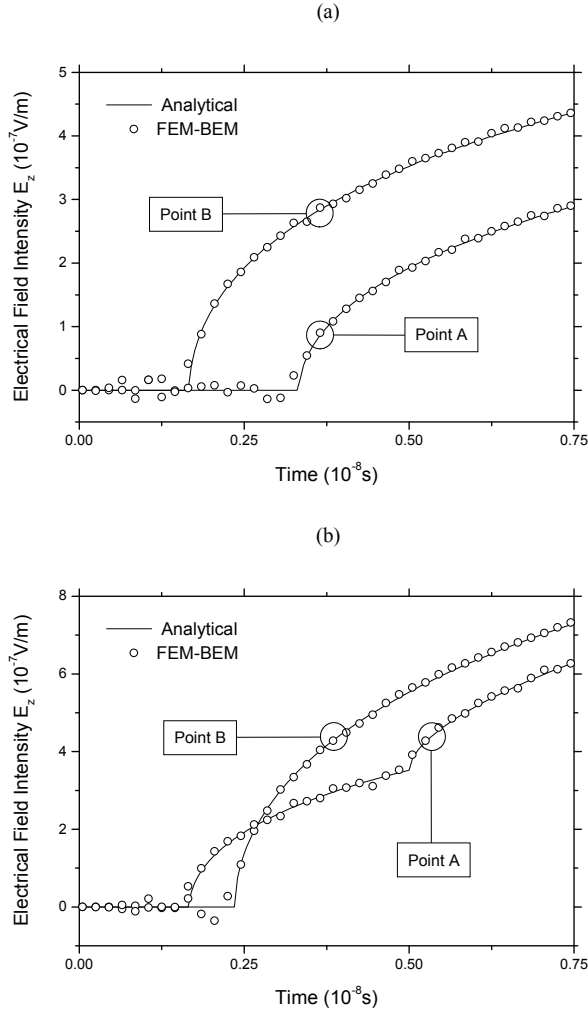


Figure 2: Time history results for the electric intensity field at points A and B considering $I(t) = t$: (a) case 1; (b) case 2.

wires are employed. For both cases, the wires are carrying time-dependent currents (i.e., $I(t) = t$ or $I(t) = t^2$) and they are located along the adopted z-axis. A sketch of the model is depicted in Fig.1. The adopted spatial discretization is also described in Fig.1: 2344 triangular finite elements and 80 boundary elements are employed in the analyses (the radius of the FEM-BEM interface is defined by $R = 1m$). For temporal discretization, the selected time-step is given by $\Delta t = 5 \cdot 10^{-11}s$. The physical properties of the medium (air) are: $\mu = 1.2566 \cdot 10^{-6}H/m$ and $\epsilon = 8.8544 \cdot 10^{-12}F/m$.

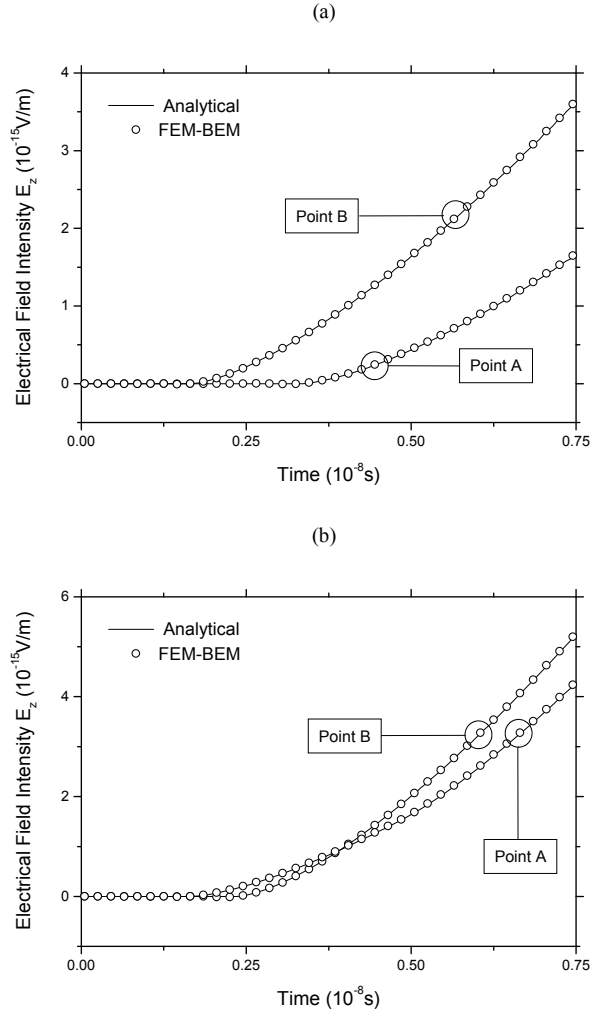


Figure 3: Time history results for the electric intensity field at points A and B considering $I(t) = t^2$: (a) case 1; (b) case 2.

Fig.2 shows the modulus of the electric field intensity obtained at points A and B (see Fig.1) considering the proposed methodology and $I(t) = t$. Analytical time histories (Machado, 2006) are also depicted in Fig.2, highlighting the good accuracy of the numerical results. In Fig.3 analogous results are presented considering $I(t) = t^2$. In Fig.4, charts are displayed, indicating the percentage of occurrence of different relaxation parameter values (evaluated according to expression (25)), in each analysis. As can be observed, for all considered cases, optimal relaxation parameters are mostly in the interval $0.7 \leq \varphi \leq 0.8$.

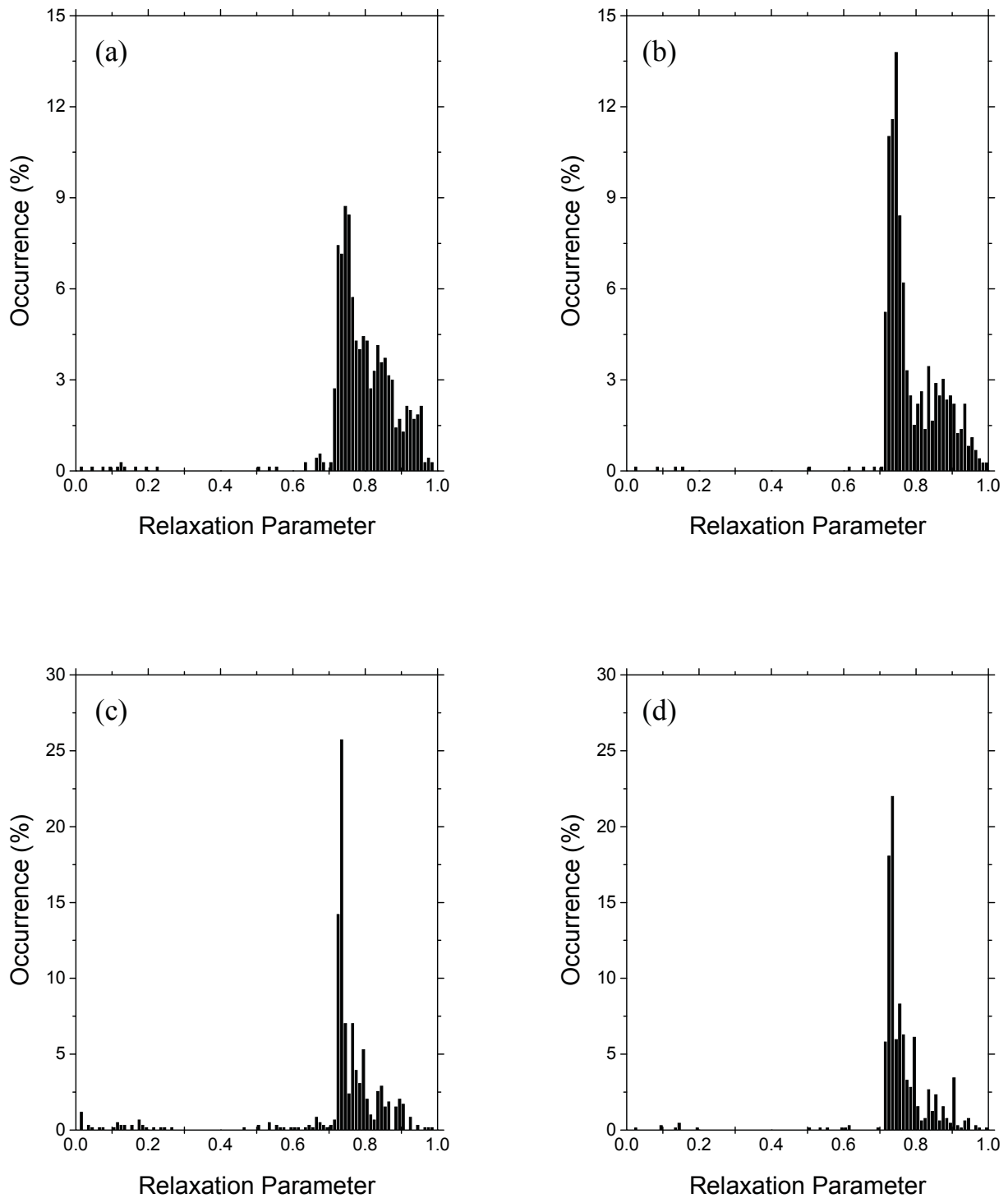


Figure 4: Percentage of occurrence of different relaxation parameter values during the analysis: (a) case 1 - $I(t) = t$; (b) case 2 - $I(t) = t$; (c) case 1 - $I(t) = t^2$; (d) case 2 - $I(t) = t^2$.

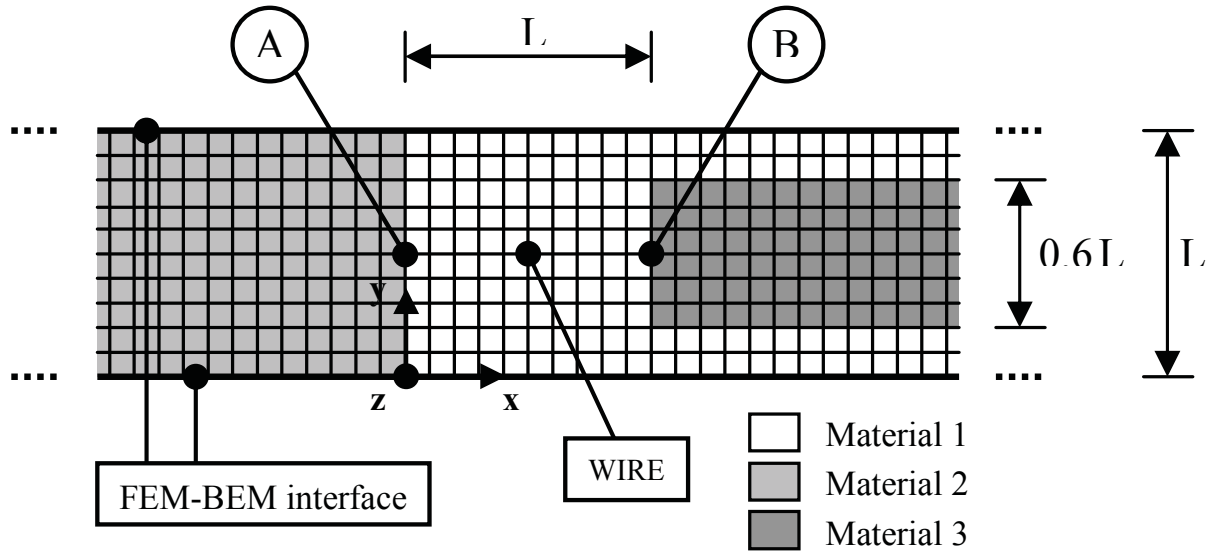


Figure 5: Sketch of the model and adopted spatial discretization for the heterogeneous medium analysis.

In fact, an optimal relaxation parameter selection is extremely case dependent. It is function of the physical properties of the model, geometric aspects, adopted spatial and temporal discretizations etc. Equation (25) provides a simple expression to evaluate this complex parameter easily and properly.

6.2 Heterogeneous medium

In this numerical application, two close walls, surrounded by air (material 1), are considered. The first wall is made of concrete (material 2: $\epsilon = 4.427 \cdot 10^{-11} F/m$) and the second wall (which is thinner) is made of steel (material 3: $\mu = 875 \cdot 10^{-6} H/m$). A long wire, carrying a time-sinusoidal current (i.e., $I(t) = A \sin(\omega t)$) is located between the walls, as indicated in Fig.5. In Fig.5, the adopted spatial discretization is also depicted: both FEM and BEM meshes are horizontally extended according to the time duration of the analysis in focus (causality). The geometry of the model is defined by: $L = 0.1 m$. The adopted time-step is $\Delta t = 5 \cdot 10^{-12} s$.

Fig.6 shows the modulus of the electric field intensity obtained at points A and B (see Fig.5) considering the proposed methodology. FEM time history results (computed considering a large enough mesh) are also depicted in Fig.6, as a reference. In Fig.7, the spatial and temporal evo-

lution of the electric field intensity is illustrated, considering the central part of the FEM-BEM coupled mesh and part of the mesh related to the FEM analysis (the FEM-BEM results are plotted over the FEM mesh, in Fig. 7). As can be observed, results are in good agreement, in spite of the relative poor spatial discretization adopted.

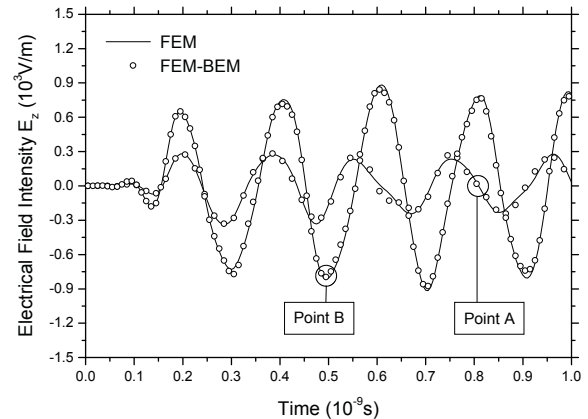


Figure 6: Time history results for the electric intensity field at points A and B.

In the present analysis, the average value for the computed optimal relaxation parameters is 0.93 and the average number of iterations per time step is 3.6 (which is a typical value).

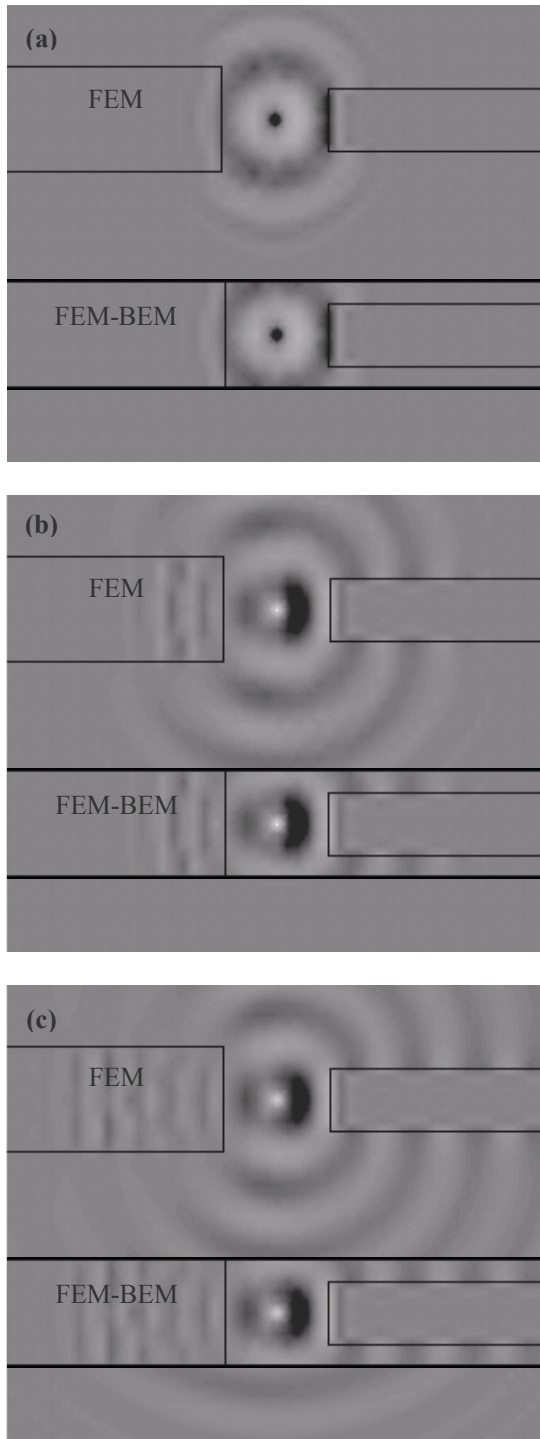


Figure 7: Electric intensity field temporal and spatial evolution considering FEM and FEM-BEM coupled analysis: (a) $t = 0.3 \cdot 10^{-9}$ s; (b) $t = 0.6 \cdot 10^{-9}$ s; (c) $t = 1.0 \cdot 10^{-9}$ s.

7 Conclusions

In this work, a time-domain FEM-BEM iterative coupling procedure is discussed. The proposed formulation is very attractive since it allows each sub-domain of the global model to be independently and optimally treated. As another advantage, the formulation allows existing codes or computer programs to be employed in coupled analyses once simple interface routines are implemented. As a consequence, quite complex electromagnetic phenomena can be properly analysed, taking into account the benefits of different numerical methods.

In order to improve the efficiency and robustness of the time-domain iterative coupling algorithm, the evaluation of optimal relaxation parameters is discussed and an easy to implement expression is presented. In fact, the effectiveness of the iterative algorithm is closely related to a proper relaxation parameter selection: an inappropriate value can drastically increase the number of iterations in the analysis or, even worse, make convergence unfeasible. Once suitable parameters are employed, convergence is achieved in quite few iterative steps (mostly, less than four iterations per time-step are necessary).

At the end of the paper, numerical applications are considered, illustrating the good level of accuracy of the proposed formulation, as well as its flexibility and applicability to model electromagnetic wave propagation through infinite-domain media, taking into account scattering from inhomogeneous objects.

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